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Differential algebraic approach in non-linear dynamical compensator design for d.c.-to-d.c. power converters

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A non-linear dynamical compensator design, based on Fliess' generalized controller canonical form for non-linear dynamical systems, is proposed for the asymptotic stabilization of output load voltage, or current, in pulse-width-modulation controlled d.c.-to-d.c. power supplies. The technique rests on the recently introduced differential algebraic approach to the study of controlled dynamical systems.

1. Introduction

In recent years, differential algebra (see Kolchin 1973, Kaplansky 1957) has been introduced as a general tool in mathematical physics and control theory. Major contributions in these application fields are mainly due to Pommaret (1987) and Fliess (1988, 1989 a, b, c, 1990). In a series of outstanding recent articles Fliess has used *differential algebra* for the study of linear and non-linear lumped, or distributed, controlled dynamical systems. A number of long standing problems in automatic control theory, such as feedback decoupling, invertibility, model matching and realization, have been conceptually clarified and generalized by Fliess in a powerful and most elegant manner. Crucially based on the extension to *differential fields* of the theorem of the primitive element (Kolchin 1973, p. 103), any controlled dynamical system, described by a set of forced ordinary differential equations, was shown to possess a generalized controller canonical form (GCCF) depending on the input and a finite number of its time derivatives (Fliess 1989 b). Such canonical form is obtainable by means of state co-ordinate transformations which are, in general, control-dependent and, possibly, including a finite number of the control input time derivatives. As a direct consequence of this result, the problem of *feedback linearization* of a controlled dynamical system is always trivially solvable, in a local manner, using non-linear dynamical feedback. The linearizing compensator is clearly suggested by the canonical form itself. However, the asymptotic stability of the linear closed loop dynamics, around an equilibrium point, crucially depends on the *minimum phase* character of the non-linear GCCF on such a point.

In this article, Fliess's GCCF (Fliess 1989 b) is used for synthesizing non-linear dynamical compensators which asymptotically stabilize, to a preselected desirable constant value, the output (load) voltage, or current, of typical configurations of pulse width modulation (PWM) controlled d.c.-to-d.c. power converters. Due to stability considerations on the non-linear compensator dynamics, the output load voltage must be indirectly controlled via input inductor current control in all the

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converter cases studied here. The non-linear compensator design is based on the *infinite frequency average model* of the PWM regulated converter (see Sira-Ramirez 1987, 1989 a, b, 1990a, b, Sira-Ramirez and Ilic 1989). In such an average model the piece-wise smooth feedback control action is represented by the *duty ratio* function of the PWM actuator. An exact dynamical feedback linearization is simply accomplished by equating the last differential equation, in Fliess's GCCF of the average PWM model, to a stabilizing linear state feedback controller equation (see Fliess, 1989b). This equality yields a dynamical compensator represented by a non-linear time-varying differential equation for the required stabilizing duty ratio function. The transformed linear average plant dynamics is chosen so as to yield asymptotically stable responses toward the desired equilibrium point. The corresponding actual discontinuous PWM controller design is then easily obtained, in a well-known fashion (Sira-Ramirez 1990 a), in terms of the derived average stabilizing controller.

The load voltage stabilization of the Cuk converter, the Boost converter, and the Buck–Boost converter cases are treated in full detail in this paper. Simulations are provided which assess the local nature of the stabilizing properties of the proposed non-linear dynamical controller design. It should be pointed out that if direct output load voltage control is attempted for any of the above mentioned converters, the average dynamic linearlizing compensator is *unstable* around the desired equilibrium point. This is due to the non-minimum phase characteristics of the GCCF of the non-linear converters plant when the output load voltage, or current, is used to generate the output error (load voltage, or current, control mode). On the other hand, indirect average output load voltage control is entirely feasible through an input inductor current control mode strategy. This is possible thanks to the minimum phase character exhibited by the input–output plant behaviour when the input inductor current is taken as the output variable to be regulated.

Section 2 presents Fliess's derivation of the GCCF form and demonstrates, via linearized analysis, that in order to obtain a stabilizing compensator design, a minimum phase plant behaviour is required. Section 3 obtains the compensators which exactly linearize the converters dynamics and presents some simulation examples. Section 4 is devoted to the conclusions. The required background on differential algebra, used in § 2, is presented in the Appendix.

2. Differential algebra approach to systems dynamics

In this section, Fliess's derivation of the GCCF for non-linear dynamical systems is presented. The results are directly taken from Fliess (1989 b), but they are presented here for the sake of self-containment. The background results on differential algebra (see also Kolchin 1973, Kaplansky 1957) are contained in the Appendix.

2.1. *Fliess's generalized controller canonical form for non-linear systems and exact dynamical feedback linearization* (*Fliess* 1989 *b*)

Let u be a differential scalar indeterminate and let k be a 'differential field', with derivation denoted by d/dt. A '*dynamics*' is defined as a 'finitely generated differentially algebraic extension' $K/k\langle u\rangle$ of the differential field $k\langle u\rangle$. The *input* u is said to be independent if u is a 'differential transcendence basis' of K/k. Suppose

$x = (x_1, x_2, \ldots, x_n)$ is a non-differential transcendence basis of $K/k\langle u\rangle$. It follows that the derivatives dx_i/dt $(i = 1, \ldots, n)$ are $k\langle u\rangle$-algebraically dependent on the components of x. Thus, there exists exactly n polynomial differential equations of the form

$$P_i(dx_i/dt, x, u, du/dt, \ldots, d^\nu u/dt^\nu) = 0; \quad i = 1, \ldots, n \tag{2.1}$$

implicitly describing the controlled dynamics. Under the assumption that such equations can be locally solved in 'normal form,' i.e. as

$$dx_i/dt = p_i(x, u, du/dt, \ldots, d^\nu u/dt^\nu); \quad i = 1, \ldots, n \tag{2.2}$$

one obtains a non-redundant description of the dynamics. Such is not the case if one uses a generator system of $K/k\langle u\rangle$ which strictly contains a transcendence basis. Any other transcendence basis, say $z = (z_1, z_2, \ldots, z_n)$ also qualifies as a 'state' and similar expressions can be obtained for the given dynamics. The components of x are $k\langle u\rangle$-algebraically dependent upon the components of z, and vice versa. Such transformations, from one state to another, involve equations dependent upon the control input u and its derivatives. It should be pointed out that even if (2.1) is in polynomial form, it may happen, in general, that (2.2) is not.

According to the theorem of the differential primitive element (Kolchin 1973), there exists an element $\xi \in K$ such that $K = k\langle u, \xi\rangle$. The (non-differential) transcendence degree n of $K/k\langle u\rangle$ is the smallest integer such that $\xi^{(n)}$ is $k\langle u\rangle$-algebraically dependent on $\xi, d\xi/dt, \ldots, d^{(n-1)}\xi/dt^{(n-1)}$. We let $q_1 = \xi$, $q_2 = d\xi/dt, \ldots,$ $q_n = d^{(n-1)}\xi/dt^{(n-1)}$. It follows that $q = (q_1, \ldots, q_n)$ is also a transcendence basis of $K/k\langle u\rangle$. One, hence, obtains a non-linear generalization of the controller canonical form

$$\left.\begin{aligned} &\frac{d}{dt}q_1 = q_2 \\ &\frac{d}{dt}q_2 = q_3 \\ &\vdots \\ &\frac{d}{dt}q_{n-1} = q_n \\ &C(\dot{q}_n, q, u, \dot{u}, \ldots, u^{(\nu)}) = 0 \end{aligned}\right\} \tag{2.3}$$

where C is the polynomial with coefficients in k. If one can locally solve for the time derivative of q_n in the last equation one obtains an explicit system of first order differential equations, known as the generalized controller canonical form (GCCF)

$$\left.\begin{aligned} &\frac{d}{dt}q_1 = q_2 \\ &\frac{d}{dt}q_2 = q_3 \\ &\vdots \\ &\frac{d}{dt}q_{n-1} = q_n \\ &\frac{d}{dt}q_n = c(q, u, \dot{u}, \ddot{u}, \ldots, u^{(\nu)}) \end{aligned}\right\} \tag{2.4}$$

'Exact dynamic feedback linearization' is simply achieved by equating the expression in the last differential equation to a (stable) linear equation in the components of q, possibly including an external reference input signal v, as follows

$$c(q, u, \dot{u}, \ddot{u}, \dots, u^{(v)}) = -\alpha_1 q_1 - \alpha_2 q_2 -, \dots, -\alpha_n q_n + \kappa v \qquad (2.5)$$

The last equation implicitly defines a dynamical non-linear state feedback law which accomplishes an exact linearization of the non-redundant dynamics. In some instances, the implicit equation (2.5) does not globally admit a state space representation and may even exhibit *impasse* points, typical of some non-linear dynamical circuit examples (see Fliess and Hassler 1989). The obtained linear system has prespecified asymptotic stability properties chosen by means of the α's. However, u and its time derivatives must be synthesized.

2.2. *Stability considerations on exact dynamical feedback linearization*

It is evident that the non-linear dynamical feedback linearization presented above is based on exact cancellation of the non-linear plant dynamics by means of the proposed controller. One intuitively expects that the cancellation may lead to internal instabilities in some special instances. That such is, indeed, the case can be easily demonstrated by performaing a straightforward linear analysis of the GCCF and of the proposed dynamical feedback scheme around a constant equilibrium point.

Consider $y = q_1$ as the *output* of the system (2.4) and let Q_1 and U be a constant equilibrium point for y and u, respectively. Let $Q = (Q_1, 0, \dots 0)$ denote, then, the state equilibrium vector, i.e. $c(Q_1, 0, \dots 0, U, 0, \dots 0) = 0$. Linearization about (Q, U) yields

$$\left.\begin{aligned}
&\frac{d}{dt} q_{\delta 1} = q_{\delta 2} \\
&\frac{d}{dt} q_{\delta 2} = q_{\delta 3} \\
&\vdots \\
&\frac{d}{dt} q_{\delta(n-1)} q_{\delta n} \\
&\frac{d}{dt} q_{\delta n} = -\gamma_1 q_{\delta 1} - \gamma_2 q_{\delta 2} -, \dots, -\gamma_n q_{\delta n} + \beta_0 u\delta + \beta_1 \dot{u}_\delta +, \dots, + \beta_v u_\delta^{(v)} \\
&y_\delta = q_{\delta 1}
\end{aligned}\right\} \qquad (2.6)$$

where $y_\delta := y - Q_1$, $q_{\delta i} := q_i - 0$, $i = 2, \dots, n$; $u_\delta(:= u - U$, and $u_\delta^{(i)} := d^{(i)}u/dt^{(i)}$, $i = 1, 2, \dots, v$. The constant coefficients in (2.6) are obtained from

$$\gamma_i = \left.\frac{\partial c}{\partial q_i}\right|_{q=Q,\, u=U}; \quad i = 1, 2, \dots, n$$

$$\beta_j = \left.\frac{\partial c}{\partial u^{(j)}}\right|_{q=Q,\, u=U}; \quad j = 1, 2, \dots, v$$

Model (2.6) can be conveniently described in terms of the rational transfer function

$$G(s) = y_\delta(s)/u_\delta(s) = \frac{\beta_0 + \beta_1 s + \ldots + \beta_v s^v}{\gamma_1 + \gamma_2 s + \ldots + \gamma_n s^n} \tag{2.7}$$

Notice that, due to minimality of the state dimension, a pole–zero cancellation in (2.7) is not possible.

Definition

The non-linear plant (2.4) is said to be *minimum phase* in a neighborhood of the equilibrium point (Q, U), if and only if the linearized transfer function $G(s)$, in (2.7), is minimum phase, i.e. poles and zeros of the transfer function $G(s)$ lie in the left half of the complex plane.

The incremental transfer function of the exactly linearized plant is easily seen to be given by

$$G_d(s) = y_\delta(s)/v_\delta(s) = \frac{1}{\alpha_1 + \alpha_2 s + \ldots + \alpha_n s^n}. \tag{2.8}$$

On the other hand, the linearized transfer function of the compensator is given by

$$\begin{aligned} u_\delta(s) &= \frac{\kappa}{\beta_0 + \beta_1 s +, \ldots, + \beta_v s^v} v_\delta(s) \\ &+ \frac{(\gamma_1 - \alpha_1)q_{\delta 1}(s) + (\gamma_2 - \alpha_2)q_{\delta 2}(s) + \ldots + (\gamma_n - \alpha_n)q_{\delta n}(s)}{\beta_0 + \beta_{1s} +, \ldots, + \beta_v s^v} \end{aligned} \tag{2.9}$$

whose stability characteristics crucially depend upon the nature of the roots of the polynomial equation

$$\beta_0 + \beta_1 s +, \ldots, + \beta_v s^v = 0 \tag{2.10}$$

which just yields the *zeros* of the linearized transfer function $G(s)$ of the plant.

We have thus proved the following proposition.

Proposition 2.1

The dynamical compensator (2.5) yields a stable closed loop linearization around a given equilibrium point if and only if the GCCF (2.4) is minimum phase at such point.

Remark

Evidently, the exactly linearized closed loop system is made locally asymptotically stable via exact pole–zero cancellation imposed by the compensator dynamics. Such a cancellation is valid only if the linearized transfer function of the plant, $G(s)$, is minimum phase. Otherwise, the controller design is unstable. Thus, in order to obtain a locally asymptotically stable behaviour of the closed loop system, toward the required equilibrium point, the open loop linearized plant transfer function $G(s)$ in (2.7) must be minimum phase. The local stability implications of these facts, corresponding to the original non-linear case (2.4)–(2.5), depicted in proposition 2.1 are obvious, by virtue of Lyapunov's first stability theorem (see for instance Verhulst 1990, Chap. 7).

3. Nonlinear controller design by exact dynamical feedback linearization in d.c.-to-d.c. power converters

3.1. *Cuk converter*

Consider the Cuk converter model (Middlebrook and Cuk 1981) shown in Fig. 1. This ubiquitous converter, which is the topological dual of the boost converter, is described by the following bilinear state equation model

$$\left.\begin{aligned} dx_1/dt &= -\omega_1 x_2 + u\omega_1 x_2 + b \\ dx_2/dt &= \omega_1 x_1 - u\omega_1 x_1 - u\omega_2 x_3 \\ dx_3/dt &= -\omega_4 x_3 + u\omega_2 x_2 \end{aligned}\right\} \qquad (3.1)$$

where, $x_1 = I_1\sqrt{L_1}$, $x_2 = V_2\sqrt{C_2}$ and $x_3 = I_3\sqrt{L_3}$ represent normalized input inductor current, transfer capacitor voltage and output inductor current variables, respectively. The quantity $b = E/\sqrt{L_1}$ is the normalized external input voltage. The converter parameters are defined as $\omega_1 = 1/\sqrt{L_1 C_2}$, $\omega_2 = 1/\sqrt{L_3 C_2}$ and $\omega_4 = R/L_3$, these are, respectively, the $L-C$ input circuit natural oscillating frequency, the $L-C$ output circuit natural oscillating frequency and the $R-L$ output circuit time constant. The variables u denotes the switch position function, which acts as a control input, taking values in the discrete set $\{0, 1\}$. We demonstrate that, in order to apply the dynamical feedback linearization design method, a convenient output variable y is represented by the normalized input inductor current $x_1(t)$.

The discontinuous feedback control strategy is usually specified on the basis of a sampled closed loop PWM control scheme of the form (see Skoog and Blankenship 1970)

$$u = \begin{cases} 1 & \text{for } t_k \leqslant t < t_k + \mu[x(t_k)]T \\ 0 & \text{for } t_k + \mu[x(t_k)]T \leqslant t < t_k + T \end{cases} \qquad (3.2)$$

where $\mu[x(t_k)]$ is known as the *duty ratio function*, which is generally represented by a piecewise smooth feedback function of the converter state (or of some related variables such as sampled output error $e(t_k) := y_d - y(t_k) = y_d - x_1(t_k)$) which satisfies the natural bounding constant: $0 < \mu[x(t_k)] < 1$, for all sampling instants t_k. T is known as the *duty cycle* determining the time elapsed between sampling instants, i.e. $t_{k+1} = t_k + T$.

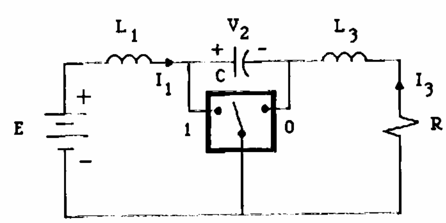

Figure 1. The Cuk converter.

Remark

The '*average PWM model*' of the controlled Cuk converter is simply obtained from (3.1) by replacing the discontinuous control function u by the duty ratio function μ (see Sira-Ramirez 1990 b). The average model has been shown to be a useful tool for non-linear controller design in discontinuous PWM controlled dynamical systems (see Sira-Ramirez 1987, 1989 a, b, 1990 a). The average model is an idealized version of the controlled system when the sampling frequency is increased without bound (i.e. when the duty cycle $T \to 0$). Its fundamental design value lies in the fact that finite but large sampling frequencies in a PWM actuator based control system can be made to produce output responses which are arbitrarily close to those of the average idealized responses. Hence, the stability properties of the average PWM controlled system can be arbitrarily closely reproduced by the actual discontinuous controller responses, provided the sampling frequency is sufficiently large. The proof of this fact, and important connections with sliding mode control (Utkin 1978), as well as a method to obtain an estimate of the required sampling frequency appear in several recently published articles (see Sira-Ramirez 1989 c, 1990 b).

The average PWM model of the Cuk converter is then given by

$$\left.\begin{aligned} dz_1/dt &= -\omega_1 z_2 + \mu\omega_1 z_2 + b \\ dz_2/dt &= \omega_1 z_1 - \mu\omega_1 z_1 - \mu\omega_2 z_3 \\ dz_3/dt &= -\omega_4 z_3 + \mu\omega_2 z_2 \end{aligned}\right\} \tag{3.3}$$

where we have used $z_i (i = 1, 2, 3)$ to denote the corresponding 'averaged' components x_i of the normalized state vector x describing the Cuk Converter.

In order to assess the feasibility of asymptotic stabilization of the average normalized output load current in the converter via the dynamical exact linearization approach, the minimum phase features of the linearized model, around an equilibrium point, must be studied, as implied by the results in § 2.2.

The 'equilibrium points' of the average model (3.4) are obtained from (3.3) assuming a *constant* value U for the duty ratio function μ (see Sira-Ramirez 1990 a)

$$\begin{aligned} &\mu = U; \quad Z_1(U) = \omega_2^2 b U^2 / \omega_1^2 \omega_4 (1-U)^2; \\ &Z_2(U) = b/\omega_1(1-U); \quad z_3(U) = \omega_2 b U / \omega_1 \omega_4 (1-U) \end{aligned} \tag{3.4}$$

The linearization of the average PWM model (3.3) around the constant equilibrium points (3.4) results in an incremental model, parameterized by U, of the form

$$\begin{aligned} \frac{d}{dt}\begin{bmatrix} z_{1\delta} \\ z_{2\delta} \\ z_{3\delta} \end{bmatrix} &= \begin{bmatrix} 0 & -(1-U)\omega_1 & 0 \\ (1-U)\omega_1 & 0 & -U\omega_2 \\ 0 & U\omega_2 & -\omega_4 \end{bmatrix}\begin{bmatrix} z_{1\delta} \\ z_{2\delta} \\ z_{3\delta} \end{bmatrix} \\ &+ \begin{bmatrix} \omega_1 z_2(U) \\ -\omega_1 Z_1(U) - \omega_2 Z_3(U) \\ \omega_2 Z_2(U) \end{bmatrix}\mu_\delta \end{aligned} \tag{3.15}$$

where $z_{i\delta}(t) = z_i(t) - Z_i(U); \quad i = 1, 2, 3; \quad \mu_\delta(t) = \mu(t) - U.$

The transfer functions relating the average incremental output to the incremental duty ratio μ_δ on the given equilibrium point depend on the chosen output variable which is to be regulated.

If the average incremental output load current, $z_{3\delta}$, is taken as the output variable deviation to be regulated, the corresponding transfer function is found to be

$$G_u(s) = \frac{z_{3\delta}(s)}{\mu_\delta(s)} = \frac{b\omega_2}{(1-U)\omega_1} \frac{s^2 - \dfrac{U^2\omega_2^2}{(1-U)\omega_4}s + (1-U)\omega_1^2}{s^3 + \omega_4 s^2 + [U^2\omega_2^2 + (1-U)^2\omega_4]s + (1-U)^2\omega_1^2\omega_4} \tag{3.6}$$

which is evidently non-minimum phase and, according to the results of § 2.2, it is not suitable for exact dynamical feedback linearization. Direct output current regulation is therefore not feasible by the proposed method. The same situation occurs if one takes the incremental average transfer capacitor voltage $z_{2\delta}$ as the output variable deviation to be regulated in the plant. In this case the corresponding transfer function is obtained as

$$G_U(s) = \frac{z_{2\delta}(s)}{\mu_\delta(s)} = -\frac{bU\omega_2^2}{(1-U)\omega_1\omega_4} \frac{s^2 + \left[(2-U)\omega_4 - \dfrac{\omega_1^2\omega_4(1-U)^2}{U\omega_2^2}\right]s - \dfrac{\omega_1^2\omega_4^2(1-U)^2}{U\omega_2^2}}{s^3 + \omega_4 s^2 + [U^2\omega_2^2 + (1-U)^2\omega_4]s + (1-U)^2\omega_1^2\omega_4}$$

Average normalized input inductor current z_1 is actually the only output variable that can be asymptotically regulated toward its equilibrium point, using the proposed exact linearization scheme. The corresponding minimum-phase transfer function relating the incremental input inductor current to the incremental duty ratio is found to be

$$G_U(s) = \frac{z_{1\delta}(s)}{\mu_\delta(s)} = \frac{b}{(1-U)} \frac{s^2 + \left[\dfrac{\omega_2^2 U + \omega_4^2}{\omega_4}\right]s + 2\omega_2^2 U}{s^3 + \omega_4 s^2 + [U^2\omega_2^2 + (1-U)^2\omega_4]s + (1-U)^2\omega_1^2\omega_4}$$

Constant average normalized input inductor current regulation indirectly accomplishes a desirable constant value for the average normalized output load current, or, equivalently, normalized output load voltage. Indeed, from the equilibrium point expressions (3.4), given a desirable constant equilibrium value Z_3 for the normalized output load current, the corresponding set point value for the normalized input inductor current Z_1 is given by

$$Z_1 = \frac{\omega_4}{b} Z_3^2 \tag{3.9}$$

We propose a stabilizing nonlinear dynamical feedback controller scheme which considers the average normalized value of the input inductor current, z_1, as the converter output that needs to be regulated. Such regulation scheme will be referred to as the 'input current control mode'. The relevant formulae leading to the non-linear dynamical controller specification are summarized below.

Control-dependent transformation for expressing the Cuk converter in generalized controller canonical form

Let Z_1 be a desirable constant average normalized input inductor current possibly computed in terms of a desirable average normalized output load current z_3 as in (3.9). It is easy to verify that $q_1 = z_1 - Z_1$ is a 'differential primitive element' that allows one to write the normalized average model (3.3) in a GCCF of the form (2.4) with $\nu = 2$.

$$\left.\begin{aligned} q_1 &= z_1 - Z_1 \\ q_2 &= -\omega_1 z_2 + \mu\omega_1 z_2 + b \\ q_3 &= -(1-\mu)^2\omega_1^2 z_1 + \dot{\mu}\omega_1 z_2 + \mu(1-\mu)\omega_1\omega_2 z_3 \end{aligned}\right\} \tag{3.10}$$

The inverse transformation is given by

$$\begin{aligned} z_1 &= q_1 + z_1 \\ z_2 &= \frac{b - q_2}{(1-\mu)\omega_1} \\ z_3 &= \frac{(1-\mu)\omega_1 q_1}{\mu\omega_2} + \frac{\dot{\mu} q_2}{\mu(1-\mu)^2\omega_1\omega_2} + \frac{q_3}{\mu(1-\mu)\omega_1\omega_2} + \frac{(1-\mu)\omega_1 z_1}{\mu\omega_2} - \frac{\dot{\mu} b}{\mu(1-\mu)^2\omega_1\omega_2} \end{aligned} \tag{3.11}$$

The Cuk converter dynamics in generalized controller canonical form

$$\left.\begin{aligned} \dot{q}_1 &= q_2 \\ \dot{q}_2 &= q_3 \\ \dot{q}_3 &= -c_1(\mu, \dot{\mu}, \ddot{\mu})q_1 - c_2(\mu, \dot{\mu}, \ddot{\mu})q_2 - c_3(\mu, \dot{\mu}, \ddot{\mu})q_3 + c_4(\mu, \dot{\mu}, \ddot{\mu}) \end{aligned}\right\} \tag{3.12}$$

with

$$\begin{aligned} c_1(\mu, \dot{\mu}, \ddot{\mu}) &= (1-\mu)^2\omega_1^2\left[\omega_4 - \frac{\dot{\mu}}{\mu(1-\mu)}\right] \\ c_2(\mu, \dot{\mu}, \ddot{\mu}) &= \left[\mu^2\omega_2^2 + (1-\mu)^2\omega_1^2 + \frac{\ddot{\mu}}{(1-\mu)} + \frac{\dot{\mu}\omega_4}{(1-\mu)} - \frac{(\dot{\mu})^2(1-3\mu)}{\mu(1-\mu)^2}\right] \\ c_3(\mu, \dot{\mu}, \ddot{\mu}) &= \left[\omega_4 - \frac{\dot{\mu}(1-3\mu)}{\mu(1-\mu)}\right] \\ c_4(\mu, \dot{\mu}, \ddot{\mu}) &= \left[\mu^2\omega_2^2 + \frac{\ddot{\mu}}{(1-\mu)} + \frac{\dot{\mu}\omega_4}{(1-\mu)} - \frac{(\dot{\mu})^2(1-3\mu)}{\mu(1-\mu)}\right]b \\ &\quad + (1-\mu)^2\omega_1^2\left[\frac{\dot{\mu}}{\mu(1-\mu)} - \omega_4\right]Z_1 \end{aligned} \tag{3.13}$$

Notice that the GCCF for the Cuk converter is linear time-varying in the transformed variables q_1, q_2 and q_3 and includes a time varying forcing term represented by $c_4(\mu, \dot{\mu}, \mu)$. The forcing term collects the influence of the desirable constant set point Z_1, as well as the constant external voltage source b. If the last differential equation in (3.12) is equated to a linear combination of the state variables, say $-\alpha_1 q_1 - \alpha_2 q_2 - \alpha_3 q_3$, as in (2.5) (with $\kappa = 0$) an asymptotically stable closed loop linear system is obtained which guarantees, for suitable chosen α_i's, that

$\lim_{t\to\infty} q_1 = 0$. This guarantees asymptotic convergence of z_1 to the desirable equilibrium point Z_1. The linearization procedure described above yields a differential equation characterizing the non-linear dynamical compensator which synthesizes the computed duty ratio $\hat{\mu}$.

Non-linear dynamical compensator in input current control mode for the average PWM controlled Cuk converter

$$\left.\begin{aligned}
\dot{\xi}_1 &= \xi_2 \\
\dot{\xi}_2 &= \frac{1-\xi_1}{q_2 - b}\left\{\left[\alpha_1 - (1-\xi_1)^2\omega_1^2\left(\omega_4 - \frac{\xi_2}{\xi_1(1-\xi_1)}\right)\right]q_1\right. \\
&\quad + \left[\alpha_2 - \left(\xi_1^2\omega_2^2 + (1-\xi_1)^2\omega_1^2 + \frac{\xi_2\omega_4}{(1-\xi_1)} - \frac{\xi_2^2(1-3\xi_1)}{\xi_1(1-\xi_1)^2}\right)\right]q_2 \\
&\quad + \left[\alpha_3 - \left(\omega_4 - \frac{\xi_2(1-3\xi_1)}{\xi_1(1-\xi_1)}\right)\right]q_3 \\
&\quad + \left[\xi_1^2\omega_2^2 + \frac{\xi_2\omega_4}{(1-\xi_1)} - \frac{\xi_2^2(1-3\xi_1)}{\xi_1(1-\xi_1)}\right]b \\
&\quad \left. + (1-\xi_1)^2\omega_1^2\left[\frac{\xi_2}{\xi_1(1-\xi_1)} - \omega_4\right]Z_1\right\} \\
\hat{\mu} &= \xi_1
\end{aligned}\right\} \quad (3.14)$$

The output $\hat{\mu}$ of the non-linear compensator is to be regarded as the specification of the needed stabilizing duty ratio function μ for the average PWM closed loop converter. However, depending on the proximity of the initial states to the desirable constant average normalized input inductor current (acting as an equilibrium set point), the actual values of $\hat{\mu}$ may violate the natural constraints imposed on the duty ratio function μ. Namely, $0 < \mu < 1$. Therefore, a limiter of the following form:

$$\mu(t) = \begin{cases} 0 & \text{for } \hat{\mu}(t) \leqslant 0 \\ \hat{\mu}(t) & \text{for } 0 < \hat{\mu}(t) < 1 \\ 1 & \text{for } \hat{\mu}(t) \geqslant 1 \end{cases} \quad (3.15)$$

has to be enforced on the output $\hat{\mu}$ of the non-linear dynamical compensator. This procedure yields the physically meaningful duty ratio function μ. In actual operation, μ may be subject to saturation, during certain intervals of time. Typically, an antireset-windup schem (see Åström and Hägglünd 1988, pp. 10–14]) would be used in combination with the non-linear controller to avoid overshooting effects on the average controlled output.

The PWM actuator induces undersirable high frequency discontinuous signals (chattering) for the converters states and output variable. In order to further approximate the average closed loop designed behaviour, a suitable Low Pass filter must then be placed at the sensing arrangement used to obtain the actual input inductor current $x_1(t)$ and the other states used for feedback purposes. One may, for instance, propose simple first order $R-C$ circuits, with a sufficiently small time constants, $1/T_{if}$ (equivalently, sufficiently small cut-off frequencies) as follows

$$df_i(t)/dt = -(1/T_{if})(f_i(t) - x_i(t)); \quad \hat{z}_i(t) = f_i(t); \quad i = 1, 2, 3 \tag{3.16}$$

One may regard the filters outputs, $\hat{z}_i(t)$ $(i = 1, 2, 3)$, as approximations to the ideal average normalized state signals $z_i(t)$ required by the non-linear dynamical feedback controller (3.10), (3.14).

The complete discontinuous non-linear dynamical feedback regulation scheme for stabilization of the normalized output load current in the Cuk converter, based on dynamical exact feedback linearization, is presented in Fig. 2. Notice that the control-dependent state coordinate transformation requires the use of $\hat{\mu}$ and $d\hat{\mu}/dt$, which represent the dynamical controller state (ξ_1, ξ_2).

A simulation example

A non-linear controller indirectly regulating the output load current via input inductor current regulation to a desirable set point was designed for the Cuk converter circuit with parameter values $R = 20\,\Omega$, $C_2 = 6{\cdot}071\,\mu$F, $L_1 = 24{\cdot}539$ mH, $L_3 = 2{\cdot}9038$ mH and $E = 20$ V. The desirable normalized constant output current was set at $Z_3 = 0{\cdot}2$. The corresponding constant operating value of μ is $U = 0{\cdot}7877$. The poles of the linearized closed loop system were chosen at: $-4000\,\text{s}^{-1}$, $-4000\,\text{s}^{-1}$, $-3000\,\text{s}^{-1}$. Figure 3 shows the average normalized controlled output current step response. Figure 4 shows the actual PWM controlled step response and the corresponding filtered output response for the actual output load current x_3. The sampling frequency for the PWM actuator was chosen as 20 kHz and the low pass filters cut-off frequency was set at $1570\,\text{rad s}^{-1}$.

3.2. *Boost converter*

Consider the Boost converter model shown in Fig. 5. This circuit is described by the bilinear state equation model

$$\left.\begin{aligned} dx_1/dt &= -\omega_0 x_2 + u\omega_0 x_2 + b \\ dx_2/dt &= \omega_0 x_1 - \omega_1 x_2 - u\omega_0 x_1 \end{aligned}\right\} \tag{3.17}$$

where, $x_1 = I\sqrt{L}$, $x_2 = V\sqrt{C}$ respresent normalized input current and output

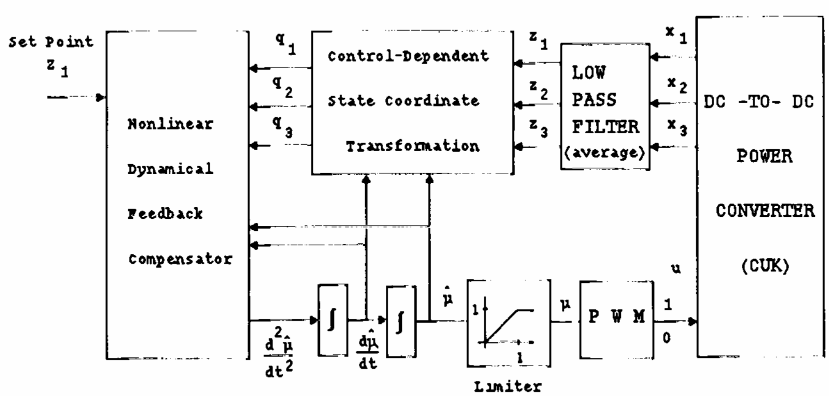

Figure 2. Non-linear PWM dynamical feedback regulation scheme for the Cuk converter.

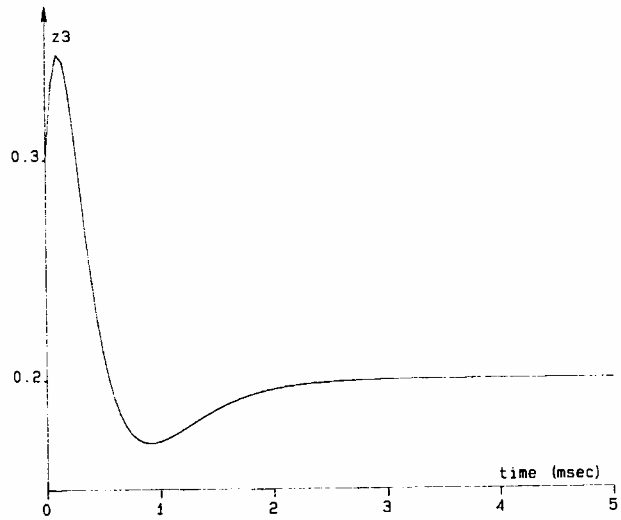

Figure 3. Average normalized output load current step response in PWM dynamically feedback controlled CUk converter.

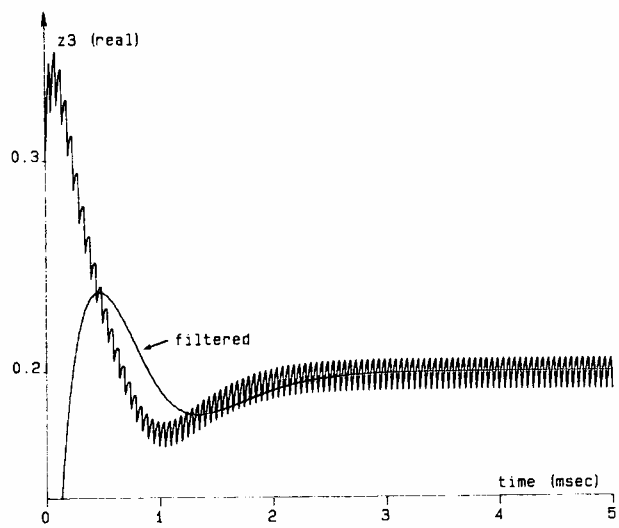

Figure 4. Actual and filtered PWM controlled normalized output load current step response in PWM dynamically feedback controlled Cuk converter.

voltage variables, respectively. The quantity $b = E/\sqrt{L}$ is the normalized external input voltage and, $\omega_0 = 1/\sqrt{LC}$ and $\omega_1 = 1/RC$ are, respectively, the LC (input) circuit natural oscillating frequency and the RC output circuit time constant. The variable u denotes the switch position function, acting as a control input, and taking values in the discrete set $\{0, 1\}$. We now summarize, according to the theory presented in the previous section, the formulae leading to a non-linear dynamical controller design for the average model (3.17).

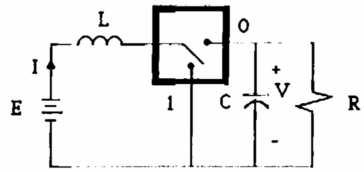

Figure 5. Boost converter.

Average Boost converter model

$$\left.\begin{aligned} dz_1/dt &= -\omega_0 z_2 + \mu\omega_0 z_2 + b \\ dz_2 dt &= -\omega_0 z_1 - \omega_1 z_2 - \mu\omega_0 z_1 \end{aligned}\right\} \quad (3.18)$$

Constant operating equilibrium points

$$\mu = U; \quad Z_1(U) = b\omega_1/[\omega_0^2(1-U)^2]; \quad Z_2(U) = b/[\omega_0(1-U)] \quad (3.19)$$

Linearized Boost converter model about the constant operation point

$$\frac{d}{dt}\begin{bmatrix} z_{1\delta}(t) \\ z_{2\delta}(t) \end{bmatrix} = \begin{bmatrix} 0 & -(1-U)\omega_0 \\ (1-U)\omega_0 & -\omega_1 \end{bmatrix}\begin{bmatrix} z_{1\delta}(t) \\ z_{2\delta}(t) \end{bmatrix} + \begin{bmatrix} b/(1-U) \\ -b\omega_1/[(1-U)^2 w_0] \end{bmatrix}\mu_\delta$$

with

$$z_{i\delta}(t) = z_i(t) - Z_i(U); \quad i = 1, 2; \quad \mu_\delta(t) = \mu(t) - U.$$

Incremental transfer function parametrized by operating point

If the average normalized output capacitor voltage, z_2, is taken as the converters output, the resulting linearized transfer function, given by

$$G_U(s) = \frac{z_{2\delta}(s)}{\mu_\delta(s)} = -\omega_0 Z_1(U)\frac{s - b[Z_1(U)]^{-1}}{s^2 + \omega_1 s + (1-U)^2\omega_0^2} \quad (3.21)$$

is non-minimum phase and hence the proposed compensator design approach is not feasible. On the other hand, if the average normalized input inductor current z_1 is taken as the output of the system the resulting incremental transfer function is indeed minimum-phase and given by

$$G_U(s) = \frac{z_{1\delta}(s)}{\mu_\delta(s)} = \omega_0 Z_2(U)\frac{s + 2\omega_1}{s^2 + \omega_1 s + (1-U)^2\omega_0^2} \quad (3.22)$$

Input inductor current must then be taken as the output variable to be regulated. One usually pursues constant average normalized input current regulation to indirectly obtain a desirable controlled constant average normalized output voltage at the load. Indeed, given a desirable constant equilibrium value Z_2 for the average normalized output load voltage, one can compute from (3.19), the corresponding set point value for the normalized average normalized input inductor current Z_1. This value is given by

$$Z_1 = \frac{\omega_1}{b} Z_2^2 \quad (3.23)$$

We now propose an 'input current control mode' stabilizing non-linear dynamical feedback controller scheme. We only summarize the relevant formulae leading to the non-linear dynamical controller specification.

Average Boost converter model for input current control mode

$$\begin{aligned} dz_1/dt &= -\omega_0 z_2 + \mu\omega_0 z_2 + b \\ dz_2/dt &= \omega_0 z_1 - \omega_1 z_2 - \mu\omega_0 z_1 \\ y &= z_1 \end{aligned} \tag{3.24}$$

Control-dependent transformation for expressing the Boost converter in generalized controller canonical form

Let Z_1 be a desirable constant average normalized input inductor current, possibly computed in terms of the desirable constant average normalized output load voltage z_2 as in (3.23). It is easy to verify that $q_1 = z_1 - Z_1$ is a '*differential primitive element*' that allows one to write the normalized average model (3.24) in a GCCF of the form (2.4) with $v = 1$.

$$\left.\begin{aligned} q_1 &= z_1 - Z_1 \\ q_2 &= -(1-\mu)\omega_0 z_2 + b \end{aligned}\right\} \tag{3.25}$$

The inverse control-dependent transformation is simply

$$\left.\begin{aligned} z_1 &= q_1 + Z_1 \\ z_2 &= \frac{b - q_2}{(1-\mu)\omega_0} \end{aligned}\right\} \tag{3.26}$$

Boost convecter in generalized controller canonical form

$$\left.\begin{aligned} \dot{q}_1 &= q_2 \\ \dot{q}_2 &= -c_1(\mu,\dot{\mu})q_1 - c_2(\mu,\dot{\mu})q_2 + c_3(\mu,\dot{\mu}) \end{aligned}\right\} \tag{3.27}$$

with

$$\begin{aligned} c_1(\mu,\dot{\mu}) &= (1-\mu)^2\omega_0^2 \\ c_2(\mu,\dot{\mu}) &= \left[\frac{\dot{\mu}}{(1-\mu)} + \omega_1\right] \\ c_3(\mu,\dot{\mu}) &= b\left[\frac{\dot{\mu}}{(1-\mu)} + \omega_1\right] - (1-\mu)^2\omega_0^2 z_1 \end{aligned} \tag{3.28}$$

Notice that the GCCF for the boost converter is linear and time-varying in the transformed varibles q_1 and q_2 and includes a time varying forcing term represented by $c_3(\mu,\dot{\mu})$. The forcing term collects the influence of the desirable set point Z_1, as well as the constant external voltage source b. If the last differential equation in (3.27) is equated to a linear combination of the state variables, say $-\alpha_1 q_1 - \alpha_2 q_2$, as in (2.5) (with $\kappa = 0$) an asymptotically stable closed loop linear system is obtained which guarantees, for suitable chosen α_i's, that $\lim_{t\to\infty} q_1 = 0$. This guarantees asymptotic convergence of z_1 to the desirable equilibrium point Z_1. This linearization procedure yields a differential equation characterizing the non-linear dynamical compensator which synthesizes the computed duty ratio $\hat{\mu}$.

Non-linear dynamical compensator in current control mode for the average PWM controlled Boost converter

$$\dot{\xi} = \frac{1-\xi}{q_2 - b}\left\{\left[a_1 - (1-\xi)^2\omega_0^2\right]q_1 + (\alpha_2 - \omega_1)q_2 + \left[\omega_1 b - (1-\xi)^2\omega_0^2 z_1\right]\right\}$$
$$\hat{\mu} = \xi \tag{3.29}$$

The complete discontinuous non-linear dynamical feedback regulation scheme for stabilization of the normalized output load voltage in the boost converter, based on dynamical exact feedback linearization, is conceptually the same as in Fig. 2. The difference lies in the fact that since (3.29) is a first order system, only the computed duty ratio function is needed for the synthesis of the control-dependent state coordinate transformation yielding the GCCF of the normalized average boost converter model (see equation 3.25).

A simulation example

A boost converter circuit with parameter values $R = 30\,\Omega$, $C = 20\,\mu F$, $L = 20$ mH and $E = 15$ V was considered for non-linear dynamical controller design. The desirable normalized constant output voltage is $Z_2 = 0.1677$ which corresponds to a constant value $U = 0{\cdot}6$ for the duty ratio μ. The corresponding set point for the average normalized input inductor current is $z_1(0.6) = 0.4419$. The poles of the linearized closed loop system were chosen at: $-1500\,s^{-1}$ and $-3000\,s^{-1}$. Figure 6 shows several normalized average state trajectories corresponding to different initial conditions set on the ideal average boost converter model controlled by the non-linear regulator of the form (3.29). The average controlled state variables, z_1 and z_2, are shown to converge toward the desirable equilibrium point. Figure 7 shows the normalized state trajectory response of the actual PWM controlled circuit superimposed on the corresponding filtered normalized state variable responses. The sampling frequency for the PWM actuator was chosen as 10 kHz and the low pass filters cut-off frequency was set at $628{\cdot}3$ rad/s^{-1}.

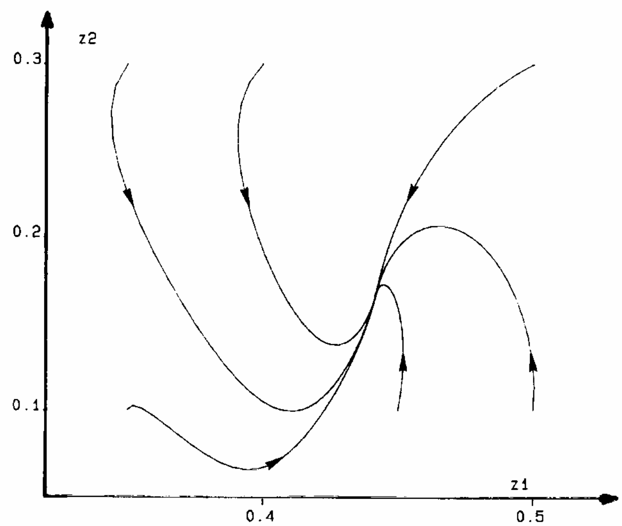

Figure 6. Average normalized state portrait for dynamically feedback PWM controlled Boost converter

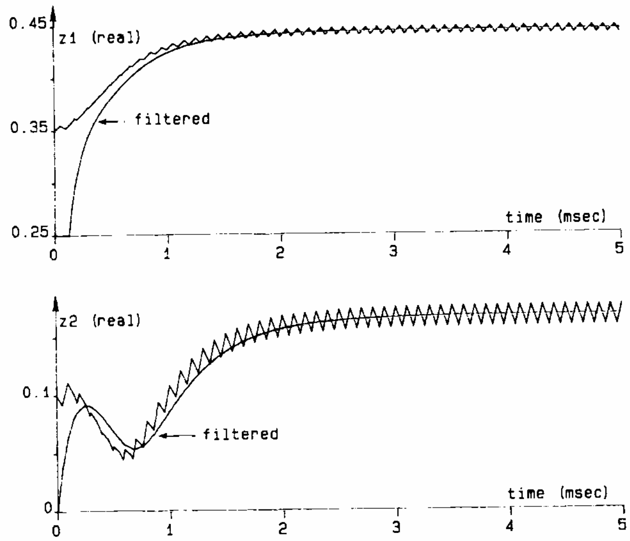

Figure 7. Actual and filtered PWM controlled normalized output load current step response in dynamically feedback controlled Boost converter.

3.3. *Buck–Boost converter*

Consider the Buck–Boost converter model shown in Fig. 8. This circuit is described by the time-invariant bilinear state equation model

$$\left.\begin{aligned} dx_1/dt &= \omega_0 x_2 - u\omega_0 x_2 + ub \\ dx_2/dt &= -\omega_0 x_1 - \omega_1 x_2 + u\omega_0 x_1 \end{aligned}\right\}. \tag{3.30}$$

where, $x_1 = I\sqrt{L}$, $x_2 = V\sqrt{C}$ represent normalized input current and output voltage variables respectively, $b = E/\sqrt{L}$ is the normalized external input voltage and it is here assumed to be a negative quantity (reversed polarity) while $\omega_0 = 1/\sqrt{(LC)}$ and $\omega_1 = 1/RC$ are, respectively, the LC (input) circuit natural oscillating frequency and the RC output circuit time constant. The switch position function, acting as a control input, is denoted by u and takes values in the discrete set $\{0, 1\}$. We now summarize the formulae leading to a nonlinear $P - I$ controller design for the average model (3.30).

Average buck-boost converter model

$$\begin{aligned} dz_1/dt &= \omega_0 z_2 - \mu_0 z_2 + \mu b \\ dz_2/dt &= -\omega_0 z_1 - \omega_1 z_2 + \mu\omega_0 z_1 \\ y &= z_2 \end{aligned} \tag{3.31}$$

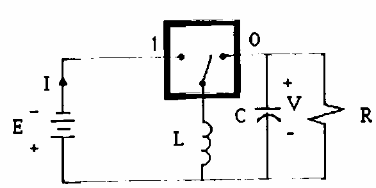

Figure 8. The Buck–Boost converter.

Constant equilibrium points

$$\mu = U; \quad Z_1(U) = bU\omega_1/[\omega_0^2(1-U)^2]; \quad Z_2(U) = -bU/[\omega_0(1-U)] \tag{3.32}$$

Linearized Boost converter model about a constant equilibrium point

$$\frac{d}{dt}\begin{bmatrix} z_{1\delta}(t) \\ z_{2\delta}(t) \end{bmatrix} = \begin{bmatrix} 0 & (1-U)\omega_0 \\ -(1-U)\omega_0 & -\omega_1 \end{bmatrix}\begin{bmatrix} z_{1\delta}(t) \\ z_{2\delta}(t) \end{bmatrix} + \begin{bmatrix} b/(1-U) \\ -bU\omega_1/[(1-U)^2 w_0] \end{bmatrix}\mu\delta \tag{3.33}$$

with

$$z_{i\delta}(t) = z_i(t) - Z_i(U); \quad i = 1, 2; \quad \mu_\delta(t) = \mu(t) - U$$

Linearized Buck–Boost converter transfer functions

If the average normalized output capacitor voltage z_2 is taken as the converters output the resulting linearized transfer function, given by

$$G_U(s) = \frac{z_{2\delta}(s)}{\mu_\delta(s)} = \omega_0 Z_1(U)\frac{s - b[Z_1(U)]^{-1}}{s^2 + \omega_1 s + (1-U)^2\omega_0^2} \tag{3.34}$$

is non-minimum phase and hence the proposed compensator design approach is not feasible. On the other hand, if the average normalized input inductor current z_1 is taken as the output of the system the resulting incremental transfer function is indeed minimum-phase and given by

$$G_U(s) = \frac{z_{1\delta}(s)}{\mu_\delta(s)} = \frac{b}{(1-U)}\frac{s + \omega_1(1+U)}{s^2 + \omega_1 s + (1-U)^2\omega_0^2} \tag{3.35}$$

Average normalized input inductor current z_1 must then be used as the output variable to be regulated. One usually pursues constant average normalized input current regulation to indirectly obtain a desirable controlled constant average normalized output voltage, z_2, at the load. One can compute from (3.32), the corresponding set point value for the normalized average normalized input inductor current Z_1. This value is given by

$$Z_1 = \frac{\omega_1}{bU} Z_2^2 \tag{3.36}$$

In this part we propose an 'input current control mode' scheme based on a non-linear dynamical controller which regulates z_1 via exact linearization. We summarize below the relevant formulae leading to the non-linear controller specification.

Average Buck–Boost converter model for input current control mode

$$\left.\begin{aligned} dz_1/dt &= \omega_0 z_2 - \mu\omega_0 z_2 + \mu b \\ dz_2/dt &= -\omega_0 z_1 - \omega_1 z_2 + \mu\omega_0 z_1 \\ y &= z_1 \end{aligned}\right\} \tag{3.37}$$

Control-dependent transformation for expressing the Buck–Boost converter in generalized controller canonical form

Let Z_1 be a desirable constant average normalized input inductor current possibly computed in terms of the desirable constant average normalized output load voltage z_2 as in (3.36). It is easy to verify that $q_1 = z_1 - Z_1$ is a '*differential primitive element*' that allows one to write the normalized average model (3.3) in GCCF of the form (2.4) with $\nu = 1$.

$$\left.\begin{aligned} q_1 &= z_1 - z_1 \\ q_2 &= (1-\mu)\omega_0 z_2 + \mu b \end{aligned}\right\} \tag{3.38}$$

The inverse control-dependent transformation is simply

$$\left.\begin{aligned} z_1 &= q_1 + Z_1 \\ z_2 &= \frac{q_2 - \mu b}{(1-\mu)\omega_0} \end{aligned}\right\} \tag{3.39}$$

The Buck–Boost converter in generalized controller canonical form

$$\left.\begin{aligned} \dot{q}_1 &= q_2 \\ \dot{q}_2 &= -c_1(\mu, \dot{\mu})q_1 - c_2(\mu, \dot{\mu})q_2 + c_3(\mu, \dot{\mu}) \end{aligned}\right\} \tag{3.40}$$

with

$$\left.\begin{aligned} c_1(\mu, \dot{\mu}) &= (1-\mu)^2\omega_0^2 \\ c_2(\mu, \dot{\mu}) &= \left[\frac{\dot{\mu}}{(1-\mu)} + \omega_1\right] \\ c_3(\mu, \dot{\mu}) &= \left[\frac{\dot{\mu}}{(1-\mu)} + \mu\omega_1\right]b - (1-\mu)^2\omega_0^2 Z_1 \end{aligned}\right\} \tag{3.41}$$

Notice that, as in the Boost case, the GCCF for the Buck–Boost converter is linear and time-varying in the transformation variables q_1 and q_2 and includes a time-varying forcing term represented by $c_3(\mu, \dot{\mu})$. This forcing term collects the influence of the desirable set point Z_1, as well as the constant external voltage source b. If the last differential equation in (3.40) is equated to a linear combination of the state variables, say $-\alpha_1 q_1 - \alpha_2 q_2$, as in (2.5) (with $\kappa = 0$) as asymptotically stable closed loop linear system is obtained which guarantees, for suitably chosen α_i's, that $\lim_{t\to\infty} q_1 = 0$. This guarantees asymptotic convergence of z_1 to the desirable equilibrium point Z_1. This linearization procedure yields a differential equation characterizing the non-linear dynamical compensator which synthesizes the computed duty ratio $\hat{\mu}$.

Non-linear dynamical compensator in current control mode for the average PWM controlled Buck–Boost converter

$$\begin{aligned} \dot{\xi} &= \frac{1-\xi}{q_2 - b}\{[\alpha_1 - (1-\xi)^2\omega_0^2]q_1 + (\alpha_2 - \omega_1)q_2 + [b\xi\omega_1 - (1-\xi)^2\omega_0^2 Z_1]\} \\ \hat{\mu} &= \xi \end{aligned} \tag{3.42}$$

As before, the discontinuous non-linear dynamical feedback regulation scheme for stabilization of the normalized output load voltage in the Buck–Boost converter is conceptually the same as that of Fig. 2. Since (3.42) is a first order dynamical system only the computed duty ratio function $\hat{\mu}$ is needed for the synthesis of the control-dependent state coordinate transformation yielding the GCCF of the normalized average Buck–Boost converter model (see equation 3.38).

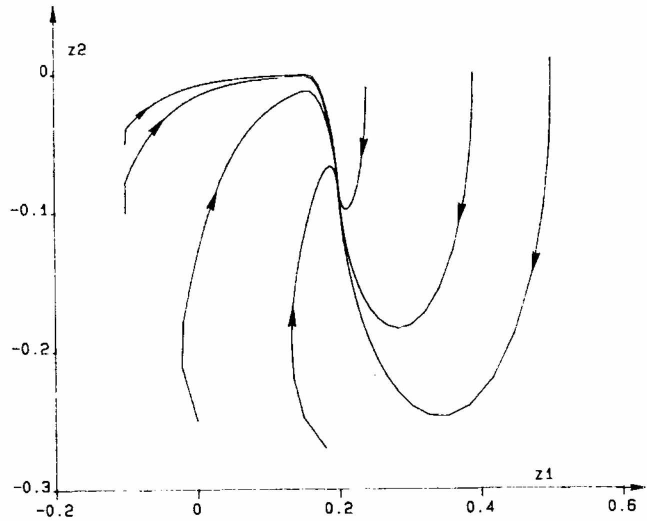

Figure 9. Average normalized state portrait for dynamically feedback PWM controlled Buck–Boost converter.

A simulation example

A Buck–Boost converter circuit with the same parameter values as in the Boost example was considered for non-linear dynamical controller design. The desirable normalized constant output voltage is $Z_2 = -0.084$ which corresponds to a constant value $U = 0{\cdot}556$ for the duty ratio μ. The corresponding set point for the average normalized input inductor current is $Z_1(0{\cdot}556) = 0{\cdot}2$. The poles of the linearized closed loop system were chosen at: $-1500\,\mathrm{s}^{-1}$ and $-3000\,\mathrm{s}^{-1}$. Figure 9

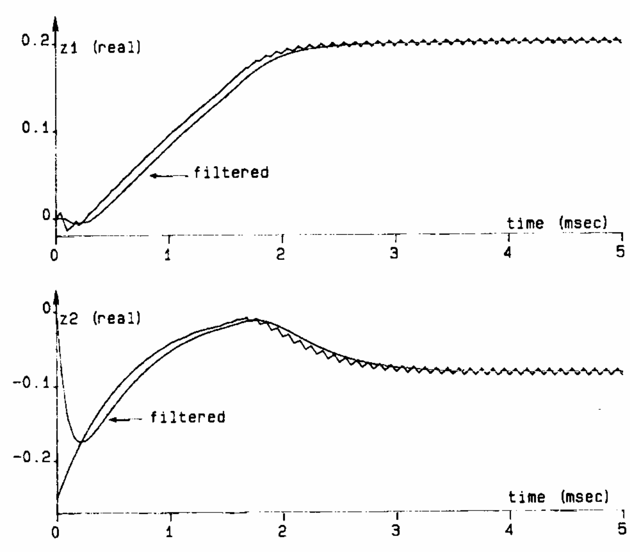

Figure 10. Actual and filtered PWM controlled normalized output load current step response in dynamically feedback controlled Buck–Boost converter.

shows several normalized average state trajectories corresponding to different initial conditions set on the ideal average Buck-Boost converter model controlled by the non-linear regulator of the form (3.41). The average controlled state variables, z_1 and z_2, are shown to converge toward the desirable equilibrium point. Figures 10 shows the normalized states trajectory response of the actual PWM controlled circuit superimposed on the corresponding filtered normalized state variable responses. The sampling frequency for the PWM actuator was chosen as 10 kHz and the low pass filters cut-off frequency was set at $628{\cdot}3\ \mathrm{rad\ s^{-1}}$.

4. Conclusions and suggestions for further research

In this article a new design method, based on exact non-linear dynamical feedback linearization, is proposed for the regulation of output load voltage, or current, in pulse-width-modulated controlled d.c.-to-d.c. power converters. The design method is based in Fliess's generalized controller canonical form (GCCF) for non-linear dynamical systems.

Computing the explicit GCCF for the different switch-mode controlled bilinear converters, such as the Boost, Buck-Boost and Cuk converters, a non-linear dynamical compensator is immediately suggested which exactly linearizes, in a local fashion, the closed loop controlled system with arbitrarily imposed stable dynamics. Such linearization, can be easily carried out on the basis of the infinite-frequency average PWM controlled model of the bilinear converter. However, we have shown, in full generality, that the local stability of the closed loop linearized system, toward a desirable reference set point, crucially depends upon the minimum-phase character of the linearized transfer function of the system model around the preselected equilibrium point. According to these result, in all the cases studied in this article, only the average input inductor current can be effectively regulated by means of the proposed technique. Thus, average output load voltage, or current, regulation is accomplished, indirectly, via regulation of the average input inductor current.

The GCCF for all three cases result in a *linear time-varying* model including a forcing term which, in general, depends on the desirable set point and the value of the external voltage input source. Particularly, the GCCF's of the Boost and the Buck–Boost converters are almost identical with a difference in the time-varying forcing term (i.e. they belong to the same equivalence class, of second order dynamical feedback linearizable systems, which has the same GCCF modulo the external forcing term).

An interesting topic, to be pursued in the future, lies in the area of practical implementation of the proposed controllers via non-linear solid state analog electronics. Further theoretical studies can be carried out for the d.c.-to-d.c. power converters establishing, for instance, the relevance of Conte's *et al.* (1988) generalized input–output representation of non-linear systems in compensator design.

ACKNOWLEDGMENTS

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Appendix

This appendix presents the background defintions on differential algebra needed to understand the first part of § 2. The material here collected closely follows the presentations given in Fliess's remarkable contributions (Fliess 1988, 1989 a, b, c, 1990) where the reader is referred for more thorough details. Some portions are directly taken from Fliess's articles.

A.1. *Some definitions and results from algebra*

A *field is a ring* whose non-zero elements form an abelian group under multiplication. The set of rational numbers $\mathbb{Q}$, the set of real numbers $\mathbb{R}$, and the set of complex numbers $\mathbb{C}$, are well known examples of fields. Also, the set of rational functions in an indeterminate x, with coefficients in $\mathbb{R}$ or $\mathbb{Q}$, constitutes a field. A field is said to be of '*characteristic zero*' if in the underlying ring not two non-zero elements yield zero as their product (finite fields are typical examples of fields with non-zero characteristic but, for instance, $\mathbb{R}$ is of characteristic zero). All fields here considered are of characteristic zero.

Let F be a field, then K is said to be an 'extension' over F, denoted as K/F, if K contains F. Thus, $\mathbb{C}/\mathbb{R}$ is an extension over $\mathbb{R}$ and $\mathbb{R}/\mathbb{Q}$ is an extension over $\mathbb{Q}$. It is easy to see that if K/F is an extension over F, then, under the field operations in K, F also qualifies as a '*vector space*'. K/F is said to be a 'finite extension' over F if K is 'finite dimensional' as a vector space over F. The dimension of K/F is called 'the degree, of the extension K/F over F. For instance, $\mathbb{C}/\mathbb{R}$ is a finite extension over $\mathbb{R}$ of degree 2, but $\mathbb{R}/Q$ is an infinite extension over $\mathbb{Q}$.

An element α of K is said to be 'algebraic' over F if there exists a polynomial P, with coefficients in F, such that $P(\alpha) = 0$. K/F is an '*algebraic extension*' over F if any element of K is algebraic over F. If there exists, at least, one element in K which is not algebraic over F, then K/F is said to be a '*transcendental extension*' over F. For example: $\mathbb{R}/\mathbb{Q}$ is transcendental over $\mathbb{Q}$, but $\mathbb{C}/\mathbb{R}$ is algebraic over $\mathbb{R}$. Let I be any finite index set with cardinality v. A set of elements $\{x_i | i \in I\}$ in K is '*F-algebraically dependent*' if and only if there exists, at least, one polynomial $P(x_1, \ldots, x_v)$, with coefficients in F, such that $P(x_1, \ldots, x_v) = 0$. A set is said to be '*F-algebraically independent*' if it is not F-algebraically dependent. F-algebraically independent sets in K, which are maximal with respect to inclusion, constitute the '*transcendence basis*' of the extension field K/F. The cardinality of such a basis is the *transcendence degree*' of K/F. Such a quantity is denoted by $\operatorname{tr} d^0 K/F$. For example, the transcendence degree of $\mathbb{C}/\mathbb{R}$ is zero. Let $F(x, y)$ denote the field of rational functions in the indeterminates x and y, with coefficients in the field F. The sets $\{x\}$, $\{y\}$ and $\{x, y\}$ are F-algebraically independent over f. However, $\{x, y\}$ is a transcendence basis of the extension field $F(x, y)/F$. In this case $\operatorname{tr} d^0 F(x, y)/F = 2$.

Let K/F be an extension over F and let α be an element in K, then $F\langle\alpha\rangle$ denotes the smallest subfield of K containing both F and α. It should be clear that $F\langle\alpha\rangle$ not always coincides with K. The subfield $F\langle\alpha\rangle$ is also called the 'field generated by F and α'. Evidently, $F\langle\alpha\rangle/F$ is an algebraic extension over F and, hence, $\operatorname{tr} d^0 F\langle\alpha\rangle/F = 0$. A field generated by F and a finite collection of elements $\{\alpha_i | i \in I\}$ in K is a '*finitely generated field*', denoted by $F\langle\alpha_1, \ldots, \alpha_v\rangle$. If $\{\alpha_i \in K | i \in I\}$ is a transcendence basis of K over F then the extension $K/F\langle\alpha_1, \ldots, \alpha_v\rangle$ is algebraic, i.e. $\operatorname{tr} d^0 K/F\langle\alpha_1, \ldots, \alpha_v\rangle = 0$.

Let K/F be a finitely generated algebraic extension over F. The 'theorem of the primitive element' states that there exists a single element $\xi \in K$, called the 'primitive element,' such that K is exactly generated by F and ξ, i.e. $K = F\langle \xi \rangle$.

A.2. *Some defintions and results from differential algebra* (Fliess 1990)

A *derivation* 'd/dt' over a field F is a linear map $d/dt : F \to F$ which satisfies the Leibnitz rule over products of elements in F. A 'differential field' is a commutative field in which a derivation is defined. The field $\mathbb{R}(s)$ of rational functions in the variable s with coefficients in $\mathbb{R}$ is a differential field when d/ds is defined as its derivation. An element in F is a 'constant' if its derivative is zero. A 'field of constants' is a differential field in which every element is a constant. $\mathbb{R}$ and $\mathbb{C}$ are trivially fields of constants.

All definitions presented in § 2.1 extend to differential fields.

Let F be a differential field, then K/F is said to be a 'differential extension' over F, if K contains F and F inherits its derivation from K, i.e. if the restriction of the derivation in K over the elements in F coincides with the derivation defined on F. As an example, consider the differential extension $\mathbb{R}(s)/\mathbb{Q}(s)$ with d/ds as the derivation.

An element α of a differential field K is said to be 'differentially algebraic' over F if the element α satisfies a polynomial differential equation with coefficients in F, i.e. $P(\alpha, d\alpha/dt, \ldots, d^{\nu}\alpha/dt^{\nu}) = 0$. K/F is a *differential algebraic extension* of F if any element of K is differentially algebraic over F. If there exists, at least, one element in K which is not differentially algebraic over F, then K/F is said to be a 'differentially transcendental extension' over F. A set of elements $\{x_i | i \in I\}$ in K is differentially F-algebraically dependent' if and only if there exists, at least, one polynomial differential equation $P(x_1, dx_1/dt, \ldots, x_2, dx_2/dt, \ldots, x_\nu, dx_\nu/dt \ldots) = 0$, with coefficients in F. A set is said to be 'differentially F-algebraically independent' if it is not differentially f-algebraically dependent. A set of differentially F-algebraic independent set of elements in K which is maximal with respect to inclusion constitutes a 'differential transcendence basis' of the differential extension K/F. The cardinality of such a basis is the 'differential transcendence degree' of K/F. Such a quantity is denoted by: diff tr d^0 K/F. For example, differentially algebraic extensions have zero differential transcendence degree. On a differential field extension K/F one defines the *non-differential transcendence degree* as the transcendence degree of K over F.

Let K be a differential extension of F and let α be an element in K, then $F\langle \alpha \rangle$ denotes the smallest differential K containing both F and α. As before $F\langle \alpha \rangle$ not always coincides with K. The subfield $F\langle \alpha \rangle$ is also called the 'field differentially generated by F and α.'

A finitely generated differential extension is differentially algebraic if and only if its nondifferential transcendence degree is finite.

Assume F is not a field of constants. Let K/F be a finitely generated differentially algebraic extension over F. The 'theorem of the differential primitive element' states that, there exists a single element $\xi \in K$, called the 'differential primitive element' such that $K = F\langle \xi \rangle$.

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