A Geometric Approach to the Feedback Control of Switch Mode DC-to-DC Power Supplies

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***Abstract*—Using the theory of variable structure systems, a geometric approach is presented for the analysis and design of control schemes in bilinear dc-to-dc switch-mode power supplies. A time scale separation property, which naturally describes desirable transient behavior of the power converter, is used to identify the slow manifold of the average system in which a stable sliding mode trajectory can be locally sustained.**

I. Introduction

THIS work is motivated by the need to explore nonlinear design strategies for switch mode dc-to-dc power supplies (SMPS), here also addressed as converters, in the context of modern control theory. The design of regulation schemes for SMPS has been traditionally based on the pulsewidth modulation (PWM) technique [1]–[4]. This technique is typically based on the use of small signal models to which linear design strategies could be directly applied [5]. Recently, the variable structure systems (VSS) theory and its associated sliding regime behavior [6] has been proposed as a design alternative for the feedback regulation of these circuits [7]–[11].

The conceptual results in this paper are derived under the assumption of infinitely fast switching frequency, and form a basis for a design scheme in which no approximations are necessary while the nonlinear aspects of the problem are fully dealt with and their intrinsic difficulties are effectively circumvented. Moreover, future studies, of more realistic nature, can benefit from the idealized results for performance evaluation and the design guidelines proposed here. It is important to remark, however, that the method of the equivalent control, which is extensively exploited here, and its associated ideal sliding mode description constitute well-founded results of VSS theory [6], [12], while the necessary technology by which a realistic approximation to such idealized behavior can be effectively accomplished, is rapidly becoming available. For instance, modern controllable electronic switches have reached switching frequencies of up to 1 MHz.

It was shown rigorously in [11] that the average behavior of the nonlinear PWM controlled systems is obtained from the system model just by replacing the discrete control input (switch position function) by a smooth analytic function of the state, known as the *duty ratio*. The ideal sliding dynamics of the VSS approach is similarly obtained by replacing the discrete control input by a smooth function known as the *equivalent control* [6]. Local integral manifolds of the average PWM controlled system qualify as local sliding surfaces on which the equivalent control totally coincides with the duty ratio. The relationships between both approaches and the use of sliding modes result in a conceptually simpler, and more systematic, technique for the analysis and design of feedback loops for SMPS control (see also [20]). Moreover, this equivalence has a definite bearing in the drastic reduction of the necessary hardware used in the PWM control option. These advantages validate a purely VSS approach for the design of feedback control strategies in dc-to-dc supplies.

Besides the conceptual and practical advantages of a VSS approach over the PWM control alternative, we propose a design scheme for the converter itself which demands component values resulting in a two-time scale separation property of the average response. The ideal sliding converter response exhibits then a slow manifold, which is locally contained on an affine variety. This affine variety locally qualifies as a stable sliding surface. The transient behavior of the average sliding motion no longer exhibits undesirable oscillatory response while the sliding regime design is considerably simplified. Moreover, a limit cycle type of behavior observed and unexplained in exact linearization design schemes [9] is avoided by clearly identifying the region of sliding mode existence associated with the proposed manifold (a feature blurred by the non-global nature of the exact linearization approach).

Section II deals with general results on sliding regimes in bilinear systems while Section III is devoted to a detailed analysis of the sliding regime problem in three popular dc-to-dc supplies: buck, boost, and buck–boost converters [13]. The detailed developments correspond to

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the boost converter and a summary of basic elements and the design of sliding regimes in the buck and buck-boost converter are also presented. Section IV contains some conclusions and suggestions for further research in this area.

II. Background and General Results

2.1. Sliding Regimes in Bilinear Systems

Consider a class of bilinear systems defined on R^n-as

$$\dot{x} = Ax + uBx \tag{2.1}$$

with $A, B\; n \times n$ matrices with constant real coefficients. The scalar control function u is assumed to take values on the discrete set $U := \{0,1\}$.

The sliding mode control [6] of (2.1) assumes that the discontinuous control law

$$u = 0.5[1 + \operatorname{sgn} s(x)], \tag{2.2}$$

and induces a discontinuous (chattering) motion in the state trajectories, known as *sliding mode*, constrained to the immediate vicinity of the smooth manifold (switching surface) specified by

$$S = \{x \in R^n : s(x) = 0\}. \tag{2.3}$$

Necessary and sufficient conditions for the existence of a sliding motion on S [6] are satisfied whenever the switching logic (2.2) and the controlled motion of (2.1) are such that

$$\lim_{s \to +0} \frac{ds}{dt} < 0; \quad \lim_{s \to -0} \frac{ds}{dt} > 0 \tag{2.4}$$

i.e., the rate of change of the sliding manifold coordinate s is such that the controlled motions invariably converge towards s in its immediate vicinity. If such a property takes place everywhere along S, the sliding motion is said to be *global*, otherwise, it will be termed *local*. All our results will be of local nature, i.e., assumed to be valid on an open neighborhood of R^n which has a nonempty intersection with S.

The strict inequalities in (2.4) avoid singular sliding motions (extensively treated in [6]). The form of conditions (2.4) imply that the vector fields corresponding to the extreme control values 0, 1, i.e., Ax and $(A+B)x$ are not tangential to S in the region of interest. The following lemma is an immediate consequence of (2.4) and the nonsingularity of the assumed sliding mode on S.

Lemma 1. If a local nonsingular sliding motion exists on S then, necessarily,

$$\left[\frac{\partial s}{\partial x}\right]^T Bx < 0 \tag{2.5}$$

locally on S.

Proof: Obvious. □

The condition (2.5) represents a local *transversality condition* [14] of the controlled vector field Bx with respect to the manifold S.

Remark: Notice that if a sliding regime existed locally on S with the reversed switching logic: $u = 0.5(1 - \operatorname{sign} s(x))$ then the transversality condition (2.5) would adopt the opposite inequality sign. In this case a redefinition of S as $\{x \in R^n : s_1(x) = -s(x) = 0\}$ would result again in a switching logic of the form (2.2) and the new transversality condition would exhibit the same inequality sign as in (2.5). Therefore, there is no loss of generality in assuming that the existing sliding mode on S is created by the switching logic (2.2) with condition (2.5) satisfied.

The smooth control function for which (2.1) adopts S as a local integral manifold is known as the *equivalent control* [6] and it is denoted by $u_{EQ}(x)$. The equivalent control is then defined from the following manifold invariance conditions:

$$s = 0; \quad \dot{s} = \left[\frac{\partial s}{\partial x}\right]^T (A + u_{EQ}(x)B)x = 0. \tag{2.6}$$

It follows from (2.6) that the equivalent control is given by

$$u_{EQ}(x) = -\frac{\left[\frac{\partial s}{\partial x}\right]^T Ax}{\left[\frac{\partial s}{\partial x}\right]^T Bx}. \tag{2.7}$$

The response of the system to the equivalent control, starting from an initial state located on the sliding region of the manifold S, is addressed to as the *ideal sliding dynamics* [15]. Such a motion is governed by the nonlinear system

$$\dot{x} = (A + u_{EQ}(x)B)x, \qquad s(x) = 0. \tag{2.8}$$

The ideal sliding dynamics is thus obtained from (2.1) just by replacing the discrete control input u by the smooth feedback control function $u_{EQ}(x)$. Notice that since (2.8) evolves on S, the condition $s(x) = 0$ in (2.6) means that there is an interdependency among the state variables of the ideal sliding system and therefore, one of the equations in (2.8) is actually redundant.

Theorem 2. For a sliding motion to locally exist on S it is necessary and sufficient that the corresponding equivalent control satisfies

$$0 < u_{EQ}(x) < 1. \tag{2.9}$$

Proof: This result is a particular case of that found in Utkin [6, p. 81] in which the extreme control values are simply 0 and 1. □

The region (or regions) R specified by conditions (2.9), where, necessarily, the transversality condition (2.5) is satisfied, determine the portion, or portions, of S where local sliding motions occur. The expressions for such an existence region are readily obtained from (2.7) and (2.9) as

$$R = R_0 \cap R_1 \cap S \tag{2.10}$$

with

$$R_0 = \left\{x \in R^n : \left[\frac{\partial s}{\partial x}\right]^T Ax > 0\right\}$$

$$R_1 = \left\{x \in R^n : \left[\frac{\partial s}{\partial x}\right]^T (A+B)x < 0\right\}. \tag{2.11}$$

It could be seen that $R_0 \cap R_1$ contains the region where

(2.5) is satisfied, i.e., the region where the transversality condition is valid. This condition is, incidentally, a necessary condition for the existence of a sliding motion [20]. For the particular case in which S is given by an affine variety of the form

$$S = \{x \in R^n: s(x) = c^T x + d = 0\} \qquad (2.12)$$

the region where a sliding motion locally exists is determined from (2.11) by simply replacing the gradient vector $[\partial s/\partial x]$ by the column vector c.

III. Sliding Mode Control of Slow Manifolds of DC-to-DC Switch Mode Power Supplies

Circuits on which switchings are performed to achieve electrical energy transfers among storage elements or, more commonly, to achieve steady-state regulation of the output variables constitute a special class of variable structure feedback systems. The control action is represented by a drastic change in the network topology as a result of the operation of a switch (transistor, diode, etc.). The commanded state trajectory typically chatters towards a stable equilibrium. The feedback portion of the controlled circuit has been traditionally designed using pulse-width-modulation control strategies [5], [13], and only recently, a variable structure control approach has been explored (see [7]–[10]. In [11] an equivalence among the PWM control approach and the VSS method has been rigorously established for general nonlinear analytic systems using elementary differential geometric concepts.

In this section, feedback control design of three popular dc-to-dc switch mode power supplies is treated using the theory of variable structure systems and their associated sliding motions. The main departure from existing studies lies in the use, as sliding manifolds, of affine varieties containing the *slow manifold* of the ideal sliding dynamics [16]. This idea has been used in a robotic controller design in [17] and it was generalized in [18] as an advantageous VSS design scheme for several areas of nonlinear control systems design. The slow manifolds are shown to be naturally associated with desirable transient characteristics of the converter response and arise from a time scale separation property among the relative value of, respectively, the natural frequency and the time constant of the LC "input" filter and RC "output" filter of the converter [13].

The proposed slow manifold constitutes a local integral manifold for the ideal sliding dynamics obtained from the assumption of a constant equivalent control. This assumption corresponds to the traditional design feature of maintaining a constant duty ratio in PWM control options [11]. The region of existence of a sliding mode turns out to be global only for the case of the buck converter, but just a local one for the remaining cases.

3.1. Sliding Motions on Affine Varieties Containing the Slow Manifold of the Boost Converter

Consider the boost converter shown in Fig. 1. Let the state variables be defined as: $x_1 = I\sqrt{L}$, $x_2 = v\sqrt{C}$, and the parameters $b = E/\sqrt{L}$, $w_0 = 1/\sqrt{LC}$ and $w_1 = 1/RC$. Then, the following bilinear system models the dynamical behavior of the circuit

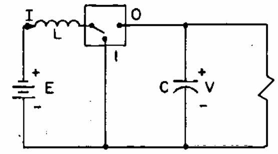

Fig. 1. Boost converter.

$$\dot{x}_1 = -w_0 x_2 + u w_0 x_2 + b$$
$$\dot{x}_2 = w_0 x_1 - w_1 x_2 - u w_0 x_1 \qquad (3.1)$$

where u represents the switch position function taking values on the discrete set $U = \{0, 1\}$.

The ideal sliding dynamics of (3.1) on a sliding surface whose corresponding equivalent control turns out to be a constant μ is described by the linear dynamical system obtained from (3.1) by replacing the discrete varible u by the constant quantity μ,

$$\dot{x}_1 = -(1-\mu) w_0 x_2 + b$$
$$\dot{x}_2 = (1-\mu) w_0 x_1 - w_1 x_2. \qquad (3.2)$$

The essential features of the transient and steady-state behavior of the average response of (3.1) to a certain VS feedback scheme, which results in a constant equivalent control, are totally specified by (3.2). In turn, the characteristic equation of (3.2), given by

$$p^2 + w_1 p + (1-\mu)^2 w_0^2 = 0 \qquad (3.3)$$

specifies, by means of the damping coefficient $d = w_1/[2(1-\mu)w_0]$, the nature of the transient behavior of (3.2). The quantity $(1-\mu)w_0$ is the natural frequency associated with the ideal sliding dynamics. Thus the damping coefficient is proportional to the ratio of the time constant of the RC "output" network w_1 and the natural oscillations frequency of the LC "input" filter w_0. When $d > 1$, the system is overdamped and generally exhibits a time scale separation property among its associated modes obtained as solutions of (3.3). The average transient response of the converter is nonoscillatory and rapidly reaches a slow (equilibrium) manifold where the motions asymptotically converge towards a stable equilibrium point. We deem this behavior as desirable not only from the "start up" viewpoint but also from the self-regulatory aspects of the circuit behavior to sudden perturbations. An indication of the validity of this assumption can be seen in [8] where the slow dynamics is fully neglected when deriving the sliding surface. This is based on an additional "current control mode" supposed to maintain the current through the inductor constant. We claim that if the converter design is properly done, a time scale separation property between the dynamics of the inductor current and the capacitor voltage is inherent. The value of the damping coefficient d is the criteria which allows an oscillation free response of the converter, while maintaining a low ripple in the inductor current. Typically, the damping coefficient should be chosen between 1.5 and 5 for an overdamped average response.

Next, the equilibrium point of (3.2) and the slow eigenspace, associated with the small eigenvalue $p_2(\mu)$ obtained from (3.3), are used to compute an explicit expression for the affine variety containing the slow manifold of the average response of (3.2). This computation is a consequence of the fact that for linear time-invariant systems (as it is the case of (3.2)), this affine variety can be obtained by rigid, parallel, translation of the slow eigenspace until containment of the equilibrium point (see [16, p. 22]). This affine variety will be proposed as a sliding surface for the variable structure feedback control of the converter circuit. The equilibrium points are given by

$$x_{1,ss} = \frac{bw_1}{(1-\mu)^2 w_0^2}; \quad x_{2,ss} = \frac{b}{(1-\mu)w_0}$$

while the slow eigenline slope is simply: $-[p_2(\mu)/(1-\mu)w_0]$. Notice that from the steady state value of x_2 the dc-gain, defined as the ratio of the steady-state output voltage $V_{ss} = \sqrt{C}x_{22,ss}$ and the input voltage E, is given simply by $1/(1-\mu)$. This qualifies the "step up" character of the boost converter.

In terms of the surface coordinate value s, the sliding surface is then given by:

$$S_\mu = \left\{ x \in R^2 : s = x_2 + \frac{p_2(\mu)}{(1-\mu)w_0} x_1 - \frac{b}{(1-\mu)w_0}\left[1 + \frac{w_1 p_2(\mu)}{(1-\mu)^2 w_0^2}\right] = 0 \right\}. \quad (3.4)$$

The proposed sliding surface is, in this case, only a *local integral manifold of the average system*, i.e., the sliding mode exists only locally (although in an unbounded portion of the R^2 plane) due to the nonglobality of the region determined by (2.10)–(2.11). Fig. 2 depicts the region of existence of a sliding motion associated with S_μ.

The variable structure control law is of the form:

$$u = \begin{cases} u^+, & \text{for } s > 0 \\ u^-, & \text{for } s < 0 \end{cases} \quad (3.5)$$

with u^+ and u^- yet to be specified. Using the definition of s given by (3.4) and the system description (3.1) for some u as in (3.5), after some algebraic manipulations, the surface coordinate is found to evolve according to

$$\frac{ds}{dt} = -\left[w_1 - \frac{(1-\mu)w_0^2}{w_1 + p_2(\mu)}(1-\mu)\right]s + \frac{w_0}{w_1 + p_2(\mu)}\left[\frac{p_2(\mu)bw_1}{(1-\mu)^2 w_0^2} + b\right](\mu - u). \quad (3.6)$$

The manifold invariance conditions (2.6) yield the defining relation for the equivalent control:

$$\frac{w_0}{w_1 + p_2(\mu)}\left[\frac{p_2(\mu)bw_1}{(1-\mu)^2 w_0^2} + b\right](\mu - u_{EQ}) = 0 \quad (3.7)$$

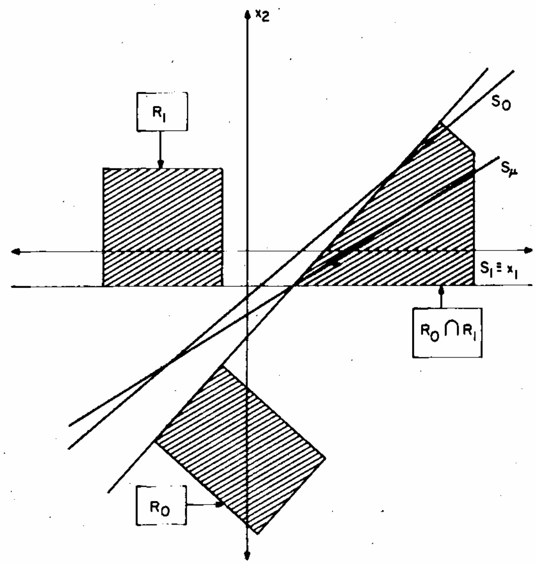

Fig. 2. Region of existence of sliding motion (boost converter).

which holds true on an open set S_μ whenever $u_{EQ} = \mu$, i.e., locally on the proposed sliding manifold the ideal sliding dynamics is indeed represented by the average system (3.2). The existence conditions (2.4), in correspondence with the control law (3.5) yield the following pair of inequalities:

$$\frac{w_0}{w_1 + p_2(\mu)}\left[\frac{p_2(\mu)bw_1}{(1-\mu)^2 w_0^2} + b\right](\mu - u^+) < 0$$

$$\frac{w_0}{w_1 + p_2(\mu)}\left[w_1 x_1 + \frac{p_2(\mu)bw_1}{(1-\mu)^2 w_0^2} + b\right](\mu - u^-) > 0. \quad (3.8)$$

It then follows from expressions (3.8) and the fact that $0 < \mu < 1$, that in the variable structure control law (3.5) u^+ and u^- should be given by 1 and 0, respectively, i.e., the control law:

$$u = 0.5(1 + \operatorname{sign} s(x)) \quad (3.9)$$

guarantees the existence of a local sliding motion on S_μ whenever

$$\left[w_1 x_1 + \frac{p_2(\mu)bw_1}{(1-\mu)^2 w_0^2} + b\right] > 0. \quad (3.10)$$

The expression obtained for the value of x_1 beyond which condition (3.10) is satisfied, coincides precisely with the one obtained from the existence conditions (2.10)-(2.11), i.e., (3.10) determines the region of existence of a sliding regime on the manifold S_μ. It can be verified, using (3.3) and (3.4), that (3.10) represents the region where the transversality condition (2.5) is valid.

The ideal sliding dynamics on S_μ is obtained by enforcing the manifold invariance conditions on the system dynamics. Using the manifold expression (3.4) and the average dynamics (3.2), the dynamics associated with the state variable representing the inductor current is governed by

$$\dot{x}_1 = p_2(\mu)\left[x_1 - \frac{bw_1}{(1-\mu)^2 w_0^2}\right] \tag{3.11}$$

which represents a linear stable motion towards the steady-state equilibrium value of the state variable x_1 representing the input inductor current. The dynamic behavior of this variable, on the *sliding manifold*, is governed by the value of the slow eigenvalue $p_2(\mu)$ which in turn is determined from our choice of the circuit component values according to (3.3). On the other hand, the dynamic behavior corresponding to the state variable x_2 on the sliding manifold is similarly obtained as

$$\dot{x}_2 = p_2(\mu)\left[x_2 - \frac{b}{(1-\mu)w_0}\right] \tag{3.12}$$

which again represents the asymptotically stable motion towards the equilibrium value of the state variable x_2 representing the capacitor voltage. The ideal sliding dynamics is thus represented by either (3.11) or (3.12) and (3.4). Notice that the component value b, representing the external voltage source, determines the equilibrium point of the ideal sliding dynamics, while $p_2(\mu)$ does not depend upon the value of b, but rather the component values w_0, w_1 and the value of μ which are chosen to obtain a desirable damping coefficient d in the ideal sliding dynamics. These facts conform the basis for a design procedure of the current dynamics itself, so as to obtain desirable regulatory features by means of a VSC approach which utilizes a purposefully designed slow manifold of the ideal sliding dynamics.

3.2. Summary of Design Procedure

1) Choose a desired voltage amplification factor $1/(1-\mu)$ (dc-gain) by proper choice of μ.

2) Choose the values of E so that a desired steady state value of the output capacitor voltage is obtained as $E/(1-\mu)$.

3) Choose component values R, C, L such that the damping coefficient d of the linear system (3.2) is greater than unity (typically, between 1.5 and 5).

4) Compute the slow eigenvalue $p_2(\mu)$ as the smaller solution (in absolute value) of (3.3). This value determines the exponential stability of the ideal sliding response. Repeat step 3) if an undesired value of $p_2(\mu)$ is obtained.

5) By using (3.4), synthesize the sliding surface by specifying the affine variety of the slow manifold of the ideal sliding dynamics.

6) Adopt as a switching strategy the VSC law given by (3.9).

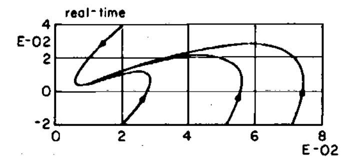

Fig. 3. Boost converter responses to $u = 0$.

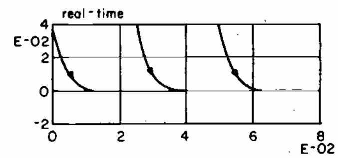

Fig. 4. Boost converter responses to $u = 1$.

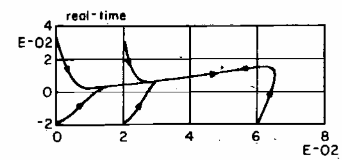

Fig. 5. Sliding mode response of the boost converter on slow manifold ($\mu = 0.5$).

Example 1

We require a dc-gain factor of 2 ($\mu = 0.5$) for a boost converter with input source of 20 V, i.e., 40 V of steady-state output voltage on a load resistor of $R = 100\ \Omega$. Choosing a damping coefficient of $d = 2$ for the ideal sliding linear dynamics, the converter parameters satisfy $w_1 = 2w_0$. Thus, for an output capacitor of $C = 0.1\ \mu$F the input inductor is $L = 4$ mH ($w_1 = 10^5$, $w_0 = 5 \times 10^4$). Steady state output load current is 0.4 A, while the average input (inductor) current is 0.8 A. Figs. 3–5 depict, respectively, the state trajectories for $u = 0$, $u = 1$ and the variable structure controlled response of such a boost converter.

3.3. Design of Sliding Modes in the Buck and Buck–Boost Converters

The reader may verify the validity of the following summary of formulas used for the specification of stable sliding motions in converters of the buck and buck–boost type. In the next paragraphs we specify equilibrium points, damping coefficients associated with the linear ideal sliding dynamics corresponding to constant equivalent control μ (equivalent to constant duty ratios in the PWM option or constant dc-gain) linear varieties, containing the slow manifolds of each converter, which can be used as sliding surfaces, and regions of existence to a sliding motion on such varieties. In each case the control law is the same as in (3.9).

Buck Converter (Fig. 6)

Define: $x_1 = I\sqrt{L}$, $x_2 = V\sqrt{C}$, $w_0 = 1/\sqrt{LC}$, $w_1 = 1\sqrt{RC}$, $b = E/\sqrt{L}$

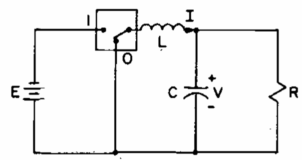

Fig. 6. Buck converter.

a) State equation model:

$$\dot{x}_1 = -w_0 x_2 + ub$$
$$\dot{x}_2 = w_0 x_1 - w_1 x_2. \tag{3.13}$$

b) Ideal sliding dynamics with constant equivalent control μ: the redundant model is obtained from (3.1) by simply replacing u by μ, $d = w_1/2w_0$.

c) Equilibrium point of ideal sliding dynamics:

$$x_{1,ss} = \frac{\mu w_1 b}{w_0^2}; \quad x_{2,ss} = \frac{\mu b}{w_0}. \tag{3.14}$$

d) DC-gain:

$$G(\mu) = \mu \tag{3.15}$$

i.e., the buck converter "steps down" and acts as a "chopper" [1], [2].

e) Eigenline slope:

$$\frac{p_2}{w_0} \tag{3.16}$$

(in this case the eigenvalues p_1 and p_2 are independent of μ).

f) Sliding surface:

$$S_\mu = \left\{ x \in R^2 \colon s = x_2 + \frac{p_2}{w_0} x_1 - \frac{b\mu}{w_0}\left[1 + \frac{p_2 w_1}{w_0^2}\right] = 0 \right\}. \tag{3.17}$$

g) Region of existence on S_μ: The transversality condition holds globally along S_μ while R_0 and R_1 in (2.11) are bounded by parallel lines in R^2. The existence conditions hold globally along S_μ (see Fig. 7).

h) Ideal sliding dynamics described by either of (3.18) and (3.17)

$$\dot{x}_1 = p_2\left[x_1 - \frac{\mu b w_1}{w_0^2}\right]$$
$$\dot{x}_2 = p_2\left[x_2 - \frac{\mu b}{w_0}\right]. \tag{3.18}$$

Example 2

Fig. 8 and 9 show, respectively, simulated responses and the sliding regime for the state trajectories of a typical, commercially available, buck converter obtained with a dc gain $\mu = 0.5$, and parameter values $L = 150$ μH, $C = 72$ nF, $R = 10$ Ω. The input voltage is chosen as $E = 400$ V, so that the steady-state value of the output capacitor voltage is 200 V. With these parameters the damping coefficient d of the ideal sliding dynamics is 2.28. Steady state output current is 20 A, and average input current 20 A.

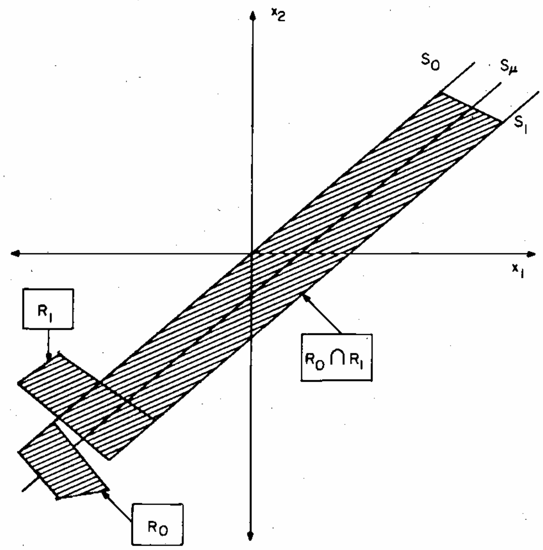

Fig. 7. Region of existence of sliding motion (Buck converter).

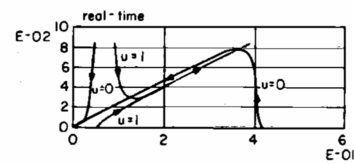

Fig. 8. Buck converter responses to $u = 0$ and $u = 1$.

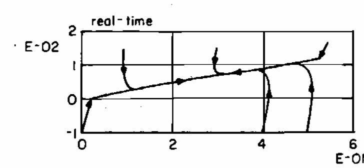

Fig. 9. Sliding mode response of the Buck converter on the slow manifold ($\mu = 0.5$).

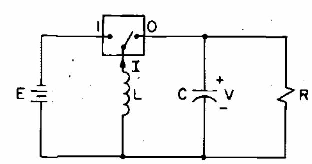

Fig. 10. Buck–boost converter.

Buck–Boost Converter (see Fig. 10)

Define: $x_1 = I\sqrt{L}$, $x_2 = V\sqrt{C}$, $w_0 = 1/\sqrt{LC}$, $w_1 = 1/RC$, $b = E/\sqrt{L}$.

a) State equation model:

$$\dot{x}_1 = +w_0 x_1 - u w_0 x_2 + ub$$
$$\dot{x}_2 = -w_0 x_1 - w_1 x_2 + u w_0 x_1. \tag{3.19}$$

b) Ideal sliding dynamics with constant equivalent control μ: the redundant model is obtained from (3.13) by simply replacing u by μ. $d = w_1/2(1-\mu)w_0$.

c) Equilibrium point of ideal sliding dynamics:

$$x_{1,ss} = \frac{\mu b w_1}{(1-\mu)^2 w_0^2}; \quad x_{2,ss} = \frac{-\mu b}{(1-\mu) w_0}. \tag{3.20}$$

d) DC-gain:

$$G(\mu) = -\mu/(1-\mu) \tag{3.21}$$

i.e., the buck-boost "steps down," in absolute value, for μ in the interval (0,0.5) and "steps up" for μ in the interval (0.5, 1).

e) Eigenline slope: The same as in the boost case.

f) Sliding surface:

$$S_\mu = \left\{ x \in R^2 \colon S = x_2 - \frac{p_2(\mu) x_1}{(1-\mu) w_0} + \frac{b\mu}{(1-\mu) w_0}\left[1 + \frac{w_1 p_2(\mu)}{(1-\mu)^2 w_0^2}\right] = 0 \right\}. \tag{3.22}$$

g) Region of existence of a sliding regime on S_μ:

$$\left[w_1 x_1 + \frac{p_2(\mu) b \mu w_1}{(1-\mu)^2 w_0^2} + b \right] > 0. \tag{3.23}$$

h) Ideal sliding dynamics is described by either of (3.24) and (3.22)

$$\dot{x}_1 = p_2(\mu)\left[x_1 - \frac{b\mu w_1}{(1-\mu)^2 w_0^2} \right]$$
$$\dot{x}_2 = p_2(\mu)\left[x_2 + \frac{\mu b}{(1-\mu) w_0} \right]. \tag{3.24}$$

IV. Conclusions and Suggestions for Further Research

In this paper we propose the use of slow manifolds associated with dc-to-dc switch-mode power supplies as sliding surfaces for the variable structure feedback control of such circuits. If the component values of the circuit are chosen so that a time scale separation property is exhibited by the average ideal sliding dynamics model, a satisfactory, i.e., non-oscillatory, transient behavior is obtained for the circuit response in an average (ideal) sense. In such a design, computation of affine varieties containing the slow manifolds is greatly facilitated, specially when the equivalent control, corresponding to such an ideal sliding mode, is a constant. This feature corresponds, entirely, to the usual desire to maintain a constant duty ratio in the traditional pulsewidth modulated control approach [5]. In the latter, this is achieved by increasing the complexity of the feedback loop. On the contrary, it is shown that a local stable sliding motion can be efficiently designed on these linear varieties with a variable structure control law that demands little in terms of feedback hardware. Notice that only sign values of s need be evaluated for a switching decision. This entitles one bit of information upon synthesis of the surface coordinate value s, a task only requiring operational amplifiers and resistors. PWM feedback options require, aside from the same elements, synthesis of a sawtooth generator of very precise features [1], [2], [19].

The geometry associated with the sliding mode control is easy to understand and could be particularly exploited in circuits which are designed to exhibit a time scale separation property. It can be generally shown that sliding modes are locally exhibited on integral manifolds of average models of pulse-width-modulation schemes. (See [7] for the linear case and [11] for the general nonlinear analytic case. Reference [4] also contains interesting equivalences of a different nature.) Thus VSS theory provides the designer with an alternative, systematic, method of geometric nature for the specification of regulation loops in dc-to-dc power converters, which is equivalent to the traditional PWM design technique in terms of average behavior.

In recently proposed design schemes [9], based on feedback linearization, the interplay between physics and the geometry associated with the circuit behavior is lost. Feedback linearization techniques, on the other hand, tend to hide, in the new coordinates, the local nature of the sliding modes, while making the feedback loop more complex due to nonlinear state coordinate transformation. The choice of linear varieties in the transformed coordinates usually leads to undesirable oscillations in which sliding mode existence conditions may be locally violated.

An interesting field of further applications of the theory of variable structure systems is the field of resonant dc-to-dc converters [1]. Some preliminary results have been reported in [10] for circuits evolving on spheres. The analysis and design methods proposed here can also be extended to the popular Cük converter [13] and more realistic SMPS models which include parasitic resistances and capacitances.

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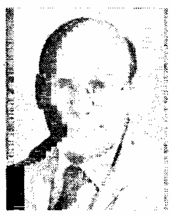

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