TABLE I
IMPROVED BOUNDS ON THE PERTURBATION E_k

$\alpha (x\,10^{-1})$	$\mu_\epsilon (x\,10^{-1})$	$\mu_{\epsilon i} (x\,10^{-1})$
2	1.151	1.418
4	1.391	1.745
6	1.471	1.875
8	1.477	1.910
10	1.443	1.890
12	1.385	1.834
14	1.310	1.752
16	1.223	1.651
18	1.126	1.534
20	1.021	1.400
22	0.905	1.250
24	0.775	1.079
26	0.627	0.877
28	0.438	0.616

TABLE II
IMPROVED BOUNDS ON THE PERTURBATION $\Sigma_i^m d_i E_i$

$\alpha (x10^{-1})$	μ_{d1}	μ_{D1}	μ_{d2}	μ_{D2}	$\mu_{d\infty}$	$\mu_{D\infty} (x10^{-1})$
2	2.302	4.082	1.628	2.887	1.628	2.887
4	2.782	5.345	1.967	3.780	1.967	3.780
6	2.942	6.124	2.080	4.330	2.080	4.330
8	2.954	6.667	2.089	4.714	2.089	4.714
10	2.886	7.071	2.041	5.000	2.041	5.000
12	2.770	7.385	1.959	5.222	1.959	5.222
14	2.620	7.638	1.853	5.401	1.853	5.401
16	2.446	7.845	1.730	5.547	1.730	5.547
18	2.252	8.018	1.592	5.669	1.592	5.669
20	2.042	8.165	1.444	5.774	1.444	5.774
22	1.810	8.292	1.280	5.863	1.280	5.863
24	1.550	8.402	1.096	5.941	1.096	5.941
26	1.254	8.498	0.887	6.009	0.887	6.009
28	0.876	8.584	0.619	6.070	0.619	6.070

(The underlined values are used in [8].)

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Nonlinear Feedback Regulator Design for the Ćuk Converter

Hebertt Sira-Ramírez and Marco Tulio Prada-Rizzo

***Abstract*—In this note the method of extended linearization is used for designing stabilizing nonlinear proportional-integral (P-I) feedback controllers which regulate to a constant set-point value either the average output inductor current, the average input inductor current, or the average transfer capacitor voltage of a pulse-width modulation (PWM) controlled Ćuk converter. The design is carried out on the basis of the frequency domain Ziegler–Nichols method, applied to a family of transfer function models of the linearized average PWM controlled circuit, parametrized by constant operating equilibrium points.**

I. INTRODUCTION

The Ćuk converter (Fig. 1) constitutes a popular nonlinear dc-to-dc switch-mode power supply comprising a maximum amount of dc power conversion advantages with a minimum number of circuit components. The original Ćuk converter, as well as most of its celebrated modifications, are usually controlled by means of finite sampling frequency pulse-width-modulation (PWM) control schemes. The various traditional control configurations consider different output variables for the feedback arrangement, namely; output inductor (load) current, input inductor current, transfer capacitor voltage, or some suitable combinations of these variables. The corresponding compensators are designed on the basis of approximate linear incremental models of either discrete or continuous-time nature (see: [1]–[3]). An alternative—and fundamentally equivalent control method—considers the use of sliding mode control about suitably specified manifolds defined in the state space of the converter (see [4]). On these manifolds the ideal sliding trajectories were shown to exhibit desirable stability properties [5]–[7]. Other recently proposed control strategies for dc-to-dc converters include: pseudolinearization [8], static feedback linearization [9], and dynamical feedback linearization from a differential algebraic viewpoint [10].

The extended linearization approach for nonlinear feedback controller design, developed by Rugh [11], [12] and Bauman and Rugh [13], constitutes a highly attractive technique based on the specification of a linear regulator which induces desirable stability characteristics on an entire family of linearized plant models parametrized by constant equilibrium points. The obtained linear design serves as the basis for (nonuniquely) specifying a nonlinear controller with the property that its linearized model, computed about the same generic operating point, coincides with the specified stabilizing regulator. The resulting nonlinear controller thus exhibits the remarkable property of "self-scheduling" with respect to operating points which may change its value due to a sudden change of the reference set point.

In this note, nonlinear P-I controllers, designed on the basis

Manuscript received April 27, 1990; revised May 24, 1991. This work was supported by the Consejo de Desarrollo Cientifico, Humanistico y Tecnologico (CDCHT) of the Universidad de Los Andes under Grant I-325-90.

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IEEE Log Number 9108144.

0018-9286/92$03.00

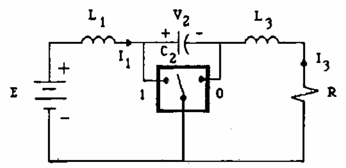

Fig. 1. The Ćuk converter.

of the extended linearization method, are proposed for the regulation to a constant set point of either average output load current, average input inductor current, or average transfer capacitor voltage in a PWM controlled converter of the Ćuk type. These three cases will be addressed, respectively, as the output current control mode, the input current control mode, and the transfer capacitor voltage control mode. The frequency domain Ziegler–Nichols method (see [14, pp. 54–58]) is used for the specification of linearized P-I regulator gains which stabilize to zero the corresponding responses of a family of parametrized transfer functions relating the particular incremental state variable, considered as an output, to the incremental duty ratio function. The nonlinear P-I controller is then obtained from the linear design in a manner entirely similar to that proposed by Rugh in [12]. Application of the extended linearization method in nonlinear controller synthesis for other kinds of dc-to-dc power converters has been carried out in [15] and [16].

It should be remarked that, for the Ćuk converter, as well as for many other dc-to-dc converters, output current, or voltage, regulation to a constant set-point value cannot be accomplished by sliding mode control defined on surfaces representing zero output error (see [5]). A constant output current or voltage control may be achieved only when a combination of the state variables is formed in an appropriate sliding plane, or, indirectly, through constant input current regulation, or, alternatively, through transfer capacitor voltage regulation [5]. For each one of the three possible control modes, the nonlinear P-I controllers proposed in this note efficiently stabilize the chosen output variable to a desirable constant operating value. The method does not present instability effects, at least in a local sense. However, depending on the system parameters and the constant set points, nonlinear P-I compensation does exhibit certain limitations inherent to the existence of a nontrivial real crossover frequency for the Nyquist plot of the family of parametrized transfer functions. In such cases, the associated infinite crossover frequency prevents the application of the Ziegler–Nichols design recipe for a P-I regulator. The designed controller is to be used in combination with a PWM actuator, generating output signals of highly discontinuous nature. For this reason, a proportional-integral-derivative (P-I-D) controller is not deemed as entirely effective due to large controller output values produced by the derivative action carried out on the discontinuously driven feedback error signal.

Section II presents, in detail, the procedure for obtaining a nonlinear P-I compensator achieving constant output current regulation for the Ćuk converter. In this section it is also presented, in a summarized fashion, the relevant formulas for obtaining nonlinear P-I regulators for average models of PWM controlled Ćuk converters for the case of both input current control mode and transfer capacitor voltage control mode. The manner in which the designed nonlinear controllers are to be used in the actual discontinuous PWM scheme is also indicated. Some simulation examples highlighting the nonlinear P-I controller performance are presented. The last section is devoted to some conclusions and suggestions for further work.

II. Design of Nonlinear P-I Controllers for DC Power Supplies of the Ćuk Type

A. Nonlinear P-I Regulation of Output Load Current in the Ćuk Converter

Consider the Ćuk converter model shown in Fig. 1. This ubiquitous converter, which is the topological dual of the boost converter [1], is described by the following bilinear state equation model:

$$\begin{aligned} \dot{x}_1 &= -\omega_1 x_2 + u\omega_1 x_2 + b \\ \dot{x}_2 &= \omega_1 x_1 - u\omega_1 x_1 - u\omega_2 x_3 \\ \dot{x}_3 &= -\omega_4 x_3 + u\omega_2 x_2 \\ y &= x_3 \end{aligned} \tag{2.1}$$

where $x_1 = I_1\sqrt{L_1}$, $x_2 = V_2\sqrt{C_2}$, and $x_3 = I_3\sqrt{L_3}$ represent normalized input inductor current, transfer capacitor voltage, and output inductor current variables, respectively. The quantity $b = E/\sqrt{L_1}$ is the normalized external input voltage. The converter parameters are defined as: $\omega_1 = 1/\sqrt{L_1C_2}$, $\omega_2 = 1/\sqrt{L_3C_2}$, and $\omega_4 = R/L_3$, These are, respectively, the LC input circuit natural oscillating frequency, the LC output circuit natural oscillating frequency, and the RL (output) circuit time constant. The variable u denotes the switch position function, which acts as a control input, taking values in the discrete set {0,1}.

The discontinuous feedback control strategy is usually specified on the basis of a sampled closed-loop PWM control scheme of the form [17]:

$$u = \begin{cases} 1 & \text{for } t_k < t \le t_k + \mu[x(t_k)]T \\ 0 & \text{for } t_k + \mu[x(t_k)]T < t \le t_k + T \end{cases} \tag{2.2}$$

where $\mu[x(t_k)]$ is known as the *duty ratio function*, which is generally represented by a smooth feedback function of the converter state [or of some related variables such as sampled output error $e(t_k) := y_d - y(t_k) = y_d - x_3(t_k)$] which satisfies the natural bounding constraint: $0 < \mu[x(t_k)] < 1$, for all sampling instants t_k. T is known as the *duty cycle* determining the time elapsed between sampling instants, i.e., $t_{k+1} = t_k + T$.

Remark: It has been rigorously shown in [18] (see also [6] and [19]) that an *average model* of a general nonlinear PWM controlled system can be obtained by assuming an *infinite sampling frequency* (i.e., letting the duty cycle $T \to 0$). The average model is simply obtained by formally substituting in the system model the discontinuous control variable u by the duty ratio function $\mu(x)$. The average trajectories, obtained as solutions of the resulting nonlinear system, satisfy the property of accurately representing all the qualitative properties of the actual PWM controlled system. This was demonstrated in [19] by showing that there always exist a sufficiently small sampling period T for which the deviations between the actual PWM controlled responses and those of the average model, under identical initial conditions, remain uniformly arbitrarily close to each other. Conversely, for each prespecified degree of error tolerance, a sampling frequency may be found such that the actual and the average trajectories differ by less than such a given tolerance bound. The error can be made even smaller if the sampling frequency is suitably increased. Moreover, from a purely geometric viewpoint, in those regions of nonsaturation of the duty ratio function μ, *integral manifolds* containing families of state responses of the average model constitute actual sliding surfaces about which the discontinuous PWM controlled trajectories exhibit sliding regimes [18]. Outside the region of nonsaturation, the trajectories of both the actual and the average PWM models entirely coincide. The average model dynamics then plays the role of the *ideal sliding dynamics* (see [20] and [21]) in the corresponding variable structure control reformulation of the PWM control strategy [18].

The average PWM controlled Ćuk converter model is thus simply obtained from (2.1) by replacing the discontinuous control function u by the duty ratio function μ.

$$\begin{aligned} \dot{z}_1 &= -\omega_1 z_2 + \mu\omega_1 z_2 + b \\ \dot{z}_2 &= \omega_1 z_1 - \mu\omega_1 z_1 - \mu\omega_2 z_3 \\ \dot{z}_3 &= -\omega_4 z_3 + \mu\omega_2 z_2 \\ y &= z_3. \end{aligned} \tag{2.3}$$

The equilibrium points of the average model are obtained from (2.3) assuming a constant value U for the duty ratio function μ

$$\mu = U; Z_1 = \frac{\omega_2^2 b U^2}{\omega_1^2 \omega_4 (1-U)^2}; Z_2(U) = \frac{b}{\omega_1(1-U)};$$

$$Z_3 = \frac{\omega_2 b U}{\omega_1 \omega_4 (1-U)}. \tag{2.4}$$

The linearization of the average PWM model (2.3) around the constant equilibrium points (2.4) results in an incremental model, parametrized by U, of the form

$$\frac{d}{dt}\begin{bmatrix} z_{1\delta} \\ z_{2\delta} \\ z_{3\delta} \end{bmatrix} = \begin{bmatrix} 0 & -(1-U)\omega_1 & 0 \\ (1-U)\omega_1 & 0 & -\omega_2 U \\ 0 & \omega_2 U & -\omega_4 \end{bmatrix} \cdot \begin{bmatrix} z_{1\delta} \\ z_{2\delta} \\ z_{3\delta} \end{bmatrix} + \begin{bmatrix} \omega_1 Z_2 \\ -\omega_1 Z_1 - \omega_2 Z_3 \\ \omega_2 Z_2 \end{bmatrix} \mu_\delta \tag{2.5}$$

where

$$z_{i\delta}(t) = z_i(t) - Z_i(U);\ i = 1,2,3;\ y_\delta(t) = y(t) - Y(U) := z_3(t) - Z_3(U);\ \mu_\delta(t) = \mu(t) - U.$$

The parametrized family of transfer functions relating the average incremental output inductor current $z_{3\delta}$ to the incremental duty ratio μ_δ is found to be

$$G_U(s) = -\frac{\omega_2 b}{\omega_1(1-U)} \cdot \frac{s^2 - \dfrac{\omega_2^2 U^2}{\omega_4(1-U)} s + (1-U)\omega_1^2}{s^3 + \omega_4 s^2 + \left[U^2\omega_2^2 + (1-U)^2\omega_1^2\right] s + \omega_4(1-U)^2\omega_1^2}. \tag{2.6}$$

The family of parametrized linear systems represented by (2.6) constitutes the basis for the frequency-response-based Ziegler–Nichols P-I controller design.

After substitution, in (2.6), of s by $j\omega$, *the phase crossover frequency* (also called the *ultimate frequency*) is found by computing the value of the frequency $W_0(U)$ that makes the imaginary part of (2.6) equal to zero (discarding, of course, the trivial solutions: $W_0(U) = 0$ and $W_0(U) = \infty$). One obtains after some straightforward calculations that the smallest solution to the resulting biquadratic equation, yielding $W_0(U)$, is

$$\begin{aligned} W_0(U) &= \sqrt{\left[\alpha(U) - \sqrt{\alpha^2(U) - \beta(U)}\right]} \\ \alpha(U) &= 0.5\frac{(2-U)}{(1-U)}\left[U^2\omega_2^2 + (1-U)^2\omega_1^2\right] \\ \beta(U) &= \omega_1^2(1-U)\left[2U^2\omega_2^2 + (1-U)^2\omega_1^2\right]. \end{aligned} \tag{2.7}$$

To guarantee the existence of a real and positive crossover frequency, the condition: $\alpha^2(U) > \beta(U)$ must be enforced on the system parameters ω_1, ω_2 and the constant duty ratio U. If this requirement is not fulfilled by the system, then the Nyquist plot of the incremental transfer function $G_U(j\omega)$ does not intersect the real axis except at $\omega = 0$ and $\omega = \infty$. In such a case the Ziegler–Nichols recipe degenerates into the specification of an arbitrary proportional controller which can be made independent of the operating point. A P-I controller is not obtained in such a case and the resulting linearized closed-loop system exhibits infinite gain margin.

The *ultimate gain* or *gain margin* $K_0(U)$ is obtained as the inverse of the absolute value of $G(jW_0(U))$. In this case, such a key design parameter is obtained as

$$K_0(U) = \frac{\omega_1\omega_4(1-U)}{\omega_2 b}\frac{|\omega_1^2(1-U)^2 - W_0^2(U)|}{|\omega_1^2(1-U) - W_0^2(U)|}. \tag{2.8}$$

According to the Ziegler–Nichols design recipe for P-I controller specification ([14, pp. 54–58]), the values of the proportional and integral term gains of the compensator $C_U(s) = K_1(U) + K_2(U)/s$, which stabilizes the entire family of linearized plant models (2.5), or (2.6), are given, respectively, by $K_1(U) = 0.4K_0(U)$ and $K_2(U) = K_0(U)W_o(U)/(4\pi)$, i.e.,

$$\begin{aligned} K_1(U) &= \frac{0.4\omega_1\omega_4(1-U)}{\omega_2 b}\frac{|\omega_1^2(1-U)^2 - W_0^2(U)|}{|\omega_1^2(1-U) - W_0^2(U)|} \\ K_2(U) &= \frac{\omega_1\omega_4(1-U)}{4\pi\omega_2 b}\frac{|\omega_1^2(1-U)^2 - W_0^2(U)|}{|\omega_1^2(1-U) - W_0^2(U)|}. \end{aligned} \tag{2.9}$$

The P-I controller $C_U(s) = K_1(U) + K_2(U)/s$ is such that it would stabilize to zero the incremental output response of the entire family of linearized plant models represented by (2.6). The extended linearization method proposes to find a nonlinear dynamical controller whose linearization around the constant equilibrium point coincides with $C_U(s)$. Following [12], it is easy to see that such a controller is of the form

$$\begin{aligned} \frac{d}{dt}\zeta(t) &= \left[\frac{\omega_1\omega_4(1-\zeta)}{4\pi\omega_2 b}\frac{|\omega_1^2(1-\zeta)^2 - W_0^2(\zeta)|}{|\omega_1^2(1-\zeta) - W_0^2(\zeta)|}W_0(\zeta)\right]e(t) \\ \hat{\mu} &= \zeta(t) + \left[\frac{0.4\omega_1\omega_4(1-\zeta)}{\omega_2 b}\frac{|\omega_1^2(1-\zeta)^2 - W_0^2(\zeta)|}{|\omega_1^2(1-\zeta) - W_0^2(\zeta)|}\right]e(t) \\ e(t) &= Z_3 - z_3(t). \end{aligned} \tag{2.10}$$

Indeed, the scalar transfer function associated to the linearization of the dynamical system (2.10), around the equilibrium point $e(U) = 0$, $\zeta(U) = \mu(U) = U$, yields precisely the designed compensator transfer function $C_U(s)$.

The output $\hat{\mu}$ of the nonlinear P-I converter is to be regarded as the specification of the needed stabilizing duty ratio function for the average PWM closed-loop converter. However, depending on the proximity of the initial states to the desirable constant average equilibrium point, the actual values of $\hat{\mu}$ may violate the natural constraints imposed on the duty ratio function μ. Therefore, a limiter of the form

$$\mu(t) = \begin{cases} 0 & \text{for } \hat{\mu}(t) \leq 0 \\ \hat{\mu}(t) & \text{for } 0 < \hat{\mu}(t) < 1 \\ 1 & \text{for } \hat{\mu}(t) \geq 1 \end{cases} \tag{2.11}$$

has to be enforced on the output $\hat{\mu}$ of the nonlinear P-I converter. This procedure yields the duty ratio function μ. In actual operation, μ may be subject to saturation during certain intervals of time. Typically, an antireset-windup scheme (see [14, pp. 10–14]) would be used in combination with the nonlinear P-I controller to avoid overshooting effects on the average controlled output.

The PWM actuator induces undesirable high frequency discontinuous signals (chattering) for the converters state and output variables. In order to suitably approximate the average closed-loop designed behavior, a low pass filter must then be placed after the sensing arrangement used to obtain the actual output inductor current $x_3(t)$ used for feedback purposes. For instance, one may propose a simple first-order RC circuit, with a sufficiently small time constant $1/T_f$ (equivalently, a sufficiently small cutoff frequency) as follows:

$$df(t)/dt = -(1/T_f)[f(t) - x_3(t)]; \qquad \hat{z}_3(t) = f(t). \tag{2.12}$$

One may regard the filter output $\hat{z}_3(t)$ as an approximation to the ideal average output current signal $z_3(t)$ required by the nonlinear P-I feedback controller (2.10).

The complete nonlinear P-I regulation scheme, based on extended linearization of an average PWM controlled Ćuk converter, is shown in Fig. 2.

Example 1: A nonlinear P-I controller regulating the output (load) current was designed for the Ćuk converter circuit with parameter values $R = 20\Omega$, $C_2 = 6.071\ \mu F$, $L_1 = 24.539$ mH, $L_3 = 2.9038$ mH, and $E = 20$ V. The constant operating value of μ was chosen to be $U = 0.6$ while the corresponding desirable normalized constant output current was $Z_3(0.6) = 0.0808$. Fig. 3 shows the average controlled output current trajectory when subject to a step change in the output current set-point value, from $Z_3(0.6) = 0.0808$ to $Z_3(U) = 0.023$ (the corresponding change in the operating point of the duty ratio was from $U = 0.6$ to $U = 0.3$). Fig. 4(a) and (b) shows, respectively, the actual PWM controlled step response of the output current and the corresponding filtered output response (the chosen set point was: $Z_3(0.6) = 0.0808$). The sampling frequency for the PWM actuator was chosen as 5 kHz and the output low-pass filter cutoff frequency was set at 0.25 kHz. (1570.7 rad/s).

B. Nonlinear P-I Regulation of Transfer Capacitor Voltage in a Ćuk Converter

Transfer capacitor voltage can also be used for feedback regulation purposes. One usually pursues constant transfer capacitor regulation to indirectly obtain the corresponding desirable constant output current, or voltage, at the load element. In this part we propose a nonlinear P-I controller scheme, similar to that in the previous section, which uses the average value of the transfer capacitor voltage z_2 for feedback purposes. The relevant formulas leading to the nonlinear P-I controller specification are summarized below. The equilibrium points are the same as in (2.4).

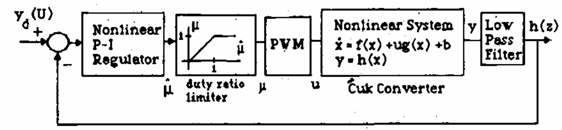

Fig. 2. A nonlinear P-I control scheme for output regulation of nonlinear PWM controlled Ćuk converter circuit.

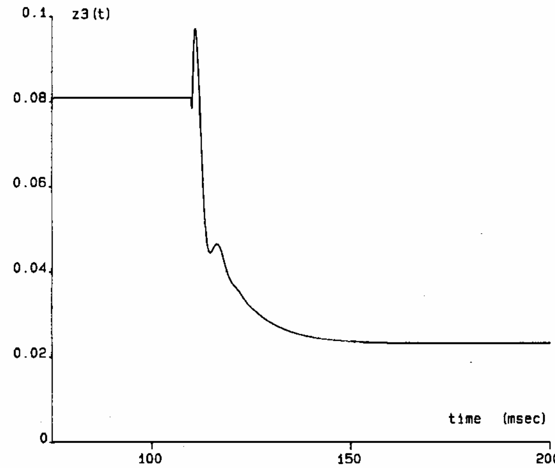

Fig. 3. Output current response to a step change in the set-point value (output current control mode).

Average Ćuk Converter Model for Transfer Capacitor Voltage Regulation:

$$\begin{aligned} \dot{z}_1 &= -\omega_1 z_2 + \mu\omega_1 z_2 + b \\ \dot{z}_2 &= \omega_1 z_1 - \mu\omega_1 z_1 - \mu\omega_2 z_3 \\ \dot{z}_3 &= -\omega_4 z_3 + \mu\omega_2 z_2 \\ y &= z_2. \end{aligned} \tag{2.13}$$

Family of Parametrized Transfer Functions Relating Incremental Transfer Capacitor Voltage to Incremental Duty Ratio:

$$G_U(s) = -\frac{\omega_2^2 bU}{\omega_1\omega_4(1-U)^2} \cdot \frac{s^2 + (2-U)\omega_4 - \dfrac{\omega_1^2\omega_4(1-U)^2}{\omega_2^2 U}s - \dfrac{\omega_1^2\omega_4^2(1-U)^2}{\omega_2^2 U}}{s^3 + \omega_4 s^2 + \left[U^2\omega_2^2 + (1-U)^2\omega_1^2\right]s + \omega_4(1-U)^2\omega_1^2}. \tag{2.14}$$

Crossover Frequency:

$$\begin{aligned} W_0 &= \sqrt{\alpha(U) + \sqrt{\alpha^2(U) + \beta(U)}} \\ \alpha(U) &= 0.5\left[U^2\omega_2^2 + (1-U)^2\omega_1^2 - (2-U)\omega_4^2\right] \\ \beta(U) &= 2\omega_1^2\omega_4^2(1-U)^2. \end{aligned} \tag{2.15}$$

Here, $\beta(U) > 0$. Hence, no restrictions exist on $\alpha(U)$ and $\beta(U)$ for $W_0(U)$ to be real.

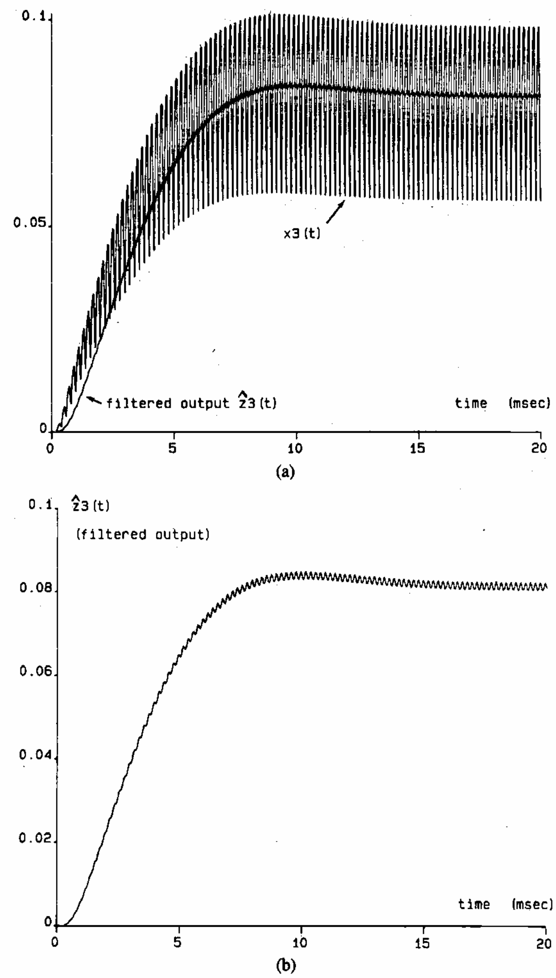

Fig. 4. (a) Actual PWM controlled step response of the output current (output current control mode). (b) Filtered PWM controlled step response of the output current (output current control mode).

Ultimate Gain:

$$K_0(U) = \frac{\omega_1\omega_4^2(1-U)^2}{b} \frac{|\omega_1^2(1-U)^2 - W_0^2(U)|}{|\omega_4^2\omega_1^2(1-U)^2 - W_0^2(U)\omega_2^2 U|}. \tag{2.16}$$

Ziegler–Nichols P-I Controller Gains for the Linearized Family of Converters:

$$K_1(U) = \frac{0.4\omega_1\omega_4^2(1-U)^2}{b} \frac{|\omega_1^2(1-U)^2 - W_0^2(U)|}{|\omega_4^2\omega_1^2(1-U)^2 - W_0^2(U)\omega_2^2 U|}$$

$$K_2(U) = \frac{\omega_1\omega_4^2(1-U)^2}{4\pi b} \cdot \frac{|\omega_1^2(1-U)^2 - W_0^2(U)|}{|\omega_4^2\omega_1^2(1-U)^2 - W_0^2(U)\omega_2^2 U|} W_0(U). \tag{2.17}$$

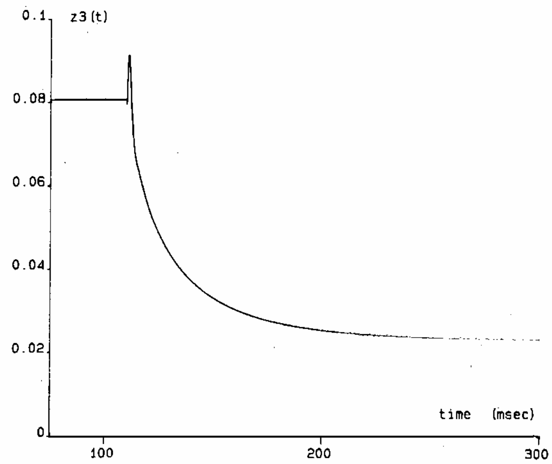

Fig. 5. Output current response to a step change in the set-point value (transfer capacitor voltage control mode).

Nonlinear P-I Controller (Transfer Capacitor Voltage Control Mode):

$$\frac{d}{dt}\zeta(t) = \frac{\omega_1\omega_4^2(1-\zeta)^2}{4\pi b} \cdot \frac{|\omega_1^2(1-\zeta)^2 - W_0^2(\zeta)|}{|\omega_4^2\omega_1^2(1-\zeta)^2 - W_0^2(\zeta)\omega_2^2\zeta|} W_0(\zeta)\, e(t)$$

$$\hat{\mu} = \zeta(t) + \frac{0.4\omega_1\omega_4^2(1-\zeta)^2}{b} \cdot \frac{|\omega_1^2(1-\zeta)^2 - W_0^2(\zeta)|}{|\omega_4^2\omega_1^2(1-\zeta)^2 - W_0^2(\zeta)\omega_2^2\zeta|} e(t). \tag{2.18}$$

Low-Pass Filter: A simple first-order low-pass filter may be proposed to yield an approximation to the ideal average output function z_2 required by the nonlinear P-I controller. Such filter is characterized by a sufficiently small time constant of value $(1/T_f)$, and a state f

$$df(t)/dt = -(1/T_f)[f(t) - x_2(t)]; \qquad \hat{z}_2(t) = f(t). \tag{2.19}$$

Example 2: A Ćuk converter circuit with the same parameter values as in Example 1 was considered for nonlinear P-I controller design regulating the normalized average transfer capacitor voltage z_2 and, thus, indirectly obtain output current regulation. The constant operating value of μ was again chosen to be $U = 0.6$ while the constant value of the corresponding desirable normalized transfer capacitor voltage is $Z_2(0.6) = 0.123$. Fig. 5 shows the average controlled output current trajectory when the transfer capacitor voltage set point is subject to a step change in the set-point value, from $Z_2(0.6) = 0.1232$ to $Z_2(U) = 0.0705$ (the corresponding change in the operating set point of the duty ratio was from $U = 0.6$ to $U = 0.3$ and the output current set-point change was as in the previous example). Fig. 6(a) shows the step response of the actual PWM output current response to a set-point value of $Z_2(0.6) = 0.1232$ and Fig. 6(b) shows the corresponding filtered output response. The sampling frequency for the PWM actuator was chosen at 5 kHz and the output low-pass filter cutoff frequency was set at 0.25 kHz.

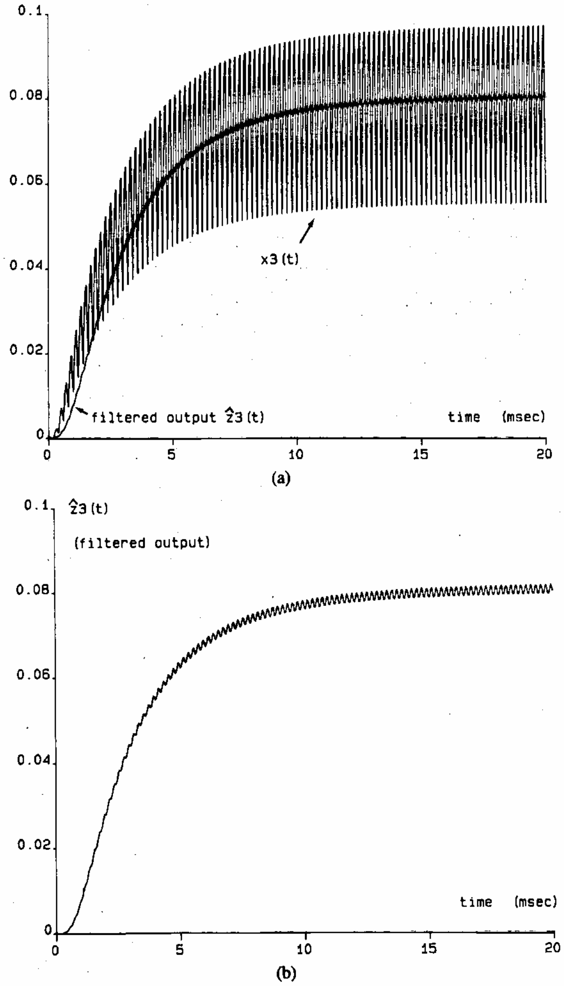

Fig. 6. (a) Actual PWM controlled step response of the output current (transfer capacitor voltage mode). (b) Filtered PWM controlled step response of the output current (transfer capacitor voltage control mode).

C. Nonlinear P-I Regulation of the Input Inductor Current of the Ćuk Converter

Input inductor current can also be used for feedback regulation purposes. One may seek constant input current regulation to indirectly accomplish a desirable constant value of the output current, or voltage, at the load. In this part, we propose a nonlinear P-I controller scheme, similar to those in the previous sections, which uses the average value of the input current z_1 for feedback purposes. The relevant formulas leading to the nonlinear P-I controller specification are summarized below. Evidently, the operating equilibrium point is the same as in (2.4).

Average Ćuk Converter Model for Input Current Control Mode:

$$\begin{aligned} \dot{z}_1 &= -\omega_1 z_2 + \mu\omega_1 z_2 + b \\ \dot{z}_2 &= \omega_1 z_1 - \mu\omega_1 z_1 - \mu\omega_2 z_3 \\ \dot{z}_3 &= -\omega_4 z_3 + \mu\omega_2 z_2 \\ y &= z_1. \end{aligned} \tag{2.20}$$

Family of Parametrized Transfer Functions Relating Incremental Input Current to Incremental Duty Ratio:

$$G_U(s) = \frac{b}{(1-U)} \cdot \frac{s^2 + \left[\dfrac{\omega_2^2 U + \omega_4^2}{\omega_4}\right] s + 2\omega_2^2 U}{s^3 + \omega_4 s^2 + \left[U^2\omega_2^2 + (1-U)^2\omega_1^2\right]s + \omega_4(1-U)^2\omega_1^2}. \tag{2.21}$$

Phase Crossover Frequency:

$$\begin{aligned} W_0(U) &= \sqrt{\alpha(U) - \sqrt{\alpha^2(U) - \beta(U)}\ \mathrm{sign}\,[\beta(U)]} \\ \alpha(U) &= 0.5\left[U(1+U)\omega_2^2 + (1-U)^2\omega_1^2 - \omega_4^2\right] \\ \beta(U) &= [\omega_2^2 U - \omega_4^2]\omega_1^2(1-U)^2 + 2\omega_2^4 U^3. \end{aligned} \tag{2.22}$$

A necessary condition for the existence of a real positive $W_0(U)$ is that $\alpha^2(U) > \beta(U)$. The condition is clearly not sufficient since, even if it is satisfied, there exists no real $W_0(U)$ in the case when $\beta(U) > 0$ and $\alpha(U) < 0$. A sufficient but not necessary condition for the existence of a real positive $W_0(U)$ is that $\beta(U) < 0$.

Ultimate Gain:

$$K_0(U) = \frac{\omega_4}{b} \frac{|\omega_1^2(1-U)^2 - W_0^2(U)|(1-U)}{|2\omega_2^2 U - W_0^2(U)|}. \tag{2.23}$$

Ziegler–Nichols P-I Controller Gains for the Linearized Family of Converters:

$$\begin{aligned} K_1(U) &= \frac{0.4\omega_4}{b} \frac{|\omega_1^2(1-U)^2 - W_0^2(U)|(1-U)}{|2\omega_2^2 U - W_0^2(U)|} \\ K_2(U) &= \frac{\omega_4}{4\pi b} \frac{|\omega_1^2(1-U)^2 - W_0^2(U)|(1-U)W_0(U)}{|2\omega_2^2 U - W_0^2(U)|}. \end{aligned} \tag{2.24}$$

Nonlinear P-I Controller:

$$\begin{aligned} \frac{d}{dt}\zeta(t) &= \frac{\omega_4}{4\pi b} \frac{|\omega_1^2(1-\zeta)^2 - W_0^2(\zeta)|(1-\zeta)W_0(\zeta)e(t)}{|2\omega_2^2\zeta - W_0^2(\zeta)|} \\ \hat{\mu} &= \zeta(t) + \frac{0.4\omega_4}{b} \frac{|\omega_1^2(1-\zeta)^2 - W_0^2(\zeta)|(1-\zeta)}{|2\omega_2^2\zeta - W_0^2(\zeta)|} e(t). \end{aligned} \tag{2.25}$$

Low-Pass Filter: A simple first-order low-pass filter may be proposed to yield an approximation to the ideal average output function z_1 required by the nonlinear P-I controller. Such filter is characterized by a sufficiently small time constant of value $(1/T_f)$, and a state f. As before, $\hat{z}_1(t)$ is taken, for all practical purposes as $z_1(t)$.

$$df(t)/dt = -(1/T_f)[f(t) - x_1(t)]; \qquad \hat{z}_1(t) = f(t). \tag{2.26}$$

Example 3: Fig. 7 shows the Nyquist plot corresponding to the linearized transfer function between the incremental input current and the incremental duty ratio for an operating point $U = 0.6$. In this case, the necessary condition for the existence of a phase crossover frequency $(\alpha(U)^2 > \beta(U))$ is not satisfied and a nonlinear P-I controller cannot be designed by the method presented here. As seen in Fig. 7, the corresponding Nyquist plot

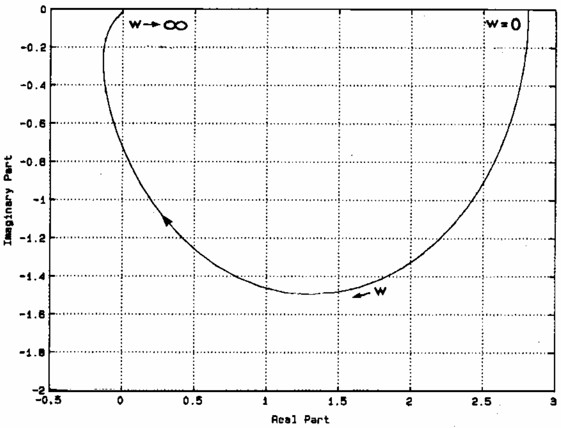

Fig. 7. Nyquist plot of linearized transfer function for Example 3.

does not intersect the $-180°$ line except at the trivial frequency values.

III. Conclusions and Suggestions for Further Research

This note has demonstrated the feasibility of output load current regulation for the Ćuk converters. Such regulation is made in either a direct load current feedback scheme, or, indirectly, through an input current, or transfer capacitor voltage, feedback schemes. The stabilizing control strategies are realized by means of nonlinear P-I controller specifications based on extended linearization of average PWM controlled models of the Ćuk converter. The stabilizing designs consider a Ziegler–Nichols type of nonlinear P-I regulators derived from a linearized family of transfer functions parametrized by constant equilibrium points of idealized (infinite-frequency) average PWM controlled converter models. The nonlinear controller scheme, as applied to the actual discontinuous PWM regulated Ćuk converter, is shown to comply with the same qualitative stabilization features imposed on the average model design, provided the output feedback signal is properly processed through a low-pass filter with a small cutoff frequency and a sufficiently high sampling frequency is used in the PWM actuator.

Nonlinear PI controllers designed by the extended linearization method exhibit, as demonstrated here by simulations, a certain degree of "self-scheduling" properties when the output voltage, or current, set-point value of the converter is suddenly changed, possibly in a purposeful manner, from a desired operating condition to a new one. In contradistinction to the classical P-I controller, based on approximate linearization, no need arises to recompute the regulator gains for the new operating point since the nonlinear characteristics of the average converter model are directly used to specify the needed nonlinear gains in an "automatic" manner. It should be pointed out, however, that due to the physical constraint of possible duty ratio saturation, the proposed nonlinear P-I controller is capable of accomplishing only local feedback stabilization. Even though simulations presented in this note demonstrate that large set-point changes can be effectively handled by the nonlinear P-I controller, this does not necessarily mean that every possible set-point perturbation will not result in a degradation of the feedback stabilization properties of the proposed controller. More research is needed, at this point, into the assessment of the regions of stabilizability ascribed to the proposed nonlinear feedback control scheme.

The proposed nonlinear control scheme can be extended to a number of the celebrated modifications of the Ćuk converter, including those with output capacitors and magnetic coupling among the input and output inductors. The cases presented in this note were carried out making extensive use of the REDUCE symbolic algebraic manipulation package. Other types of classical compensating networks can also be proposed. In particular, the use of the analytical design theory, developed in Newton *et al.* [22], and its associated integral-square error minimization, could be considered as an alternative to the Ziegler–Nichols design recipe for the nonlinear P-I controller specification. The feasibility of such an alternative approach has to be demonstrated through further work.

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Concurrency and State Feedback in Discrete-Event Systems

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Abstract—**In parallel and / or distributed architecture, strict concurrency (simultaneity of events) may have critical implications for the validity of a proposed supervisory control law. We consider this problem in the Ramadge–Wonham framework of controlled automata. We derive a simpler description of the concept of weak interaction introduced in earlier work, together with new necessary and sufficient conditions for state-feedback control to remain valid in the presence of strict concurrency.**

I. Introduction

Ramadge and Wonham [1], [2] have proposed a qualitative theory for the control of a discrete-event system (DES), and introduced two control techniques; supervisory control and state-feedback control. In the case that control specifications are given by control-invariant predicates P, there exists a state-feedback called a permissive feedback, by which P remains true at reachable states invariantly [2]. Several concepts and properties of state feedbacks were studied in [3]–[6]. A permissive feedback $f_{\max}$ for P is called a maximally permissive feedback (MPF) if no permissive feedback is more permissive than $f_{\max}$ in the sense that in the closed system, controllable events can be enabled whenever P remains true at the state after their firings. There always exists a unique MPF for P in the Ramadge–Wonham framework [2]. And in the more general structure of control patterns, it has been shown in [5], [6] that a necessary and sufficient condition for the uniqueness is weak interaction of the set of permissive feedbacks for P.

Recently, the importance of parallel and/or distributed architecture is well recognized in engineering fields such as computers and communications. And concurrency is one of the main problems in such architecture [7]–[9]. It also has very crucial effects on control in general. For example, in a concurrent DES, the given predicate is not always true at reachable states invariantly if a state feedback is designed without taking concurrency into account. Moreover, the unique existence of the MPF is no longer true even for the Ramadge–Wonham framework. From a control-theoretic point of view, it is very important to study effects of concurrency on control constructively and systematically. Li and Wonham [4] have shown that sufficient conditions for no effect of concurrency on the state-feedback control are controllability and concurrent well-posedness.

In this note, we discuss another description of weak interaction, and prove the equivalence of weak interaction and concurrent well-posedness in a concurrent DES. A brief review of basic notations on the concurrent DES and state feedbacks is given in Section II. A simpler description of weak interaction is obtained in Section III by introducing a basis feedback. In Section IV, we show several necessary and sufficient conditions for concurrency to have no effect on control, and have the equivalence of concurrent well-posedness and weak interaction.

II. Discrete-Event Systems and State-Feedbacks

A serial DES G is described by a 4-tuple

$$G := (\Sigma, X, \delta, x_0)$$

where Σ is the event set, X is the state set, a partial function $\delta: \Sigma \times X \to X$ is the transition function, and $x_0 \in X$ is the initial state. We shall write $\delta(\sigma, x)!$ if $\delta(\sigma, x)$ is defined, that is, σ may be enabled at x. Let Σ^* be the set of all finite sequences of elements in Σ, including the empty sequence ϵ. The function δ can be extended to Σ^* in the usual fashion [10]: for any $x \in X$ and any $w \in \Sigma^*$

$$\delta(\epsilon, x) := x,$$
$$\delta(w\sigma, x) := \delta(\sigma, \delta(w, x))$$

whenever $x' = \delta(w, x)!$ and $\delta(\sigma, x')!$.

We will now describe a concurrent DES G_{con}. Its (extended) event set Σ_{con} is the power set of Σ, that is, $\Sigma_{con} = 2^{\Sigma}$ [4]. An event $e = \{\sigma_1, \sigma_2, \cdots, \sigma_m\} \in \Sigma_{con}$ represents the simultaneous occurrence of $\sigma_1, \sigma_2, \cdots, \sigma_m$. Σ can be regarded as a subset of Σ_{con} by identifying $\{\sigma\} \in \Sigma_{con}$ with $\sigma \in \Sigma$. Let $\nu(e)$ be the set of concatenations of all events in e. We assume that $\delta(w_1, x) = \delta(w_2, x)$ for any $w_1, w_2 \in \nu(e)$ and $x \in X$ if $\delta(w_1, x)!$. Note that $\delta(w, x)$ is undefined for any $w \in \nu(e)$ if $\delta(w_0, x)$ is undefined for some $w_0 \in \nu(e)$. However, our assumption holds whenever G can be modeled by a Petri net [11], [12] or a state-variable description of the type of [4]. Let $\delta_{con}: \Sigma_{con} \times X \to X$ of G_{con} be the state transition function of G_{con}. For any $x \in X$ and $e \in \Sigma_{con}$, $\delta_{con}(e, x)$ is defined if $\delta(w, x)!$ for some $w \in \nu(e)$ and $\delta(\sigma, x)!$ for any $\sigma \in e$. And $\delta_{con}(e, x) := \delta(w, x)$. This function is well defined by the above assumption, and extended to Σ^*_{con} in the same fashion as δ. For convenience, we shall use δ instead of δ_{con}. Then G_{con} is given by a 4-tuple

$$G_{con} := (\Sigma_{con}, X, \delta, x_0).$$

A control mechanism for G_{con} and G is as follows [1], [2]. We decompose Σ into two subsets Σ_c and Σ_u of controllable and uncontrollable events, respectively, where $\Sigma_u = \Sigma - \Sigma_c$. Then the set Γ of control patterns for both G and G_{con} is defined by

$$\Gamma := \{\gamma | \gamma \in 2^{\Sigma} \text{ and } \Sigma_u \subseteq \gamma\}.$$

We say that $e \in \Sigma_{con}$ is enabled by γ if $e \subseteq \gamma$, and disabled by γ if $e \not\subseteq \gamma$. A (static) state-feedback for G and G_{con} is defined by a function $f: X \to \Gamma$. Let $\boldsymbol{F}$ be the set of all state feedbacks. We define an operation +(sum) and a partial order "$\leq$" on $\boldsymbol{F}$: for any $f_1, f_2 \in \boldsymbol{F}$

$$(f_1 + f_2)(x) := (f_1(x)) \cup (f_2(x))$$
$$f_1 \leq f_2 \qquad \text{if and only if } f_1(x) \subseteq f_2(x) \text{ for any } x \in X.$$

We shall write $f/G = (\Sigma, X, \delta_f, x_0)$ for the closed system with a DES G and a state feedback f, where the state transition

Manuscript received February 23, 1990; revised April 26, 1991.

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IEEE Log Number 9108145.