Switched Control of Bilinear Converters Via Pseudolinearization

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Abstract —In this paper a new method is proposed for designing regulating feedback control laws in bilinear networks such as the boost and the buck–boost dc-to-dc power supplies. A pseudolinearization approach is taken on the average pulsewidth modulated controlled model with physically meaningful local state coordinate transformations. For such transformations, the perturbed average model is independent of the operating point and exhibits a linear structure in Brunovsky's canonical form. The feedback regulation scheme is thus considerably simplified from the conceptual viewpoint.

Keywords — DC-to-DC power converters, pseudolinearization, nonlinear circuits.

I. Introduction

FEEDBACK regulation of switched-controlled bilinear networks is accomplished by means of pulsewidth-modulation (PWM) control techniques or, equivalently, by means of variable structure feedback (VSF) strategies inducing stabilizing sliding regimes (see [1]–[7]).

In [8] a general relationship is established among average PWM-controlled responses and ideal VSF controlled trajectories. This equivalence asserts that integral manifolds of average PWM-controlled networks qualify as sliding surfaces on which the corresponding *equivalent control* [9] coincides with the *duty ratio* associated with the PWM control scheme. In general, average PWM models are obtained from a nonlinear system model simply by substituting the switch position function by the duty ratio function. Similarly, the ideal sliding dynamics is obtained by replacing the discontinuous control function by the smooth equivalent control. The underlying assumption made in obtaining both average models is the high (infinite) frequency switching assumption.

In this article, a new approach is presented for the feedback control of bilinear switched networks such as the boost and buck–boost dc-to-dc power converters. The method is based on pseudolinearization [10] of the average PWM-controlled network on a controllable submanifold of stable operating points. A physically meaningful local diffeomorphic state coordinate transformation, expressible in terms of stored energy and consumed power, is found which turns the average perturbed model into a Brunovsky canonical form independent of the operating point. The linearized model is then used for the specification of a stabilizing feedback loop whose control action regulates the duty ratio around its nominal operating value. A similar, but conceptually different, approach was taken in [4] for the variable structure control of the boost converter using feedback linearization (see Hunt *et al.* [11]).

Section II presents the pseudolinearization of the boost and buck–boost converters. Section III develops a feedback control method based on the linearized perturbed model. Section IV contains the conclusions of the article.

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II. Pseudolinearization of Converter Models

2.1. The Pseudolinearization Method

The following presentation closely follows that of Reboulet and Champetier [10].

Consider the nonlinear single-input system:

$$dx/dt = f(x) + \nu g(x) \qquad (2.1)$$

with $x \in R^n$, f and g smooth local vector fields defined on an open neighborhood X of R^n and ν is a, possibly discontinuous, scalar control function ν: $R^n \to R$. The set of stable operating points is expressed as

$$C_{x,\nu} := \{(x_0, \nu_0) \in R^{n+1} \text{ s.t. } f(x_0) + \nu_0 g(x_0) = 0\} \quad (2.2)$$

with local projection on X given by

$$C_x = \{x_0 \in R^n \text{ s.t. } \exists \nu_0 \text{ s.t. } f(x_0) + \nu_0 g(x_0) = 0\}. \quad (2.3)$$

Let $F(x_0), G(x_0)$, respectively, denote the Jacobian matrices, $\partial f/\partial x$ and $\partial g/\partial x$, of f and g evaluated at x_0. Then, in the neighborhood of an operating point (x_0, ν_0), the dynamic behavior of the system is approximately linear and described by

$$d(\delta x)/dt = F(x_0, \nu_0)\,\delta x + g(x_0)\,\delta\nu \qquad (2.4)$$

where $F(x_0, \nu_0) := F(x_0) + \nu_0 G(x_0)$ while $\delta x \in R^n$, and $\delta\nu \in R$. One assumes that for any $(x_0, \nu_0) \in C_{x,\nu}$, the pair $[F(x_0, \nu_0), g(x_0)]$ is controllable.

It was shown in [10] that, for a given ν, the set of operating points $\mathscr{C}_{x,\nu}$, in the neighborhood of which the linear tangent model of system (2.1) is controllable, constitutes a one-dimensional submanifold of R^{n+1} and it is open in $C_{x,\nu}$. $\mathscr{C}_x$ denotes the projection of $\mathscr{C}_{x,\nu}$ on the x-state space. Evidently, $\mathscr{C}_x$ is contained in C_x. Notice that, necessarily, at every point $(x_0, \nu_0) \in \mathscr{C}_{x,\nu}$, the vector $(x_0) \neq 0$.

Lemma 2.1 [10]: Let there exist a local C^1-diffeomorphism $\phi = (\phi_1, \cdots, \phi_n)^T$ from R^n into R^n such that $\mathscr{C}_{x,\nu}$ is given by $\phi_2(x) = \cdots = \phi_n(x) = 0$, then for any 1-form α given along $\mathscr{C}_{x,\nu}$ there locally exists a function T of class ^{1} such that

$$\alpha = dT|_{\mathscr{C}_{x,\nu}} \tag{2.5}$$

., the 1-form α is locally integrable along $\mathscr{C}_{x,\nu}$. □

Theorem 2.2 [10]: Let the nonlinear system (2.1) have a controllable tangent model (2.4) along $C_{x,\nu}$ and let $\mathscr{C}_{x,\nu}$ be given as in Lemma 2.1 for some $\mathscr{C}^1$-diffeomorphism ϕ, then there exist mappings $z = T(x)$, $v = T_{n+1}(x, v) = \alpha(x)$ $\beta(x)\nu$, with $\beta(x) \neq 0$ in $\mathscr{C}_x$, such that *the tangent model in the transformed system* in the z-state space:

$$d(\delta z_i)/dt = \delta z_{i+1}, \qquad i = 1, 2, \cdots, n-1$$
$$d(\delta z_n)/dt = \delta v \tag{2.6}$$

is independent of the operating point. □

To obtain the tangent model (2.6) the following equations must be locally satisfied on $C_{x,\nu}$:

$$\alpha_1 F^{i-1}(x_0, \nu_0) g(x_0) = 0, \qquad i = 1, 2, \cdots, n-1 \tag{2.7}$$

$$\alpha_i = \alpha_1 F^{i-1}(x_0, \nu_0), \qquad i = 2, 3, \cdots, n \tag{2.8}$$

$$\alpha_{n+1} = \alpha_n [F(x_0, \nu_0), g(x_0)]. \tag{2.9}$$

The problem then becomes one of finding 1-forms $\alpha_1, \cdots, \alpha_n$ (and α_{n+1}) satisfying (2.7)–(2.9) at any point of $_x$ (and of $C_{x,\nu}$) such that there exist mappings $T_i(x)$ $= 1, 2, \cdots, n)$ and $T_{n+1}(x, \nu)$ such that

$$\alpha_i = dT_{i|C_x}, \qquad i = 1, 2, \cdots, n$$
$$\alpha_{n+1} = dT_{n+1}|_{C_{x,\nu}}. \tag{2.10}$$

The procedure for finding the appropriate transformations consists of three basic steps: i) choose any arbitrary covector α_1 along the direction determined by (2.7), ii) compute $\alpha_2, \cdots, \alpha_n$ from (2.8) and (2.9), and iii) integrate $(i = 1, 2, \cdots, n)$ along C_x and α_{n+1} along $C_{x,\nu}$ to obtain one mappings $T_1, \cdots, T_{n+1}$.

Remark 1: The pseudolinearization method constitutes in *extension* of the results on global feedback linearization [1]. This extension allows one to find locally diffeomorphic state coordinate transformations leading to dynamical systems, not necesarily linear, whose tangent models are controllable and independent of the operating point. Evidently, if the system is globally feedback linearizable, the transformations obtained by pseudolinearization may be made to conincide with those of exact feedback linearization developed by Hunt, Su, and Meyer [11] (see [10] for some examples). In such a case, the tangent model corresponding to the linearized system is, itself, independent of the operating point and describable in Brunovsky-controllable canonical form. It is in this respect that the pseudolinearization method contains that of feedback linearization.

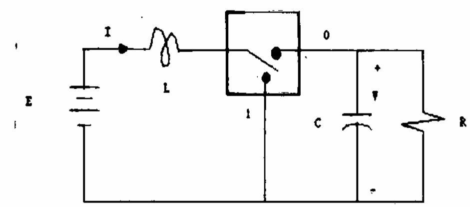

Fig. 1. Boost converter.

For systems in which the exact linearization conditions of [11] fail, and therefore the system is not globally feedback linearizable, the pseudolinearization method allows for a clear and straightforward identification of the region where a local diffeomorphic state coordinate transformation exists which produces a system with a tangent model in Brunovsky-controllable canonical form (i.e., independent of the coordinates of the operating point located on an equally clearly identified manifold of stable and controllable operating points). A linear design method seeking stabilization around any of the operating points results in no need of "gain scheduling" as long as the system trajectories remain close to the manifold of operating points and within the region of validity of the pseudolinearization method (i.e., in the region of controllability of the tangent model). □

In the next section the pseudolinearization method is applied to obtain a feedback control method for the regulation of bilinear switchmode power converters. This class of circuits is, generally speaking, not globally feedback linearizable.

2.2. Pseudolinearization of Average PWM-Controlled Switchmode Power Supplies

In this section the problem is posed of obtaining tangent models of average PWM-controlled systems, representing dc-to-dc switchmode power supplies, which are controllable and independent of the operating point. Such networks are represented, in their traditional state space, by bilinear dynamical models (Wood [13], Sira-Ramirez [2]). The boost ("up") and the buck–boost ("up–down") converters are treated and a linearized model in Brunovsky canonical form is obtained which is trivially independent of the operating point. The set of controllable equilibrium points $C_{x,\nu}$, C_x and the sets $\mathscr{C}_{x,\nu}$ and $\mathscr{C}_x$ are clearly identified and linearizing transformations are readily obtained following the procedure indicated in the last section. These transformations, aside from leading to locally exactly linearized models, turn out to have an interesting physical meaning in terms of the total stored energy and the consumed power.

Boost Converter

Consider the boost converter model shown in Fig. 1:

$$dy_1/dt = b - \omega_0 y_2 + u\omega_0 y_2$$
$$dy_2/dt = \omega_0 y_1 - \omega_1 y_2 - u\omega_0 y_1 \tag{2.11}$$

where $y_1 = \sqrt{L}i$, $y_2 = \sqrt{C}V$, $b = E/\sqrt{L}$, $\omega_0 = 1/\sqrt{LC}$, $\omega_1 =$

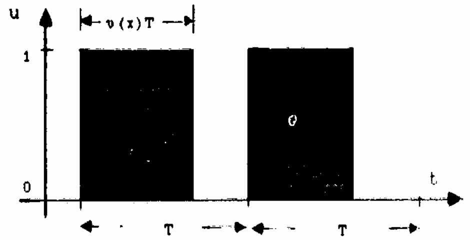

Fig. 2. Typical duty cycle and duty ratio.

$1/RC$, and u denotes the switch position function, acting as a control input, and taking values in the discrete set $U=\{0,1\}$. The vector fields f and g are given by $f=(b-w_0y_2)\partial/\partial y_1+(w_0y_1-w_1y_2)\partial/\partial y_2$; $g=w_0y_2\partial/\partial y_1-w_0y_1\partial/\partial y_2$ where $\partial/\partial y_1$ and $\partial/\partial y_2$ are the unit direction vectors in the tangent space of R^2.

An idealized PWM control strategy is defined as one in which the switchings to the $u=1$ position are assumed to occur at the beginning of each period known as the *duty cycle* and turning to the $u=0$ position once within the duty cycle according to a switching policy determined by a smooth, or possibly continuous piecewise smooth, feedback function of the average state vector x, known as the *duty ratio*, and denoted by $\nu(x)$. The duty cycle is assumed to be periodical with infinitesimally small period (i.e., infinitely large frequency) and the duty ratio is the fraction of the duty cycle on which the switch position is at $u=1$. Hence $0<\nu(x)<1$ (see Fig. 2). Under the high frequency assumption, it was rigorously demonstrated in [3] that the *average PWM model* is given by the nonlinear model:

$$dx_1/dt=b-[1-\nu(x)]\omega_0x_2$$
$$dx_2/dt=[1-\nu(x)]\omega_0x_1-\omega_1x_2. \qquad (2.12)$$

If an equilibrium point x_0 exists then $v_0=v(x_0)$ is a constant scalar satisfying $0<v_0<1$. The equilibrium point of (2.12), if it exists for the given $v(x)$, is given by

$$x_{10}=\omega_1b[\omega_0(1-\nu_0)]^{-2};\qquad x_{20}=b[\omega_0(1-\nu_0)]^{-1}. \qquad (2.13)$$

Then, eliminating v_0 in (2.13), it follows from the definitions of Section 2.1 that the sets $C_{x,\nu}$ and C_x are given by

$$C_{x,\nu}=\{(x_0,\nu_0)\in R^3:\ x_{10}=\omega_1b^{-1}x_{20}^2, 0<\nu_0<1\} \qquad (2.14)$$
$$C_x=\{x_0\in R^2:\ \exists\nu_0\in(0,1)\text{ in }R,\text{ s.t. }x_{10}=\omega_1b^{-1}x_{20}^2\} \qquad (2.15)$$

while the set of operating points where (2.12) has controllable tangent models, is of the form:

$$\mathscr{C}_{x,\nu}=\{(x_0,\nu_0)\in R^3:\ x_{10}=\omega_1b^{-1}x_{20}^2,$$
$$\omega_1b\omega_0^{-2}<x_{10}<\infty, b\omega_0^{-1}<x_{20}<\infty; 0<\nu_0<1\} \qquad (2.16)$$

while its projection on X is given by

$$\mathscr{C}_x=\{x_0\in R^2:\ \exists\nu_0\in(0,1)\text{ in }R,\text{ s.t. }x_{10}=\omega_1b^{-1}x_{20}^2,$$
$$\omega_1b\omega_0^{-2}<x_{10}<\infty, b\omega_0^{-1}<x_{20}<\infty\} \qquad (2.17)$$

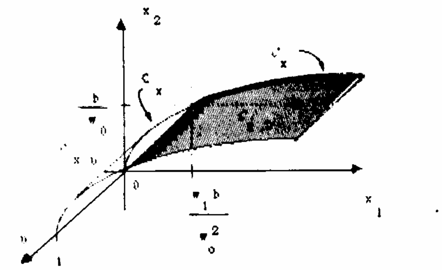

Fig. 3. Submanifold of controllable state operating points in R^3, and its projection on R^2, for the boost converter.

These sets are all shown in Fig. 3.

In the neighborhood of an operating point (x_0,ν_0) the dynamic behavior of the average system (2.12) may be considered as linear in the form (2.4) with

$$F(x_0,\nu_0)=\begin{vmatrix}0 & -(1-\nu_0)\omega_0\\ (1-\nu_0)\omega_0 & -\omega_1\end{vmatrix}$$
$$g(x_0)=\begin{vmatrix}\omega_0 & x_{20}\\ -\omega_0 & x_{10}\end{vmatrix}. \qquad (2.18)$$

The pair $[F(x_0,\nu_0),g(x_0)]$ is controllable everywhere in R^2 except on $x_2=0$ and $x_1=-b(2\omega_1)^{-1}$. None of these uncontrollability sets intersects $\mathscr{C}_x$. Notice that, from (2.15), C_x and $\mathscr{C}_x$ are contained by the level set $\phi_2(x)=0$ with ϕ_2 being a local C^1-diffeomorphism given by

$$\phi_2(x)=bx_1-\omega_1x_2^2=0. \qquad (2.19)$$

Hence any 1-form α given along $\mathscr{C}_x$ is integrable.

Let α_i be 1-forms over $\mathscr{C}_x$, $\alpha_i=dT_i|_{\mathscr{C}_x}$, $i=1,2$. To obtain a tangent model, independent of the operating point the α_i's must satisfy, on $\mathscr{C}_{x,\nu}$, equations (2.7)–(2.9).

According to (2.7), $\alpha_1g(x_0)=0$ on $\mathscr{C}_x$. This results, after integration of the corresponding partial differential equation, in $\alpha_1=x_{10}\,dx_1+x_{20}\,dx_2$ from where

$$z_1=T_1(x)=0.5(x_1^2+x_2^2). \qquad (2.20)$$

Using (2.8) and (2.18) we obtain after some manipulations involving the expressions for the equilibrium points (2.13) that the 1-form α_2 is: $\alpha_2=\alpha_1F(x_0,\nu_0)=b\,dx_1-2\omega_1x_{20}\,dx_2$. Integrating this expression along C_x we obtain T_2 as

$$z_2=T_2(x)=bx_1-\omega_1x_2^2. \qquad (2.21)$$

Finally, using (2.9),

$$\begin{aligned}\alpha_3&=\alpha_2[F(x_0,\nu_0),g(x_0)]\\ &=-2(1-\nu_0)\omega_0\omega_1x_{20}\,dx_1\\ &\quad+[-b(1-\nu_0)\omega_0+2\omega_1^2x_{20}]\,dx_2\\ &\quad+[b\omega_0x_{20}+2\omega_1\omega_0x_{10}x_{20}]\,dv\end{aligned}$$

and after further straightforward manipulations involving (2.13), one obtains

$$\begin{aligned}\nu=z_3=T_3(x,\nu)&=(b^2-b\omega_0x_2-2\omega_1\omega_0x_1x_2+2\omega_1^2x_2^2)\\ &\quad+(b\omega_0x_2+2\omega_1\omega_0x_1x_2)\nu\\ &=:\alpha(x)+\beta(x)\nu. \qquad (2.22)\end{aligned}$$

The transformation $T_1(x)$ represents the total average stored energy by the circuit while its rate of change, $T_2(x)$, is simply the average input power minus the average output power, which we term the "total average consumed power." T_3 is then the rate of change of the total average consumed power.

Evidently,

$$dT_1/dt = T_2; \quad dT_2/dt = T_3 =: v$$

or

$$dz_1/dt = z_2; dz_2/dt = z_3 \equiv v \tag{2.23}$$

i.e., the transformed system is already in Brunovsky-controllable canonical form. The tangent model to (2.23) is thus trivially independent of the operating point, and in z-state space such model is expressed as

$$d(\delta z_1)/dt = \delta z_2; \quad d(\delta z_2)/dt = \delta z_3 \equiv \delta v. \tag{2.24}$$

The inverse transformation $x = T^{-1}(z)$ does not globally exist in R^3, thus indicating the local character of the diffeormorphism T. However, in the region of interest, $\mathscr{C}_x$, T is indeed a local diffeomorphism with inverse computed as

$$x_1 = b(2w_1)^{-1}\left[1+(2w_1b^{-1})^{-2}(2z_1+w_1^{-1}z_2)\right]^{1/2}$$

$$x_2 = \left\{b^2(2w_1^2)^{-1}\left\{\left[1+(2w_1b^{-1})^{-2}(2z_1+w_1^{-1}z_2)\right]^{1/2}-1\right\} - w_1^{-1}z_2\right\}^{1/2}. \tag{2.25}$$

Notice, that in the z-state space a controller design for stabilization of the transformed system (2.23) around a desirable transformed equilibrium point (z_1^*, z_2^*) is very simple. Firstly, under steady-state conditions $z_2^* = 0$ and for a given desirable set point z_1^*, the required feedback control law for (2.23) is $v(z) = -m_1(z_1 - z_1^*) - m_2 z_2$ with $m_1, m_2 > 0$ producing desired stable closed-loop eigenvalues. The required nonlinear controller ν is obtained from (2.22) as

$$\nu = \beta^{-1}(x)[v(z) - \alpha(x)]|_{x=T^{-1}(z)}$$
$$= \beta^{-1}(T^{-1}(z))[v(z) - \alpha(T^{-1}(z))]. \tag{2.26}$$

In Section IV a control scheme, based on the above regulation scheme and on that of the "method of dynamic inverses," (see [12]), are proposed for the feedback regulation of dc-to-dc power converters.

Buck-Boost Converter

Consider the buck-boost converter model shown in Fig. 4:

$$dy_1/dt = \omega_0 y_2 + u(b - \omega_0 y_2)$$
$$dy_2/dt = -\omega_0 y_1 - \omega_1 y_2 + u\omega_0 y_1 \tag{2.27}$$

where $y_1 = \sqrt{L}i$, $y_2 = \sqrt{C}V$, $b = E/\sqrt{L}$, $\omega_0 = 1/\sqrt{LC}$, $\omega_1 = 1/RC$, and u denotes the switch position function, acting as a control input, and taking values in the discrete set $U = \{0,1\}$. The vector fields f and g are given by $f = w_0 y_2\, \partial/\partial y_1 + (-w_0 y_1 - w_1 y_2)\, \partial/\partial y_2$; $g = (b - w_0 y_2)\, \partial/\partial y_1 + w_0 y_1\, \partial/\partial y_2$ where $\partial/\partial y_1$ and $\partial/\partial y_2$ are the unit direction vectors in the tangent space of R^2.

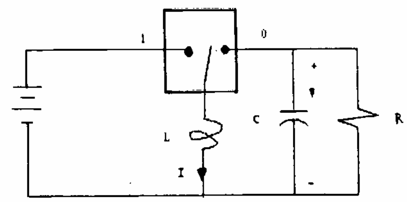

Fig. 4. Buck-boost converter.

The *average PWM model*, with duty ratio $\nu(x)$ obtained as a smooth feedback function of the average state vector x, is given by the nonlinear system:

$$dx_1/dt = [1-\nu(x)]\omega_0 x_2 + b\nu(x)$$
$$dx_2/dt = -[1-\nu(x)]\omega_0 x_1 - \omega_1 x_2. \tag{2.28}$$

If an equilibrium point x_0 exists then $\nu_0 = \nu(x_0)$ is a constant scalar satisfying $0 < \nu_0 < 1$. The equilibrium point of (2.28), if it exists for the given $v(x)$, is given by

$$x_{10} = \omega_1 b \nu_0 [\omega_0(1-\nu_0)]^{-2}$$
$$x_{20} = -b\nu_0[\omega_0(1-\nu_0)]^{-1}. \tag{2.29}$$

Then, eliminating ν_0 in (2.29), it follows from the definitions of Section 2.1 that the sets $C_{x,\nu}$ and C_x are given by

$$C_{x,\nu} = \{(x_0, \nu_0) \in R^3 \colon x_{10} = \omega_1 b^{-1} x_{20}(x_{20} - b\omega_0^{-1}),\ 0 < \nu_0 < 1\} \tag{2.30}$$

$$C_x = \{x_0 \in R^2 \colon \exists \nu_0 \in (0,1) \text{ in } R,\ \text{s.t. } x_{10} = \omega_1 b^{-1} x_{20}(x_{20} - b\omega_0^{-1})\} \tag{2.31}$$

while the set of operating points where (2.27) has controllable tangent models, is given by

$$\mathscr{C}_{x,\nu} = \{(x_0, \nu_0) \in R^3 \colon x_{10} = \omega_1 b^{-1} x_{20}(x_{20} - b\omega_0^{-1}),\ 0 < x_{10} < \infty, -\infty < x_{20} < 0;\ 0 < \nu_0 < 1\} \tag{2.32}$$

and its projection on the x-state space is

$$\mathscr{C}_x = \{x_0 \in R^2 \colon \exists \nu_0 \in (0,1) \text{ in } R,\ \text{s.t. } x_{10} = \omega_1 b^{-1} x_{20}(x_{20} - b\omega_0^{-1}),\ 0 < x_{10} < \infty, -\infty < x_{20} < 0\}. \tag{2.33}$$

These sets are all shown in Fig. 5.

In the neighborhood of an operating point (x_0, ν_0) the dynamic behavior of the average system (2.27) may be considered as linear in the form (2.4) with

$$F(x_0, \nu_0) = \begin{vmatrix} 0 & (1-\nu_0)\omega_0 \\ -(1-\nu_0)\omega_0 & -\omega_1 \end{vmatrix}$$

$$g(x_0) = \begin{vmatrix} b - \omega_0 x_{20} \\ \omega_0 x_{10} \end{vmatrix}. \tag{2.34}$$

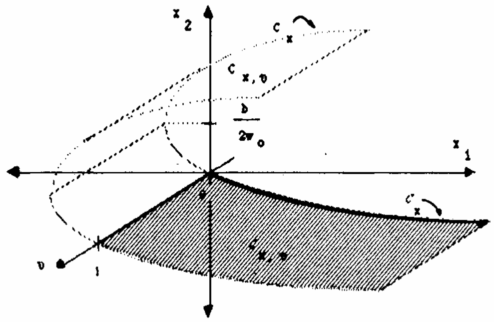

Fig. 5. Submanifold of controllable stable operating points in R^3, and its projection on R^2, for the buck–boost converter.

The pair $[F(x_0,\nu_0), g(x_0)]$ is controllable everywhere in R^2 except on $x_2 = 0$ and $x_1 = -b(2\omega_1)^{-1}$. None of these uncontrollability sets intersects $\mathscr{C}_x$. Notice that, from (2.31), C_x and $\mathscr{C}_x$ are contained by the level set $\phi_2(x) = 0$ with ϕ_2 being a local C^1-diffeomorphism given by

$$\phi_2(x) = bx_1 - \omega_1 x_2(x_2 - b\omega_0^{-1}) = 0. \quad (2.35)$$

Hence any one-form α given along $\mathscr{C}_x$ is integrable.

Let α_i be one-forms over $\mathscr{C}_x$, $\alpha_i = dT_i|_{\mathscr{C}_x}$, $i = 1,2,\cdots,T$. To obtain a tangent model, independent of the operating point the α_i's must satisfy, on $\mathscr{C}_{x,\nu}$, equations (2.7)–(2.9).

According to (2.7), $\alpha_1 g(x_0) = 0$ on $\mathscr{C}_x$. This results in $\alpha_1 = x_{10}\,dx_1 + (x_{20} - b\omega_0^{-1})\,dx_2$ from where

$$z_1 = T_1(x) = 0.5\left[x_1^2 + (x_2 - b\omega_0^{-1})^2\right]. \quad (2.36)$$

Using (2.8) and the expression (2.34) we obtain after some manipulations involving the expressions for the equilibrium points (2.29) that the 1-form α_2 is given by

$$\alpha_2 = \alpha_1 F(x_0,\nu_0) = b\,dx_1 - \left[2\omega_1 x_{20} - b\omega_1\omega_0^{-1}\right] dx_2.$$

Integrating this expression along C_x we obtain T_2 as

$$x_2 = T_2(x) = bx_1 - \omega_1 x_2(x_2 - b\omega_0^{-1}). \quad (2.37)$$

Finally, using (2.9),

$$\begin{aligned}\alpha_3 &= \alpha_2[F(x_0,\nu_0), g(x_0)]\\ &= 2(1-\nu_0)\omega_0\omega_1 x_{20}\,dx_1\\ &\quad + \left[-b(1-\nu_0)\omega_0 + 2\omega_1^2 x_{20}\right] dx_2\\ &\quad + \left[b^2 - b\omega_0 x_{20} - 2\omega_1\omega_0 x_{10}x_{20} + b\omega_1 x_{10}\right] d\nu.\end{aligned}$$

Integrating this expression along $\mathscr{C}_x$ one obtains after further manipulations involving (2.29):

$$\begin{aligned}T_3(x,\nu) = &\left[(1-\omega_1^2\omega_0^{-2})b\omega_0 x_2 - b\omega_1 x_1 x\right.\\ &\left.+ 2\omega_1\omega_0 x_1 x_2 + 2\omega_1^2 x_2^2\right]\\ &+ \left[b^2 - b\omega_0 x_2 - 2\omega_1\omega_0 x_1 x_2 + b\omega_1 x_1\right]\nu. \quad (2.38)\end{aligned}$$

The transformation $T_1(x)$ represents the total average stored energy by the circuit with capacitor voltage measured with respect to its average steady-state value when $\nu(x) = 0.5$. The rate of change of the total average stored energy, $T_2(x)$, is, similarly, the average input power minus the average output power with capacitor voltage measured relative to its steady-state value when $\nu(x) = 0.5$.

As before,

$$\begin{aligned}&dT_1/dt = T_2; \quad dT_2/dt = T_3 =: v; \quad \text{or,}\\ &dz_1/dt = z_2; \quad dz_2/dt = z_3 \equiv v \qquad (2.39)\end{aligned}$$

i.e., the transformed system is already in Brunovsky-controllable canonical form. The tangent model to (2.39) is thus independent of the operating point, and in z-state space such model is expressed as

$$d(\delta z_1)/dt = \delta z_2; \quad d(\delta z_2)/dt = \delta z_3 \equiv \delta v. \quad (2.40)$$

The inverse transformation $x = T^{-1}(z)$ does not globally exist in R^3, thus indicating the local character of the diffeormorphism T. As in the previous case, in the region of interest, $\mathscr{C}_x$, T is indeed a local diffeomorphism. The computation of the inverse transformation is cumbersome in this case as a full fourth-order polynomial equation must be solved. This, however, does not have any substantial bearing on the specification of a feedback control loop for the buck–boost converter as x-variables can always be used as part of a nonlinear regulating feedback loop (compare Figs. 6 and 7).

In the z-state space a controller design for stabilization around a desirable transformed equilibrium point (z_1^*, z_2^*) is again very simple. Under steady-state conditions $z_2^* = 0$, and for a desirable set point z_1^*, the required feedback control law for (2.39) is $v(z) = -m_1(z_1 - z_1^*) - m_2 z_2$ with $m_1, m_2 > 0$, chosen according to a desirable asymptotically exponentially stable dynamics of the transformed system, described in z coordinates, towards the chosen equilibrium point.

IV. Feedback Regulation on the Basis of the Average Linearized Model

A control scheme based on the method of dynamic inverses (Meyer and Cicolani [12]) is proposed for the feedback control of dc-to-dc power converters. Using the pseudolinearization results of Section III a simplified regulator is specified which generates a corrective control to the linearized plant, as seen through the "trimmap", in a servo-model-following scheme (see Figs. 6 and 7).

For the converters presented above, the system in transformed variables is to be controlled asymptotically towards the operating point $(z_1^*, 0)$ by means of a suitable regulator. An ideal reference model, or command generator model, with state variables z_1^0, z_2^0, is proposed which asymptotically exponentially converges towards the desired equilibrium point in transformed coordinates. This "model servo" is governed by

$$dz_1^0/dt = z_2^0; \quad dz_2^0/dt = v^0 = -m_1(z_1^0 - z_1^*) - m_2 z_2^0 \quad (2.41)$$

where v^0 is the "model control law" guaranteeing asymptotic stability towards $(z_1^*, 0)$ whenever m_1, m_2 are chosen as suitable positive constants. Error signals e_1, e_2 are generated by comparison of the servo model reference state

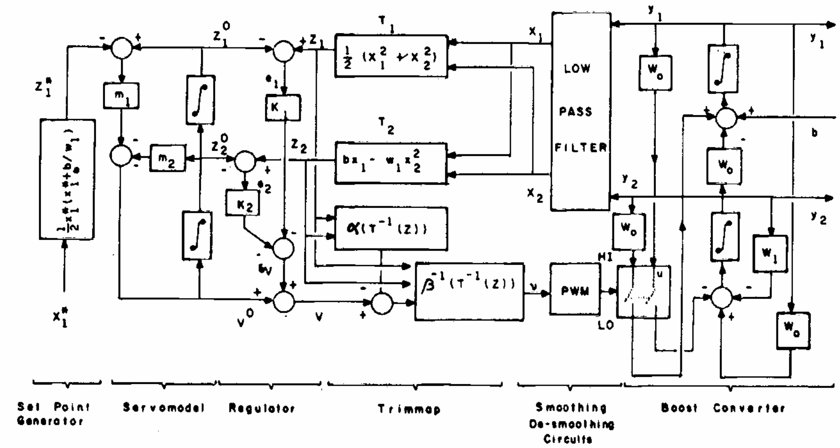

Fig. 6. Feedback control scheme for the boost converter.

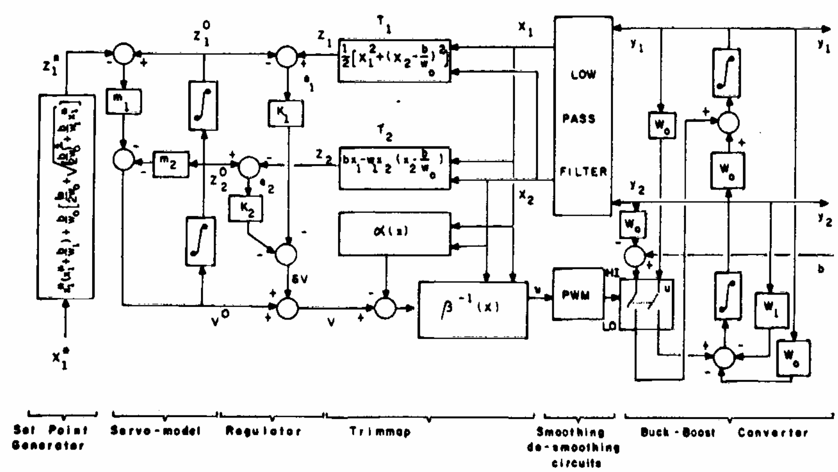

Fig. 7. Feedback control scheme for the buck–boost converter.

trajectories with the corresponding transformed states of the linearized system z_1, z_2, i.e.,

$$e_1 = z_1 - z_1^0; \quad e_2 = z_2 - z_2^0. \tag{2.42}$$

The error state vector $[e_1, e_2]^T$ thus evolves according to

$$de_1/dt = e_2; \quad de_2/dt = v - v^0 = \delta v(e). \tag{2.43}$$

The regulator design is hence reduced to specifying a control law, $\delta v(e)$, for the error system (2.43). This may be accomplished by a simple pole assignment scheme, such as

$$\delta v(e) = -k_1 e_1 - k_2 e_2 = k_1(z_1 - z_1^0) - k_2(z_2 - z_2^0) \tag{2.44}$$

with k_1 and $k_2 > 0$ rendering desired stable closed-loop eigenvalues. Setting of the initial states of the servo model to be nearly equal to those of the transformed system, makes the error components $e_1, e_2 \to 0$ asymptotically. This, in turn, guarantees exact tracking of the model servo states by the transformed plant variables. Once the perturbed control law δv is designed, one obtains the linear controller from (2.43) as

$$v = v^0 + \delta v = (k_1 - m_1)z_1^0 + (k_2 - m_2)z_2^0 + m_1 z_1^* - k_1 z_1 - k_2 z_2. \tag{2.45}$$

The required corresponding nonlinear controller, ν, for the average PWM system is obtained from v as in (2.26).

Remark 2: The regulated error system (2.43), (2.44) can be assigned any arbitrary stable closed-loop dynamics. The only possible limitation in the converters case is constituted by the nonglobal character of the linearization scheme. This determines the need to obtain closed-loop state trajectories, of the transformed system, bounded away from the transformed uncontrollability region of the linearized model (see comments following (2.18), (2.34)). Hence, closed-loop eigenvalues leading to highly oscillatory responses are not desirable, particularly if the initial

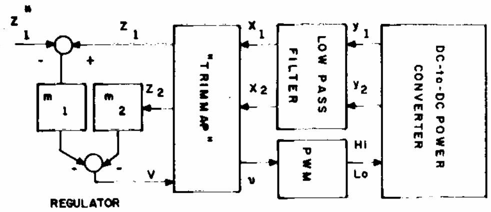

Fig. 8. Direct linear feedback control scheme for locally linearized converters.

error vector components have large amplitudes. A combination of sufficiently damped dynamics and suitable initial states setting of the servo model guarantee fast regulation properties of the closed-loop converter (see Sira-Ramirez and Ilic-Spong [5] for further justification of desirable time scale separation properties of VSF closed-loop converters). □

The feedback regulation scheme synthesizes, after appropriate inversion, a desirable duty ratio geared to achieve steady state stability towards the equilibrium point of the average model. The obtained smooth duty ratio is translated then into the appropriate sequence of on–off pulses by means of a pulsewidth modulator. The observed state variables of the plant are processed through a low pass filter to obtain the average (i.e., chattering-free), smooth, values of the state functions. The actual states of the regulated plant undergo small high-frequency oscillations around the regulated equilibrium point.

Figs. 6 and 7 show the control schemes for the boost and buck–boost converters, respectively. Here x_1^* represents a desirable average value for the steady-state input current acting as a set point. Notice that for a given average steady-state input current x_1^*, the corresponding steady-state capacitor voltage x_2^* is fixed by the manifold conditions (2.15) or (2.31). The steady-state set points, in x-state space, must be translated, by means of $T(x)$, to corresponding steady-state values z_1^*, z_2^* in z-state space. In fact, z_2^* is invariably zero, in the converters case, according to (2.19), (2.21), and (2.35), (2.37), respectively.

If in (2.45) one designs the perturbed system regulator by choosing $k_1 = m_1$ and $k_2 = m_2$ the linear model following regulation scheme reduces to

$$v(z) = -m_1(z_1 - z_1^*) - m_2 z_2. \qquad (2.46)$$

In this case the servo model is completely replaced by a direct stabilizing controller guaranteeing the asymptotic stability of the linearized plant towards $(z_1^*, 0)$. Under the same general limitations pointed out in the previous remark, this scheme obviates the model servo construction and hence the dependence of the control scheme on its initial states. Fig. 8 shows a block diagram of this alternative regulation scheme.

Remark 3: It should be stressed that the feedback approach presented in this article is widely different to the regulation scheme presented in [5]. In [5], by appropriate design of the system components, a time scale separation property was induced among the state variables describing the average PWM-controlled system. The *slow manifold* corresponding to the average PWM-controlled system with known, prescribed, constant duty ratio, was used as a *sliding surface* for the nonlinear converters. On that slow sliding manifold, the resulting controlled state variables ideally behaved linearly and decoupled, with arbitrarily chosen exponential decay. The sliding motion was shown not to be global and the region of sliding mode existence was clearly identified. The technique proposed in [5] established a Variable Structure Control alternative, with equivalent properties, to the traditional, and heuristically designed, PWM feedback control schemes for dc-to-dc converters.

In the pseudolinearization approach, the duty ratio is assumed to be any arbitrary stabilizing *nonlinear* feedback function for the average PWM system in a family of such control laws. The duty ratio, evidently, becomes constant after the trajectories have converged to a controllable equilibrium point of the nonlinear average system. On the *manifold of controllable equilibrium points*, the pseudolinearization method applies and a simplified *smooth* feedback regulation loop is designed on the basis of the new state coordinates. The manifold of equilibrium points is here explicitly identified. This manifold, incidentally, has no relationship whatsoever with the slow sliding manifold used in [5], except that, in general, they have a point in common; an operating equilibrium point. The resulting scheme, based in pseudolinearization, is totally independent of the particular equilibrium point achieved by the system (a property evidently not sheared by the scheme in [5]). It should be obvious that locally stabilizing pseudolinearization-based feedback control laws do not, necessarily, demand for sliding mode control considerations. □

V. Conclusions and Suggestions for Further Research

A pseudolinearization approach has been applied for the regulator design of dc-to-dc switchmode power supplies of bilinear nature such as the boost and the buck–boost converters. The integration of appropriately chosen one-forms along smooth one-dimensional controllable steady-state submanifolds of the average PWM-controlled converter leads to the specification of local linearizing diffeomorphic state coordinate transformations. These transformations, besides being physically meaningful in terms of total average stored energy and total average consumed power, yield a controllable tangent model expressible in Brunovsky's canonical form. The independency of the tangent model, with respect to the average operating point, allows the design of a generally smooth feedback control scheme specifying the desirable duty ratio, computed according to an externally specified set point value for the steady-state input current (or, equivalently, steady-state output voltage). The practical aspects of the implementation of the ideas presented in this paper will be fully reported elsewhere.

As topics for further research, the reader is invited to explore applications of the proposed method on the wellknown Cuk converter, and some of its most popular modifications. Also, detailed studies and practical limitations of the proposed technique on more realistic models of actual converters, including, for instance, inductor resistence, hard nonlinearities such as inductor current limitations, and parasitics in the switching elements, constitute an interesting topic for further work.

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