Nonlinear P-I Controller Design for Switchmode dc-to-dc Power Converters

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***Abstract*—In this article, extended linearization techniques are proposed for the design of nonlinear proportional-integral (P-I) controllers stabilizing, to a constant value, the average output voltage of pulse-width modulation (PWM) switch-regulated dc-to-dc converters. The Ziegler–Nichols method is employed for the P-I controller specification, as applied to a family of parametrized transfer function models of the linearized average converter behavior around a constant operating equilibrium point of the average PWM controlled circuit. The boost and the buck-boost converters are specifically treated and the regulated performance is illustrated through computer simulation experiments.**

***Keywords*: dc-to-dc power converters, extended linearization, nonlinear P-I controllers, pulse-width modulation.**

I. Introduction

PULSE-width modulation (PWM) control schemes usually regulate dc-to-dc power converters in a variety of different arrangements, involving a high sampling frequency of the required feedback signals. The available regulator design techniques are based on approximate linear incremental models of discrete-time nature (see Severns and Bloom [1], Middlebrook and Cuk [2], Csaki *et al.* [3], etc.). Nonlinear control schemes have been recently proposed for this class of circuits, which do not necessarily resort to the discrete-time approximation scheme. Instead, the properties of suitable nonlinear continuous average models, obtained by imposing an infinite sampling frequency assumption, are conveniently exploited. The proposed design schemes are, among others: Sliding mode control strategies, with switching surfaces prescribed on the basis of properties associated to the "ideal sliding dynamics" (see Venkataramanan *et al.* [4] and Sira-Ramirez [5]); discontinuous control strategies, such as PWM, based on singular perturbation considerations related to time-scale separation properties of the converters' average responses and their associated slow manifolds (see Sira-Ramirez and Ilic [6] and Sira-Ramirez [7]); regulation schemes based on exact linearization of the average PWM circuit model (Sira-Ramirez and Ilic [8]); and pseudolinearization techniques applied on the nonlinear continuous average PWM converter model (see Sira-Ramirez [9]). A comparison of the various proposed designs methods, contained in [5]–[9], is carried out in Section 3.3 of this paper.

A new nonlinear regulator design scheme is proposed in this article for the stabilization of output variables in dc-to-dc power converters. The extended linearization control technique, developed by Rugh and his co-workers (see [10]–[12]), is applied to nonlinear infinite frequency average models of the PWM controlled converters. Feedback controller design by means of the extended linearization approach constitutes a highly attractive nonlinear design technique with potential for many practical applications. The method is based on the specification of a linear regulator inducing desirable stability characteristics on the behavior of an entire family of linearized plant models. The family of linear models is parametrized by constant operating points of a smooth surface of equilibrium points defined in the input–output space of the system. The linear design constitutes the basis for (nonuniquely) prescribing a nonlinear regulator that exhibits the fundamental property that its linearized model, computed about the same generic operating point, coincides with the specified stabilizing controller. The obtained nonlinear regulator is known to exhibit the advantageous property of "self-scheduling" with respect to constant reference operating equilibria which may be subject to sudden (and purposeful) changes in their nominal values.

Nonlinear P-I controllers are proposed here for the regulation of the output voltage in dc-to-dc power converters such as the boost and the buck-boost power supplies. The frequency domain version of the Ziegler–Nichols prescription [13] is used for the determination of the linearized P-I regulator gains, which result in a stable family of closed loop parametrized transfer functions relating the incremental output voltage to the incremental duty ratio function of the converter. The nonlinear P-I controller is directly specified from the linear design in a manner entirely similar to that proposed in [18]. In contrast to sliding mode control techniques, it should be remarked that constant output voltage regulation cannot be successfully accomplished by means of sliding mode behavior induced on surfaces representing zero output errors. Within such a control technique, constant output voltage control may be achieved only when a combination of the state variables is formed in a sliding line or, alternatively, indirectly through constant input current regulation [5]. Similarly, it is easy to verify that direct use of PWM controllers, processing the same error signals, do perform constant output voltage regulation for a limited (but

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unnaturally restricted) range of desirable set points. Quite on the contrary, the nonlinear P-I controller here proposed efficiently handles the output voltage regulation problem, in a direct manner, without noticeable instability effects, valid at least in a local sense. Since the designed regulator is to be used in combination with an actual PWM actuator, producing highly discontinuous signals, a proportional-integral-derivative (P-I-D) controller does not seem feasible due to large controller output values produced by the derivative action performed on the discontinuous error voltages.

A brief review of nonlinear P-I controller design, by means of the extended linearization technique, is presented in Section II. The class of systems treated there corresponds to general time-invariant bilinear systems. In Section III we present the procedure for synthesizing nonlinear P-I regulators for dc-to-dc converters of the boost and buck-boost type. These nonlinear P-I controllers are designed on the basis of the average models of the PWM controlled converters. Here, the form in which the designed nonlinear P-I controllers are to be used in the actual discontinuous PWM feedback scheme is also indicated. This section also presents some computer simulation experiments illustrating the performance of the nonlinear P-I controller. The last section contains some conclusions and suggestions for further work.

II. Background Results

The extended linearization technique is reviewed in this section and applied to time invariant discontinuously controlled bilinear systems of the form

$$dx/dt = Ax + u(Bx+g) + h$$
$$y = cx \tag{2.1}$$

with $x \in R^n$, g and h are constant n-dimensional vectors, while A, B, and c are matrices of appropriate dimensions. The variable u represents a switch position function acting as the control signal and taking values on the binary set $\{0,1\}$. The output y is assumed to be a scalar function.

The feedback control strategy regulating the system is assumed to be of the discontinuous type and specified on the basis of a sampled closed loop PWM control scheme of the form

$$u = \begin{cases} 1, & \text{for } t_k \leqslant t < t_k + \mu[x(t_k)]T \\ 0, & \text{for } t_k + \mu[x(t_k)]T \leqslant t < t_k + T \end{cases} \tag{2.2}$$

where $\mu[x(t_k)]$ is the *duty ratio function*, which, generally speaking, is constituted by a smooth feedback function of the state variables (or of some related variables such as sampled output error $e(t_k) := y_d - y(t_k) = y_d - cx(t_k)$) satisfying the following natural bounding constraint: $0 < \mu[x(t_k)] < 1$, for all sampling instants t_k. T is the *duty cycle* prescribing the time period between the regularly spaced sampling instants, i.e., $t_{k+1} = t_k + T$.

Remark 1: An *average model* of (2.1)–(2.2) can be obtained by assuming an *infinite sampling frequency* (i.e., letting the duty cycle $T \to 0$) on (2.2) as it has been rigorously shown in Sira-Ramirez [14] (see also [7] and Sira-Ramirez [15]). The fundamental property of the resulting average model trajectories is to accurately represent all the qualitative properties of the actual PWM controlled system (2.1)–(2.2). This was demonstrated in [15] by showing that there always exists a sufficiently small sampling period T for which the deviations between the actual PWM controlled responses and those of the average model, under identical initial conditions, remain uniformly arbitrarily close to each other. Conversely, for each prespecified degree of error tolerance, a sufficiently high sampling frequency may be found such that the actual and the average trajectories differ by less than such a given tolerance bound. The error can be made even smaller if the sampling frequency is suitably increased. Moreover, from a purely geometric viewpoint, in those regions of nonsaturation of the duty ratio function μ, *integral manifolds* containing families of state responses of the average model constitute actual sliding surfaces about which the discontinuous PWM controlled trajectories may exhibit sliding regimes [14]. Outside the region of nonsaturation, the trajectories of both the actual and the average PWM models entirely coincide. The average model dynamics plays then the role of the *ideal sliding dynamics* (see Utkin [16] and Sira-Ramirez [17]) in the corresponding variable structure control reformulation of the PWM control strategy [14].

One formally obtains the average model of system (2.1)–(2.2) by simply substituting the duty ratio feedback function μ in place of the actual switch control function u [14]. In order to differentiate the actual state vector x from the average states, we shall denote the average of the actual state vector x by means of the vector z:

$$dz/dt = Az + \mu(Bz+g) + h$$
$$y = cz. \tag{2.3}$$

By means of Z, we denote an equilibrium state vector for the average system (2.3). If such an equilibrium state exists then it must, necessarily, correspond to a constant value of the duty ratio feedback function μ. We could express such a value by $\mu(Z)$. We prefer, however, to denote such a constant input value as U. It is also easy to see that for a given U, the equilibrium state vector can, in turn, be expressed as a function of U by means of a function $Z(U)$. This value $Z(U)$ coincides with the previously given equilibrium state Z if and only if the matrix $(A+UB)$ is invertible. In such a case we obtain

$$Z(U) = (A+UB)^{-1}(Ug+h). \tag{2.4}$$

We prefer to use this last convention, in the same spirit of [10]–[12], rather than expressing U as a function of the equilibrium state Z. Notice that preliminary feedback can always render an invertible matrix $A+UB$ if such is not the case, for a particular U, in the original average system (2.3).

The traditional linearization of (2.3) about a given equilibrium point $(U, Z(U))$ results in

$$dz_\delta/dt = [A+UB]z_\delta + [BZ(U)+g]\mu_\delta$$
$$y_\delta = cz_\delta \tag{2.5}$$

with $z_\delta(t) := z(t) - Z(U)$, $y_\delta(t) := y(t) - Y(U) := y(t) - cZ(U)$, and $\mu_\delta(t) := \mu(t) - U$.

The linearized system (2.5) actually constitutes a family of linearizations of (2.3) parametrized by the constant input equilibrium point U. Taking Laplace transforms in (2.5) one obtains, under zero initial conditions, the family of parametrized scalar transfer functions relating the incremental output transform $y_\delta(s)$ to the incremental input transform $\mu_\delta(s)$ as

$$G_U(s) := y_\sigma(s)/\mu_\sigma(s) = c(sI-[A+UB])^{-1}[BZ(U)+g]. \tag{2.6}$$

The description of the linearized system as a proper rational transfer function is only valid in the region of nonsaturation of the duty ratio function, i.e., for $0<U<1$.

The extended linearization approach suggests, at this point, the use of a P-I controller which accomplishes stabilization to zero for the incremental output response associated to an arbitrary element of the parametrized family of systems represented by (2.6) (see also Rugh [18]). The Ziegler–Nichols design recipe (see [13], p. 55) can be readily used for determining the P-I gains upon determination of the design parameters known as the *ultimate frequency*, here denoted by $W_0(U)$ (or, equivalently, *the ultimate period*, defined as $P_0(U) = 2\pi/W_0(U)$) and the *ultimate gain*, denoted by $K_0(U)$, corresponding to system (2.6). These parameters are obtained from the following relations:

$$\operatorname{Arg} G_U(jW_0(U)) = -\pi;\ K_0(U) = |G_U(jW_0(U))|^{-1}. \tag{2.7}$$

These design parameters, in turn, specify the gains of the P-I controller which efficiently regulates the incremental error function $e_\delta(U) = 0 - y_\delta(U)$ to zero. Such a P-I controller is described by its transfer function $C_U(s)$ as

$$C_U(s) = \mu_\delta(s)/e_\delta(s) = K_1(U) + K_2(U)/s. \tag{2.8}$$

The above gains are easily computed in terms of the ultimate frequency and the ultimate gain as prescribed by the Ziegler–Nichols design rules:

$$K_1(U) = 0.4K_0(U),\ K_2(U) = K_1(U)W_0(U)/1.6\pi. \tag{2.9}$$

Following [18], a nonlinear P-I controller may then be specified by considering the dynamical regulator:

$$\begin{aligned} d\zeta(t)/dt &= K_2[\zeta(t)]e(t) \\ m(t) &= \zeta(t) + K_1[\zeta(t)]e(t) \end{aligned} \tag{2.10}$$

where $m(t)$ is the nonlinear controller output signal that is to be taken as the specification of the duty ratio function, for the PWM actuator, only in the region where such an output signal $m(t)$ does not violate the saturation limits naturally imposed to the duty ratio function as $0<\mu<1$. We refer to m as the computed duty ratio function.

Linearization of the nonlinear state equations, describing the dynamical controller represented by (2.10), around the operating point $e(U)=0$, $\zeta(U)=U$, produces an incremental model whose transfer function entirely coincides with (2.8). The operation of the average nonlinear controlled system (2.3) in the vicinity of the given equilibrium point $(U, Z(U))$ thus exhibits the same stability characteristics that the linearized P-I controller (2.8) imposes on the family of linearized plants represented by (2.6). By the results commented upon in Remark 1, the behavior of the actual discontinuous PWM controlled system (2.1)–(2.2) in conjunction with the designed nonlinear P-I controller will exhibit the same qualitative stability characteristics caused by the stabilizing design on the average PWM closed loop system, provided a sufficiently high sampling frequency is used in (2.2).

A physically meaningful duty ratio $\mu(t)$ must not violate its natural bounding constraints: $0<\mu(t)<1$. One may then obtain the actual duty ratio function μ by properly bounding the computed controller output signal $m(t)$ by means of a limiter, as follows:

$$\mu(t) = \begin{cases} 1 \text{ for } m(t) > 1 \\ m(t) \text{ for } 0 < m(t) < 1 \\ 0 \text{ for } m(t) < 0. \end{cases} \tag{2.11}$$

This bounding scheme may cause saturation effects on the PWM actuator, especially for those initial conditions which are far from the required equilibrium point. The use of anti-reset windup schemes ([13], pp. 10–14) may be attempted in such cases. We do not give further consideration to this topic here since it implies only a minor modification of the control scheme proposed here.

The high-frequency discontinuous trajectories induced by the PWM actuator on the system state and output variables (known as "chattering") requires some additional processing to further approximate the proposed control scheme to that of the designed average system. The feedback design presented above is based on the infinite frequency averaged output values; one can thus approximate the ideal smooth performance by using a suitably designed low-pass filter at the system output before feeding this signal back to the P-I controller. This procedure approximates the characteristics of the idealized design when the cut-off frequency of the filter and its associated phase lag is made sufficiently small.

III. P-I Controller Design for dc-to-dc Power Converters

3.1. Boost Converter

Consider the boost converter model shown in Fig. 1. This converter is described by the following bilinear system of controlled differential equations:

$$\begin{aligned} dx_1/dt &= -\omega_0 x_2 + u\omega_0 x_2 + b \\ dx_2/dt &= \omega_0 x_1 - \omega_1 x_2 - u\omega_0 x_1 \\ y &= x_2 \end{aligned} \tag{3.1}$$

where $x_1 = I\sqrt{L}$, $x_2 = V\sqrt{C}$ represent normalized input current and output voltage variables, respectively. The quantity $b = E/\sqrt{L}$ is the normalized external input voltage, and $\omega_0 = 1/\sqrt{LC}$ and $\omega_1 = 1/RC$ are, respectively, the LC (input) circuit natural oscillating frequency and the RC (output) circuit time constant. The variable u denotes the switch position function, acting as a control input, and taking values in the discrete set $\{0,1\}$. System (3.1) is of the same form as (2.1), with $g=0$ and $h=[b\ \ 0]'$. We now summarize, according to the theory presented in the previous section, the formulas leading to a nonlinear P-I controller design for the average model of (3.1).

Average boost converter model:

$$\begin{aligned} dz_1/dt &= -\omega_0 z_2 + \mu\omega_0 z_2 + b \\ dz_2/dt &= \omega_0 z_1 - \omega_1 z_2 - \mu\omega_0 z_1 \\ y &= z_2. \end{aligned} \tag{3.2}$$

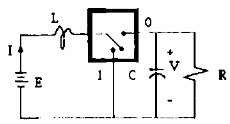

Fig. 1. Boost converter.

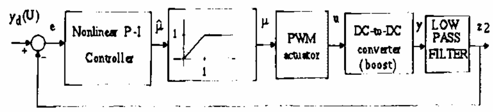

Fig. 2. A nonlinear P-I control scheme for the output voltage regulation of the boost converter.

Constant operating equilibrium points:

$$\mu = U;\ Z_1(U) = b\omega_1 / \left[\omega_0^2(1-U)^2\right]$$
$$Z_2(U) = b / \left[\omega_0(1-U)\right]. \tag{3.3}$$

Parametrized family of linearized systems about the constant operating points:

$$\frac{d}{dt}\begin{bmatrix} z_{1\delta}(t) \\ z_{2\delta}(t) \end{bmatrix} = \begin{bmatrix} 0 & -\omega_0(1-U) \\ \omega_0(1-U) & -\omega_1 \end{bmatrix}\begin{bmatrix} z_{1\delta}(t) \\ z_{2\delta}(t) \end{bmatrix} + \begin{bmatrix} b/(1-U) \\ -b\omega_1 U/\left[\omega_0(1-U)^2\right] \end{bmatrix}\mu_\delta$$
$$y_\delta(t) = z_{2\delta}(t) \tag{3.4}$$

with

$$z_{i\delta}(t) = z_i(t) - Z_i(U);\ i = 1,2$$
$$y_\delta(t) = y(t) - Y(U) := z_2(t) - Z_2(U);\ \mu_\delta(t) = \mu(t) - U.$$

Family of parametrized transfer functions:

$$G_U(s) = \frac{y_\delta(s)}{\mu_\delta(s)} = -\omega_0 Z_1(U)\frac{s - b[Z_1(U)]^{-1}}{s^2 + \omega_1 s + \omega_0^2(1-U)^2}. \tag{3.5}$$

Crossover frequency:

$$W_0(U) = \sqrt{2}\omega_0(1-U). \tag{3.6}$$

Ultimate period and ultimate gain:

$$P_0(U) := 2\pi / W_0(U) = \sqrt{2}\pi / \left[\omega_0(1-U)\right]$$
$$K_0(U) = \omega_0(1-U)^2 / b. \tag{3.7}$$

Ziegler–Nichols P-I controller gains for the linearized family of converters:

$$K_1(U) = 0.4\omega_0(1-U)^2/b;\ K_2(U) = \omega_0^2(1-U)^3/(2\sqrt{2}\pi b). \tag{3.8}$$

Nonlinear P-I controller:

$$d\zeta(t)/dt = \left[\omega_0^2(1-\zeta(t))^3/(2\sqrt{2}\pi b)\right]e(t)$$
$$\mu(t) = \zeta(t) + \left[0.4\omega_0(1-\zeta(t))^2/b\right]e(t)$$
$$e(t) = y_d(U) - y(t) = Z_2(U) - z_2(t). \tag{3.9}$$

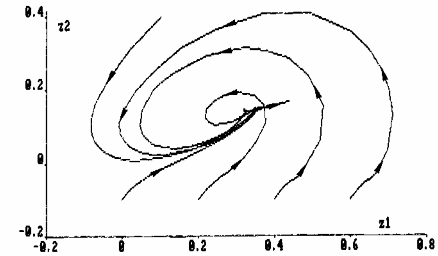

Fig. 3. State trajectories of the ideal average boost converter model controlled by a nonlinear P-I regulator.

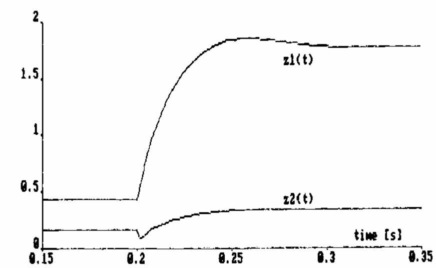

Fig. 4. Average controlled state response of the boost converter subject to a sudden change in the set point value.

In this case, upon integration of the differential equation for $\zeta(t)$, an explicit expression can be obtained for the nonlinear P-I controller in terms of the error signal integral.

$$\mu(t) = \zeta(t) + \left[0.4\omega_0(1-\zeta(t))^2/b\right]e(t)$$
$$\zeta(t) = 1 - \frac{(1-\zeta_0)}{\sqrt{1 + 2k(1-\zeta_0)^2\int_0^t e(\sigma)\,d\sigma}}$$

with

$$\zeta(0) = \zeta_0 \quad \text{and} \quad k = \frac{\omega_0^2}{2\sqrt{2}pb}$$
$$e(t) = y_d(U) - y(t) = Z_2(U) - z_2(t).$$

Low-Pass filter: A first-order low-pass filter may be used to yield an approximation to the ideal average output function z_2 required by the nonlinear P-I controller. Such a filter is characterized by a sufficiently small time constant of value $(1/T_c)$, and a state f.

$$df(t)/dt = -(1/T_c)(f(t) - x_2(t)),\ z_2(t) = f(t). \tag{3.10}$$

The proposed regulation scheme corresponding to the boost converter is shown in Fig. 2.

3.4. A Simulation Example

A boost converter circuit with parameter values $R = 30\ \Omega$, $C = 20\ \mu F$, $L = 20$ mH, and $E = 15$ V was considered for nonlinear P-I controller design. The constant operating value of μ was chosen to be $U = 0.6$ while the corresponding desirable normalized constant output voltage turned out to be $Z_2(0.6) = 0.1677$. Fig. 3 shows several state trajectories corresponding to different initial conditions set on the ideal

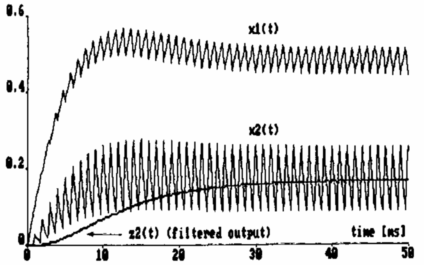

Fig. 5. State response of the actual PWM-controlled boost converter.

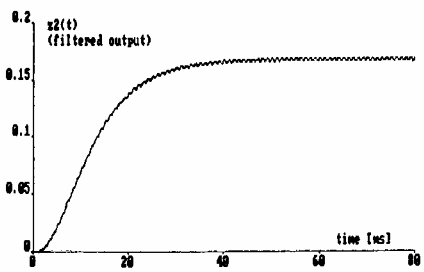

Fig. 6. Filtered output response of the actual PWM boost converter controlled by a nonlinear P-I regulator.

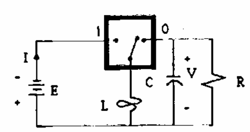

Fig. 7. Buck–boost converter.

average boost converter model controlled by the nonlinear P-I regulator of the form (3.9). The figure represents the projection of the closed loop system three-dimensional state onto the $z_1 - z_2$ average state plane. The average controlled state variables, z_1 and z_2, are shown to converge toward the desirable equilibrium point represented by $Z_1(U) = 0.4419$ and $Z_2(U) = 0.1677$. Fig. 4 shows the average controlled variables evolution when subject to a 100% step change in the output set point value, from 0.1677 to 0.3354 (the corresponding change in the operating point of the duty ratio was from 0.6 to 0.8). Figs. 5 and 6 show, respectively, the state response of the actual (i.e., discontinuous) PWM controlled circuit and the filtered output response. The sampling frequency for the PWM actuator was chosen as 1 kHz and the output low-pass filter cut-off frequency was set at 0.1 rad/s.

3.3. Buck–Boost Converter

Consider the buck-boost converter model shown in Fig. 7. This device is described by the following constant bilinear state equation model:

$$\begin{aligned} dx_1/dt &= \omega_0 x_2 - u\omega_0 x_2 + ub \\ dx_2/dt &= -\omega_0 x_1 - \omega_1 x_2 + u\omega_0 x_1 \\ y &= x_2 \end{aligned} \tag{3.11}$$

where $x_1 = i\sqrt{L}$, $x_2 = V\sqrt{C}$ represent normalized input current and output voltage variables, respectively, $b = E/\sqrt{L}$ is the normalized external input voltage (here assumed to be a negative quantity, i.e., reversed polarity) while $\omega_0 = 1/\sqrt{LC}$ and $\omega_1 = 1/RC$ are, respectively, the LC (input) circuit natural oscillating frequency and the RC (output) circuit time constant. The switch position function, acting as a control input, is denoted by u and takes values in the discrete set $\{0,1\}$. System (3.11) is of the same form as (2.1), with $h = 0$ and $g = [b \quad 0]'$. We now summarize the formulas leading to a nonlinear P-I controller design for the average model of (3.11).

Average buck–boost converter model:

$$\begin{aligned} dz/dt &= \omega_0 z_2 - \mu\omega_0 z_2 + \mu b \\ dz_2/dt &= -\omega_0 z_1 - \omega_1 z_2 + \mu\omega_0 z_1 \\ y &= z_2. \end{aligned} \tag{3.12}$$

Constant equilibrium points:

$$\begin{aligned} &\mu = U;\ Z_1(U) = bU\omega_1/\left[\omega_0^2(1-U)^2\right]; \\ &Z_2(U) = -bU/\left[\omega_0(1-U)\right]. \end{aligned} \tag{3.13}$$

Parameterized family of linearized systems about the constant operating points:

$$\begin{aligned} \frac{d}{dt}\begin{bmatrix} z_{1\delta}(t) \\ z_{2\delta}(t) \end{bmatrix} &= \begin{bmatrix} 0 & \omega_0(1-U) \\ -\omega_0(1-U) & -\omega_1 \end{bmatrix} \begin{bmatrix} z_{1\delta}(t) \\ z_{2\delta}(t) \end{bmatrix} \\ &\quad + \begin{bmatrix} b/(1-U) \\ b\omega_1 U/\left[\omega_0(1-U)^2\right] \end{bmatrix} \mu_\delta \\ y_\delta(t) &= z_{2\delta}(t) \end{aligned} \tag{3.14}$$

with

$$z_{i\delta}(t) = z_i(t) - Z_i(U);\ i = 1,2$$
$$y_\delta(t) = y(t) - Y(U) := z_2(t) - Z_2(U);\ \mu_\delta(t) = \mu(t) - U.$$

Family of parametrized transfer functions:

$$G_U(s) = \frac{y_\delta(s)}{\mu_\delta(s)} = \omega_0 Z_1(U)\frac{s - b[Z_1(U)]^{-1}}{s^2 + \omega_1 s + \omega_0^2(1-U)^2}. \tag{3.15}$$

Crossover frequency:

$$W_0(U) = \omega_0(1-U)(1+1/u)^{1/2} \tag{3.16}$$

Ultimate period and ultimate gain:

$$\begin{aligned} P_0(U) &:= 2\pi/W_0(U) = 2\pi/\left[\omega_0(1-U)(1+1/u)^{1/2}\right] \\ K_0(U) &= \left[\omega_0(1-U)^2\right]/(b|U). \end{aligned} \tag{3.17}$$

Ziegler–Nichols P-I controller gains for the linearized family of converters:

$$\begin{aligned} K_1(U) &= \left[0.4\omega_0(1-U)^2\right]/(|b|U) \\ K_2(U) &= \left[\omega_0^2(1-U)^3(1+1/U)^{1/2}\right]/(4\pi|b|U). \end{aligned} \tag{3.18}$$

Nonlinear P-I controller:

$$\begin{aligned} d\zeta(t)/dt &= \left[\omega_0^2(1-\zeta(t))^3(1+1/\zeta(t))^{1/2}\right] \\ &\qquad /[4\pi|b|\zeta(t)]e(t) \\ \mu(t) &= \zeta(t) + \left\{\left[0.4\omega_0(1-\zeta(t))^2\right]/(|b|\zeta(t))\right\}e(t) \\ e(t) &= y_d(U) - y(t) = Z_2(U) - z_2(t) \end{aligned} \tag{3.19}$$

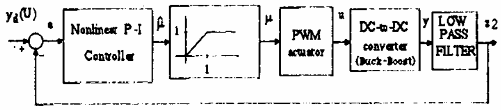

Fig. 8. A nonlinear P-I control scheme for the output voltage regulation of the buck–boost converter.

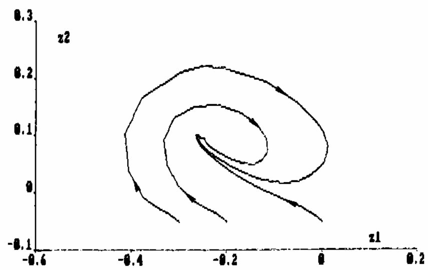

Fig. 9. State trajectories of the ideal average buck–boost converter model controlled by a nonlinear P-I regulator.

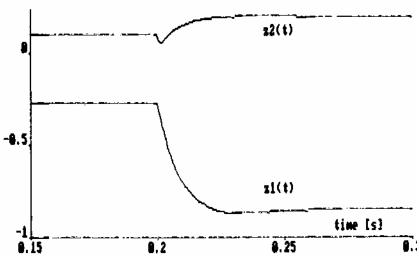

Fig. 10. Average controlled state response of the buck–boost converter subject to a sudden change in the set point value.

(in this case an explicit closed-form expression for $\zeta(t)$ is impossible to obtain).

Low-Pass filter: A simple first-order low-pass filter may be proposed to yield an approximation to the ideal average output function z_2 required by the nonlinear P-I controller. Such a filter is characterized by a sufficiently small time constant of value $(1/T_c)$ and a state variable f.

$$df(t)/dt = -(1/T_c)(f(t) - x_2(t)); \; z_2(t) = f(t). \quad (3.40)$$

The proposed regulation scheme corresponding to the buck–boost converter is shown in Fig. 8.

3.4. A Simulation Example

A buck–boost converter circuit with the same parameter values as in the previous example was considered for nonlinear P-I controller design. The constant operating value of μ was again chosen to be $U = 0.6$ while the corresponding desirable normalized constant output voltage turned out to be $Z_2(0.6) = 0.1006$. Fig. 9 shows several state trajectories corresponding to different initial conditions set on the ideal average buck-boost converter model controlled by the nonlinear P-I regulator of the form (3.19). Fig. 9 represents the projection of the closed loop system three-dimensional state onto the $z_1 - z_2$ average state plane. The average controlled

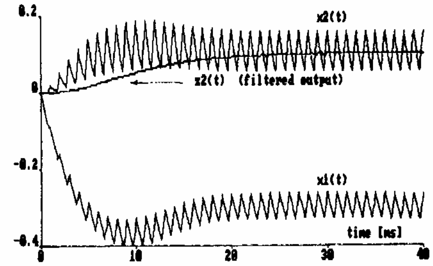

Fig. 11. State response of the actual PWM controlled buck–boost converter.

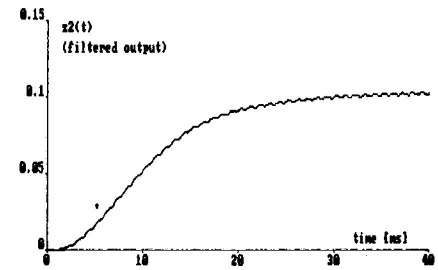

Fig. 12. Filtered output response of the actual PWM buck–boost converter controlled by a nonlinear P-I regulator.

state variables, z_1 and z_2, are shown to converge toward the desirable equilibrium point represented by $Z_1(U) = -0.2652$ and $Z_2(U) = 0.1006$. Fig. 10 shows the ideal average controlled state variables evolution when subject to a 100% step change in the output set point value, from 0.1006 to 0.2012 (the corresponding change in the operating point of the duty ratio was from 0.6 to 0.75.) Figs. 11 and 12 show, respectively, the state response of the actual (i.e., discontinuous) PWM controlled circuit and the filtered output response. The sampling frequency for the PWM actuator was chosen as 1 kHz and the output low-pass filter cut-off frequency was set at 0.1 rad/s.

3.5. Discussion and Review of Some Controller Design Techniques for dc-to-dc Converters

An overview and comparison of some recently published techniques for regulating dc-to-dc power converters seems to be in order at this point, especially those appearing in [5]–[9], published by the author of this paper.

Regulation circuits for dc-to-dc power supplies have been traditionally designed by a combination of approximate linearization exercised on discretized converter models, and PWM actuators used as simple proportional controllers [1]–[3], [20]. The techniques are analytically cumbersome and valid over a rather limited range of regulating conditions. In [5]–[9], nonlinear design techniques were proposed on the basis of rigorous analytical developments with some discussion about practical implementation from two fundamental viewpoints: sliding mode control and PWM. The sliding mode approach, extensively used in [5], explores the fundamental advantages and limitations of control schemes based on active switching of the converter about linear sliding "surfaces" that guarantee either constant input current or constant output voltage regulation for the different converters. It was shown, by using Lyapunov stability analysis of the resulting nonlinear ideal sliding dynamics, that some of these schemes result in stable sliding motions (particularly those pursuing regulation of input current to a constant value) and some do result in unstable sliding motions (basically those pursuing regulation of output voltage to a constant value). The class of proposed stable sliding modes were particularly simple to achieve since only one state variable of the converter need be measured. Sliding regimes, on the other hand, are known to be robust with respect to parameter variations and are easy to implement from a hardware standpoint. However, only indirect constant output voltage control can be stably achieved by constant input current regulation in all of the converter cases. The associated transients may be significant and thus undesirable for certain applications. In order to overcome some of these limitations, sliding mode control performed on affine switching lines was proposed in [6] and [7]. The resulting controlled action naturally led to a decoupling of the input current and output voltage dynamics with predetermined exponential convergence of the state variables toward the equilibrium point. In this respect, [6] and [7] showed that time scale separation properties could be made inherent to the converter by appropriate circuit parameter specifications. This could be advantageously exploited, in case of input source or parameter perturbations, for diminishing undesirable effects of transients toward the stable equilibrium point. However, the techniques in [6] and [7] call for the accurate measuring of all state variables of the converter circuit and more elaborate hardware for the switching line synthesis as compared to the methods in [5]. The sliding mode techniques basically tried to achieve linearization of the circuit dynamics in the range of existence of the sliding motion. But, of course, this can also be achieved without use of sliding modes. With the advent of the geometric theory of nonlinear systems and the spur of theoretical results dealing with the possibilities of feedback regulation for nonlinear plants, a new avenue of nonlinear controller design techniques was initiated in [8], [9], and the present paper for dc-to-dc power supplies. Such techniques are based on the possibilities of feedback regulation of the converter variables by means of exact linearization [21], pseudolinearization [22], and extended linearization [11], [12] of the average PWM controlled converter model. Aside from showing in [8] and [9] that modern nonlinear feedback design methods based in the geometric theory of nonlinear systems could be easily handled from an analytical viewpoint for the converters case, the simplicity of regulating a linear system in Brunovsky controllable canonical form is highly attractive from both the conceptual and practical standpoints. Of course, the limitations of the approach are referred to the non-globality of the state and input coordinate transformations needed to achieve linearization, and the hardware complexity for synthesizing the required state-space coordinate transformations. Even though such limitations are being overcome, day by day, with the advances of modern electronics, the approach in [8] is still highly dependent upon the constant operating point of the converter and its lack of desirable self-scheduling properties may be significant. In [9], a controller design based in pseudolinearization of the average PWM model is proposed so as to free the linear design from the equilibrium point dependence present in the technique developed in [8]. (For a discussion, see [9, p. 864, remark 3].) The present paper gives a more concise and complete answer to the nonlinear feedback regulation of state variables in dc-to-dc power converters, yet with a traditional flavor. Only averaged input–output data is needed, classical linear design techniques are exploited, and a convenient degree of self-scheduling, characteristic of extended linearization, is also achieved. This is highly desirable in cases where the reference output voltage is subject to sudden changes, a feature not considered before in [5]–[9]. This article thus constitutes a partial completion of the picture of utilization of the available nonlinear feedback regulator design techniques for dc-to-dc power converters. Each nonlinear controller design method proposed thus far implicitly bears the advantages and limitations of the underlying nonlinear design technique.

IV. Conclusions and Suggestions for Further Research

Output voltage regulation in PWM controlled dc-to-dc power supplies of the boost and the buck-boost types has been demonstrated through use of nonlinear controllers designed on the basis of extended linearization techniques in conjunction with classical control methods. The stabilizing design considers nonlinear proportional-integral regulators derived from a linearized family of transfer functions parametrized by constant equilibrium points of idealized (infinite-frequency) average PWM controlled converter models. The nonlinear controller scheme is shown to comply with the same qualitative stabilization features imposed on the average model, provided the output signal used for feedback purposes is properly low-pass filtered and a suitably high sampling frequency is used for the PWM actuator. As a research topic that deserves further work, one could investigate the local character of the stabilizing properties of the proposed nonlinear P-I regulation scheme.

The proposed control scheme can be extended to the Cuk converter and to its various modifications. Application of the method to higher order converters requires the use of symbolic algebraic manipulation packages such as REDUCE, MAPLE, or MATEMATICA. Other classical compensator design techniques can also be proposed. In particular, the use of the frequency domain observer–controller regulator, or even the classical analytical design theory [19] and its associated integral-square error minimization, could be an alternative to the Ziegler–Nichols design recipe for the nonlinear P-I controller specification. The feasibility of these approaches remains to be demonstrated. Actual laboratory implementation of the nonlinear controllers looks promising, but elaborate with present day technology. Efforts should be conducted in such a direction.

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