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Design of P–I controllers for DC-to-DC power supplies via extended linearization

HEBERTT SIRA-RAMIREZ†

In this article, the method of 'extended linearization' is used for designing stabilizing non-linear proportional–integral (P–I) controllers which regulate to a constant value either the average output voltage or the average input current of PWM controlled DC-to-DC power converters such as the boost, the buck–boost and the Cuk converters. The design is carried out on the basis of the Ziegler–Nichols P–I controller design method applied to a family of linearized transfer function models of converters parametrized by constant operating equilibrium points of the average PWM controlled circuit.

1. Introduction

The regulation of switch-mode DC-to-DC power converters is usually accomplished by means of finite sampling frequency pulse-width-modulation (PWM) control schemes in various arrangements with regulators whose design is based on approximate linear incremental models of either a discrete or continuous-time nature (see Severns and Bloom 1985, Middlebrook and Cuk 1981, Csaki *et al.* 1983, etc.). Recently, non-linear control schemes have been proposed that do not resort to incremental models or to discrete-time approximation schemes, but exploit the properties of non-linear continuous average models obtained by an infinite sampling frequency assumption. Among such schemes one finds:

(*a*) variable structure control and its associated 'sliding mode' control strategies with switching surfaces designed on the basis of the 'ideal sliding dynamics' properties (Venkataramanan *et al.* 1983, Sira-Ramirez 1987);

(*b*) discontinuous control methods, such as PWM, based on considerations about the time-scale separation properties of the converters average responses and their associated slow manifold (such as in the work of Sira-Ramirez and Ilic 1988 and Sira-Ramirez 1988 a);

(*c*) pseudolinearization techniques applied on the non-linear continuous average PWM converter model (Sira-Ramirez 1989 b); and

(*d*) control schemes based on exact linearization of the average PWM circuit model (Sira-Ramirez and Ilic-Spong 1989).

In this article, a new non-linear controller design method is proposed for the regulation of either the output load voltage or the input inductor current state variables in DC-to-DC converters. Such cases are addressed, respectively, as the voltage control mode and the current control mode. The design method is based on the application of the extended linearization controller design technique extensively

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† Deparamento Sistemas de Control, Escuela de Ingenieria de Sistemas, Universidad de Los Andes, Merida, Venezuela.

developed by Rugh and his co-workers (see the References section), to non-linear average PWM controlled models of the converters. The extended linearization approach for feedback controller design constitutes a highly attractive non-linear design technique based on the specification of a linear regulator which induces desirable stability characteristics on an entire family of linearized plant models parametrized by constant equilibrium points. Such a family may be represented by a smooth surface defined in the input–output space of the system. The obtained linear design serves as the basis for specifying (usually in a non-unique manner) a non-linear controller with the property that its linearized model, computed about the same generic operating point, coincides with the specified stabilizing regulator. The resulting non-linear controller thus exhibits the remarkable property of 'self-scheduling' with respect to operating points which may change its value due to a sudden change of the reference set point.

We propose here the use of non-linear P–I controllers for regulating either the output load voltage or the input inductor current of DC-to-DC power supplies such as the boost, the buck–boost and the Cuk converters. The frequency domain Ziegler–Nichols method (Åström and Hagglünd 1988, pp. 54–58) is used for the specification of the linearized P–I regulator gains which stabilize the family of parametrized transfer functions. In the output voltage control mode, such a transfer function relates the incremental output load voltage to the incremental duty ratio function of the converter. In the current control mode, the transfer function relates the incremental input inductor current to the incremental duty ratio function of the converter. The non-linear P–I controller is obtained from the linear design in a manner similar to that proposed by Rugh (1987). It should be remarked that, for the three converters, constant output voltage regulation cannot be accomplished by sliding mode control defined on surfaces representing zero output voltage error (Sira-Ramirez 1987). A constant output voltage control may be achieved only when a combination of the state variables is formed in an appropriate sliding line, or else indirectly through constant input current regulation (Sira-Ramirez 1987). Similarly, it is easy to verify that even if direct use of PWM controllers does achieve constant output voltage regulation for a limited range of desirable set points, such restrictions turn out to be rather unnatural. On the contrary, the non-linear P–I controller proposed in this article efficiently handles the output voltage regulation problem for the three converters without instability effects, at least in a local sense. However, input inductor current control by non-linear P–I compensation does exhibit certain limitations inherent in the non-existence of a non-trivial real crossover frequency for the family of transfer functions for certain sets of parameter values and constant set points. In such cases, the associated infinite crossover frequency prevents the application of the Ziegler–Nichols design recipe for a P–I regulator.

It should also be pointed out that since the designed controller is to be used in combination with an actual PWM actuator of a highly discontinuous nature, a proportional–integral–derivative (P–I–D) controller is not feasible owing to large controller output values produced by the derivative action carried out on the discontinuous error signal. This difficulty could only be prevented through the use of high-order low-pass filters on the measured output signal used for feedback purposes. This topic, and the associated saturation problems induced on the PWM actuator, are not investigated in this article.

Section 2 of this article presents a brief overview of the extended linearization technique as applied to the design of non-linear P–I regulators for general time-invariant bilinear systems. Section 3 presents, in detail, the procedure for obtaining a non-linear P–I compensator for the Cuk converter. In this section the relevant formulae for obtaining non-linear P–I regulators for average models of PWM controlled converters of the boost and the buck–boost type are also presented in summary form. Such formulae specify P–I controllers for both the output voltage control mode case and the input inductor current control mode case. Here, the manner is also indicated in which the designed non-linear controllers are to be used in the actual discontinuous PWM scheme. This section presents some simulation examples highlighting the non-linear P–I controller performance. The last section is devoted to some conclusions and suggestions for further work.

2. Background results

In this section we briefly review the extended linearization technique (Baumann and Rugh 1986) as applied to time-invariant discontinuously controlled systems of the form:

$$\left.\begin{aligned} dx/dt &= f(x) + u[b(x) + \gamma] + \eta \\ y &= h(x) \end{aligned}\right\} \tag{2.1}$$

with x being an element of $\mathbb{R}^n$, γ and η constant n-dimensional vectors, $f(x)$ and $b(x)$ smooth vector fields defined on an open set of $\mathbb{R}^n$, and h a smooth scalar output function. The variable u represents the control signal taking values on the discrete set $\{0, 1\}$.

The discontinuous feedback control strategy is usually specified on the basis of a sampled closed-loop PWM control scheme of the form (Skoog and Blankenship 1970):

$$u = \begin{cases} 1 & \text{for } t_k < t < t_k + \mu[x(t_k)]T \\ 0 & \text{for } t_k + \mu[x(t_k)]T < t < t_k + T \end{cases} \tag{2.2}$$

where $\mu[x(t_k)]$ is known as the *duty ratio function*, which is generally represented by a smooth feedback function of the state (or of some related variables such as sampled output error $e(t_k) := y_d - y(t_k) = y_d - h[x(t_k)]$) which satisfies the natural bounding constraint: $0 < \mu[x(t_k)] < 1$, for all sampling instants t_k. T is known as the *duty cycle* determining the time elapsed between sampling instants, i.e. $t_{k+1} = t_k + T$.

Remark

It has been rigorously shown by Sira-Ramirez (1989 a, see also Sira-Ramirez 1988 a, 1989 c) that an *average model* of (2.1), (2.2) can be obtained by assuming an *infinite sampling frequency* (i.e. letting the duty cycle $T \to 0$) on (2.2). The average model is simply obtained by formally substituting the discontinuous control variable u in (2.1) by the duty ratio function $\mu(x)$. The average trajectories, obtained as solutions of the resulting non-linear system, satisfy the property of accurately representing all the qualitative properties of the actual PWM controlled system (2.1), (2.2). This was demonstrated in Sira-Ramirez (1989 c) by showing that there always exist a sufficiently small sampling period T for which the deviations between the actual PWM controlled responses and those of the average model, under identical initial conditions, remain uniformly arbitrarily close to each other. Conversely, for each prespecified degree of error tolerance, a sampling frequency may be found such that the actual and the average trajectories differ by less than such a given tolerance bound. The error can be made even smaller if the sampling frequency is suitably increased. Moreover, from a purely geometric viewpoint, in those regions of non-saturation of the duty ratio function μ, *integral manifolds* containing families of state responses of the average model constitute actual sliding surfaces about which the discontinuous PWM controlled trajectories exhibit sliding regimes (Sira-Ramirez 1989 a). Outside the region of non-saturation, the trajectories of both the actual and the average PWM models entirely coincide. The average model dynamics then play the role of the *ideal sliding dynamics* (see Utkin, 1978 and Sira-Ramirez 1988 b) in the corresponding variable structure control reformulation of the PWM control strategy (see Sira-Ramirez 1989 a).

The average model of (2.1), (2.2) is thus formally obtained by substituting the duty ratio feedback function μ in place of the actual switch control function u. However, in the resulting system we shall denote the averaged state vector x by means of the vector z:

$$\left.\begin{aligned} dz/dt &= f(z) + \mu[b(z) + \gamma] + \eta \\ y &= h(z) \end{aligned}\right\} \tag{2.3}$$

Let Z be a constant equilibrium state vector for (2.3). If such an equilibrium state exists then it must, necessarily, correspond to a constant value U of the duty ratio feedback function μ. We could express such a value U as a function of Z by, say, $U = \mu(Z)$. We prefer, however, in the same spirit as Rugh (1986), to express Z as a function of U by $Z(U)$. This value $Z(U)$ would coincide with the previously given equilibrium state Z if and only if the jacobian matrix $[A(U) + UB(U)] := \partial f/\partial z + U\partial b/\partial z$, evaluated at the equilibrium point $(U, Z(U))$ is invertible. In such a case, the implicit function theorem guarantees the existence of a unique solution $Z(U)$ of:

$$f(Z(U)) + U[b(Z(U)) + \gamma] + \eta = 0 \tag{2.4}$$

Notice that preliminary feedback can always render an invertible jacobian matrix $A(U) + UB(U)$ if such is not the case, for a particular U, in the original average system (2.3).

The linearization of (2.3) about a given equilibrium point $(U, Z(U))$ results in:

$$\left.\begin{aligned} \frac{dz_\delta}{dt} &= [A(U) + UB(U)]z_\delta + [b(Z(U)) + \gamma]\mu_\delta \\ y_\delta &= cz_\delta := \left[\frac{\partial h}{\partial z}\right]^{\mathrm{T}} Z_\delta \end{aligned}\right\} \tag{2.5}$$

with: $z_\delta(t) := z(t) - Z(U)$; $y_\delta(t) := y(t) - Y(U) := y(t) - cZ(U)$; $\mu_\delta(t) := \mu(t) - U$.

Equation (2.5) constitutes a family of linearizations of (2.3) parametrized by the constant input equilibrium point U. Taking Laplace transforms in (2.5) one obtains, under zero initial conditions, the family of parametrized scalar transfer functions relating the incremental output transform $y_\delta(s)$ to the incremental duty ratio input transform $\mu_\delta(s)$:

$$G_U(s) := \frac{y_\delta(s)}{\mu_\delta(s)} = c(sI - [A(U) + UB(U)])^{-1}[b(Z(U)) + \gamma] \tag{2.6}$$

This transfer function description of the linearized system is only valid in the region of non-saturation of the duty ratio function, i.e. for $0 < U < 1$.

At this stage, the extended linearization approach suggests the use of a P–I controller which stabilizes to zero the incremental output response corresponding to a generic element of the parametrized family of systems represented by (2.6) (see also Rugh 1987). For this purpose, the Ziegler–Nichols method (Åström and Hagglünd 1988, pp. 54–58) can be readily used upon determination of the *ultimate frequency* (also called phase cross-over frequency), $W_0(U)$, (or, equivalently, *the ultimate period*, defined as: $P_0(U) = 2\pi/W_0(U)$) and the *ultimate gain* (also called the gain margin), $K_0(U)$, corresponding to (2.6). These parameters are defined by the following relations:

$$\operatorname{Arg} G_U(jW_0(U)) = -\pi; \quad K_0(U) = |G_U(jW_0(U))|^{-1} \tag{2.7}$$

These two basic parameters, in turn, specify the gains of a P–I controller which is to act on the incremental error function $e_\delta(U) = 0 - y_\delta(U)$. Such a P–I controller is described by its transfer function $C_U(s)$ as:

$$C_U(s) = \mu_\delta(s)/e_\delta(s) = K_1(U) + K_2(U)/s \tag{2.8}$$

Using the well-known Ziegler–Nichols rules, the above gains are easily computed in terms of the ultimate frequency and the ultimate gain as:

$$K_1(U) = 0{\cdot}4K_0(U), \quad K_2(U) = \frac{K_1(U)W_0(U)}{1{\cdot}6\pi} \tag{2.9}$$

A non-linear P–I controller may then be proposed, following Rugh (1987), by considering:

$$\left.\begin{aligned} \frac{d\xi(t)}{dt} &= K_2[\zeta(t)]e(t) \\ \hat{\mu}(t) &= \zeta(t) + K_1[\zeta(t)]e(t) \end{aligned}\right\} \tag{2.10}$$

where $\hat{\mu}(t)$ is the non-linear controller output signal. This signal is to be taken as the specification of the duty ratio function, μ, for the PWM actuator only in the region where the output signal $\hat{\mu}(t)$ does not violate the condition: $0 < \hat{\mu} < 1$.

It is easy to see that linearization of the non-linear state equations (2.10), around the operating point $e(U) = 0$, $\hat{\mu}(U) = \zeta(U) = U$, produces an incremental model whose transfer function coincides with (2.8). The behaviour of the average non-linear controlled system (2.3) in the vicinity of a given equilibrium point $(U, Z(U))$ thus exhibits the same stability characteristics that the linearized P–I controller (2.8) imposes on the family of linearized plants represented by (2.6). By the results commented upon in the Remark, the response of the actual discontinuous PWM controlled system (2.1)–(2.2) in conjunction with the designed non-linear P–I controller will exhibit the same qualitative stability characteristics caused by the stabilizing design on the average PWM closed loop system, provided a sufficiently high sampling frequency is used in (2.2).

Since the computed duty ratio $\hat{\mu}(t)$ must not violate its natural bounding constraints, $0 < \hat{\mu}(t) < 1$, one obtains the actual duty ratio μ by bounding the computed controller output signal $\hat{\mu}(t)$ through a limiter, as follows:

$$\mu(t) = \begin{cases} 0 & \text{for } \hat{\mu}(t) < 0 \\ \hat{\mu}(t) & \text{for } 0 < \mu(t) < 1 \\ 1 & \text{for } \hat{\mu}(t) > 1 \end{cases} \tag{2.11}$$

This bounding process of the computed duty ratio function $\hat{\mu}(t)$ may cause saturation effects on the PWM actuator for those initial conditions far from the required equilibrium point. If such operational requirements cannot be avoided, the use of anti-reset windup schemes may be used (Åström and Hagglünd 1988, pp. 10–14). We do not give further consideration to this topic here since it implies only a minor modification of the proposed control scheme.

The PWM actuator induces a high frequency discontinuous motion (chattering) on the systems state and output variables. Since the feedback design presented above is based on the infinite frequency averaged output values, one can obtain an approximation to the ideal smooth performance by placing a low-pass filter at the system output before feeding this signal back through the P–I controller. This procedure approximates the characteristics of the idealized design when the cut-off frequency of the filter and its associated phase lag is made sufficiently small.

The complete non-linear P–I regulation scheme, based on the extended linearization of an average PWM controlled plant of non-linear nature, is shown in Fig. 1.

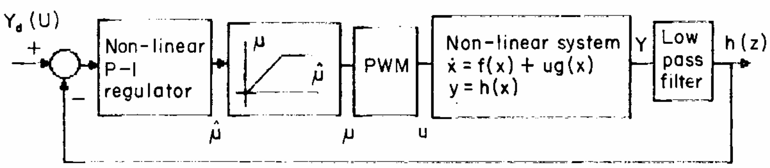

Figure 1. Non-linear P–I control scheme for output regulation of a non-linear PWM controlled system.

3. Design of P–I controllers for DC-to-DC power supplies

3.1. *Cuk converter*

Consider the Cuk converter model shown in Fig. 2. This circuit is the outcome of a deliberate design effort geared to obtain as many desirable DC-to-DC conversion properties as possible with a numinum of component elements. This converter, invented by Dr Slodoban Cuk, is described by the following bilinear state equation model:

$$\left.\begin{aligned} \frac{dx_1}{dt} &= -\omega_1 x_2 + u\omega_1 x_2 + b \\ \frac{dx_2}{dt} &= \omega_1 x_1 - u\omega_1 x_1 - u\omega_2 x_3 \\ \frac{dx_3}{dt} &= -\omega_4 x_3 + u\omega_2 x_2 \\ y &= x_3 \end{aligned}\right\} \tag{3.1}$$

where, $x_1 = I_1\sqrt{L_1}$, $x_2 = V_2\sqrt{C_2}$ and $x_3 = I_3\sqrt{L_3}$ represent normalized input inductor current, transfer capacitor voltage and output inductor current variables, respectively. The quantity $b = E/\sqrt{L_1}$ is the normalized external input voltage. The converter parameters are defined as: $\omega_1 = 1/\sqrt{L_1 C_2}$, $\omega_2 = 1/\sqrt{L_3 C_2}$ and $\omega_4 = R/L_3$.

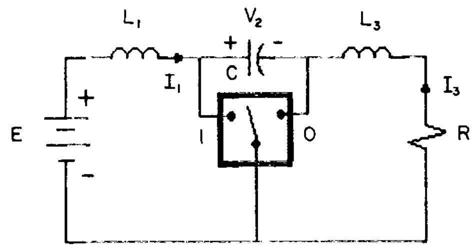

Figure 2. Cuk converter.

These are, respectively, the *LC* input circuit natural oscillating frequency, the *LC* output circuit natural oscillating frequency and the *RL* (output) circuit time constant. The variable u denotes the switch position function, which acts as a control input, taking values in the discrete set $\{0, 1\}$. System (3.1) is of the same form as (2.1), with the vector field $f(x)$ being of the form Ax, and the vector field $b(x)$ being of the form Bx. The constant vectors γ and η are of the form $\gamma = 0$ and $\eta = [b \quad 0 \quad 0]^T$. We now present, according to the theory contained in the previous section, the formulae leading to a non-linear stabilizing P–I controller design for the average model of (3.1).

The average PWM controlled Cuk converter model is simply obtained from (3.1) by replacing the discontinuous control function u by the duty ratio function μ.

$$\left.\begin{aligned} \frac{dz_1}{dt} &= -\omega_1 z_2 + \mu\omega_1 z_2 + b \\ \frac{dz_2}{dt} &= \omega_1 z_1 - \mu\omega_1 z_1 - \mu\omega_2 z_3 \\ \frac{dz_3}{dt} &= -\omega_4 z_3 + \mu\omega_2 z_2 \\ y &= z_3 \end{aligned}\right\} \tag{3.2}$$

The equilibrium points of the average model are obtained from (3.2) assuming a constant value, U, for the duty ratio function μ:

$$\left.\begin{aligned} &\mu = U; \quad Z_1(U) = \omega_2^2 bU^2/\omega_1^2\omega_4(1-U)^2; \quad Z_2(U) = b/\omega_1(1-U) \\ &Z_3(U) = \omega_2 bU/\omega_1\omega_4(1-U) \end{aligned}\right\} \tag{3.3}$$

The linearization of the average PWM model (3.2) around the constant equilibrium points (3.3) results in an incremental model, parametrized by U, of the form:

$$\frac{d}{dt}\begin{bmatrix} z_{1\delta} \\ z_{2\delta} \\ z_{3\delta} \end{bmatrix} = \begin{bmatrix} 0 & -(1-U)\omega_1 & 0 \\ \omega_1(1-U) & 0 & -\omega_2 U \\ 0 & \omega_2 U & -\omega_4 \end{bmatrix}\begin{bmatrix} z_{1\delta} \\ z_{2\delta} \\ z_{3\delta} \end{bmatrix} + \begin{bmatrix} \omega_1 Z_2(U) \\ -\omega_1 Z_1(U) - \omega_2 Z_3(U) \\ \omega_2 Z_2(U) \end{bmatrix}\mu_\delta \tag{3.4}$$

where:

$$z_{i\delta}(t) = z_i(t) - Z_i(U); \quad i = 1, 2, 3; \quad y_\delta(t) = y(t) - Y(U) := z_3(t) - Z_3(U);$$
$$\mu_\delta(t) = \mu(t) - U$$

The parametrized family of transfer functions relating the average incremental output inductor current $z_{3\delta}$ to the incremental duty ratio μ_δ is found to be:

$$G_U(s) = \frac{\omega_2 b}{\omega_1(1-U)} \frac{s^2 - \dfrac{\omega_2^2 U^2}{\omega_4(1-U)} s + (1-U)\omega_1^2}{s^3 + \omega_4 s^2 + (U^2\omega_2^2 + (1-U^2)\omega_1^2)s + \omega_4(1-U)^2\omega_1^2} \tag{3.5}$$

The family of parametrized linear systems represented by (3.5) is used for the frequency-response-based Ziegler–Nichols P–I controller design.

After substitution in (3.5) of s by $j\omega$, the phase cross-over frequency is found by computing the value of the frequency $W_0(U)$ that makes the imaginary part of (3.5) equal to zero (discarding, of course, the trivial solutions: $W_0(U) = 0$, and $W_0(U) = \infty$). One obtains after some straightforward calculations:

$$\left.\begin{aligned} W_0(U) &= \sqrt{\alpha(U)}[1 - (1 - \beta^2(U)/\alpha^2(U))^{1/2}]^{1/2} \\ \alpha(U) &= 0{\cdot}5(2-U)(1-U)^{-1}[U^2\omega_2^2 + (1-U)^2\omega_1^2] \\ \beta^2(U) &= \omega_1^2(1-U)[2U^2\omega_2^2 + (1-U)^2\omega_1^2] \end{aligned}\right\} \tag{3.6}$$

To guarantee the existence of a real crossover frequency, the condition: $\beta^2(U)/\alpha^2(U) < 1$ must be enforced on the system parameters ω_1, ω_2 and the constant duty ratio U. If this requirement is not fulfilled by the system then the Nyquist plot of the incremental transfer function $G_U(j\omega)$ does not intersect the real axis except at $\omega = 0$ and $\omega = \infty$. In such a case the Ziegler–Nichols recipe degenerates into the specification of an arbitrary proportional controller which can be made independent of the operating point. A P–I controller is not obtained in such a case and the resulting linearized closed loop system exhibits infinite gain margin.

According to (2.7), the ultimate gain $K_0(U)$ is obtained as the inverse of the absolute value of $G(jW_0(U))$. In this case, such a key design parameter is obtained as:

$$K_0(U) = \frac{\omega_1\omega_4(1-U)}{\omega_2 b} \frac{|\omega_1^2(1-U)^2 - W_0^2(U)|}{|\omega_1^2(1-U) - W_0^2(U)|} \tag{3.7}$$

According to the Ziegler–Nichols design recipe (2.9) for P–I controller specification, the values of the proportional and integral term gains which stabilize the entire family of linearized plant models (3.4), or (3.5) are given, respectively, by:

$$\left.\begin{aligned} K_1(U) &= \frac{0{\cdot}4\omega_1\omega_4(1-U)}{\omega_2 b} \frac{|\omega_1^2(1-U)^2 - W_0^2(U)|}{|\omega_1^2(1-U) - W_0^2(U)|} \\ K_2(U) &= \frac{\omega_1\omega_4(1-U)}{4\pi\omega_2 b} \frac{|\omega_1^2(1-U)^2 - W_0^2(U)|}{|\omega_1^2(1-U) - W_0^2(U)|} W_0(U) \end{aligned}\right\} \tag{3.8}$$

The P–I controller $C_U(s) = K_1(U) + K_2(U)/s$ is such that it would stabilize to zero the output response of the entire family of linearized plant models represented by (3.5). The non-linear P–I controller whose linearization around the constant equilibrium point coincides with (3.8) is given, according to (2.10), by:

$$\left.\begin{aligned} \frac{d}{dt}\zeta(t) &= \left[\frac{\omega_1\omega_4(1-\zeta)}{4\pi\omega_2 b}\frac{|\omega_1^2(1-\zeta)^2 - W_0^2(\zeta)|}{|\omega_1^2(1-\zeta) - W_0^2(\zeta)|}W_0(\zeta)\right]e(t) \\ \hat{\mu} &= \zeta(t) + \left(\frac{0{\cdot}4\omega_1\omega_4(1-\zeta)}{\omega_2 b}\frac{|\omega_1^2(1-\zeta)^2 - W_0^2(\zeta)|}{|\omega_1^2(1-\zeta) - W_0^2(\zeta)|}\right)e(t) \\ e(t) &= Z_3(U) - z_3(t) \end{aligned}\right\} \qquad (3.9)$$

The output $\hat{\mu}$ of the non-linear P–I converter is to be regarded as the specification of the needed stabilizing duty ratio function for the average PWM closed-loop converter. However, depending on the proximity of the initial states to the desirable constant average load current (acting as a set point), the actual values of $\hat{\mu}$ may violate the natural constraints imposed on the duty ratio function μ. Therefore, a limiter of the form (2.11) has to be enforced on the output of the non-linear P–I converter. This procedure yields the actual duty ratio function. In actual operation, μ may be subject to saturation during certain intervals of time. Typically, an antireset-windup scheme (see Åström and Hagglünd 1988, pp. 10–14) would be used in combination with the non-linear P–I controller to avoid overshooting effects on the average controlled output.

The PWM actuator induces undesirable high frequency discontinuous signals (chattering) for the converters state and output variables. In order suitably to approximate the average closed-loop designed behaviour, a low pass filter must then be placed at the sensing arrangement used to obtain the actual output inductor current $x_3(t)$ used for feedback purposes. One may, for instance, propose a simple first-order RC circuit, with a sufficiently small time constant, $1/T_f$ (equivalently, a sufficiently small cut-off frequency) as follows:

$$\frac{df(t)}{dt} = -\left(\frac{1}{T_f}\right)(f(t) - x_3(t)); \quad z_3(t) = f(t) \qquad (3.10)$$

One may regard the filter output, $z_3(t)$, as the average output current value required by the non-linear P–I controller (3.9).

The complete non-linear P–I regulation scheme based on extended linearization of an average PWM controlled Cuk converter would be that of Fig. 1, replacing the block representing the non-linear system (2.1) by a block representing the Cuk converter.

3.2. *Boost converter*

3.2.1. *Non-linear P–I regulation for the case of the output voltage control mode in a boost converter.* Consider the boost converter model shown in Fig. 3. Ths circuit is

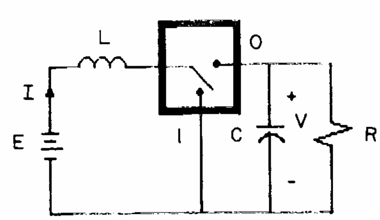

Figure 3. Boost converter.

described by the bilinear state equation model:

$$\left.\begin{aligned} \frac{dx_1}{dt} &= -\omega_0 x_2 + u\omega_0 x_2 + b \\ \frac{dx_2}{dt} &= \omega_0 x_1 - \omega_1 x_2 - u\omega_0 x_1 \\ y &= x_2 \end{aligned}\right\} \tag{3.11}$$

where, $x_1 = I\sqrt{L}$, $x_2 = V\sqrt{C}$ represent normalized input current and output voltage variables, respectively. The quantity $b = E/\sqrt{L}$ is the normalized external input voltage and $\omega_0 = 1/\sqrt{LC}$ and $\omega_1 = 1/RC$ are, respectively, the LC (input) circuit natural oscillating frequency and the RC (output) circuit time constant. The variable u denotes the switch position function, acting as a control input, taking values in the discrete set $\{0, 1\}$. System (3.1) is of the same form as (2.1), with $f(x)$ a linear vector field of the form $f(x) = Ax$, while $b(x)$ is of the form $b(x) = Bx$; the constant vectors γ and η are given by: $\gamma = 0$ and $\eta = [b \quad 0]^T$. We now summarize the formulae leading to a non-linear P–I controller design for the average model of (3.11).

Average boost converter model for the output voltage control mode

$$\left.\begin{aligned} \frac{dz_1}{dt} &= -\omega_0 z_2 + \mu\omega_0 z_2 + b \\ \frac{dz_2}{dt} &= \omega_0 z_1 - \omega_1 z_2 - \mu\omega_0 z_1 \\ y &= z_2 \end{aligned}\right\} \tag{3.12}$$

Constant equilibrium points

$$\mu = U; \quad Z_1(U) = \frac{b\omega_1}{[\omega_0^2(1-U)^2]}; \quad Z_2(U) = \frac{b}{[\omega_0(1-U)]} \tag{3.13}$$

Parametrized family of linearized systems about the constant equilibrium points

$$\left.\begin{aligned} \frac{d}{dt}\begin{bmatrix} z_{1\delta}(t) \\ z_{2\delta}(t) \end{bmatrix} &= \begin{bmatrix} 0 & -\omega_0(1-U) \\ \omega_0(1-U) & -\omega_1 \end{bmatrix} \begin{bmatrix} z_{1\delta} \\ z_{2\delta} \end{bmatrix} + \begin{bmatrix} \dfrac{b}{(1-U)} \\ \dfrac{-b\omega_1}{[\omega_0(1-U)^2]} \end{bmatrix} \mu_\delta \\ y_\delta(t) &= z_{2\delta}(t) \end{aligned}\right\} \tag{3.14}$$

with

$$z_{i\delta}(t) = z_i(t) - Z_i(U); \quad i = 1, 2; \quad y_\delta(t) = y(t) - Y(U) := z_2(t) - Z_2(U);$$

$$\mu_\delta(t) = \mu(t) - U$$

Family of parametrized transfer functions relating incremental output voltage to incremental duty ratio

$$G_U(s) = y_\delta(s)/\mu_\delta(s) = -\omega_0 Z_1(U)\frac{s - b[Z_1(U)]^{-1}}{s^2 + \omega_1 s + \omega_0^2(1-U)^2} \tag{3.15}$$

Crossover frequency

$$W_0(U) = \sqrt{2}\omega_0(1-U) \tag{3.16}$$

Ultimate period and ultimate gain

$$P_0(U) := \frac{2\pi}{W_0(U)} = \frac{\sqrt{2}\pi}{[\omega_0(1-U)]}; \quad K_0(U) = \frac{\omega_0(1-U)^2}{b} \tag{3.17}$$

Ziegler–Nichols P–I controller gains for the linearized family of converters

$$K_1(U) = \frac{0{\cdot}4\omega_0(1-U)^2}{b}; \quad K_2(U) = \frac{\omega_0^2(1-U)^3}{(2\sqrt{2}\pi b)} \tag{3.18}$$

Non-linear P–I controller (output voltage control mode)

$$\left.\begin{aligned} \frac{d\zeta(t)}{dt} &= \left[\frac{\omega_0^2(1-\zeta(t))^3}{(2\sqrt{2}\pi b)}\right]e(t) \\ \hat{\mu}(t) &= \zeta(t) + \left[\frac{0{\cdot}4\omega_0(1-\zeta(t))^2}{b}\right]e(t) \\ e(t) &= y_d(U) - y(t) = Z_2(U) - z_2(t) \end{aligned}\right\} \tag{3.19}$$

Low pass filter

A simple first-order low pass filter may be proposed to yield an approximation to the ideal average output function z_2 required by the non-linear P–I controller. Such a filter is characterized by a sufficiently small time constant of value $(1/T_f)$, and a state f.

$$\frac{df(t)}{dt} = -\left(\frac{1}{T_f}\right)(f(t) - x_2(t)); \quad z_2(t) = f(t) \tag{3.20}$$

3.2.2. *Simulation example.* A boost converter circuit with parameter values $R = 30\ \Omega$, $C = 20\ \mu$F, $L = 20$ mH and $E = 15$ V was considered for non-linear P–I controller design. The constant operating value of μ was chosen to be $U = 0{\cdot}8$ while the corresponding desirable normalized constant output voltage turned out to be $Z_2(0{\cdot}6) = 0{\cdot}3354$. Figure 4 shows several state trajectories corresponding to different initial

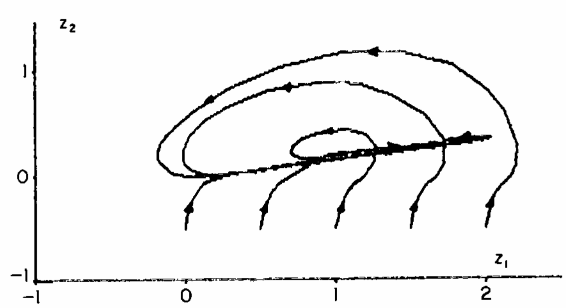

Figure 4. Normalized state trajectories of the ideal average boost converter model controlled by a non-linear P–I regulator.

conditions set on the ideal average boost converter model controlled by the non-linear P–I regulator of the form (3.19). Figure 4 represents the projections of the closed loop system three-dimensional state vector trajectories onto the z_1–z_2 average state coordinate plane. The average controlled state variables, z_1 and z_2, are shown to converge toward the desirable equilibrium point $Z_1(0{\cdot}8) = 1{\cdot}7683$ and $Z_2(0{\cdot}8) = 0{\cdot}3354$. Figure 5 shows the average controlled variables evolution when subject to a step change in the output set point value, from $Z_2(0{\cdot}8) = 0{\cdot}3354$ to $Z_2(U) = 0{\cdot}1677$ (the corresponding change in the operating point of the duty ratio was from $U = 0{\cdot}8$ to $U = 0{\cdot}6$). Figures 6(a) and 6(b) show the state response of the actual PWM controlled

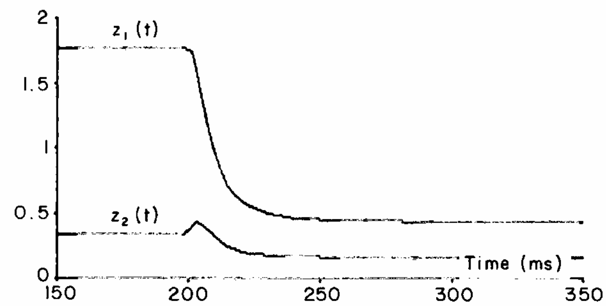

Figure 5. Average controlled state response of the boost converter subject to a 100% step change in the set point value.

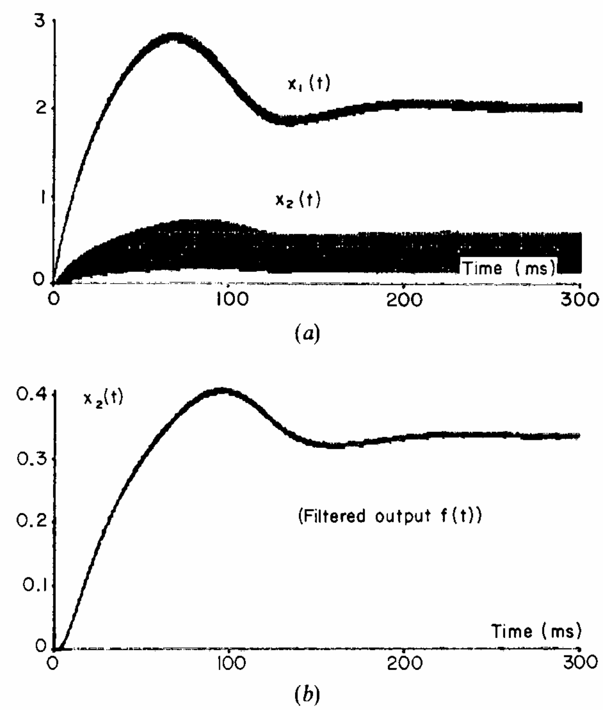

Figure 6. (a) Normalized state response of the actual PWM controlled boost converter. (b) Filtered output response of the actual PWM controlled boost converter with a non-linear P–I compensation scheme.

circuit and the filtered output response, respectively. The sampling frequency for the PWM actuator was chosen as 1 kHz and the output low pass filter cut-off frequency was set at 0·1 rad s^{-1}.

3.2.3. *Non-linear P–I regulation for the case of the input current control mode of the boost converter.* Input inductor current can also be sensed, and appropriately filtered, for feedback regulation purposes. One usually pursues constant input current regulation to obtain a desirable controlled constant output voltage at the load indirectly. In this section, we propose a non-linear P–I controller scheme similar to the one in the previous section, which uses the average value of the input current for feedback purposes. As before, we only summarize the relevant formulae leading to the non-linear P–I controller specification. Evidently, the equilibrium points are the same as in (3.13).

Average boost converter model for input current control mode

$$\left.\begin{aligned} \frac{dz_1}{dt} &= -\omega_0 z_2 + \mu\omega_0 z_2 + b \\ \frac{dz_2}{dt} &= \omega_0 z_1 - \omega_1 z_2 - \mu\omega_0 z_1 \\ y &= z_1 \end{aligned}\right\} \tag{3.21}$$

Family of parametrized transfer functions relating incremental input current to incremental duty ratio

$$G_U(s) = \frac{y_\delta(s)}{\mu_\delta(s)} = \omega_0 Z_2(U)\frac{s + 2\omega_1}{s^2 + \omega_1 s + \omega_0^2(1-U)^2} \tag{3.22}$$

The *damping ratio* ξ, defined by $\xi = \omega_1/(2\omega_0)$, is independent of U.

Crossover frequency

$$W_0(U) = \omega_0(1-U)\left[1 - \frac{8\xi^2}{(1-U)^2}\right]^{1/2} \tag{3.23}$$

The Ziegler–Nichols method is thus applicable only for $0 < U < 1 - 2\sqrt{2}\xi$.

Ultimate period and ultimate gain

$$\left.\begin{aligned} P_0(U) &:= \frac{2\pi}{W_0(U)} = \frac{\sqrt{2}\pi}{\left\{\omega_0(1-U)\left[1 - \frac{8\xi^2}{(1-U)^2}\right]^{1/2}\right\}} \\ K_0(U) &= \frac{\omega_1^2(1-U)}{b} \end{aligned}\right\} \tag{3.24}$$

Ziegler–Nichols P–I controller gains for the linearized family of converters

$$\left.\begin{aligned} K_1(U) &= \frac{0{\cdot}4\omega_1^2(1-U)}{b} \\ K_2(U) &= \frac{\omega_1^2(1-U)[\omega_0^2(1-U)^2 - 2\omega_1^2]^{1/2}}{(4\pi b)} \end{aligned}\right\} \tag{3.25}$$

Non-linear P–I controller

$$\left.\begin{aligned} \frac{d\zeta(t)}{dt} &= \{(4\pi b)^{-1}\omega_1^2(1-\zeta)[\omega_0^2(1-\zeta)^2 - 2\omega_1^2]^{1/2}\}e(t) \\ \hat{\mu}(t) &= \zeta(t) + \left[\frac{0{\cdot}4\omega_1^2(1-\zeta)}{b}\right]e(t) \\ e(t) &= y_d(U) - y(t) = Z_2(U) - z_2(t) \end{aligned}\right\} \tag{3.26}$$

Low pass filter

A simple first-order low pass filter may be proposed to yield an approximation to the ideal average output function z_1 required by the non-linear P–I controller. Such a filter is characterized by a sufficiently small time constant of value $(1/T_f)$, and a state f.

$$\frac{df(t)}{dt} = -\left(\frac{1}{T_f}\right)(f(t) - x_1(t)); \quad z_1(t) = f(t) \tag{3.27}$$

3.3. *Buck–boost converter*

3.3.1. *Non-linear P–I regulation for the case of the output voltage control mode in a buck–boost converter.* Consider the buck–boost converter model shown in Fig. 7. This circuit is described by the time-invariant bilinear state equation model:

$$\left.\begin{aligned} \frac{dx_1}{dt} &= \omega_0 x_2 - u\omega_0 x_2 + ub \\ \frac{dx_2}{dt} &= -\omega_0 x_1 - \omega_1 x_2 + u\omega_0 x_1 \\ y &= x_2 \end{aligned}\right\} \tag{3.28}$$

where, $x_1 = I\sqrt{L}$, $x_2 = V\sqrt{C}$ represent normalized input current and output voltage variables, respectively; $b = E/\sqrt{L}$ is the normalized external input voltage and it is

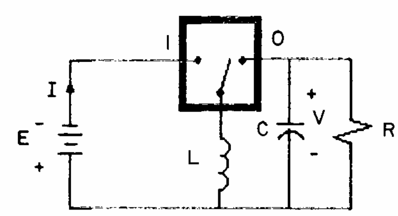

Figure 7. Buck–boost converter.

here assumed to be a negative quantity (reversed polarity) while, $\omega_0 = 1/\sqrt{LC}$ and $\omega_1 = 1/RC$ are, respectively, the LC (input) circuit natural oscillating frequency and the RC (output) circuit time constant. The switch position function, acting as a control input, is denoted by u and takes values in the discrete set $\{0, 1\}$. System (3.11) is of the same form as (2.1), with $f(x)$ of the form: $f(x) = Ax$, and $b(x)$ of the form: $b(x) = Bx$, $\eta = 0$ and $\gamma = [b \quad 0]^T$. We now summarize the formulae leading to a non-linear P–I controller design for the average model of (3.28).

Average buck–boost converter model for the output voltage control mode

$$\left.\begin{aligned} \frac{dz_1}{dt} &= \omega_0 z_2 - \mu\omega_0 z_2 + \mu b \\ \frac{dz_2}{dt} &= -\omega_0 z_1 - \omega_1 z_2 + \mu\omega_0 z_1 \\ y &= z_2 \end{aligned}\right\} \tag{3.29}$$

Constant equilibrium points

$$\mu = U; \quad Z_1(U) = \frac{bU\omega_1}{[\omega_0^2(1-U)^2]}; \quad Z_2(U) = \frac{-bU}{[\omega_0(1-U)]} \tag{3.30}$$

Parametrized family of linearized systems about the constant operating points

$$\left.\begin{aligned} \frac{d}{dt}\begin{bmatrix} z_{1\delta}(t) \\ z_{2\delta}(t) \end{bmatrix} &= \begin{bmatrix} 0 & \omega_0(1-U) \\ -\omega_0(1-U) & -\omega_1 \end{bmatrix}\begin{bmatrix} z_{1\delta} \\ z_{2\delta} \end{bmatrix} + \begin{bmatrix} \dfrac{b}{(1-U)} \\ \dfrac{b\omega_1}{[\omega_0(1-U)^2]} \end{bmatrix}\mu_\delta \\ y_\delta(t) &= z_{2\delta}(t) \end{aligned}\right\} \tag{3.31}$$

with:

$$z_{i\delta}(t) = z_i(t) - Z_i(U); \quad i = 1, 2; \quad y_\delta(t) = y(t) - Y(U) := z_2(t) - Z_2(U);$$
$$\mu_\delta(t) = \mu(t) - U$$

Family of parametrized transfer functions relating incremental output voltage to incremental duty ratio

$$G_U(s) = \frac{y_\delta(s)}{\mu_\delta(s)} = \omega_0 Z_1(U)\frac{s - b[z_1(U)]^{-1}}{s^2 + \omega_1 s + \omega_0^2(1-U)^2} \tag{3.32}$$

Crossover frequency

$$W_0(U) = \omega_0(1-U)\left(1 + \frac{1}{U}\right)^{1/2} \tag{3.33}$$

Ultimate period and ultimate gain

$$\left.\begin{aligned} P_0(U) &:= \frac{2\pi}{W_0(U)} = \frac{2\pi}{\left[\omega_0(1-U)\left(1+\frac{1}{U}\right)\right]^{1/2}} \\ K_0(U) &= \frac{[\omega_0(1-U)^2]}{(|b|U)} \end{aligned}\right\} \tag{3.34}$$

Ziegler–Nichols P–I controller gains for the linearized family of converters

$$\left.\begin{aligned} K_1(U) &= \frac{[0\cdot 4\omega_0(1-U)^2]}{(|b|U)} \\ K_2(U) &= \frac{\left[\omega_0^2(1-U)^3\left(1+\frac{1}{U}\right)\right]^{1/2}}{(4\pi|b|U)} \end{aligned}\right\} \tag{3.35}$$

Non-linear P–I controller

$$\left.\begin{aligned} \frac{d\zeta(t)}{dt} &= \frac{\left[\omega_0^2(1-\zeta(t))^3\left(1+\frac{1}{\zeta(t)}\right)\right]^{1/2}}{[4\pi|b|\zeta(t)]}e(t) \\ \hat{\mu}(t) &= \zeta(t) + \left\{\frac{[0\cdot 4\omega_0(1-\zeta(t))^2]}{(|b|\zeta(t))}\right\}e(t) \\ e(t) &= y_d(U) - y(t) = Z_2(U) - z_2(t) \end{aligned}\right\} \tag{3.36}$$

Low pass filter

$$\frac{df(t)}{dt} = -\left(\frac{1}{T_f}\right)(f(t) - x_2(t)); \quad z_2(t) = f(t) \tag{3.37}$$

3.3.2. *Simulation example.* A buck–boost converter circuit with the same parameter values as in the previous example was considered for non-linear P–I controller design. The constant operating value of μ was again chosen to be $U = 0{\cdot}75$ while the corresponding desirable normalized constant output voltage turned out to be $Z_2(0{\cdot}75) = 0{\cdot}2012$. Figure 8 shows several state trajectories corresponding to different initial conditions set on the ideal average buck–boost converter model controlled by the non-linear P–I regulator of the form (3.36). Figure 8 represents the projections of the closed-loop system three-dimensional state vector trajectories onto the z_1–z_2 average state coordinates plane. The average controlled state variables, z_1 and z_2, are shown to converge toward the desirable equilibrium point represented by $Z_1(0{\cdot}75) = -0{\cdot}8482$ and $Z_2(0{\cdot}75) = 0{\cdot}2012$. Figure 9 shows the ideal average controlled state variables evolution when subject to a step change in the output set point value, from $Z_2(0{\cdot}75) = 0{\cdot}2012$ to $Z_2(U) = 0{\cdot}1006$ (the corresponding change in the operating point of the duty ratio was from $U = 0{\cdot}75$ to $U = 0{\cdot}6$). Figures 10(*a*) and 10(*b*) show the state response of the actual (i.e. discontinuous) PWM controlled circuit and the filtered output response, respectively. The sampling frequency for the PWM actuator

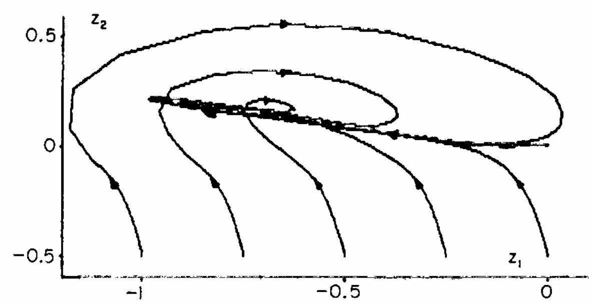

Figure 8. Normalized state trajectories of the ideal average buck–boost converter model controlled by a non-linear P–I regulator.

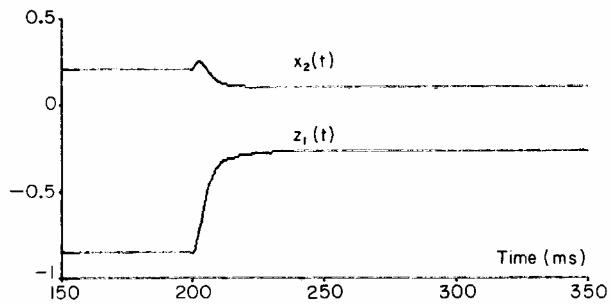

Figure 9. Average controlled state response of the buck–boost converter subject to a 100% step change in the set point value.

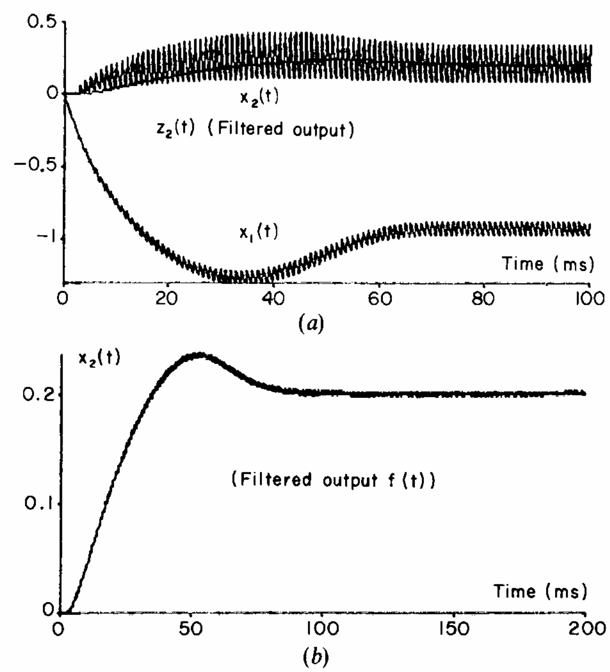

Figure 10. (*a*) Normalized state response of the actual PWM controlled buck–boost converter. (*b*) Filtered output response of the actual PWM controlled buck–boost converter with a non-linear P–I compensation scheme.

was chosen as 1 kHz and the output low pass filter cut-off frequency was set at 0·1 rad s^{-1}.

3.3.3. *Non-linear P–I regulation for the case of the input current control mode in a buck–boost converter*

Average buck–boost converter model for the input current control mode

$$\left.\begin{aligned} \frac{dz_1}{dt} &= \omega_0 z_2 - \mu\omega_0 z_2 + \mu b \\ \frac{dz_2}{dt} &= -\omega_0 z_1 - \omega_1 z_2 + \mu\omega_0 z_1 \\ y &= z_1 \end{aligned}\right\} \quad (3.38)$$

Family of parametrized transfer functions relating incremental input current to incremental duty ratio

$$G_U(s) = \frac{y_\delta(s)}{\mu_\delta(s)} = \frac{b}{1-U}\frac{s+\omega_1(1+U)}{s^2+\omega_1 s+\omega_0^2(1-U)^2} \quad (3.39)$$

The *damping ratio* ξ is defined by $\xi = \omega_1/(2\omega_0)$.

Crossover frequency

$$W_0(U) = \omega_0(1-U)[1-4\xi^2(1+U)(1-U)^{-2}]^{1/2} \quad (3.40)$$

The Ziegler–Nichols design method is valid only for values of ξ and U that satisfy $\xi < 0{\cdot}5(1-U)(1+U)^{-1/2}$.

Ultimate period and ultimate gain

$$\left.\begin{aligned} P_0(U) &:= \frac{2\pi}{W_0(U)} = \frac{2\pi}{\{\omega_0(1-U)[1-4\xi^2(1+U)(1-U)^{-2}]^{1/2}\}} \\ K_0(U) &= \frac{\omega_1(1-U)}{b} \end{aligned}\right\} \quad (3.41)$$

Ziegler–Nichols P–I controller gains for the linearized family of converters

$$\left.\begin{aligned} K_1(U) &= \frac{0{\cdot}4\omega_1(1-U)}{b} \\ K_2(U) &= \frac{[\omega_0\omega_1(1-U)^2[1-4\xi^2(1+U)(1-U)^{-2}]^{1/2}]}{(4\pi b)} \end{aligned}\right\} \quad (3.42)$$

Non-linear P–I controller

$$\left.\begin{aligned} \frac{d\zeta(t)}{dt} &= \{(4\pi b)^{-1}\omega_0\omega_1(1-\zeta)^2[1-4\xi^2(1+\zeta)(1-\zeta)^{-2}]^{1/2}\}e(t) \\ \hat{\mu}(t) &= \zeta(t) + \left\{\frac{0{\cdot}4\omega_1(1-\zeta)}{b}\right\}e(t) \\ e(t) &= y_d(U) - y(t) = Z_1(U) - z_1(t) \end{aligned}\right\} \quad (3.43)$$

Low pass filter

$$\frac{df(t)}{dt} = -\left(\frac{1}{T_f}\right)(f(t) - x_2(t)); \qquad z_1(t) = f(t) \tag{3.44}$$

4. Conclusions and suggestions for further research

This article has demonstrated the feasibility of output load voltage regulation for buck and buck–boost converters. Such regulation is made in either a direct output voltage feedback scheme, or, indirectly, through an input current feedback scheme. The stabilizing control strategy is accomplished by means of a non-linear P–I controller specification based on extended linearization of average PWM controlled models of the DC-to-DC power supplies. The output current regulation problem for a Cuk converter was also investigated and solved by the same technique. The stabilizing design considers non-linear P–I regulators derived from a linearized family of transfer functions parametrized by constant equilibrium points of idealized (infinite-frequency) average PWM controlled converter models. The non-linear controller scheme, as applied to the actual discontinuous PWM regulated converter, is shown to comply with the same qualitative stabilization features imposed on the average model design, provided the output feedback signal is properly processed through a low pass filter and a sufficiently high sampling frequency is used in the PWM actuator. The local character of the stabilizing properties of the proposed non-linear P–I regulation schemes are left as a research topic which requires further elaborate work.

The proposed non-linear control scheme can be extended to a number of other DC-to-DC power supply configurations such as the various celebrated modifications of the Cuk converter, including those with output capacitors and magnetic coupling between the input and output inductors. Application of the method to higher-order converters will undoubtedly require the use of symbolic algebraic manipulation packages such as MACSYMA or REDUCE. As a matter of fact, the third-order Cuk converter design example, presented in this article, was carried out making extensive use of the REDUCE package. Other types of classical compensating networks can also be proposed. In particular, the use of the analytical design theory developed in Newton *et al.* (1967), and its associated integral-square error minimization, could be considered as an alternative to the Ziegler–Nichols design recipe for the non-linear P–I converter specification. The feasibility of such an alternative approach has to be demonstrated through further work.

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