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VARIABLE STRUCTURE CONTROL OF FLEXIBLE JOINT MANIPULATORS

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Abstract

In this article we study the robust control of robot manipulators with flexible joints. Our results are based on the nonlinear model developed in ref. [17] which is globally linearizable via diffeomorphic state coordinate transformation and static nonlinear state feedback. An outer-loop control based on the sliding mode theory of variable structure systems is proposed for robust tracking. A nontrivial eighth-order example, representing a two-link elastic joint manipulator, is presented.

1. Introduction

Most of the theoretical control results for robot manipulators have been based on models which assume perfect structural rigidity. Recently it has been experimentally confirmed [22] for a large class of robots, such as those with harmonic drive transmission, that the joint flexibility, if uncontrolled, is a major limiting factor to higher performance. For control systems designed using rigid dynamic models the best one can do is to "detune" the closed-loop system so as not to excite the unmodeled resonant flexible models of the system. The drawback to this is that these unmodeled resonant modes lie within the frequency range of the desired closed-loop bandwidth. In other words, detuning the system to avoid excitation of the joint flexibility is possible only at a cost of degraded performance. The alternative, which we pursue in this article, is to base the controller design on more realistic models which account for the joint flexibility explicitly. Our results in this article are based on the flexible joint model of ref. [17] which is shown in that reference to be globally feedback linearizable via diffeomorphic coordinate transformation and nonlinear static state feedback. The global linearizability of the system in this manner in a nonlinear "inner loop" allows the design of an "outer-loop" controller which robustly stabilizes the linearized dynamics around an equilibrium point or equilibrium trajectory [16]. Variable structure control and its associated sliding regimes [23–25] are used for the outer-loop controller design. A linear switching (sliding) surface is proposed on which a globally stable sliding motion is created for the linearized system. Inversion of the linearizing state coordinate transformation provides a nonlinear sliding manifold where the original coordinates exhibit a globally stable behavior of robust nature.

The article is organized as follows. Section 2 presents the model from ref. [17] for manipulators with flexible joints. Section 3 discusses the global state space diffeomorphism and nonlinear inner-loop control law that reduces the system to Brunovsky canonical form. Section 4 presents a variable structure control approach for the design of the outer-loop tracking controller. Section 5 discusses robustness issues and Section 6 presents a controller design example for a two-link manipulator with flexible joints. Conclusions and some suggestions for further research are presented at the end of the article.

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2. Modeling

We consider an n-link manipulator with flexible joints. For simplicity, we treat the case of revolute joints actuated by DC motors, and model the flexibility of the ith joint as a linear torsional spring with spring constant $k_i = k$ for all i. The same techniques and results, however, are applicable for prismatic joints and for nonlinear springs. The results here, in fact, are unchanged if we allow a spring with any force/displacement characteristic $\phi(\Delta\theta)$, where $\Delta\theta$ is the angular displacement of the spring as long as the spring characteristic is describable by a diffeomorphism $\phi: R \to R$.

Because of the additional degrees of freedom introduced by the elastic coupling of the motor shaft to the links, we model the rotor of each actuator as a "fictitious link," that is, as an additional rigid body in the chain with its own inertia. Thus the manipulator consists of n "actual" links and n "fictitious" or rotor links. Referring to Figure 1, let $\mathbf{q} = (q_1, \ldots, q_{2n})^T$ be a set of generalized coordinates for the system where

$$q_{2i-1} = \text{the angle of link } i, \qquad i = 1, \ldots, n, \quad (2.1)$$

$$q_{2i} = -\frac{1}{m_i}\theta_i, \qquad i = 1, \ldots, n, \quad (2.2)$$

where θ_i is the angular displacement of rotor i and m_i is the gear ratio. In this case then $q_{2i} - q_{2i-1}$ is the elastic displacement of link i.

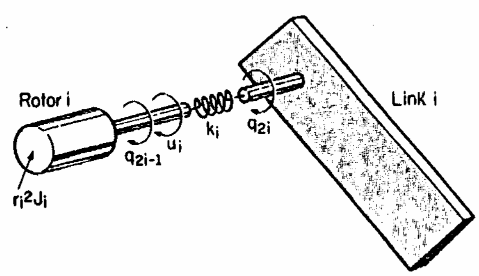

Figure 1. Elastic joint

We next define the n-dimensional vectors $\mathbf{q}_1$ and $\mathbf{q}_2$ as

$$\mathbf{q}_1 = (q_1, q_3, \ldots, q_{2n-1})^{\mathrm{T}}, \qquad \mathbf{q}_2 = (q_2, q_4, \ldots, q_{2n})^{\mathrm{T}}.$$

Under the modeling assumptions of [17] the equations of motion are found from the Euler–Lagrange equations as

$$D(\mathbf{q}_1)\ddot{\mathbf{q}}_1 + \mathbf{c}(\mathbf{q}_1\dot{\mathbf{q}}_1) + K(\mathbf{q}_1 - \mathbf{q}_2) = 0, \quad (2.3)$$

$$J\ddot{\mathbf{q}}_2 - K(\mathbf{q}_1 - \mathbf{q}_2) = \mathbf{u}. \quad (2.4)$$

The $n \times n$ matrix $D(\mathbf{q}_1)$ is symmetric, positive definite for each $\mathbf{q}_1$. J is a diagonal matrix with the actuator inertias (reflected to the link) along the main diagonal. The vector $\mathbf{c}(\mathbf{q}_1, \dot{\mathbf{q}}_1)$ contains Coriolis, centripetal, and gravitational forces and torques, $K = \mathrm{diag}(k_1, \ldots, k_n)$ is the diagonal matrix of joint stiffness constants, and $\mathbf{u} = (u_1, \ldots, u_n)^{\mathrm{T}}$ is the input vector of generalized forces produced by the actuators.

In case all joints are perfectly rigid it is shown in ref. [17] that the model (2.4)–(2.5) reduces to the usual rigid joint model

$$(D(q_1) + J)\ddot{q}_1 + \mathbf{c}(q_1, \dot{q}_1) = \mathbf{u}. \quad (2.5)$$

3. Feedback Linearization

It is well known that the rigid robot equation (2.5) may be globally linearized and decoupled by nonlinear feedback. This is just the familiar inverse dynamics or computed torque control scheme which transforms (2.5) into a set of double integrator equations which can then be controlled by adding an "outer-loop" control [16].

The above technique of inverse dynamics control is now understood as a special case of a more general procedure for transforming a nonlinear system to a linear system, known as *external* or *feedback linearization*.

Definition 3.1:

A nonlinear system

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \sum_{i=1}^{n} \mathbf{g}_i(\mathbf{x})u_i = \mathbf{f}(\mathbf{x}) + G(\mathbf{x})\mathbf{u} \quad (3.1)$$

is said to be *feedback linearizable* in a neighborhood U_0 of the origin if there is a diffeomorphism $T\colon U_0 \to R^n$ and nonlinear feedback

$$\mathbf{u} = \alpha(\mathbf{x}) + \beta(\mathbf{x})\mathbf{v} \quad (3.2)$$

such that the transformed state

$$\mathbf{y} = T(\mathbf{x}) \quad (3.3)$$

satisfies the linear system

$$\dot{\mathbf{y}} = A\mathbf{y} + B\mathbf{v}, \quad (3.4)$$

where (A, B) is a controllable linear system.

Necessary and sufficient conditions for a system of the form (3.1) to be feedback linearizable are given in ref. [6]. In ref. [17] the model (2.3)–(2.4) was shown to be globally feedback linearizable. This result is extremely important for control design. It is known [16] that the structure of feedback linearizable systems allows the design and implementation of highly robust nonlinear control algorithms, such as the sliding mode design detailed here.

We will briefly summarize the feedback linearization result for the model (2.3)–(2.4) before proceeding with the robust outer-loop control. We first write the system (2.3)–(2.4) in state space form by defining state variables

$$\mathbf{x}_1 = \mathbf{q}_1, \quad \mathbf{x}_2 = \dot{\mathbf{q}}_1, \quad \mathbf{x}_3 = \mathbf{q}_2, \quad \mathbf{x}_4 = \dot{\mathbf{q}}_2. \quad (3.5)$$

Then from (2.3)–(2.4) we have

$$\dot{\mathbf{x}}_1 = \mathbf{x}_2, \quad (3.6)$$

$$\dot{\mathbf{x}}_2 = -D(\mathbf{x}_1)^{-1}\{\mathbf{c}(\mathbf{x}_1, \mathbf{x}_2) + K(\mathbf{x}_1 - \mathbf{x}_3)\}, \quad (3.7)$$

$$\dot{\mathbf{x}}_3 = \mathbf{x}_4, \quad (3.8)$$

$$\dot{\mathbf{x}}_4 = J^{-1}K(\mathbf{x}_1 - \mathbf{x}_3) + J^{-1}\mathbf{u}. \quad (3.9)$$

Since the nonlinearities enter into the second equation above, while the control appears only in the last equation, it is not obvious that the system is linearizable nor can u immediately be chosen to cancel the nonlinearities as in the case of the rigid equation (2.5).

Consider now the nonlinear state space change of coordinates

$$\mathbf{y}_1 = T_1(\mathbf{x}) = \mathbf{x}_1, \quad (3.10)$$

$$\mathbf{y}_2 = T_2(\mathbf{x}) = \dot{T}_1 = \mathbf{x}_2, \quad (3.11)$$

$$\mathbf{y}_3 = T_3(\mathbf{x}) = \dot{T}_2 = -D(\mathbf{x}_1)^{-1}\{\mathbf{c}(\mathbf{x}_1, \mathbf{x}_2) + K(\mathbf{x}_1 - \mathbf{x}_3)\}, \quad (3.12)$$

$$\begin{aligned}\mathbf{y}_4 = T_4(\mathbf{x}) = \dot{T}_3 &= -\frac{d}{dt}\left[D(\mathbf{x}_1)^{-1}\right]\{\mathbf{c}(\mathbf{x}_1, \mathbf{x}_2) + K(\mathbf{x}_1 - \mathbf{x}_3)\} \\ &\quad - D(\mathbf{x}_1)^{-1}\left\{\frac{\partial \mathbf{c}}{\partial \mathbf{x}_1}\mathbf{x}_2 + \frac{\partial \mathbf{c}}{\partial \mathbf{x}_2}\left(-D(\mathbf{x}_1)^{-1}(\mathbf{c}(\mathbf{x}_1, \mathbf{x}_2) + K(\mathbf{x}_1 - \mathbf{x}_3))\right) + K(\mathbf{x}_2 - \mathbf{x}_4)\right\} \\ &:= f_4(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) + D(\mathbf{x}_1)^{-1}K\mathbf{x}_4, \end{aligned} \quad (3.13)$$

where for simplicity we define the function f_4 to be everything in the definition of $\mathbf{y}_4$ above except the last term, which is $D^{-1}K\mathbf{x}_4$. Note that $\mathbf{x}_4$ appears only in this last term so that f_4 depends only on $\mathbf{x}_1$, $\mathbf{x}_2$, and $\mathbf{x}_3$.

The above mapping is actually a global diffeomorphism. Its inverse is found by inspection to be

$$\mathbf{x}_1 = \mathbf{y}_1, \tag{3.14}$$

$$\mathbf{x}_2 = \mathbf{y}_2, \tag{3.15}$$

$$\mathbf{x}_3 = \mathbf{y}_1 + K^{-1}(D(\mathbf{y}_1)\mathbf{y}_3 + \mathbf{c}(\mathbf{y}_1,\mathbf{y}_2)), \tag{3.16}$$

$$\mathbf{x}_4 = K^{-1}D(\mathbf{y}_1)(\mathbf{y}_4 - f_4(\mathbf{y}_1,\mathbf{y}_2,\mathbf{y}_3)). \tag{3.17}$$

The linearizing control law can now be found from the condition

$$\dot{\mathbf{y}}_4 = \mathbf{v}, \tag{3.18}$$

where $\mathbf{v}$ is a new control unit. Computing $\dot{\mathbf{y}}_4$ from (3.13) and suppressing function arguments for brevity yields

$$\begin{aligned}\nu = {} & \frac{\partial f_4}{\partial \mathbf{x}_1}\mathbf{x}_2 - \frac{\partial f_4}{\partial \mathbf{x}_2}D^{-1}(\mathbf{c} + K(\mathbf{x}_1 - \mathbf{x}_3)) + \frac{\partial f_4}{\partial \mathbf{x}_3}\mathbf{x}_4 \\ & + \frac{d}{dt}[D^{-1}]K\mathbf{x}_4 + D^{-1}K\left(J^{-1}K(\mathbf{x}_1 - \mathbf{x}_3) + J^{-1}\mathbf{u}\right) \\ := {} & -\beta^{-1}(\mathbf{x})\alpha(\mathbf{x}) + \beta^{-1}(\mathbf{x})\mathbf{u},\end{aligned} \tag{3.19}$$

where $\beta^{-1}(\mathbf{x})\alpha(\mathbf{x})$ denotes all the terms in (3.19) but the last term, which involves the input $\mathbf{u}$, and $\beta^{-1} := D^{-1}KJ^{-1}$.

Solving the above expression for u yields

$$\mathbf{u} = \alpha(\mathbf{x}) + \beta(\mathbf{x})\nu \tag{3.20}$$

With the nonlinear change of coordinates (3.10)–(3.13) and nonlinear feedback (3.20) the system (3.6)–(3.9) now has the linear block form

$$\begin{aligned}\dot{\mathbf{y}} &= \begin{pmatrix} 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \\ 0 & 0 & 0 & 0 \end{pmatrix}\mathbf{y} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ I \end{pmatrix}\mathbf{v} \\ &:= A\mathbf{y} + B\mathbf{v},\end{aligned} \tag{3.21}$$

where $I = n \times n$ identity matrix, $0 = n \times n$ zero matrix, $\mathbf{y}^{\mathrm{T}} = (\mathbf{y}_1^{\mathrm{T}}, \mathbf{y}_2^{\mathrm{T}}, \mathbf{y}_3^{\mathrm{T}}, \mathbf{y}_4^{\mathrm{T}}) \in R^{4n}$, and $\mathbf{v} \in R^n$. The system (3.21) is (modulo a reordering of the equations) said to be in *Brunovsky canonical form*.

The nonlinear control law (3.20) is not completely determined until the function $\mathbf{v}$ is specified. Since $\mathbf{v}$ is the control input to the linear system (3.21) it can be designed to control the linear system (3.21). For example, linear optimal control methods or pole placement technique can be used to design $\mathbf{v}$. In the next section we will discuss a variable structure design for $\mathbf{v}$ in order to guarantee robustness to parametric uncertainty in the inner-loop control (3.20).

Remark:

It is easy to determine from the linear system (3.21) what the response of the system in the $\mathbf{y}_i$ coordinate system will be. The corresponding response of the original coordinates $\mathbf{x}_i$ is not necessarily easy to determine since the nonlinear coordinate transformation (3.10)–(3.13) must be inverted to find the $\mathbf{x}_i$. However, in this case the transformed coordinates $\mathbf{y}_i$ are themselves physically meaningful. Inspecting (3.10)–(3.13), we see that the variables $\mathbf{y}_1$, $\mathbf{y}_2$, $\mathbf{y}_3$, and $\mathbf{y}_4$ are n-vectors representing, respectively, the link positions, velocities, accelerations, and jerks (derivative of the acceleration). Since the motion trajectory of the manipulator is typically specified in terms of these quantities [7], they are natural variables to use for control.

4. A Variable Structure Control Approach

Variable structure control of systems through sliding regimes has been the subject of extensive investigations for the last 30 years. The reader is referred to the work of Utkin [23–25] and Itkis [19] for complete details on the historical aspects of the approach and its wide range of applications. For the application of variable structure theory to the control of rigid manipulators the reader is referred to ref. [15].

The approach is based on the specification of a sliding surface or sliding manifold where the system state is forced to evolve thanks to active switchings among available feedback control laws. These laws are designed to bring the state trajectory to the vicinity of the surface and maintain it there in an "overshoot and correct" fashion. The static relationships among the state variables describing the surface are imposed on the dynamic equations of the controlled system. As a consequence of this, the state variables used as dependent variables to describe the surface lose their character of independent state variables and can be discarded in the description of the resulting reduced order manifold-constrained dynamics. The resulting state equations specify a behavior which is largely independent of the original system parameters and depending instead on the design parameters used for the synthesis of the sliding-surface equations. If a sliding motion can be created and the manifold conditions are satisfied in an average sense, the surface parameters totally determine the dynamic evolution of the controlled system. The sliding surface is thus designed to bestow desirable characteristics to the average ideally manifold-constrained motions of the system. In what follows we assume that both the original vectors $\mathbf{x}_i$, and the transformed vectors $\mathbf{y}_i$ may be used for feedback, and we consider the robust control problem.

Following [19], we consider the transformed system:

$$\begin{aligned}\dot{\mathbf{y}}_1 &= \mathbf{y}_2, \\ \dot{\mathbf{y}}_2 &= \mathbf{y}_3, \\ \dot{\mathbf{y}}_3 &= \mathbf{y}_4, \\ \dot{\mathbf{y}}_4 &= \mathbf{v}.\end{aligned} \tag{4.1}$$

This system represents n uncoupled fourth-order subsystems of the form:

$$\begin{aligned} \dot{y}_{1i} &= y_{2i}, \\ \dot{y}_{2i} &= y_{3i}, \\ \dot{y}_{3i} &= y_{4i}, \\ \dot{y}_{4i} &= \nu_i. \end{aligned} \tag{4.2}$$

On each subsystem a switching function is defined as

$$s_i = \sum_{j=1}^{3} m_{ij} y_{ji} + y_{4i}, \tag{4.3}$$

then the set

$$S_i = \left\{ \mathbf{y} \in R^4 \colon s_i = \sum_{j=1}^{3} m_{ij} y_{ji} + y_{4i} = 0 \right\} \tag{4.4}$$

defines a *sliding surface* for the ith subsystem and establishes an *algebraic relationship* among the subsystem state variables. If, in particular, we choose y_{4i} to act as the dependent variable in the submanifold definition in (4.4), then y_{4i} does not qualify as a state variable under ideal sliding-motion conditions (i.e., if the surface equation is exactly satisfied) and it is to be discarded from the system description in sliding mode. The function s_i is addressed as the *surface coordinate function* and it is useful to obtain a differential equation describing its evolution in terms of the undiscarded state variables. The ith subsystem equations can be written as

$$\dot{y}_{ji} = y_{j+1,i}, \qquad j = 1,2,3,\ i = 1,\dots,n, \tag{4.5}$$

$$\dot{s}_i = m_{i3} s_i + \sum_{j=1}^{3} \left[m_{i(j-1)} - m_{i3} m_{ij} \right] y_{ji} + \nu_i, \qquad m_{i0} := 0. \tag{4.6}$$

If a sliding motion can be created on $s_i = 0$ by means of an appropriate VSC law,

$$\nu_i = \begin{cases} \nu_i^+ & \text{for } s_i > 0, \\ \nu_i^- & \text{for } s_i < 0, \end{cases} \tag{4.7}$$

the resulting reduced order dynamics, ideally evolving on the sliding surface, are described by

$$\begin{aligned} \dot{y}_{ji} &= y_{j+1,i}, \qquad j = 1,2,\ i = 1,\dots,n, \\ \dot{y}_{3i} &= -\sum_{j=1}^{3} m_{ij} y_{ji}. \end{aligned} \tag{4.8}$$

Each subsystem is then totally governed by the design coefficients of the switching *surface* m_{ij} and therefore the stability of the controlled system depends on the choice of these design parameters. In the case at hand, stability is guaranteed whenever the m_{ij} coefficients are chosen as

$$m_{ij} > 0,\ j = 1,2,3, \qquad m_{i3} m_{i2} - m_{i1} > 0,\ i = 1,\dots,n. \tag{4.9}$$

The smooth control input which makes S_i into an invariant surface for the state trajectories is obtained from the following invariance conditions:

$$s_i = 0, \qquad \dot{s}_i = 0 \tag{4.10}$$

as

$$\nu_{i,EQ} = -\sum_{j=1}^{3} \left[m_{i(j-1)} - m_{i3} m_{ij} \right] y_{ji}. \tag{4.11}$$

This control is known as the *equivalent control* [23]. Its existence constitutes a necessary condition for the creation of a sliding motion on the sliding surface.

A necessary and sufficient condition for the existence of a sliding regime on S_i is that the following existence conditions be satisfied:

$$\lim_{s_i \to +0} \dot{s}_i < 0, \qquad \lim_{s_i \to -0} \dot{s}_i > 0. \tag{4.12}$$

It follows from (4.11) and (4.12) that these conditions are equivalent to

$$\begin{aligned} \nu_i^+ &> \nu_{i,EQ}, \\ \nu_i^- &< \nu_{i,EQ}. \end{aligned} \tag{4.13}$$

A variable structure control law that satisfies the existence conditions is synthesized by

$$\nu_i = -\lambda_i |\nu_{i,EQ}| \operatorname{sign} s_i, \qquad \lambda_i > 1. \tag{4.14}$$

A second possibility is to prescribe a linear control law with variable structure gains of the form:

$$\nu_i = -\sum_{i=1}^{3} \psi_{ij} y_{ji}, \tag{4.15}$$

with ψ_{ij} taking two possible values $\{\psi_{ij}^+, \psi_{ij}^-\}$. Substituting the controller expression in the differential equation for s_i and imposing the existence conditions (4.12), the following variable structure gain switchings law is obtained:

$$\psi_{ij} = \begin{cases} \psi_{ij}^+ > m_{i(j-1)} - m_{i3} m_{ij} & \text{for } s_i y_{ji} > 0, \\ \psi_{ij}^- < m_{i(j-1)} - m_{i3} m_{ij} & \text{for } s_i y_{ji} < 0, \end{cases} \tag{4.16}$$
$$j = 1,2,3, \qquad i = 1,2,\dots,n.$$

The above approach generates, upon inversion of the linearizing transformation, a set of nonlinear sliding *surfaces* and a nonlinear variable structure control law whose switchings are also determined by nonlinear condi-

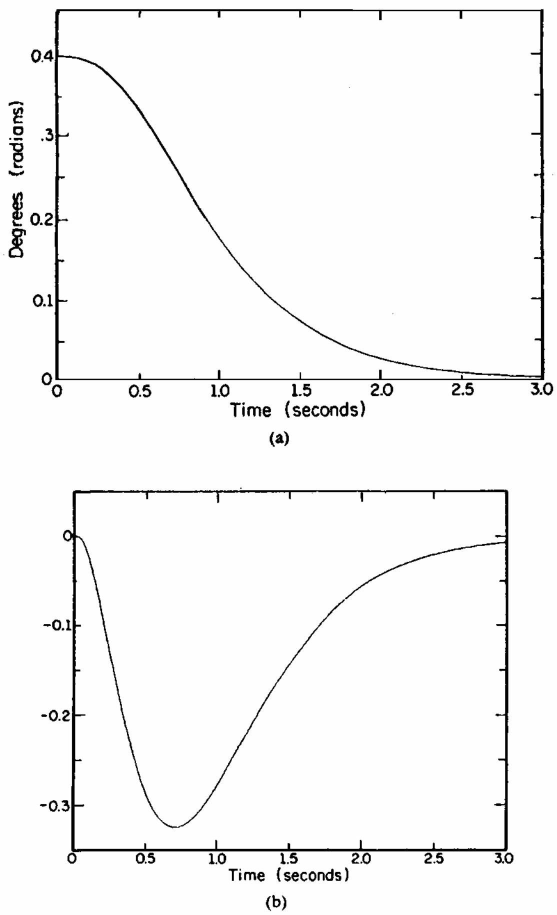

Figure 4. (a) Link 2 position (radians). (b) Link 2 velocity (radians/sec)

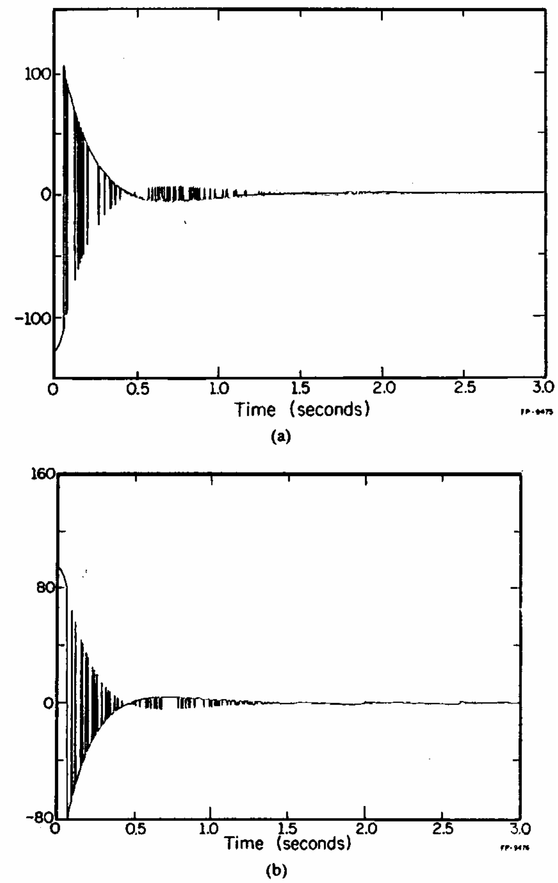

Figure 5. (a) Outer-loop control ν_1. (b) Outer-loop control ν_2

outer loop model (5.3), where a sudden change of 100% in the nominal values of the perturbation terms ϕ and ψ was allowed during sliding motion. The SIMNON nonlinear simulation package was used in the simulations. The insensitivity of the controlled motion to the parametric perturbations is clearly demonstrated by these simulations.

7. Conclusions and Suggestions for Further Research

In this article the concepts of global feedback linearization and variable structure control have been combined to design a robust control law for flexible joint manipulators. Diffeomorphic state coordinate transformation and nonlinear static state feedback account for the exact linearization in an inner-loop stage, while an outer-loop variable structure control law is designed to achieve robustness to parameter uncertainty and external disturbances.

The simplicity of the linear Brunovsky form not only allows for a straightforward specification of stable sliding surfaces and switching laws but also allows the demonstration of the robustness of the variable structure controller in a transparent manner.

It should be pointed out that in order to achieve the robustness to parameter uncertainty, the proposed scheme requires for its implementation the measurement or estimation of the transformed state variables, which in this case are the vectors of joint position, velocity, acceleration, and jerk. It is, of course, a tautology that any state feedback control algorithm requires feedback of the system state. An interesting topic for future research would be to include a robust state estimator as part of the overall control scheme to overcome the difficulties of measuring directly the joint acceleration and jerk.

Since the Brunovsky form used here is independent of the joint stiffness, the proposed control algorithm may become ill-conditioned for manipulators with very stiff joints. This is because, for very stiff joints, the system is "nearly" a system of coupled second-order equations rather than fourth-order equations. In this case, the results of refs. [11], [12], and [18] can be used to exploit the resulting two-time-scale behavior. It would be of interest here to investigate the properties of the variable structure controller here as the joint stiffness $k \to \infty$.

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Dr. Sira-Ramirez and Dr. Spong's photographs and biographies are unavailable.