INT. J. CONTROL, 1992, VOL. 56, NO. 1, 1–21

Dynamical sliding mode control strategies in the regulation of nonlinear chemical processes

HEBERTT SIRA-RAMÍREZ†

In this article, the use of variable structure feedback control strategies is proposed for the asymptotic stabilization of nonlinear dynamic systems describing chemical processes. The proposed discontinuous controller is dynamic in nature and it effectively eliminates the traditional bang–bang nature of the control input signals and the associated chattering responses. The controller is based on recent results of the differential algebraic approach to system dynamics; in particular Fliess's generalized observability canonical form is used in the derivation of the dynamical discontinuous controller. Some illustrative examples, including simulations, are provided.

1. Introduction

Far-reaching understanding of dynamic controlled systems has recently benefitted from research efforts accomplished by Fliess who proposed the use of powerful techniques, based on differential algebra (see Fliess 1989 a, b, a, b, c), in the study of systems dynamics. Fliess's remarkable studies have definitely contributed to revise and clarify traditionally well-established concepts in the theory of dynamic controlled systems, arising from Kalman's fundamental state space approach. Among such revisions, it has been found that the concept of *state* only has a local validity and, hence, a more general setting is necessary to explain, detect and possibly circumvent typical difficulties such as: impasse points and nonminimum-phase regions, associated to the explicit state space variable description of certain classes of non-linear systems. Implicit controlled ordinary differential equations account for a more general, and enlightening, setting from which a unified treatment, with far reaching implications, is possible. The approach yields the revolutionary reformulation of basic concepts such as controllability, observability, invertibility, model matching, realization, exact linearization and decoupling. Within this viewpoint, natural canonical forms for linear and non-linear controlled systems are allowed to exhibit explicitly time derivatives of the control input functions on the state and output equations. In the case of linear systems only, elimination of these input derivatives, from the state equations is possible via control-dependent state coordinate transformations. In this manner the original Kalman formulation is elegantly recovered (Fliess 1990 a, Diop 1991). The differential algebraic approach has also been successfully extended by Fliess to discrete-time systems, differential-difference systems and infinite dimensional systems described by partial differential equations.

The differential algebraic approach has also greatly improved the applicability of discontinuous feedback strategies leading to sliding motions (Utkin 1978). This is accomplished by eliminating some of its traditional disadvantages while

Received 15 April 1991. Revised 7 August 1991.

† Departamento Sistemas de Control, Universidad de Los Andes, Mérida, Venezuela.

enhancing its outstanding robustness and efficiency advantages over smooth stabilization strategies in non-linear systems (Fliess and Messager 1990, Sira-Ramírez 1991, 1993, Sira Ramírez and Lischinsky-Arenas 1991, Sira-Ramirez *et al.* 1990). Aside from the potential for possible conceptual advances in the theory of the sliding mode control alternative, the differential algebraic approach naturally allows for defining dynamical variable structure feedback strategies in nonlinear systems control. Dynamical sliding mode strategies definitely result in substantially smoothed control input signals and, hence, the possibility of chattering-free controlled responses without high gain approximations or additional tuning efforts on some controller parameters (Slotine and Li 1991). This is particularly important in the control of objects in which large input vibrations, or discontinuities, cannot be simply allowed, while still demanding enhanced robustness features on the proposed controller. Such is the case of some electromechanical systems including d.c. motors, robotic manipulators, and some other devices subject to wear and tear (Sira-Ramírez *et al.* 1990). Also, it is easy to recognize that hard discontinuities in the control input signals cannot simply be provided by a large class of actuators such as pneumatic and mechanically driven valves and dispensers, extensively used by many industries today.

In this article we treat the asymptotic output stabilization problem via dynamical discontinuous variable structure feedback control, or sliding mode control, and explore the possibilities of its application to chemical process control (for some typical non-linear control problems, treated by modern techniques, in such an area the reader is referred to Kravaris and Chung (1987), Kravaris and Palanki (1988), Kravaris and Wright (1989), Kravaris and Daoutidis (1988) and Limecq and Kantor (1990). It should be remarked that chemical process control has been an area in which, traditionally, the sliding mode control techniques could not be applied. The very nature of the regulation process, with the required smoothness of the input variables and the inherent limitations in today's actuators in the chemical industry, did not allow naturally for discontinuous feedback strategies. Moreover, the typical chattering responses, in sliding mode, of the discontinuously controlled trajectories seem rather inappropriate for the delicate, and precise, regulation of some of the output variables. All these without counting the typically slow response of some of these processes in which overshooting and 'instantaneous' correction caused by incipient errors, typical of sliding regimes, is simply not possible. These facts naturally precluded, for a long time, the use of traditional sliding mode control in the regulation of chemical processes. On the other hand, the fundamental robustness, simplicity, and ease of implementation associated with sliding mode control could not be taken advantage of in an area where the need for controllers with such characteristics is always in high demand. At the same time the lack of precise knowledge of model parameters and the ever-present perturbations affecting the performance of the regulation schemes in chemical process control led one to suspect that sliding mode control could be a useful alternative provided some of its natural incompatibilities with chemical processes were somehow overcome. However, as pointed out before, the use of differential algebraic results in non-linear regulated dynamical systems allows for the introduction of dynamical feedback control strategies which result in smooth, yet sufficiently robust control inputs generated by integrated 'internal' sliding motions taking place in the state space of the dynamical controller and not in the state space of the controlled plant.

The synthesis of the dynamical sliding mode regulator is entirely based on Fliess's local generalized observability canonical form (LGOCF) for non-linear systems (see Fliess 1989 b). In Section 2 of this article, we present the dynamical sliding mode control solution to the output stabilization problem. In Section 3, we present two application examples that illustrate the performance of the proposed dynamical variable structure feedback strategies in chemical process control. The examples are concerned with the control of continuously stirred tank reactors (CSTR) in which controlled isothermal, liquid-phase, multicomponent chemical reactions are accomplished via bounded molar feed rate control. The design examples presented include computer simulations. Concluding remarks, and proposals for further work, are collected at the end of the article in Section 4. The Appendix deals with the special case of exactly linearizable non-linear systems. In this special instance, the developments of Section 2 inevitably lead to a chattering controller. The solution to this inconvenience rests on the consideration of the GOCF of the 'extended system' (Nijmeijer and Van der Schaft 1990).

2. Dynamical variable structure feedback control in asymptotic output stabilization problems

2.1. *Fliess's generalized observability canonical form of non-linear systems*

Consider the following n-dimensional *state space realization* of a single-input single-output non-linear analytic system of the form

$$\left.\begin{aligned} \dot{x} &= f(x, u) \\ y &= h(x) \end{aligned}\right\} \tag{2.1}$$

According to recent results by Conte *et al.* (1988), there exist, under quite mild assumptions, non-uniquely defined input-dependent state coordinate transformations which eliminate the state vector x from any representation of the form (2.1). In general, this procedure yields an input–output representation, possibly given in *implicit* form, for the above system as:

$$C(y^{(d)}, \dots, \dot{y}, y, u, \dot{u}, \dots, u^{(\alpha)}) = 0 \tag{2.2}$$

where d is defined as the smallest integer satisfying the following rank condition:

$$\operatorname{rank}\begin{bmatrix} \dfrac{\partial h(x)}{\partial x} \\ \dfrac{\partial \dot{h}(x)}{\partial x} \\ \vdots \\ \dfrac{\partial h^{(d-\alpha)}(x, u)}{\partial x} \\ \vdots \\ \dfrac{\partial h^{(d-1)}(x, u, \dot{u}, \dots, u^{(\alpha)})}{\partial x} \end{bmatrix} = \operatorname{rank}\begin{bmatrix} \dfrac{\partial h(x)}{\partial x} \\ \dfrac{\partial \dot{h}(x)}{\partial x} \\ \vdots \\ \dfrac{\partial h^{(d)}(x, u, \dot{u}, \dots, u^{(\alpha)})}{\partial x} \end{bmatrix}$$

By defining $y^{(i-1)} = \xi_i$; $i = 1, ..., d$, and, under the assumption that $\partial C/\partial y^{(d)}$ is locally non-zero, one obtains the following explicit local generalized observability canonical form (LGOCF) for the system given (see Fliess 1989 b):

$$\left.\begin{aligned} \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= \xi_3 \\ &\cdots \\ \dot{\xi}_{d-1} &= \xi_d \\ \dot{\xi}_d &= c(\xi, u, \dot{u}, ..., u^{(\alpha)}) \\ y &= \xi_1 \end{aligned}\right\} \tag{2.3}$$

where it is assumed that $\alpha(=d-r) \geqslant 1$ (for the case $\alpha = 0$, see the Appendix) with r being the relative degree or relative order of the output function y with respect to the scalar control input u (this integer r is roughly defined as the minimum number of times that the output signal has to be differentiated, with respect to time, for the control input to explicitly appear in the output derivative expression (Isidori 1989, p. 145).) We refer to ξ_i $(i = 1, ..., n)$ as the generalized phase variable coordinates. Notice that if $d < n$, then the state realization (2.1) is non-minimal. We henceforth assume, for simplicity, that d equals n $(d = n)$.

Under such circumstances, the input-dependent state coordinate transformation required is then given by the full rank map (Conte *et al.* 1988):

$$\xi = T(x, u, \dot{u}, ..., u^{(\alpha)}) = \begin{bmatrix} h(x) \\ \dot{h}(x) \\ \vdots \\ h^{(r)}(x, u) \\ \vdots \\ h^{(n-1)}(x, u, \dot{u}, ..., u^{(\alpha-1)}) \end{bmatrix}$$

This implicit function theorem is responsible for the invertibility of the above map. Thus, the new state coordinates are represented by the scalar output function y and its first $n-1$ time derivatives. An input–output representation follows trivially from the state space representation (2.3) in transformed coordinates.

Let $y = 0$ be a constant equilibrium point for the system output y, in (2.3), and let $\theta := \mathrm{col}(0, 0, ..., 0)$ denote the state equilibrium vector for such a dynamical system. Suppose, furthermore, that the corresponding locally unique solution for u of the nonlinear equation $c(\theta, u, 0, ..., 0) = 0$, is given by $u = U$.

Under the assumption that d is exactly equal to n, we say that the nonlinear system (2.1) is locally minimum phase at the given equilibrium point (θ, U), if the autonomous differential equation:

$$c(\theta, u, \dot{u}, ..., u^{(\alpha)}) = 0 \tag{2.4}$$

is locally asymptotically stable to such an equilibrium point. System (2.4) is known as the zero dynamics (Fliess 1990 c).

Remark: An equivalent definition of non-minimum phase system about an equilibrium point, for a single input single output case, rests on the classical definition using scalar transfer functions. System (2.1) is said to be locally minimum phase about an equilibrium point—given by $f(x(U), U) = 0$, with $h(x(U)) = 0$—if, both the numerator and denominator polynomials of the scalar input–outupt transfer function of the linearization exhibit roots located in the open left half of the complex plane. □

2.2. *Dynamical output stabilization in nonlinear systems by exact linearization*

The asymptotic output stabilization problem consists in specifying a feedback controller, possibly of dynamic nature, described in general by a locally explicit time-varying scalar ordinary differential equation, which accepts as input functions the generalized phase variables coordinates ξ_i, and is capable of producing, as a solution output signal, a scalar function u. The control signal u has the virtue of locally forcing the system output, $y = \xi_1$, to converge asymptotically toward the desired constant reference value of zero.

Let $p(s) = s^n + \gamma_{n-1}s^{n-1} + \ldots + \gamma_1 s + \gamma_0$ be a Hurwitz polynomial. Consider imposing a linear time-invariant autonomous behaviour on the output function dynamics $y(t)$, and its first $n-1$ time derivatives, represented by the controlled state realization (2.3), in such a manner that $p(s)$ exactly represents the characteristic polynomial of the closed loop system, i.e.

$$y^{(n)}(t) + \gamma_{n-1}y^{(n-1)}(t) + \ldots + \gamma_1 y^{(1)}(t) + \gamma_0 y(t) = 0 \tag{2.5}$$

By virtue of the definition of the transformed state ξ, the reproduction of the asymptotically stable behaviour, represented by (2.5), may be clearly accomplished by the nonlinear system given, as long as the control input function $u(t)$ satisfies the non-linear implicit, time-varying, ordinary differential equation (Fliess, 1989 a):

$$c(\xi, u, \dot{u}, \ldots, u^{(\alpha)}) = -\sum_{i=0}^{n-1} \gamma_i \xi_{i+1} \tag{2.6}$$

This scalar differential equation implicitly defines a linearizing dynamical feedback controller which accomplishes asymptotic output error stabilization to zero, in a manner entirely prescribed by the set of constant design coefficients $\{\gamma_0, \gamma_1, \ldots, \gamma_{n-1}\}$. Evidently, the synthesis of such a dynamical controller requires that the signals represented by the components of the vector ξ are fed, as inputs, to such a controller. We assume that this may actually be carried out, explicitly, in terms of the original state coordinates x.

The asymptotic equilibrium point of the controlled (i.e. closed loop) system:

$$\left.\begin{aligned} \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= \xi_3 \\ &\cdots \\ \dot{\xi}_{n-1} &= \xi_n \\ \dot{\xi}_n &= -\sum_{i=0}^{n-1} \gamma_i \xi_{i+1} \\ y &= \xi_1 \end{aligned}\right\} \tag{2.7}$$

is simply given by $\xi = \theta$. Hence, under such an equilibrium condition, i.e. under perfect stabilization of y to the value 0, the resulting dynamic controller exhibits the following remaining, or hidden, dynamics (Fliess 1990 c):

$$c(\theta, u, \dot{u}, \ldots, u^{(\alpha)}) = 0 \tag{2.8}$$

It follows that the feasibility of the linearizing approach, in terms of the stability characteristics of the dynamic controller and that of the resulting closed loop system, is intimately related to the minimum phase character of the underlying non-linear system (2.1), i.e. to the nature of the stability of (2.8) around its local equilibrium value $u = U$.

It should be evident that the dynamic controller (2.6) has an interpretation in terms of an inverse system which processes, as inputs, the output function y and its time derivatives, in the form of the (input) vector ξ, and produces, in turn, as an output function, the scalar control signal u which is responsible, in turn, for leading and maintaining the output y of the original non-linear system (2.1), on the desired equilibrium value of 0. Under the assumption that, locally, $\partial c/\partial u^{(\alpha)}$ is non-zero, then no *impasse* points need be considered (Fliess and Hassler 1990, and Fliess *et al.* 1990).

2.3. *Dynamical output stabilization in nonlinear systems via sliding regimes*

The fundamental idea behind the use of the zero level set of a scalar function $s:\mathbb{R}^n \to \mathbb{R}$ of the state x, here denoted by $S = \{x : s(x) = 0\}$, as a switching manifold, or sliding surface, is to force the controlled motions, even if in a feedback discontinuous manner, to adopt S as an integral manifold. The sliding manifold is designed in such a manner as to induce a constrained dynamics which is desirable in some well-defined sense. The available feedback control actions acting outside the switching manifold, and leading the controlled trajectories toward the designed sliding surface, could be fixed, or not, from the outset. In any case, two different control polices are used on each one of the open regions of $\mathbb{R}^n$ separated by S. Upon reaching S, fast switching take plcae in the immediate vicinity of S, among the available feedback control laws, triggered by incipient errors caused by the 'overshoot and correct' control actions, which try to keep the state motions constrained to S. One of the advantages of the sliding mode control approach is that the resulting constrained motions become intrinsically independent of the original system dynamics, and of its associated parameters which, typically, are not precisely known (i.e. robustness is achieved). Moreover, the resulting constrained dynamics become basically dependent only upon the features imposed on the designed switching manifold, which are entirely up to the designer's will and needs. The reader is referred to the extensive literature on this topic for more details and applications (Utkin 1978, Slotine and Li 1991, Sira-Ramírez, 1987, 1988, 1989 a, b, c, 1990).

The following proposition describes a simple manner of obtaining a sliding motion for a one-dimensional system. This simple result accounts for much of the remaining developments in this article.

Proposition 2.1: *Consider the one dimensional discontinuous controlled system:*

$$\dot{s} = -\mu s + v \tag{2.9}$$

where the variable v acts as an external control input. Let μ and W be strictly positive quantities. Then, the discontinuous feedback control policy:

$$v = -\mu W \operatorname{sgn}(s)$$

globally creates a sliding regime on $s = 0$. Here 'sgn' stands for the signum function, *defined as*:

$$\left.\begin{array}{l} \operatorname{sgn} s = +1 \text{ if } s > 0 \\ \operatorname{sgn} s = 0 \quad \text{if } s = 0 \\ \operatorname{sgn} s = -1 \text{ if } s < 0 \end{array}\right\} \tag{2.10}$$

Furthermore, any trajectory starting on the value $s = s(0)$, *at time* $t = 0$, *reaches the condition* $s = 0$ *in finite time* T, *given by*: $T = \mu^{-1} \ln[1 + |s(0)|/W]$.

Proof. Immediate upon checking that globally: $s\, \mathrm{d}s/\mathrm{d}t < 0$ for $s \neq 0$, which is a well-known condition for the existence of a sliding mode (Utkin 1978). The second part follows from the linearity of the two intervening system structures. □

Let the set of real coefficients $\{m_0, \ldots, m_{n-2}\}$ be such that the following polynomial is Hurwitz:

$$s^{n-1} + m_{n-2}s^{n-2} + \ldots + m_1 s + m_0 \tag{2.11}$$

Consider the system (2.1), or its GOCF. Define an auxiliary output variable s, in terms of the components of the generalized phase coordinates vector ξ:

$$s = \sum_{i=1}^{n} m_{i-1}\xi_i := m^{\mathrm{T}}\xi \tag{2.12}$$

with $m_{n-1} = 1$.

A dynamical variable structure feedback controller is readily obtained for the dynamical system (2.1) if we impose on the evolution of the auxiliary output variable s the discontinuous dynamics considered in Proposition 2.1. Let A be an $n \times n$ matrix in companion form with the last row constrituted by zeros. From (2.3), (2.9) and (2.12) one obtains:

$$c(\xi, u, u^{(1)}, \ldots, u^{(\alpha)}) = -m^{\mathrm{T}}[\mu I + A]\xi - \mu W \operatorname{sgn} m^{\mathrm{T}}\xi \tag{2.13}$$

which is to be viewed as an implicit scalar differential equation with discontinuous right-hand-side. On each one of the regions $s = m^{\mathrm{T}}\xi > 0$, and $s = m^{\mathrm{T}}\xi < 0$, a different 'feedback structure' is valid and the implicit controlled differential equation is to be solved for the controller u, on the basis of knowledge of the transformed state ξ. Since s was shown to exhibit a sliding regime on the discontinuity surface $s = 0$, Filippov's continuation method (Filippov 1988), or, equivalently, the method of the equivalent control (Utkin 1978), must be used for defining the idealized solutions of (2.13) on the switching manifold given by the condition: $s = m^{\mathrm{T}}\xi = 0$.

According to the method of the equivalent control, the discontinuous motions on the sliding surface $s = 0$ can be described in an idealized fashion, by the invariance conditions $s = 0$ and $\mathrm{d}s/\mathrm{d}t = 0$. The conditions: $\mathrm{d}s/\mathrm{d}t = 0$, allows the definition of a virtual control action, known as equivalent control, which would be responsible for locally smoothly maintaining the evolution of the state variables on any member of the family of manifolds $s = \text{const.}$, should the motions started precised on any one such manifold. If the condition $s = 0$ is locally valid for the controlled system, then at least one of the transformed variables, say ξ_n, no longer qualifies as a state variable, since it is expressible in terms of the remaining $n - 1$ state variables. The resulting autonomous dynamics, ideally constrained to the switching manifold $s = 0$ and 'controlled' by the equivalent control, is known as the ideal sliding dynamics. It follows from (2.12) and the invariance conditions, $s = 0$, $\mathrm{d}s/\mathrm{d}t = 0$, that such a, non-redundant, ideal sliding dynamics is given by:

$$\left.\begin{aligned} \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= \xi_3 \\ &\vdots \\ \dot{\xi}_{n-1} &= -m_{n-2}\xi_{n-1} - m_{n-3}\xi_{n-2} - \dots - m_0\xi_1 \end{aligned}\right\} \tag{2.14}$$

The dynamic system (2.14) exhibits an asymptotically stable motion toward the origin of generalized phase coordinates ξ, with eigenvalues uniquely specified by the prescribed set of coefficients $\{m_0, \dots, m_{n-2}\}$. In particular, the output function $y = h(x) = \xi_1$ asymptotically converges to zero. From the invariance conditions considered, the equivalent control, denoted by u_{EQ}, is defined as the solution of the implicit differential equation:

$$c(\xi, u_{EQ}^{(1)}, \dots, u_{EQ}^{(\alpha)}) = -m^T A\xi \tag{2.15}$$

In equilibrium conditions for the closed loop system, the definition of the equivalent control coincides with that of the zero dynamics. The proposed variable structure control scheme (2.13) produces an asymptotically stable controlled motion of the output y, provided the original system (2.1) is minimum phase.

The determination of the minimum phase character of the system (2.1) can be carried out by studying the stability of the nonlinear zero dynamics around each of its equilibrium points or, alternatively, by linearization about the operating point of the system equations and computation of the input–output scalar transfer function.

Remark: Two important advantages about the dynamical variable structure controller respresented by (2.13) can be readily established. The first one is the fact that the output function $y = h(x)$ asymptotically approaches 0 with substantially reduced or smoothed out, 'chattering'. Notice that at least n integrators stand between the output variable y and the regulated chattering behaviour of the auxiliary output variable s. Therefore, with respect to the static variable structure controller alternative (Sira-Ramírez 1989 c, 1990), $n - r$ additional integrations contribute to smooth out the controlled output signal further. Secondly, and this is possibly the most important advantage, a canonical generalized phase variable representation for the dynamical controller (2.13) indicates that the control input u is the outcome of at least α $(= n - r)$ integrations performed on a non-linear function of the discontinuous actions that leads the auxiliary output function s to 0. This means substantially smoothed control actions which do not result in a bang–bang behaviour for the actuator, something that cannot be avoided in the static controller alternative. Notice that none of the above smoothing features applies to the case of exactly linearizable systems $(\alpha = 0, n = r)$, as treated in this section. This special case is dealt with, in full detail, in the Appendix. □

3. Some examples of sliding mode controlled chemical processes

In this Section we apply the results of Section 2 to two chemical processes control examples. Even though the examples presented are academic, they are truly representative of some chemical processes which can not be controlled by static sliding modes. The examples deal with the output stabilization of continuously stirred tank reactors (CSTR) in which controlled isothermal, liquid-phase, multicomponent chemical reactions are accomplished via molar feed rate control. The mathematical models for this application of the theory are taken directly from Kravaris and Palanki (1988).

Example 1: Dynamical variable structure controller design for total concentration control in a continuously stirred tank reactor. Consider the following simple non-linear dynamical model of a controlled CSTR in which an isothermal, liquid-phase, multicomponent chemical reaction takes place (Kravaris and Palanki 1988):

$$\left.\begin{aligned}\dot{x}_2 &= (1 + D_{a1})x_1 + u\\ \dot{x}_2 &= D_{a1}x_1 - x_2 - D_{a2}x_2^2\\ y &= x_1 + x_2 - Y\end{aligned}\right\} \tag{3.1}$$

Where x_1 represents the normalized (dimensionless) concentration C_P/C_{P0} of a certain species P in the reactor, with $Y = C_{P0}$ being the desired concentration of the species P and Q measured in mol m^{-3}. The state variable x_2 represents the normalized concentration C_Q/C_{P0} of the species Q. The control variable u is defined as the ratio of the per-unit volumetric molar feed rate of species P, denoted by N_{PF}, and the desired concentration C_{P0}, i.e. $u = N_{PF}/(FC_{P0})$ where F is the volumetric feed rate in cubic metres per second. The constants D_{a1} and D_{a2} are respectively defined as $k_1 V/F$ and $k_2 VC_{P0}/F$ with V being the volume of the reactor, in inverse cubic metres, and k_1 and k_2 are the first order rate constants in inverse seconds.

It is assumed that the species Q is highly acidic while the reactant species R is neutral. In order to avoid corrosion problems in the downstream equipment, it is desired to regulate the total concentration y to a prescribed set-point value specified by the constant Y. It is assumed that the control variable u is naturally bounded in the closed interval $[0, U_{\max}]$ reflecting the bounded (physical) limits of molar feed rate of the species P.

It is easy to verify that for the given system (3.1), the rank of the following 2×2 matrix (Conte *et al.* 1988):

$$S = \begin{bmatrix}\dfrac{\partial y}{\partial x}\\[2mm] \dfrac{\partial \dot{y}}{\partial x}\end{bmatrix} = \begin{bmatrix}1 & 1\\ -1 & -(1 + 2D_{a2}x_2)\end{bmatrix} \tag{3.2}$$

is everywhere equal to two, except on the line $x_2 = 0$. Natural physical considerations lead us to restricting x_2 to values greater than zero. The singularity of (3.2) is therefore devoid of any physical significance.

A stable constant equilibrium point for this system is given by:

$$u = U;\quad x_1(U) = \frac{U}{(1 + D_{a1})};\quad x_2(U) = \frac{1}{2D_{a2}}\left[-1 + \left\{1 + \frac{4D_{a1}D_{a2}U}{(1 + D_{a1})}\right\}^{1/2}\right] \tag{3.3}$$

It is easy to verify, by computing the linearized transfer function on the given equilibrium point, that the above system is indeed minimum phase. The following input-dependent state coordinate transformation:

$$\left.\begin{aligned}\xi_1 &= x_1 + x_2 - Y\\ \xi_2 &= -x_1 - x_2 - D_{a2}x_2^2 + u\end{aligned}\right\} \tag{3.4}$$

allows one to obtain a GOCF for the system in the form given by (2.3). The inverse of this transformation is simply written as

$$\left.\begin{aligned} x_1 &= \xi_1 - \left[\frac{u-(\xi_1+\xi_2+Y)}{D_{a2}}\right]^{1/2} \\ x_2 &= \left[\frac{u-(\xi_1+\xi_2+Y)}{D_{a2}}\right]^{1/2} \end{aligned}\right\} \tag{3.5}$$

Notice that the quantity inside the square root in (3.5) is never 0.

In transformed coordinates, the system is given by:

$$\left.\begin{aligned} \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= -2(1+D_{a1})\xi_1 - (3+2D_{a1})\xi_2 - 2D_{a1}D_{a2}(\xi_1+Y)\left[\frac{u-(\xi_1+\xi_2+Y)}{D_{a2}}\right]^{1/2} \\ &\quad + 2D_{a2}^2\left[\left(\frac{u-(\xi_1+\xi_2+Y)}{D_{a2}}\right)^3\right]^{1/2} + 2(1+D_{a1})u + \dot{u} \\ y &= \xi_1 \end{aligned}\right\} \tag{3.6}$$

which is in LGOCF. i.e. (3.6) straightforwardly leads to the following input-output representation of the given system (3.1):

$$\left.\begin{aligned} &\ddot{y} + 2(1+D_{a1})y + (3+2D_{a1})\dot{y} + 2D_{a1}D_{a2}(y+Y)\left[\frac{u-(y+\dot{y}+Y)}{D_{a2}}\right]^{1/2} \\ &- 2D_{a2}^2\left[\left(\frac{u-(y+\dot{y}+Y)}{D_{a2}}\right)^3\right]^{1/2} = 2(1+D_{a1})u + \dot{u} \end{aligned}\right\} \tag{3.7}$$

The hidden, or zero dynamics associated to the output nulling in (3.7) is given, according to (2.4), by:

$$\dot{u} + 2(1+D_{a1})u - 2D_{a1}D_{a2}Y\left(\frac{u-Y}{D_{a2}}\right)^{1/2} + 2D_{a2}^2\left[\left(\frac{u-Y}{D_{a2}}\right)^3\right]^{1/2} = 0$$

It is easy to show, by means of an approximate linearization analysis of the previous equation around the equilibrium point $u = U$, that the such a constant equilibrium point, corresponding to $Y = X_1(U) + X_2(U)$ as computed from (3.3), is locally asymptotically stable. The system is, hence, minimum phase.

Consider the following auxiliary output function, with $m_0 > 0$:

$$s = \xi_2 + m_0\xi_1 \tag{3.8}$$

Notice that if s is zeroed, in finite time, by means of a sliding mode controlled strategy such that as in (2.9), then, under such sliding conditions, the respoinse of the output function $y = \xi_1$ is ideally governed by the asymptotically stable linear autonomous dynamics

$$\dot{\xi}_1 = -m_0\xi_1 \tag{3.9}$$

Imposing on s the asymptotically stable discontinuous controlled dynamics given by $\mathrm{d}s/\mathrm{d}t = -\mu[s + W\,\mathrm{sgn}\,(s)]$, one readily obtains the following stabilizing dynamic variable structure feedback controller

$$\dot{u} = -2(1+D_{a1})u + [2(1+D_{a1}) - \mu m_0]\xi_1 + 2(1+D_{a1})Y + (3 + 2D_{a1} - m_0 - \mu)\xi_2 + 2D_{a1}D_{a2}(\xi_1 + Y)\left[\frac{u-(\xi_1+\xi_2+Y)}{D_{a2}}\right]^{1/2} - 2D_{a2}^2\left[\left[\frac{u-(\xi_1+\xi_2+Y)}{D_{a2}}\right]^3\right]^{1/2} - \mu W \operatorname{sgn}(\xi_2 + m_0\xi_1) \quad (3.10)$$

In original coordinates, the sliding surface is, evidently, a control input dependent manifold. The proposed dynamical variable structure controller (3.9) adopts, then, the following expression:

$$\dot{u} = (1-\mu-m_0)u - (1-\mu)(1-m_0)(x_1+x_2) + 2D_{a1}D_{a2}x_1x_2 - (3-\mu-m_0)D_{a2}x_2^2 - 2D_{a2}^2x_2^3 + \mu m_0 Y - \mu W \operatorname{sgn}[-(x_1+x_2) - D_{a2}x_2^2 + u + m_0x_1 + x_2 - Y] \quad (3.11)$$

Simulation results for Example 1

Simulations were performed for a reactor characterized by the following parameters:

$$D_{a1} = 1{\cdot}0; \quad D_{a2} = 1{\cdot}0$$

The simulated control task considered an output stabilization problem for the total normalized concentration y in system (3.1). This was accomplished by means of a dynamic variable structure feedback controller of the form (3.9), or (3.10). Specifically, it was required to steer the outupt y to a constant reference value, $Y = 3$. The variable structure system parameters were chosen as $\mu = 1$, $m_0 = 2$. Figure 1 portrays the time response of the dynamical discontinuously controlled output and the time responses of the corresponding controlled state variables. Figure 2 shows the corresponding (smooth) continuous control input trajectory

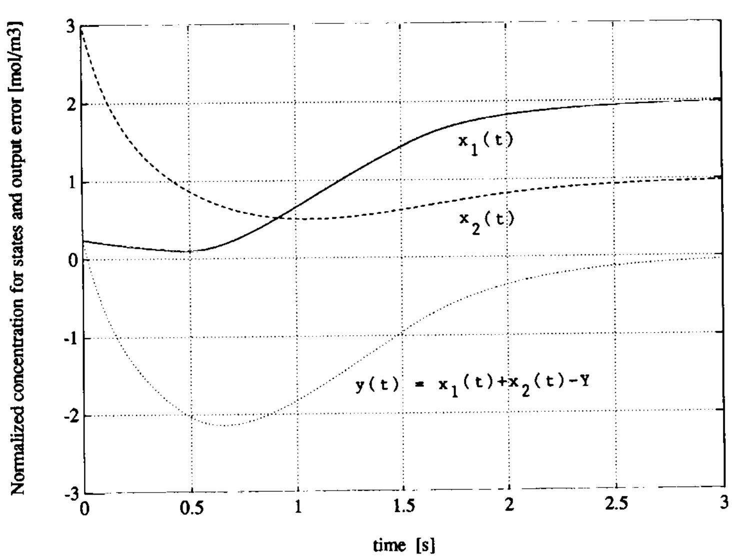

Figure 1. State and output variables responses of dynamically sliding mode controlled CSTR (Example 1).

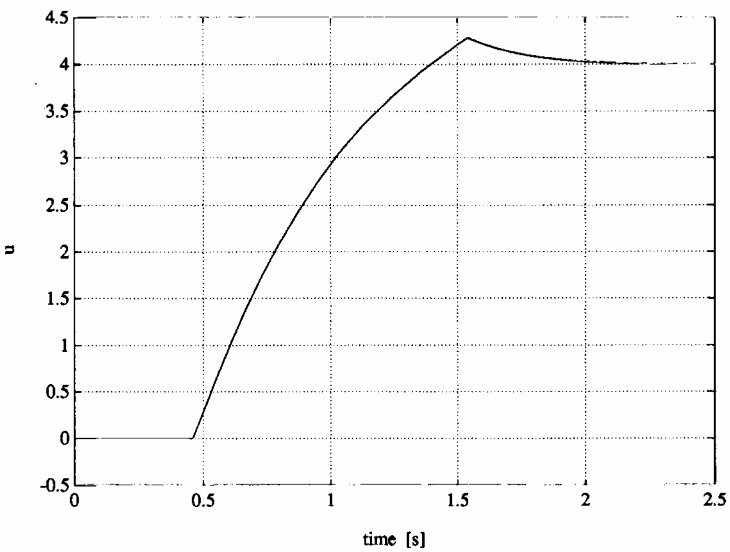

Figure 2. Dynamically generated control input trajectory (Example 1).

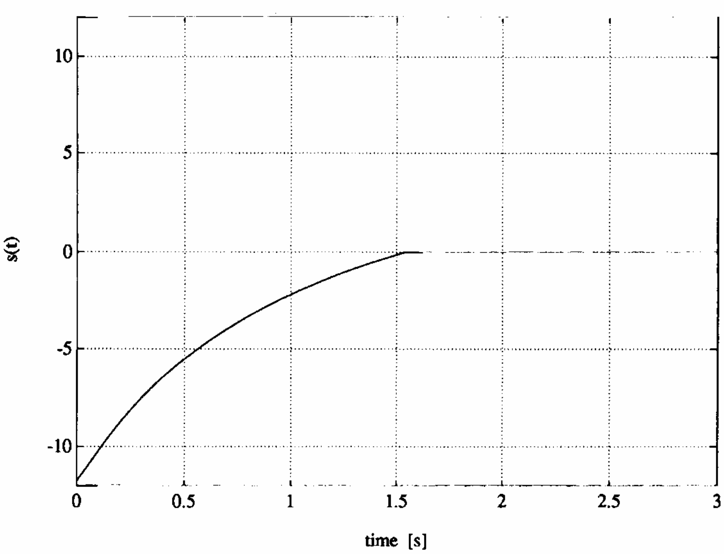

Figure 3. Time response of auxiliary output function undergoing sliding regime.

$u(t)$. Realistically, a very small amplitude chattering should be present in the simulation after the auxiliary output s reaches the value of zero, i.e. about $t = 1{\cdot}5$ s (see Fig. 3). However, the Runge–Kutta integration routine and its automatic averaging features further smooths out the simulated dynamic controller response. If sampling of the sliding surface values is carried out, a more realistic response is obtained, which is not substantially different from the one presented here, especially if very high frequency sampling is performed. Initial conditions were chosen so that

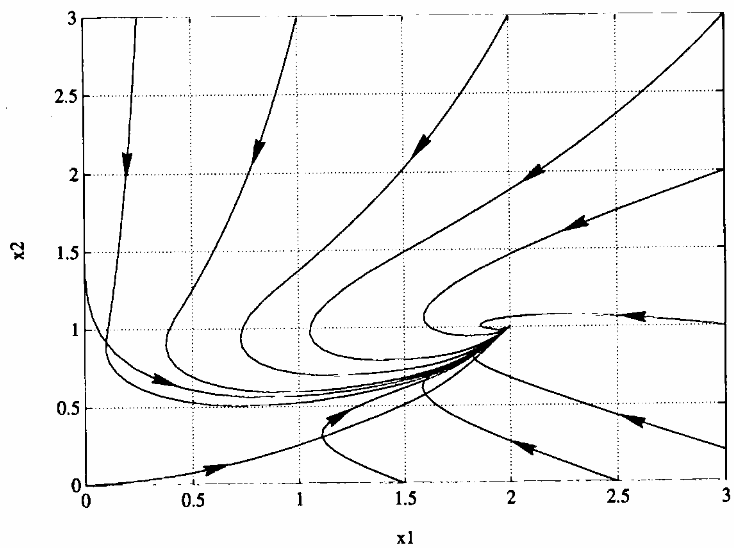

Figure 4. Phase portrait of sliding model contorlled CSTR (Example 1).

some 'saturation' was exhibited by the control input at the lower bound of zero (negative inputs having no physical meaning whatsoever, impling that the actual input must be zeroed under such conditions). This was done just to test the stabilizing features of the dynamic controller under such temporary (open loop) saturation conditions. A family of asymptotically stable state trajectories, obtained for widely different initial conditions, is shown in the phase portrait depicted in Fig. 4.

Example 2: Dynamical controller design for concentration control in a continuously stirred tank reactor. Consider the following nonlinear third order dynamic model, taken from Kravaris and Palanki (1988), of a controlled CSTR in which an isothermal, liquid-phase, multicomponent chemical reaction takes place:

$$\left.\begin{aligned} \dot{x}_1 &= 1-(1+D_{a1})x_1+D_{a2}x_2^2 \\ \dot{x}_2 &= D_{a1}x_1-x_2-(D_{a2}+D_{a3})x_2^2+u \\ \dot{x}_3 &= D_{a3}x_2^2-x_3 \\ y &= x_3 \end{aligned}\right\} \tag{3.12}$$

where x_1 represents the normalized concentration C_A/C_{AF} of a species A in the reactor, with C_{AF} being the feed concentration of the species A measured in mol m^{-3}. The state variable x_2 represents the normalized concentration C_B/C_{AF} of a species B in the reactor. The state variable x_3 represents the normalized concentration C_C/C_{AF} of a certain species C in the reactor. The control variable u is defined as the ratio of the per-unit volumetric molar feed rate of species B, denoted by N_{BF}, and the feed concentration C_{AF}. i.e., $u = N_{bF}/(FC_{AF})$ where F is the volumetric feed rate in m^3 s^{-1}. The constants D_{a1}, D_{a2} and D_{a3} are, respectively, defined as k_1V/F, k_2VC_{AF}/F and k_3VC_{AF}/F with V being the volume of the reactor, in m^3, and k_1, k_2 and k_3 stand for first order rate constants, in s^{-1}.

It is desired to regulate the normalized concentration C_C/C_{AF} to a prescribed set-point value specified by the constant Y. It is assumed that the control variable u is naturally bounded in the closed interval $[0, U_{\max}]$ reflecting the physical limits of molar feed rate of the species B.

We next summarize a dynamic variable structure feedback controller design, carried out in a manner entirely similar to that of the previous example.

It is easy to verify that for the system (3.12), the rank of the following 3×3 observability matrix **S** (Conte *et al.* 1988):

$$\mathbf{S} = \begin{bmatrix} & 0 & 1 \\ 0 & 2D_{a3}x_2 & -1 \\ 2D_{a1}D_{a3}x_2 & [2D_{a1}D_{a3}x_1 - 6D_{a3}x_2 - 6D_{a3}(D_{a2}+D_{a3})x_2^2 + 2D_{a3}u] & 1 \end{bmatrix} \tag{3.13}$$

is everywhere equal to three except on $x_2 = 0$, which, again, represents a singularity condition devoid of any physical significance.

Stable constant equilibrium point at $u = U$ in a minimum phase region

$$\left.\begin{aligned} x_1(U) &= \frac{1 + D_{a2}[x_2(U)]^2}{1 + D_{a1}} \\ x_2(U) &= (1+D_{a1})\left[\frac{-1+\left\{1+4\left(U+\dfrac{D_{a1}}{1+D_{a1}}\right)\left(\dfrac{D_{a2}+D_{a3}+D_{a1}D_{a3}}{1+D_{a1}}\right)\right\}^{1/2}}{2(D_{a2}+D_{a3}+D_{a1}D_{a3})}\right] \\ x_3(U) &= D_{a3}[x_2(U)]^2 \end{aligned}\right\} \tag{3.14}$$

Input-dependent state coordinate transformation to generalized observability canonical form and its inverse transformation

$$\left.\begin{aligned} \xi_1 &= x_3 \\ \xi_2 &= D_{a3}x_2^2 - x_3 \\ \xi_3 &= 2D_{a1}D_{a3}x_1x_2 - 3D_{a3}x_2^2 - 2D_{a3}(D_{a2}+D_{a3})x_2^3 + 2D_{a3}x_2u + x_3 \end{aligned}\right\} \tag{3.15}$$

$$\left.\begin{aligned} x_1 &= \frac{\xi_3 + 3(\xi_1+\xi_2) + 2D_{a3}(D_{a2}+D_{a3})\left(\dfrac{\xi_1+\xi_2}{D_{a3}}\right)^{3/2} - 2D_{a3}\left(\dfrac{\xi_1+\xi_2}{D_{a3}}\right)^{1/2}u - \xi_1}{2D_{a1}D_{a3}\left(\dfrac{\xi_1+\xi_2}{D_{a3}}\right)^{1/2}} \\ x_2 &= \left(\frac{\xi_1+\xi_2}{D_{a3}}\right)^{1/2} \\ x_3 &= \xi_1 \end{aligned}\right\} \tag{3.16}$$

Generalized observability canonical form

$$\left.\begin{aligned}\dot{\xi}_1 &= \xi_2\\ \dot{\xi}_2 &= \xi_3\\ \dot{\xi}_3 &= c(\xi, u, \dot{u})\\ y &= \xi_1\end{aligned}\right\} \tag{3.17}$$

where, for simplicity, and to facilitate the computation of the dynamical controller in original coordinates, we write the nonlinear function $c(\xi, u, u)$ in terms of the original coordinates x, as a function $g(x, u, u)$, given by:

$$\begin{aligned}g(x, u, \dot{u}) = {}& [2D_{a1}D_{a3}x_1 - 6D_{a3}x_2 - 6D_{a3}(D_{a2}+D_{a3})x_2^2 + 2D_{a3}u][D_{a1}x_1 - x_2\\ & - (D_{a2}+D_{a3})x_2^2 + u]2D_{a1}D_{a3}[1-(1+D_{a1})x_1 + D_{a2}x_2^2]x_2\\ & + 2D_{a3}x_2\dot{u} + D_{a3}x_2^2 - x_3\end{aligned} \tag{3.18}$$

Auxiliary output function to induce asymptotically stable sliding regime

$$\begin{aligned}s &= \xi_3 + 2\zeta\omega_n\xi_2 + \omega_n^2\xi_1\\ &= 2D_{a1}D_{a3}x_1x_2 - 2D_{a3}(D_{a2}+D_{a3})x_2^3 + 2D_{a3}x_2u\\ &\quad + (2\zeta\omega_n - 3)D_{a3}x_2^2 + (1 - 2\zeta\omega_n + \omega_n^2)x_3\end{aligned} \tag{3.19}$$

Ideal sliding dynamics

$$\left.\begin{aligned}\dot{\xi}_1 &= \xi_2\\ \dot{\xi}_2 &= -2\zeta\omega_n\xi_2 - \omega_n^2\zeta_1 =: -m_1\xi_2 - m_0\xi_1\end{aligned}\right\} \tag{3.20}$$

Asymptotic stabilizing dynamical feedback controller in original coordinates

$$\begin{aligned}\dot{u} = -\frac{1}{x_2}\Bigg\{ & 2[D_{a1}x_1 + (-2 + 0{\cdot}5\mu + \zeta\omega_n)x_2 - 2(D_{a2}+D_{a3})x_2^2]u + u^2 + D_{a1}x_2\\ & - \frac{(1-\mu)}{2D_{a3}}(1 - 2\zeta\omega_n + \omega_n^2)x_3 + D_{a1}^2x_1^2 + D_{a1}[(\mu + 2\zeta\omega_n) - 5 - D_{a1}]x_1x_2\\ & + [7 - 3(\mu + 2\zeta\omega_n) + 2\zeta\omega_n\mu + \omega_n^2]\frac{x_2^2}{2} + [D_{a1}D_{a2} + (D_{a2}+D_{a3})(6 - \mu - 2\zeta\omega_n)]x_2^3\\ & - 4D_{a1}(D_{a2}+D_{a3})x_1x_2^2 + 3(D_{a2}+D_{a3})^2x_2^4 - \frac{\mu\omega_n^2}{2D_{a3}}Y + \frac{\mu\mathbf{W}}{2D_{a3}}\operatorname{sgn}(s)\Bigg\}\end{aligned} \tag{3.21}$$

Notice that *impasse* points are located everywhere on the plane $x_2 = 0$.

It is easy to verify by means of a linearized analysis, involving some tedious but straightforward calculations, that the zero dynamics, associated to the ideal sliding motions, is asymptotically stable toward the physically meaningful equilibrium point.

Simulation results for Example 2

Simulations were peformed for an output stabilization problem defined on system (3.13) using the dynamic variable structure controller (3.21) for a reactor characterized by the following parameters:

$$D_{a1} = 3{\cdot}0; \quad D_{a2} = 0{\cdot}5; \quad D_{a3} = 1{\cdot}0$$

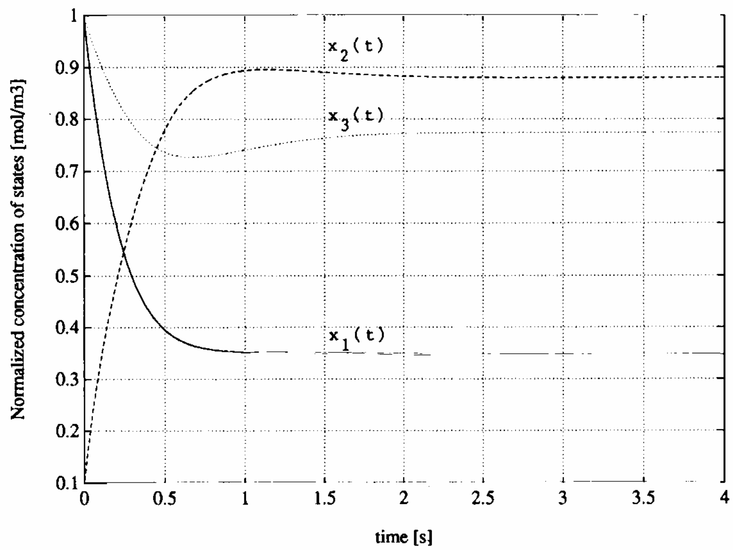

Figure 5. State variables responses of dynamically sliding mode controlled CSTR (Example 2).

It was required to regulate the total normalized concentration y to a constant reference value $Y = 0{\cdot}7737$. The chosen equilibrium values for the state variables, according to Kravaris and Palanki (1989), were given by $x_1(U) = 0{\cdot}3467$, $x_2(U) = 0{\cdot}8796$, $x_3(U) = 0{\cdot}7737$. The sliding surface parameters were chosen as $\xi = 0{\cdot}9$, $\omega_n = \sqrt{10}$. The variable structure controller parameters associated to the dynamics imposed on the auxiliary output function s were chosen as $\mu = 2\,s^{-1}$, $W = 1$. Figure 5 depicts the asymptotically stable time responses of the three controlled state varaibles, converging toward their respective equilibrium values. Figure 6 portrays the time response of the dynamical controlled output smoothly approaching the expected equilibrium point. Figure 7 shows the corresponding continuous feedback control input trajectory delivered, as an output, by the dynamical variable structure controller. The same comments, regarding the smoothness of the control input trajectory, made in the simulation results of the previous example apply to this case.

4. Conclusions

In this work, dynamical variable structure feedback compensators which accomplish asymptotic output stabilization were examined for some nonlinear chemical control processes. Generally speaking, this class of discontinuous controllers is readily obtainable for any nonlinear system once its describing differential equations are placed in Fliess's local generalized observability canonical form. Such a canonical form naturally leads, via the definition of a suitable auxiliary output function, to a dynamic variable structure linearizing controller which zeroes, in an asymptotic fashion, the output function considered. The differential algebraic approach to variable structure controller design, however, requires full state feedback and it entails dealing, in general, with the complexity of non-linear time-varying implicit dynamic controllers, which may not be globally defined. Some of the

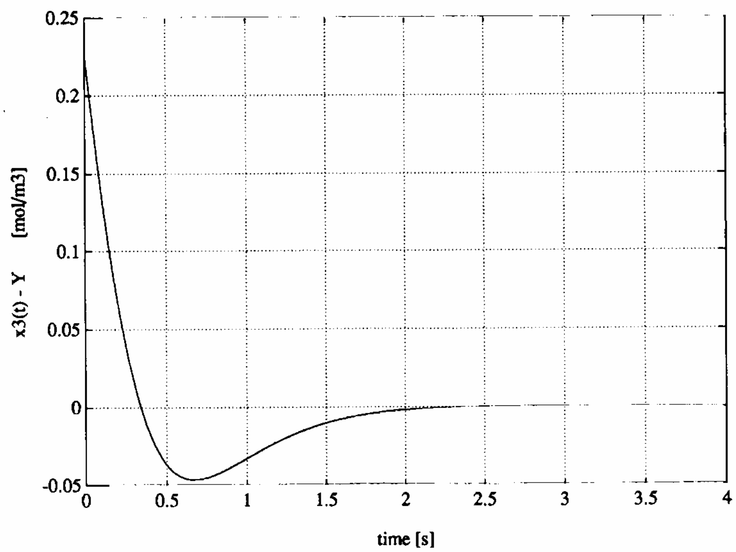

Figure 6. Output variable response of dynamically sliding mode controlled CSTR (Example 2).

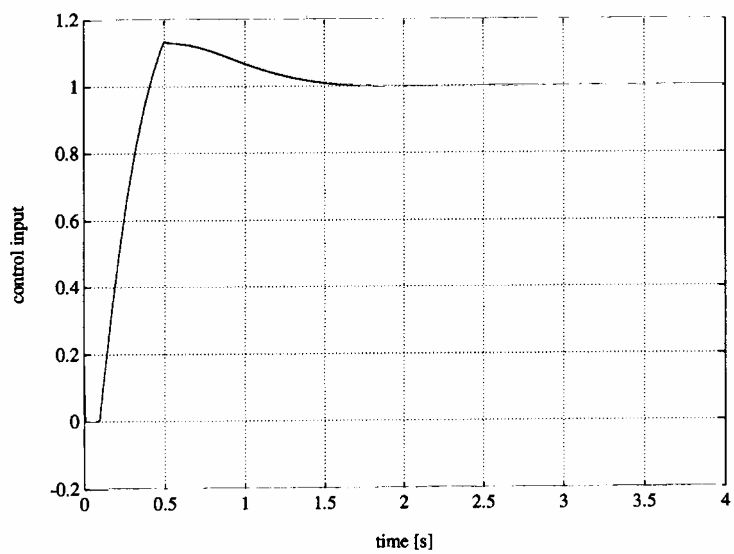

Figure 7. Dynamically generated control input trajectory Example 2).

associated difficulties are known to include the presence of *impasse* points and equilibrium points non-minimum phase regions. In such pathological cases, the usual remedy indicates that the use discontinuities in the control signal is appropriate. This prescription has been shown to produce the required results, without disturbing side effects (see Fliess *et al.* 1990). Illustrative examples were presented in this article, along with some encouraging simulation results, dealing with the discontinuous regulation of continuously stirred tank reactor outputs toward constant operating points. The chattering responses, otherwise typical of sliding mode controlled systems, are effectively suppressed and the control input signals obtained are smooth enough. The basic feature is that sliding modes actually take place in the state space of the dynamical controller and not on the process state space itself. Thus, the input function obtained is continuous and only exhibits bounded discontinuities (i.e. first order discontinuities) in its time derivative. As topics for further research, the proposed non-linear variable structure dynamic compensator could be actually implemented in an experimental CSTR by using non-linear analogue electronics. The results can be easily extended to the class of right invertible multivariable systems.

ACKNOWLEDGMENT

This work was supported by the Consejo de Desarrollo Cientifico, Humanistico y Tecnológico of the Universidad de Los Andes under Research Grant I-325-90.

Appendix

In this Appendix, we propose to use the concept of the 'extended system' (Nijmeijer and Van der Schaft 1990, p. 190), to obtain a dynamic variable structure feedback controller exhibiting smoothing features for the control input signal synthesized. This alternative arises, and as a special need, in systems which are exactly linearizable by local diffeomorphisms—representing local state coordinate transformations—and state-dependent redefinition (i.e. feedback) of the control input (Isidori 1989). If one follows the developments of Section 2.2, one easily realizes that, for this case, the obtained variable structure controller is static and none of the argued advantages of the dynamic discontinuous controller are thus achievable.

Suppose that system (2.1) is relative degree n. In this case, the local GOCF is given by:

$$\left.\begin{aligned} \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= \xi_3 \\ &\cdots \\ \dot{\xi}_{n-1} &= \xi_n \\ \dot{\xi}_n &= c(\xi, u) \\ y &= \xi_1 \end{aligned}\right\} \qquad \text{(A 1)}$$

Consider the 'extended system' of (2.1):

$$\left.\begin{aligned} \dot{x} &= f(x, u) \\ \dot{u} &= v \\ y &= h(x) \end{aligned}\right\} \qquad \text{(A 2)}$$

where v is an auxiliary external input signal and the original input variable u is now an added state to the system. As can easily be seen, the extended system, aside from being linear in the new control input v, is relative degree $n + 1$ with respect to the auxiliary input v and hence exactly linearizable by a diffeomorphic state coordinate transformation, whenever the original system is also exactly linearizable. Computing

Isidori's normal canonical form (or the GOCF, since in this case they coincide) of the system (A 2) by means of the following state coordinate transformation:

$$\xi = \begin{bmatrix} \xi \\ \xi_{n+1} \end{bmatrix} = \begin{bmatrix} T(x) \\ c(T(x), u) \end{bmatrix} = \begin{bmatrix} h(x) \\ \dot{h}(x) \\ \cdots \\ h^{(n-1)}(x) \\ c(T(x), u) \end{bmatrix} \tag{A. 3}$$

one obtains:

$$\left.\begin{aligned} \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= \xi_3 \\ &\cdots \\ \dot{\xi}_{n-1} &= \xi_n \\ \dot{\xi}_n &= \xi_{n+1} \\ \dot{\xi}_{n+1} &= \frac{\partial c(\xi, u)}{\partial \xi}\bigg|_{u=\vartheta(\xi)} (A\xi + b\xi_{n+1}) + \frac{\partial c(\xi, u)}{\partial u}\bigg|_{u=\vartheta(\xi)} v \\ y &= \xi_1 \end{aligned}\right\} \tag{A 4}$$

where A is an $n \times n$ matrix in companion form, with the last row constituted by zeros, the n dimensional column vector b has all entries zero except the last, which is equal to one. The function $\vartheta(\xi)$ is the solution, with respect to u, of the equation $c(\xi, u) = \xi_{n+1}$ (i.e. $c(\xi, \vartheta(\xi)) = \xi_{n+1}$) which is guaranteed to exist by virtue of the implicit theorem and the fact that the original system (2.1) is assumed to be relative degree n and hence, necessarily, $\partial c/\partial u \neq 0$.

On the basis of (A 4) one can now define the auxiliary output function s as:

$$s(\xi) = \xi_{n+1} + m_{n-1}\xi_n + m_{n-2}\xi_{n-1} + \ldots + m_0\xi_1 \tag{A 5}$$

such that the following polynomial, characterizing the linearized closed loop motions on $s(\xi) = 0$:

$$s^n + m_{n-1}s^{n-1} + m_{n-2}n^{n-2} + \ldots + m_0 \tag{A 6}$$

is Hurwitz. One then imposes, as it was done before, a discontinuous, asymptotically stable dynamics on s which guarantees a sliding regime on $s(\xi) = 0$, of the form:

$$\mathrm{d}s/\mathrm{d}t = -\mu[s + W \operatorname{sgn}(s)] \tag{A 7}$$

Substituting (A 5) into (A 7), using (A 4), and reverting the resulting algebraic equation to original state and input coordinates, one obtains the following dynamical variable structure contoller, with smoothed features for the control input signal u and asymptotically regulating the controlled output motions toward zero:

$$u = -\left[\frac{\partial c(T(x),u)}{\partial u}\right]^{-1} \left\{ \frac{\partial c(\xi,u)}{\partial \xi}\bigg|_{\xi=T(x)} [AT(x) + bc(T(x), u)] + \sum_{i=1}^{n+1} (m_{i-2} + \mu m_{i-1}) h^{(i-1)}(x) + \mu W \operatorname{sgn}(s) \right\} \tag{A 8}$$

with $m_n = 1$, $m_{-1} = 0$ and $h^{(n)}(x) = c(T(x), u)$.

References

CONTE, G., MOOG, C. H., and PERDON, A., 1988, Un théoreme sur la répresentation entrée-sortie d'un système linéaire. *Compte Rendu hebdomadaire des Séances de l'Académie des Sciences, Paris*, **307**, Serie I, 363–366.

DIOP, S., 1991, Elimination in control theory. *Mathematics of Control, Signal and Systems*, **4**, 17–32.

FILIPPOV, A. F., 1988, *Differential Equations with Discontinuous Right Hand Sides* (Dordrecht: Kluwer Academic).

FLIESS, M., 1989 a, Generalisation non lineaire de la forme canonique de commande et linearisation par Bouclage, *Compte Rendu hebdomadaire des Séances de l'Académie des Sciences, Paris*, **308**, Serie I, 377–379; 1989 b Nonlinear control theory and differential algebra. *Modeling and Adaptive Control*, edited by Ch. I. Byrnes and A. Khurzhansky (*Lecture Notes in Control and Information Sciences*, Vol. 105) (Berlin: Springer-Verlag); 1990 a, Generalized controller canonical forms for linear and nonlinear dynamics. *IEEE Transactions on Automatic Control*, **35**, 994–1001; 1990 b, State variable representation revisited, application to some control problems. *Perspectives in Control Theory*, edited by B. Jakubczyk, K. Malanowsky and W. Respondek (Boston: Birkhäuser); 1990 c, What the Kalman state variable representation is good for. *29th IEEE Conference on Decision and Control*, Honolulu, Hawaii. Vol. 3, pp. 1282–1287.

FLIESS, M., CHANTRE, P., ABU EL ATA, S., and COÏC, A., 1990, Discontinuous predictive control, inversion and singularities. Application to a heat exchanger. *9th International Conference on Analysis and Optimization of Systems*, Antibes (*Lecture Notes in Control and Information Sciences*) (Berlin: Springer-Verlag).

FLIESS, M., and HASLER, M., 1989, Questioning the classic state-space description via circuit examples. *Mathematical Theory of Networks and Systems* (MTNS-89), edited by M. A. Kaashoek, A. C. M. Ram and J. H. van Schuppen (*Progress in Systems and Control*) (Boston: Birkhäuser).

FLIESS, M., and MESSAGER, F., 1990 Vers une stabilisation non linéaire discontinue. *Analysis and Optimization of Systems*, edited by A. Bensoussan and J. L. Lions, (*Lecture Notes in Control and Information Sciences*, Vol. 144) (Berlin: Springer-Verlag), pp. 778–787.

ISIDORI, A., 1989, *Nonlinear Control Systems*, second edition (New York: Springer-Verlag).

KRAVARIS, C., and CHUNG, C. B., 1987, Nonlinear state feedback synthesis by global input/output linearization. *AIChE Journal*, **33**, 592–603.

KRAVARIS, C., and DAOUTIDIS, P., 1988, Synthesis of feedforward/state feedback controllers for nonlinear processes. *AIChE Journal*, **35**, 1602–1616.

KRAVARIS, C., and PALANKI, S., 1988, Robust nonlinear state feedback under structured uncertainty. *AIChE Journal*, **34**, 1119–1127.

KRAVARIS, C., and WRIGHT, R., 1989, Deadtime compensation for nonlinear processes. *AIChE Journal*, **35**, 1535–1542.

LIMECQ, L. C., and KANTOR, J. C., 1990, Nonlinear output feedback control of an exothermic reactor. *Computers in Chemical Engineering*, **14**, 427–437.

NIJMEIJER, H., and VAN DER SCHAFT, A., 1990, *Nonlinear Dynamical Control Systems* (New York: Springer-Verlag).

SIRA-RAMÍREZ, H., 1987, Variable structure control of nonlinear systems. *International Journal of Systems Science*, **18**, 1673–1689; 1988, Differential geometric methods in variable structure control. *International Journal of Control*, **48**, 1359–1391; 1989 a, Nonlinear variable structure systems in sliding mode: the general case. *IEEE Transactions on Automatic Control*, **34**, 1186–1188; 1989 b, Distributed sliding mode control in systems described by quasilinear partial differential equations. *Systems and Control Letters*, **13**, 177–181; 1989 c, Sliding regimes in general nonlinear systems: a relative degree approach. *International Journal of Control*, **50**, 1487–1506; 1990, Structure at infinity, zero dynamics and normal forms of systems undergoing sliding motions, *International Journal of Systems Science*, **21**, 665–674; 1991, Nonlinear dynamically feedback controlled descent on a non atmosphere-free planet: a differential algebraic approach. *Control—Theory and Advanced Technology*, **7**, 301–320; 1993, The differential algebraic approach in nonlinear dynamical feedback controlled landing maneuvers. *IEEE Transactions on Automatic Control*, to be published.

References

Conte, G., Moog, C. H., and Perdon, A., 1988, Un théoreme sur la répresentation entrée-sortie d'un système linéaire. *Compte Rendu hebdomadaire des Séances de l'Académie des Sciences, Paris*, **307**, Serie I, 363–366.

Diop, S., 1991, Elimination in control theory. *Mathematics of Control, Signal and Systems*, **4**, 17–32.

Filippov, A. F., 1988, *Differential Equations with Discontinuous Right Hand Sides* (Dordrecht: Kluwer Academic).

Fliess, M., 1989 a, Generalisation non lineaire de la forme canonique de commande et linearisation par Bouclage, *Compte Rendu hebdomadaire des Séances de l'Académie des Sciences, Paris*, **308**, Serie I, 377–379; 1989 b Nonlinear control theory and differential algebra. *Modeling and Adaptive Control*, edited by Ch. I. Byrnes and A. Khurzhansky (*Lecture Notes in Control and Information Sciences*, Vol. 105) (Berlin: Springer-Verlag); 1990 a, Generalized controller canonical forms for linear and nonlinear dynamics. *IEEE Transactions on Automatic Control*, **35**, 994–1001; 1990 b, State variable representation revisited, application to some control problems. *Perspectives in Control Theory*, edited by B. Jakubczyk, K. Malanowsky and W. Respondek (Boston: Birkhäuser); 1990 c, What the Kalman state variable representation is good for. *29th IEEE Conference on Decision and Control*, Honolulu, Hawaii. Vol. 3, pp. 1282–1287.

Fliess, M., Chantre, P., Abu el Ata, S., and Coïc, A., 1990, Discontinuous predictive control, inversion and singularities. Application to a heat exchanger. *9th International Conference on Analysis and Optimization of Systems*, Antibes (*Lecture Notes in Control and Information Sciences*) (Berlin: Springer-Verlag).

Fliess, M., and Hasler, M., 1989, Questioning the classic state-space description via circuit examples. *Mathematical Theory of Networks and Systems* (MTNS-89), edited by M. A. Kaashoek, A. C. M. Ram and J. H. van Schuppen (*Progress in Systems and Control*) (Boston: Birkhäuser).

Fliess, M., and Messager, F., 1990 Vers une stabilisation non linéaire discontinue. *Analysis and Optimization of Systems*, edited by A. Bensoussan and J. L. Lions, (*Lecture Notes in Control and Information Sciences*, Vol. 144) (Berlin: Springer-Verlag), pp. 778–787.

Isidori, A., 1989, *Nonlinear Control Systems*, second edition (New York: Springer-Verlag).

Kravaris, C., and Chung, C. B., 1987, Nonlinear state feedback synthesis by global input/output linearization. *AIChE Journal*, **33**, 592–603.

Kravaris, C., and Daoutidis, P., 1988, Synthesis of feedforward/state feedback controllers for nonlinear processes. *AIChE Journal*, **35**, 1602–1616.

Kravaris, C., and Palanki, S., 1988, Robust nonlinear state feedback under structured uncertainty. *AIChE Journal*, **34**, 1119–1127.

Kravaris, C., and Wright, R., 1989, Deadtime compensation for nonlinear processes. *AIChE Journal*, **35**, 1535–1542.

Limecq, L. C., and Kantor, J. C., 1990, Nonlinear output feedback control of an exothermic reactor. *Computers in Chemical Engineering*, **14**, 427–437.

Nijmeijer, H., and van der Schaft, A., 1990, *Nonlinear Dynamical Control Systems* (New York: Springer-Verlag).

Sira-Ramírez, H., 1987, Variable structure control of nonlinear systems. *International Journal of Systems Science*, **18**, 1673–1689; 1988, Differential geometric methods in variable structure control. *International Journal of Control*, **48**, 1359–1391; 1989 a, Nonlinear variable structure systems in sliding mode: the general case. *IEEE Transactions on Automatic Control*, **34**, 1186–1188; 1989 b, Distributed sliding mode control in systems described by quasilinear partial differential equations. *Systems and Control Letters*, **13**, 177–181; 1989 c, Sliding regimes in general nonlinear systems: a relative degree approach. *International Journal of Control*, **50**, 1487–1506; 1990, Structure at infinity, zero dynamics and normal forms of systems undergoing sliding motions, *International Journal of Systems Science*, **21**, 665–674; 1991, Nonlinear dynamically feedback controlled descent on a non atmosphere-free planet: a differential algebraic approach. *Control—Theory and Advanced Technology*, **7**, 301–320; 1993, The differential algebraic approach in nonlinear dynamical feedback controlled landing maneuvers. *IEEE Transactions on Automatic Control*, to be published.

Sira-Ramírez, H., Ahmad, S., and Zribi, M., 1990, Dynamical feedback control of robotic manipulators with joint flexibility. Technical Report TR-EE-90-70, School of Electrical Engineering, Purdue University, West Lafayette, Indiana 47907 (to appear in *IEEE Transactions on Systems, Man and Cybernetics*).

Sira-Ramírez, H., and Lischinsky-Arenas, P., 1991, The differential algebraic approach in nonlinear dynamical compensator design for DC-to-DC power converters. *International Journal of Control*, **54**, 111–133.

Slotine, J. J. E., and Li, W., 1991, *Applied Nonlinear Control* (Englewood Cliffs, NJ: Prentice Hall).

Utkin, V., 1978, *Sliding Regimes and their Applications to Variable Structure Systems* (Moscow: MIR Publishers).