Variable Structure Controller Design for Spacecraft Nutation Damping

HEBERTT SIRA-RAMIREZ AND THOMAS A. W. DWYER, III

***Abstract*—Variable structure systems theory is used to design an automatic controller for active nutation damping in momentum biased stabilized spacecraft. Robust feedback stabilization of roll and yaw angular dynamics is achieved with prescribed qualitative characteristics which are totally independent of the spacecraft defining parameters.**

I. Introduction

The use of variable structure systems theory (hereafter called VSS) is receiving increased attention in the aerospace field as referenced in [1], [2]. The technique, extensively developed in the Soviet Union and Eastern Europe for a number of years [3], permits the use of a lower order system model for generating control commands, and is robust with respect to external disturbances as well as vehicle configuration and mass properties: indeed, the latter typically are needed only for estimates of the required level of control effort for the attainment of a desired "sliding motion" trajectory. The required accompanying switching logic, used for overshoot correction, is based only on the designer-selected sliding motion, as well as on invariant kinematic equations.

Recent results on the use of VSS for spacecraft slewing maneuvers are found in [1], [2], where global nonlinear methods are employed and in [4] where passive damping mechanisms during slewing are taken into account.

In this note, multivariable but linear VSS theory is used for active nutation damping. Nutation is defined as the rotational periodic motion exhibited by spacecraft when control or environmental disturbance torques perturb its stable spin-free equilibrium position (see also Wertz *et al.* [5]).

Nutation damping is accomplished in a passive manner by energy dissipation mechanisms such as: fluidic friction, used in connection with one or two degree of freedom penduli, generation of eddy currents arising from relative motion between a conducting plate and a magnet, or free rolling ball-in-the tube viscous friction dampers. On the other hand, active nutation dampers involve the use of feedback control in order to exercise

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H. Sira-Ramirez is with the Department of Aeronautical and Astronautical Engineering and the Coordinated Science Laboratory, University of Illinois, Urbana, IL 61801, on leave from the Control Systems Department, Universidad de Los Andes, Merida, Venezuela.

T. A. W. Dwyer, III is with the Space Systems Laboratory, Department of Aeronautical and Astronautical Engineering, University of Illinois, Urbana, IL 61801.

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an appropriate change of the spacecraft angular momentum. The actuators used for this control task include: magnetic coils, gas jets, momentum reaction wheels, and control moment gyros. Either continuous or bang-bang control can be used by judicious selection among such actuators. The recent development of pulsed width, pulsed frequency thrusters [6] also permits variable amplitude bang bang torque generation which is best suited for variable structure control (VSC).

Using VSS theory, the problem of designing an active nutation damping control system for a momentum biased stabilized spacecraft is addressed on the basis of a linearized model of the angular rate. A sliding surface is prescribed as the intersection of two linear manifolds which represent desirable static relationships among the state variables, in terms of reduced-order controlled motion stability.

The multivariable aspects of the problem lead to an input precedence arrangement, known as the method of hierarchy of controls (MHC) [3], by which the inherent conflict arising from "individual surface" reachability is resolved among the inputs. The geometric aspects of torque generation by a pair of roll-yaw thrusters, offset at a fixed angle with respect to a pair of yaw thrusters, induce a natural hierarchical arrangement among the inputs, thus facilitating the feedback gain design problem via MHC. A design example is presented here.

II. Nutation Control via VSS Theory

The small angular rate nutation model, describing the oscillatory motion of the momentum bias vector about the nominal spin axis, in terms of roll (ϕ) and yaw (ψ) [5], [7], is as follows:

$$\frac{d^2}{dt^2}\begin{bmatrix}\phi\\ \psi\end{bmatrix}+\begin{bmatrix}0 & w_n-w_0\\ w_0-w_n & 0\end{bmatrix}\frac{d}{dt}\begin{bmatrix}\phi\\ \psi\end{bmatrix}+w_0w_n\begin{bmatrix}\phi\\ \psi\end{bmatrix}=\begin{bmatrix}T_1/I\\ T_3/I\end{bmatrix} \tag{2.1}$$

where w_n is the natural frequency of the harmonic nutational oscillations along the roll and yaw axes in the absence of external torques T_1, T_3, and w_0 stands for the orbital rate of the spacecraft. It is assumed that the moments of inertia about the roll and yaw axes are equal, with value I. Control torques T_1, T_3 are provided by a pair of roll-yaw thrusters offset at a fixed angle α together with a pair of yaw thrusters (see Fig. 1), i.e.,

$$\begin{bmatrix}T_1\\ T_3\end{bmatrix}=\begin{bmatrix}\cos\alpha & 0\\ -\sin\alpha & 1\end{bmatrix}\begin{bmatrix}u_1\\ u_3\end{bmatrix}=:Au \tag{2.2}$$

where $u^T = [u_1, u_3]$ denotes the torque vector with components generated by the offset and yaw thrusters, respectively.

State variables are defined as $x_1 = \phi$; $x_2 = \dot\phi$; $x_3 = \psi$; $x_4 = \dot\psi$ and (2.1) is then rewritten as:

$$\dot x_1 = x_2$$

$$\dot x_2 = -(w_n-w_0)x_4 - w_0w_nx_1 + \frac{u_1\cos\alpha}{I}$$

$$\dot x_3 = x_4$$

$$\dot x_4 = -(w_0-w_n)x_2 - w_0w_nx_3 - \frac{u_1\sin\alpha}{I} + \frac{u_3}{I}. \tag{2.3}$$

A variable structure controller design requires the specification of two sliding surfaces S_1, S_2 (one for each control input) defined in terms of the state vector x, as

$$S_1 = \{x \in R^4 \colon s_1(x) = 0\}$$

$$S_3 = \{x \in R^4 \colon s_3(x) = 0\} \tag{2.4}$$

such that the evolution of the dynamic system on the intersection manifold $S = S_1 \cap S_3$ results in a reduced-order system with desirable stability characteristics. The desired second-order system will be supposed to

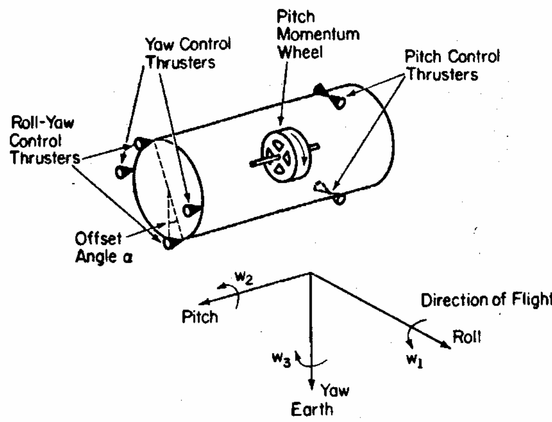

Fig. 1.

evolve according to:

$$\dot x_1 = -m_{11}x_1 - m_{13}x_3$$

$$\dot x_3 = -m_{31}x_1 - m_{33}x_3 \tag{2.5}$$

with freely selected eigenvalues:

$$\lambda_{1,2}=\frac{1}{2}\left[-(m_{11}+m_{33})\pm\sqrt{(m_{11}+m_{33})^2-4(m_{11}m_{33}-m_{13}m_{31})}\right] \tag{2.6}$$

then, it follows from (2.5) and the first and third equations in (2.3) that the desired surfaces are linear varieties (linear manifolds) defined by

$$S_1 = \{x \in R^4 \colon s_1(x) = x_2 + m_{11}x_1 + m_{13}x_3 = 0\}$$

$$S_3 = \{x \in R^4 \colon s_3(x) = x_4 + m_{31}x_1 + m_{33}x_3 = 0\}. \tag{2.7}$$

If the system is forced to evolve on S, the design parameters m_{ij} $i, j = 1.3$ uniquely determine the dynamic response and qualitative characteristics of the controlled system, according to (2.6). As a result, none of the original system parameters affects the controlled motion on the intersection of the sliding surfaces. Consequently, a robust design is achieved guaranteeing nutation damping with prescribed stability characteristics.

The evolution of the sliding surface coordinates s_1, s_3 is governed, according to (2.3), by the following differential equations, obtained by differentiating the expressions for $s_1(x)$ and $s_3(x)$ found in (2.7):

$$\dot s_1 = m_{11}s_1 + [m_{13}-(w_n-w_0)]s_3 + [m_{31}(w_n-w_0) - w_0w_n - m_{11}^2 - m_{13}m_{31}]x_1 + [(w_n-w_0)m_{33} - m_{11}m_{13} - m_{33}m_{13}]x_3 + \frac{u_1\cos\alpha}{I}$$

$$\dot s_3 = [m_{31}-(w_0-w_n)]s_1 + m_{33}s_3 + [m_{11}(w_0-w_n) - m_{31}m_{11} - m_{33}m_{31}]x_1 + [m_{13}(w_0-w_n) - w_0w_n - m_{31}m_{13} - m_{33}^2]x_3 - \frac{u_1\sin\alpha}{I} + \frac{u_3}{I}. \tag{2.8}$$

If the invariance conditions

$$s_i = 0;\ \dot s_i = 0\colon\ i = 1,\ 3 \tag{2.9}$$

are satisfied, the evolution of the controlled system ideally takes place on the intersection of the sliding surfaces S_1, S_3 and no excursions occur from this intersection manifold. Moreover, the smooth torques which, ideally, would generate invariant motions, known as the equivalent torques or equivalent controls [3], are found from (2.8) and (2.9) to be as shown:

$$\begin{bmatrix}u_{1,EQ}\\ u_{3,EQ}\end{bmatrix} = -IA^{-1}\begin{bmatrix}m_{31}(w_n-w_0)-w_0w_n-m_{11}^2-m_{13}m_{31} & m_{33}(w_n-w_0)-m_{11}m_{13}-m_{33}m_{13}\\ m_{11}(w_0-w_n)-m_{31}m_{11}-m_{33}m_{31} & m_{13}(w_0-w_n)-w_0w_n-m_{31}m_{13}-m_{33}^2\end{bmatrix}\begin{bmatrix}x_1\\ x_3\end{bmatrix}. \tag{2.10}$$

The equivalent controls are not to be used in practice due to their inherent sensitivity with respect to plant parameter variations and external disturbances. Instead, a variable structure control (VSC) law

$$u_i=\begin{cases} u_i^+(x) & \text{for } s_i(x)>0 \\ u_i^-(x) & \text{for } s_i(x)<0 \end{cases} \qquad i=1,\ 3 \tag{2.11}$$

is prescribed with the following objectives: 1) to obtain sliding surface reachability and 2) to impose the invariance conditions on the surface by means of an "overshoot and correct" approach. If the objectives are met and the invariance conditions are satisfied on S even in an approximate manner (due to unavoidable imperfections such as small delays in the actuators, plant and parametric perturbations and sensor errors), then a sliding motion is said to have been created on the surface S.

An arbitrary precedence relation is usually established among the controllers, according to the MHC. To motivate the need for such arrangement, one may consider the necessary and sufficiency conditions for "individual surface" reachability by each corresponding control input [8]:

$$\lim_{s_i\to 0^+} \dot{s}_i<0;\ \lim_{s_i\to 0^-} \dot{s}_i>0; \qquad i=1,\ 3. \tag{2.12}$$

Conditions (2.12) are equivalent to the following more compact set of conditions [8]:

$$\lim_{s_i\to 0} s_i\dot{s}_i<0; \qquad i=1,\ 3. \tag{2.13}$$

Using (2.8) one obtains:

$$s_1\dot{s}_1=m_{11}s_1^2+[m_{13}-(w_n-w_0)]s_1s_3+[m_{31}(w_n-w_0)-w_0w_n-m_{13}m_{31}]s_1x_1 + [(w_n-w_0)m_{33}-m_{11}m_{13}-m_{33}m_{13}]s_1x_3+u_1s_1\cos\alpha/I \tag{2.14}$$

$$s_3\dot{s}_3=[m_{31}-(w_0-w_n)]s_3s_1+m_{33}s_3^2+[m_{11}(w_0-w_n)-m_{31}m_{11}-m_{33}m_{31}]s_3x_1 + [m_{13}(w_0-w_n)-w_0w_n-m_{31}m_{13}-m_{33}^2]s_3x_3-u_1s_3\sin\alpha/I+u_3s_3/I. \tag{2.15}$$

Due to the dynamic and control coupling among the surface coordinates, a conflict may arise regarding reachability of the sliding surface S. Indeed, if each control input u_i is independently designed to make the state trajectory reach the corresponding "individual sliding surface" S_i, conditions (2.13) may not be satisfied in general, unless some "centralized" assignments are enforced. The MHC resolves off-line the multivariable aspects of the reachability problem by imposing an arbitrary arrangement of the control inputs and computing the required VSC feedback gains which guarantee sliding surface reachability. The scheme also results in an individual surface "queue" towards sliding. In our case, however, the geometry associated with the actuator location naturally dictates the necessary precedence relation among the inputs for direct application of the MHC.

The supposed precedence $u_3 \rightarrow u_1$ is adopted. Following the MHC a variable structure control law is assigned to the torque input u_1 such that the reachability condition (2.13) is satisfied for s_1, under the assumption that s_3 already satisfies the ideal sliding conditions (2.9). A variable structure control law is then specified for the torque input u_3, which robustly satisfies condition (2.13) in spite of the values of the surface coordinate s_1 and the possible control actions to be taken by u_1 in its pursuit of reachability of S_1.

From (2.14), it follows that if a linear VSC law of the form $u_1 = -K_{11}x_1 - K_{13}x_3$ is used, with

$$K_{11}=\begin{cases} K_{11}^+>I[m_{31}(w_n-w_0)-w_0w_n-m_{11}^2-m_{13}m_{31}]\sec\alpha & \text{for } s_1x_1>0 \\ K_{11}^-<I[m_{31}(w_n-w_0)-w_0w_n-m_{11}^2-m_{13}m_{31}]\sec\alpha & \text{for } s_1x_1<0 \end{cases}$$

$$K_{13}=\begin{cases} K_{13}^+>I[m_{33}(w_n-w_0)-m_{11}m_{13}-m_{33}m_{13}]\sec\alpha & \text{for } s_1x_3>0 \\ K_{13}^-<I[m_{33}(w_n-w_0)-m_{11}m_{13}-m_{33}m_{13}]\sec\alpha & \text{for } s_1x_3<0 \end{cases} \tag{2.16}$$

then the reachability condition (2.13) is satisfied for s_1 under the

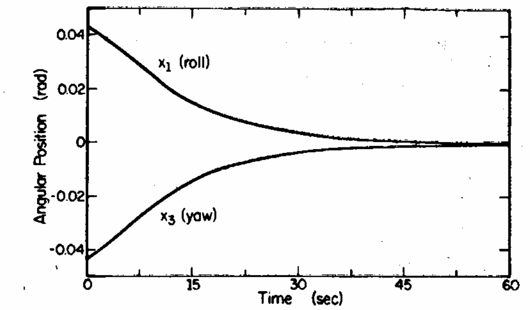

Fig. 2.

assumption that $s_3 = 0$. It also follows from (2.15) that if a VSC law $u_3 = -K_{31}x_1 - K_{33}x_3 - K_3s_1$ is used, with

$$K_{31}=\begin{cases} K_{31}^+>I[m_{11}(w_0-w_n)-m_{31}m_{11}-m_{33}m_{31}]+K_{11}^+\sin\alpha & \text{for } s_3x_1>0 \\ K_{31}^-<I[m_{11}(w_0-w_n)-m_{31}m_{11}-m_{33}m_{31}]+K_{11}^-\sin\alpha & \text{for } s_3x_1<0 \end{cases}$$

$$K_{33}=\begin{cases} K_{33}^+>I[m_{13}(w_0-w_n)-w_0w_n-m_{31}m_{13}-m_{33}^2]+K_{13}^+\sin\alpha & \text{for } s_3x_3>0 \\ K_{33}^-<I[m_{13}(w_0-w_n)-w_0w_n-m_{31}m_{13}-m_{33}^2]+K_{13}^-\sin\alpha & \text{for } s_3x_3<0 \end{cases} \tag{2.17}$$

$$K_3=\begin{cases} K_3^+>I[m_{31}-(w_0-w_n)] & \text{for } s_3s_1>0 \\ K_3^-<I[m_{31}-(w_0-w_n)] & \text{for } s_3s_1<0 \end{cases}$$

then condition (2.13) is satisfied for s_3 in spite of the value of s_1 and of the value of the feedback control law used by u_1 in its efforts to attain reachability of S_1.

It should be noted, from (2.14), (2.15), that in the opposite hierarchical ordering ($s_1 \rightarrow s_3$) the input u_1 cannot compensate for the control actions of u_3, while reachability of S_3 may be precluded by the uncompensated control actions of u_1. This situation does not arise in the previously adopted hierarchical scheme.

Use of the control laws (2.16), (2.17) on the system (2.3) results in the strict inequality

$$s_1\dot{s}_1+s_3\dot{s}_3<0 \tag{2.18}$$

locally around S. This implies that the function $V(s_1, s_3) = 1/2(s_1^2 + s_3^2)$ is made into a Lyapunov function by the VSC laws. This demonstrates attractivity of the sliding manifold S in a local sense.

Example: Using the SIMNON package [9], simulations were performed on a CTS spacecraft model [7] with the following defining parameters:

$$I=1327.28\ \text{kg-m}^2;\ h=20\ \text{N-m-s};\ w_0=727\times 10^{-7}\ \text{rad/s};$$

$$w_n=h/I=0.01506;\ \alpha=10^\circ$$

$$m_{11}=m_{33}=0.12;\ m_{13}=m_{31}=0.03$$

$$K_{11}^+=-13.380,\ K_{11}^-=-26.652;\ K_{13}^+=-0.643,\ K_{13}^-=-13.916;$$

$$K_{31}^+=-7.630,\ K_{31}^-=-20.903;\ K_{33}^+=-14.381,\ K_{33}^-=-27.654$$

$$K_3^+=65.020,\ K_3^-=54.402.$$

Fig. 2 shows the time responses of the roll and yaw angles; Fig. 3 depicts the corresponding angular velocities, while the "bang-bang modulated" torques u_1, u_3, applied to the spacecraft for nutation damping, are shown in Figs. 4 and 5, respectively, with the corresponding equivalent torques. Fig. 6 contains the time responses of the surface coordinates s_1, s_3.

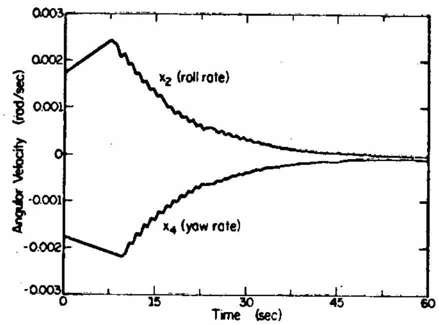

Fig. 3.

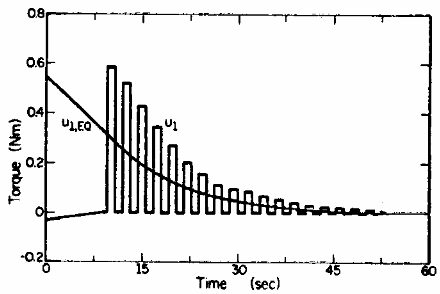

Fig. 4.

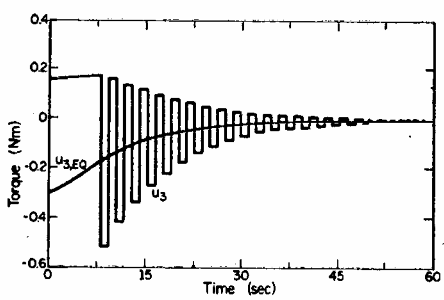

Fig. 5.

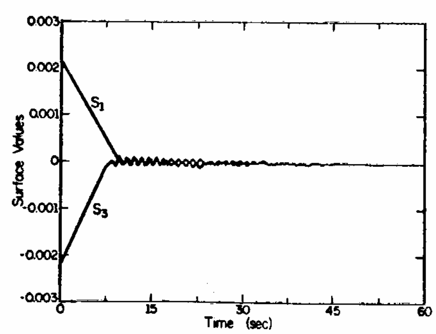

Fig. 6.

III. Conclusions

A VSC scheme has been proposed for active nutation damping in momentum biased stabilized spacecraft with multiple torque inputs. It was found that a natural hierarchy of controls is imposed by the spatial arrangement of the torque effectors. This facilitates the off-line resolution of possible conflicts among input objectives in their quest for individual sliding surface reachability. Once the sliding surface is reached, and sliding conditions are satisfied, the controlled motions are asymptotically exponentially stable according to prespecified qualitative characteristics. The robustness of VSC with respect to plant parameter variations and external perturbations is studied in [11], following [12].

Nutation as a desired spacecraft maneuver has been little investigated in the literature, except for [5] and [10]. The ability of VSC laws to render controlled behavior with characteristics not present in any of the intervening closed-loop structures makes them attractive for applications where geometric performance constraints are to be imposed. Induced nutation via VSC as a means of periodic scanning maneuvers is a possible area for further research. The outstanding robustness properties of VSC also remain to be explored in the control of flexible spacecraft and energy dissipation control strategies for large space stations.

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