Dynamical Feedback Control of Robotic Manipulators with Joint Flexibility

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***Abstract*—Dynamical feedback control strategies are proposed for the asymptotic stabilization and asymptotic output tracking problems, associated with the operation of flexible joint manipulators. Smooth dynamical linearizing feedback controllers, as well as dynamical sliding mode regulators are derived within the context of Fliess' generalized observability canonical form (GOCF). The GOCF is obtained by means of a state elimination procedure, carried out on the system of differential equations describing the manipulator dynamics. The remarkable feature of this new approach lies in the fact that a truly effective smoothing of the sliding mode controlled responses is possible while substantially reducing the chattering in the control input torque. Simulation examples are given that illustrate the performance of the proposed controllers.**

I. Introduction

ROBOTIC MANIPULATORS are usually required to perform tasks that involve the end effector to asymptotically track a prespecified trajectory or to stabilize itself about some constant operating point. However, as it is well known, most robotic manipulators exhibit joint flexibility, as a result of which tracking precision is severely limited in comparison with the performance that one would obtain from a perfectly rigid joint robot manipulator. A number of nonlinear static feedback control schemes have been proposed that address the joint flexibility issue. Most of these schemes make use of static feedback in several control scenarios ranging from singular perturbations to adaptive control, modern nonlinear control theory and energy-based Lypunov control schemes (see [1]–[9]). For an excellent survey on the available literature on the control of flexible joint robots, the reader is referred to Spong [10]. Recently, Khorasani [11] proposed the use of a dynamical state feedback controller, developed in DeLuca *et al.* [12], for flexible manipulators with strongly observable parasitics. Khorasani's dynamical controller is represented by a linear time-varying system excited by nonlinear functions of the state variables. The design is carried out on the basis of singular perturbation arguments on an error correction approach related to an equivalent rigid joint manipulator. The approach described in this paper considers a fully nonlinear time-varying dynamical feedback controller for the stabilization and tracking problems for flexible joint manipulators. The controller is obtained by means of exact linearization for Fliess' GOCF [13] of the error system dynamics. A global input-dependent state coordinate transformation, in a manner described by Conte *et al.* [14], is used that directly leads to Fliess' GOCF for the flexible joint manipulator model. From this viewpoint, singular perturbation arguments, and the associated approximation schemes, are, therefore, not necessary for us to consider.

A considerable number of researchers have studied the asymptotic tracking problem for nonlinear systems from different perspectives. An appealing approach is based on elementary differential geometric methods that are summarized in Isidori's outstanding book [15]. In Isidori's treatment, clear connections are established with the concept of the inverse system, and that of the zero dynamics using the recently introduced notion of relative degree, or relative order, and the associated *normal* canonical form for nonlinear systems (see [15] and also Byrnes and Isidori [16] and Nijmeijer and Van der Schaft [17]). An adaptive control approach was also recently proposed by Isidori and Sastry [18] and by Sastry and Kokotovic [19].

Recently, outstanding contributions to the theory of dynamical controlled systems have been made by Prof. M. Fliess and his co-workers. These contributions have been obtained using powerful techniques based on *Differential Algebra* (see [20]–[22]). Fliess' ideas have contributed to revise, and clarify, deeply rooted concepts in the theory of dynamical controlled systems, stemming from Kalman's fundamental state space approach. Among such revisions, it has been found that the concept of state only has a local validity. A more general setting is then necessary to explain and circumvent typical difficulties associated to the state variable description of certain nonlinear systems, such as impasse points, nonminimum phase regions, and other singularities. Implicit ordinary differential equations account for a more general setting from which a unified, and far reaching, treatment of nonlinear control problems is possible. This approach has succeeded in clearly establishing basic concepts such as controllability, observability, invertibility, model matching, realization, exact linearization and decoupling (see also DiBenedetto *et al.* [23]). Within this viewpoint *canonical forms* for nonlinear controlled systems are allowed to explicitly exhibit time derivatives of the control input functions on the state and output equations. Only in the case of linear systems, elimination of these input derivatives, from the state equations, is possible via control

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dependent state coordinate transformations. In this manner the original Kalman formulation is elegantly recovered (see Fliess [24] and also Diop [25]).

In this paper, we treat the asymptotic output tracking problem from the perspective of dynamical feedback linearization and dynamical variable structure control. The approach is entirely based on Fliess' generalized observability canonical form (GOCF) for nonlinear systems (see [13]). Dynamical variable structure controllers using differential algebra results were originally proposed by Fliess and Messager in [26]. Based on Fliess' GOCF, these controllers have also been used by Sira-Ramírez in a variety of aerospace control problems [27], [28] and in the design of feedback controllers for dc-to-dc power converters [29].

This paper is organized as follows: in Section II, we first study the smooth version of the output tracking problem in order to relate it to the concept of the inverse system and set the ground for our main results. We then proceed to present a dynamical sliding mode control approach as a viable feedback alternative exhibiting attractive features such as robustness and, more importantly, certain level of smoothness in the resulting generated input and output trajectories. This level of smoothness is shown to depend on the difference between the order of the system and its relative degree. This last feature makes the approach especially suitable for controlling robotic manipulators with joint flexibility. In Section III, we present the two dynamical feedback control schemes for the stabilization and trajectory tracking for single-link flexible joint robots. In this section we also present some illustrative simulations that assess the effectiveness of the dynamical feedback control schemes. Concluding remarks, and proposals for further work, are collected in Section IV.

II. Asymptotic Output Tracking via Dynamical Feedback Control

A. Smooth Reference Output Tracking for Nonlinear Systems

Consider the following n-dimensional *minimal state space realization* of a single-input single-output nonlinear analytic system of the form:

$$\dot{x} = f(x,u)$$
$$y = h(x). \tag{1}$$

According to Conte *et al*'s results [14], under mild conditions, there exists a, nonuniquely defined, input-dependent state coordinate transformation that eliminates the state vector x for any representation of the form (1), and allows the finding of a, possibly implicit, input–output representation for this system in the form:

$$C\left(y^{(d)}, \cdots, \dot{y}, y, u, \dot{u}, \cdots, u^{(a)}\right) = 0 \tag{2}$$

where, d is defined as the integer satisfying the following rank condition:

$$\text{rank}\frac{\partial\left(h; \dot{h}; \cdots; h^{(d-1)}\right)}{\partial x} = \text{rank}\frac{\partial\left(h; \dot{h}; \cdots; h^{(d)}\right)}{\partial x}.$$

By defining $y^{(i-1)} = \eta_i$; $i = 1, \cdots, d$, and, under the assumption that $\partial C/\partial y^{(d)}$ is locally nonidentically zero, one locally obtains the following explicit GOCF for the given system (see Fliess [20]):

$$\begin{aligned}
\dot{\eta}_1 &= \eta_2 \\
\dot{\eta}_2 &= \eta_3 \\
&\cdots \\
\dot{\eta}_{d-1} &= \eta_d \\
\dot{\eta}_d &= c\left(\eta, u, \dot{u}, \cdots u^{(a)}\right) \\
y &= \eta_1
\end{aligned} \tag{3}$$

with $\eta := (\eta_1, \cdots, \eta_d)$ and $a = d - r$, which is assumed to be strictly positive integer, with r being the *relative degree* of the output function y with respect to the scalar control input u. The integer r is roughly defined as the minimum number of times the output signal y has to be differentiated for the control input u to explicitly appear in the output derivative expression (see [15, pp. 145]). Notice that if $d < n$, then the state realization (1) is nonminimal. We henceforth assume, for the sake of argument and simplicity, that d equals n.

Remark: Notice that the output y and its first $n-1$ time derivatives can be computed from (1) as explicit functions of the state vector x and the control input u together with $n-r-1$ of its time derivatives. If the rank condition written above holds (for $d = n$) then, by virtue of the implicit function theorem one can locally solve for the state vector x, from the set of n nonlinear output derivative equations, in terms of $y^{(j)}$ $(j = 0, 1, \cdots, n-1)$ and $u^{(i)}$ $(i = 0, 1, \cdots, n-r-1)$. Substituting the obtained expressions for the state vector components on the nth time derivative of y one obtains (2). The redefinition of the state vector in terms of generalized canonical coordinates, $\eta_i = y^{(i-1)}$, yields (3).

Let $y = Y$ be a constant equilibrium point for the system output y in system (3), and let $\eta = H := \text{col}(Y, 0, 0, \cdots, 0)$ be the corresponding state equilibrium vector. Suppose, furthermore, that the corresponding locally unique solution for u of the nonlinear equation $c(H, u, 0, \cdots, 0) = 0$, is given by $u = U$. We assume that the equilibrium point (H, U) is an asymptotically stable equilibrium point.

Definition: Under the assumption that d is exactly equal to n, we say that the nonlinear system (1) is locally *minimum phase* at the given equilibrium point (H, U), if the linearization of the autonomous differential equation:

$$c\left(\eta, u, \dot{u}, \cdots, u^{(a)}\right) = 0 \tag{4}$$

around $(H, U,)$, is asymptotically stable to zero.

Remark: Notice that in case the integer d is smaller than n, the above definition of a minimum phase system has to be substantially modified. In such a case, asymptotic stability of (4) around the equilibrium point is not sufficient to guarantee a corresponding stable unobservable dynamics in the representation (1). □

Let $y_R(t)$ be a prescribed reference output function differentiable at least n times with respect to t. The asymptotic output tracking problem consists in specifying a dynamical controller, possibly described by an implicit time-varying scalar ordinary differential equation, which accepts as input functions: 1) the output reference signal $y_R(t)$, together with a finite number of its time derivatives $y_R^{(i)}(t)$ $(i = 1, \cdots, n-1)$ and 2) the state coordinates η_i of the plant, and it is capable of producing, as a solution output signal, a scalar function u, which locally forces the system output $y = \eta_1$ to asymptotically converge toward the desirable reference output $y_R(t)$.

Define a tracking error function $e(t)$ as the difference between the actual system output $y(t)$ and the reference output signal $y_R(t)$:

$$e(t) = y(t) - y_R(t). \tag{5}$$

By definition, the transformed coordinate function η_i coincides with the $(i-1)$th time derivative of the output, that is $\eta_i = y^{(i-1)}$ for $i = 1, 2, \cdots, n-1$. We then have

$$e^{(i)}(t) = \eta_{i+1} - y_R^{(i)}(t); \qquad 0 \le i \le n-1 \tag{6}$$

$$e^{(n)}(t) = \dot{\eta}_n - y_R^{(n)}(t) = c\Big(\eta, u, \dot{u}, \cdots u^{(a)}\Big) - y_R^{(n)}. \tag{7}$$

Let $p(s) = s^n + \gamma_{n-1}s^{n-1} + \cdots + \gamma_1 s + \gamma_0$ be a Hurwitz polynomial. Consider imposing a linear time-invariant autonomous dynamics on the tracking error function by forcing $e(t)$ to satisfy:

$$e^{(n)}(t) + \gamma_{n-1}e^{(n-1)}(t) + \cdots + \gamma_1 e^{(1)}(t) + \gamma_0 e(t) = 0. \tag{8}$$

By virtue of (6) and (7), it follows that (8) may be rewritten as

$$\dot{\eta}_n - y_R^{(n)}(t) + \sum_{i=1}^{n} \gamma_{i-1}\Big[\eta_i - \gamma_R^{(i-1)}(t)\Big] = 0 \tag{9}$$

that is

$$c\Big(\eta, u, \dot{u}, \cdots u^{(a)}\Big) = y_R^{(n)}(t) - \sum_{i=1}^{n} \gamma_{i-1} \cdot \Big[\eta_i - y_R^{(i-1)}(t)\Big]. \tag{10}$$

This scalar time-varying differential equation implicitly defines, as a solution, the control input function u, which accomplishes asymptotic output tracking error stabilization to zero, in a manner entirely prescribed by the set of constant design coefficients $\{\gamma_0, \gamma_1, \cdots, \gamma_{n-1}\}$.

By defining $e_i = e^{(i-1)}$ $(i = 1, 2, \cdots, n)$, as components of an error vector $\mathbf{e}$, we alternatively express the error system (8) and the resulting dynamical controller (10), respectively, as

$$\begin{aligned}\dot{e} &= e_2\\ \dot{e}_2 &= e_3\\ &\cdots\\ \dot{e}_{n-1} &= e_n\\ \dot{e}_n &= c\Big(\xi_R(t) + \mathbf{e}, u, \dot{u}, \cdots u^{(a)}\Big) - y_R^{(n)}(t)\\ &= -\sum_{i=1}^{n} \gamma_{i-1}e_i\end{aligned} \tag{11}$$

and

$$c\Big(\xi_R(t) + \mathbf{e}, u, \dot{u}, \cdots u^{(a)}\Big) = y_R^{(n)}(t) - \sum_{i=1}^{n} \gamma_{i-1}e_i \tag{12}$$

with

$$\begin{aligned}\xi_R(t) &= \mathrm{col}\Big(y_R(t), y_R^{(1)}(t), \cdots, y_R^{(n-1)}(t)\Big)\\ e &= \mathrm{col}(e_1, e_2, \cdots, e_n).\end{aligned} \tag{13}$$

The asymptotic equilibrium point of the tracking error system (11) is simply given by $e_1 = e_2 = \cdots = e_n = 0$. Hence, under such an equilibrium condition, i.e., under perfect tracking, the resulting dynamical controller exhibits the following remaining, or "hidden," dynamics:

$$c\Big(\xi_R(t), u, \dot{u}, \cdots u^{(a)}\Big) = y_R^{(n)}(t). \tag{14}$$

It should be evident that the dynamical controller (14) has an interpretation in terms of an *inverse system* that takes, as inputs, the desired reference output function, $y_R(t)$, and a finite number of its time derivatives, and produces, as an output function, the scalar control input u. This function u is responsible, in turn, for forcing the original nonlinear system (1) (or (3)) to reproduce the desired output function $y_R(t)$. Under the assumption that, locally, $\partial c/\partial u^{(a)}$ is nonzero, then no *impasse* points need be locally considered (see Fliess and Hassler [30] and Fliess *et al.* [31]).

Assume, just for a moment, that $y_R(t)$ adopts a constant value Y_R in (14), and let $\Xi_R := \mathrm{col}(Y_R, 0, 0, \cdots, 0)$. Suppose, furthermore, that the corresponding locally unique solution of $c(\Xi_R, u, 0, \cdots, 0) = 0$, is given by $u = U_R$. If the underlying nonlinear system (3) is locally *minimum phase* at (Y_R, U_R), then, the resulting differential equation,

$$c\Big(\eta, u, \dot{u}, \cdots, u^{(a)}\Big) = 0 \tag{15}$$

linearized around the constant equilibrium point, is asymptotically stable to zero. It then follows that the stability characteristics of the dynamical controller (12), and (14), for arbitrary but bounded output reference functions $y_R(t)$, is intimately related to the minimum phase character of the underlying nonlinear system (3).

Remark: Notice that (15) has exactly the same form as the differential equation obtained from (3), under the assumption of a constant equilibrium condition for the transformed state variable coordinates η. □

B. A Dynamical Variable Structure Control Approach to Asymptotic Reference Output Tracking in Nonlinear Systems

Let μ and Ω represent strictly positive quantities and let us denote "sgn" for the *signum* function, and we define:

$$\begin{cases}\mathrm{sgn}\ \omega = 1 & \text{if } \omega > 0\\ \mathrm{sgn}\ \omega = 0 & \text{if } \omega = 0\\ \mathrm{sgn}\ \omega = -1 & \text{if } \omega < 0\end{cases} \tag{16}$$

The following proposition is quite basic in our forthcoming developments:

Proposition: The one dimensional discontinuous system:

$$\dot{\omega} = -\mu(\omega + \Omega\ \mathrm{sgn}\ \omega) \tag{17}$$

globally exhibits a sliding regime on $\omega = 0$. Furthermore, any trajectory starting on the initial value $\omega = \omega(0)$, at time $t = 0$, reaches the condition $\omega = 0$ in finite time T, given by:

$$T = \mu^{-1} \ln[1 + |\omega(0)|/\Omega]. \tag{18}$$

Proof: Obvious upon checking that globally: $\omega d\omega/dt < 0$ for $\omega \neq 0$, which is a well known condition for sliding mode existence (see [32]). The second part follows easily from the linearity of the two intervening system "structures." □

Let the set of real coefficients $\{m_0, \cdots, m_{n-2}\}$ be such that the following polynomial, in the complex variable "s," is Hurwitz:

$$s^{n-1} + m_{n-2}s^{n-2} + \cdots + m_1 s + m_0. \tag{19}$$

Consider the auxiliary scalar output variable ω, defined in terms of the output tracking error coordinates e_i as

$$\omega = \sum_{i=1}^{n} m_{i-1} e^{(i-1)} = \sum_{i=1}^{n} m_{i-1} e_i; \quad \text{with} \quad m_{n-1} = 1. \tag{20}$$

If we impose on the evolution of the auxiliary output variable ω, the discontinuous dynamics considered in (17), one obtains, from (11) and (20):

$$\begin{aligned} \dot\omega &= \dot e_n + \sum_{i=1}^{n-1} m_{i-1} e_{i+1} \\ &= -\mu \cdot \left[\sum_{i=1}^{n} m_{i-1} e_i + \Omega \,\mathrm{sgn}\left(\sum_{i=1}^{n} m_{i-1} e_i\right)\right] \end{aligned} \tag{21}$$

Using (17) one obtains the following discontinuous dynamical feedback controller in terms of an implicit ordinary differential equation:

$$\begin{aligned} c\left(\xi_R + \boldsymbol{e}, u, \dot u, \cdots u^{(a)}\right) &= y_R^{(n)} - \sum_{i=1}^{n-1} m_{i-1} e_{i+1} \\ &\quad -\mu \cdot \left[\sum_{i=1}^{n} m_{i-1} e_i + \Omega \,\mathrm{sgn}\left(\sum_{i=1}^{n} m_{i-1} e_i\right)\right] \end{aligned} \tag{22}$$

which is to be viewed as an implicit differential equation with discontinuous right hand side. On each one of the regions $\omega > 0$, and $\omega < 0$, a different "structure' is valid and the corresponding implicit differential equation is to be independently solved for the controller u, on the basis of knowledge of the error vector $\boldsymbol{e}$ and the reference output signal. Since ω was shown to exhibit a sliding regime on the discontinuity surface $\omega = 0$, Filippov's continuation method (see [33]), or, equivalently, the method of the equivalent control [32], needs to be invoked for defining the idealized solutions of (22) on the switching manifold $\omega = 0$.

According to the method of the equivalent control, the discontinuous motions on the sliding surface $\omega = 0$ can be described, in an idealized fashion, by the following *invariance conditions:* $\omega = 0$ and $d\omega/dt = 0$. These conditions allow, in turn, the definition of a *virtual* control action, known as the equivalent control, which would be responsible for locally smoothly maintaining the evolution of the state variables on the manifold $\omega = 0$, should the motions precisely start on this manifold. The resulting autonomous dynamics for the controlled output tracking error, ideally constrained to the switching manifold and "controlled" by the equivalent control, denoted by u_{EQ}, is generally known as the ideal sliding dynamics. It follows from (11) and (20) that such an ideal sliding dynamics is given by

$$\begin{aligned} \dot e_1 &= e_2 \\ \dot e_2 &= e_3 \\ &\cdots \\ \dot e_{n-1} &= -\sum_{i=1}^{n-1} m_{i-1} e_i \end{aligned} \tag{23}$$

which represents an exponentially asymptotically stable motion toward the origin of the error vector coordinates, with eigenvalues uniquely specified by the prescribed set of coefficients $\{m_0, \cdots, m_{n-2}\}$. In particular, the output tracking error function $e_1 = \eta_1 - y_R(t)$ asymptotically converges to zero. Using the invariance conditions $\omega = 0$, $d\omega/dt = 0$, on (22), it follows that the equivalent control u_{EQ} is defined as the solution of the implicit time-varying differential equation:

$$\begin{aligned} c\left(\xi_R(t) + \boldsymbol{e}, u_{EQ}, \dot u_{EQ}, \cdots u_{EQ}^{(a)}\right) &= \\ & y_R^{(n)}(t) - \sum_{i=1}^{n} [m_{i-2} + \mu m_{i-1}] e_i; \\ m_{-1} &= 0 \end{aligned} \tag{24}$$

In view of (19) and (23), the tracking error vector $\boldsymbol{e}$ asymptotically converges to zero. Under perfect tracking conditions $(e_1 = \cdots = e_n = 0)$, the equivalent control u_{EQ}, is then given by the solution of

$$c\left(\xi_R(t), u_{EQ}, \dot u_{EQ}, \cdots, u_{EQ}^{(a)}\right) = y_R^{(n)}(t) \tag{25}$$

i.e., once more we find that the concept of the minimum phase property plays a crucial role in the solution of the tracking problem. In this case, it bears a definite influence in the rightful definition, and existence, of the equivalent control function.

Remark: Two important advantages can be readily established about the dynamical variable structure controller represented by (22). The first one is the fact that the output tracking error function $e(t)$ asymptotically approaches zero with substantially reduced, or smoothed out "chattering." Notice that there exist at least n integrators between the tracking error variable $e_1 = e(t)$ and the regulated chattering behavior of the auxiliary output variable ω. Therefore, with respect to a *static* variable structure controller alternative, based on Isidori's normal canonical form approach (see [34], [35]), $n - r$ additional integrations contribute to further smooth out the controlled tracking error signal $e(t)$. Secondly, and this is possibly the most important advantage of the approach, a traditional canonical phase variable representation for the dynamical controller (22) indicates that the control input u is the outcome of at least a $(= n - r)$ integrations, performed on a nonlinear function of the discontinuous actions that lead the auxiliary output ω to zero. This means substantially smoothed control actions that do not demand from the actuator a "bang-bang" behavior. This chattering at the input variable cannot be effectively avoided in the static controller alternative with the same quality of response (see the simulation examples below). However, a certain amount of input chattering can still be suitably avoided by using a "boundary layer" approach that amounts to replacing the discontinuous controller by a high gain saturating controller (see [36]–[38]). □

III. Dynamical Feedback Control of Flexible Joint Manipulators

In this Section we apply the results of Section II to stabilization and tracking problems defined for a single-link flexible joint manipulator.

A. A Mathematical Model for a Single-Link Flexible Joint Robot

The dynamical equations governing the behavior of a single-link flexible joint robot are traditionally obtained from Lagrangian dynamics considerations. Let q denote the angular position of the link (see Fig. 1) of half length L and mass m and let q_m be the angular position of the motor. The differential equations governing the controlled motions are given by

$$\begin{aligned} \tau &= D_m\ddot{q}_m + B_m\dot{q}_m + K_s(q_m - q) \\ 0 &= D\ddot{q} + B\dot{q} + mgL\sin q + K_s(q - q_m) \end{aligned} \tag{26}$$

where D denotes the inertia of the link, D_m denotes the motor inertia; the flexible joint stiffness coefficient is K_s and the motor viscous damping and the link viscous damping are B_m and B, respectively. The gravitational acceleration is denoted by g.

Define, $\rho^2 = 1/K_s$, which is *not* to be taken as a small constant related to singular perturbation techniques. The state variables were defined as the motor's angular position $x_1 = q_m$, the corresponding angular velocity $x_2 = dq_m/dt$, the elastic force $x_3 = K_s(q - q_m)$ and $x_4 := (dq/dt - dq_m/dt)/\rho$. The state variable representation is then obtained as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -a_5x_2 + a_1x_3 + a_1u \\ \dot{x}_3 &= x_4/\rho \\ \dot{x}_4 &= \left[-a_2a_3\sin\left(\rho^2x_3 + x_1\right) - a_4x_3 - a_7x_2 \right. \\ &\quad \left. - a_6\rho x_4 - a_1u\right]/\rho \end{aligned} \tag{27}$$

with: $a_1 = 1/D_m$; $a_2 = 1/D$; $a_3 = mgL$; $a_4 = a_1 + a_2$; $a_5 = B_m/D_m$;. $a_6 = B/D$; $a_7 = a_6 - a_5$; $u = \tau$.

We shall consider, as the system output y, the motor position x_1 instead of the link position $z = x_1 + \rho^2x_3$, based on the following considerations:

1) Taking the link position z as the output of the system, the system is relative degree four and, hence, it is exactly linearizable by nonlinear *static* state feedback using link position, velocity, acceleration and "jerk" [3]. These variables are usually very hard to measure in practice. Moreover, any "outer loop" discontinuous feedback control scheme, say of the sliding mode type or the pulse width modulation type, results in unacceptable chattering [39]–[41].

2) By using motor position as the output, the system now becomes relative degree two and the GOCF will involve second order time derivatives of the control input torque. We will show that a dynamical feedback control scheme, based on the GOCF, is not only entirely feasible from the nature of the state variables that have to be measured, but it also yields a nonlinear second order dynamical system acting as the controller. This system naturally integrates (i.e., smooths) the synthesized input torque. The method substantially eliminates chattering input torques to the manipulator thus giving a much more acceptable performance in both tracking and stabilization tasks.

A constant stable steady state operating point, achieved by constant input torque $u = U$, is given by

$$\begin{aligned} &u = U; \qquad x_1(U) = \rho^2U + \sin^{-1}\left(\frac{U}{a_3}\right); \qquad x_2 = 0; \\ &x_3(U) = -U; \qquad x_4(U) = 0; \qquad z(U) = \sin^{-1}\left(\frac{U}{a_3}\right). \end{aligned} \tag{28a}$$

Notice that violation of the restriction $a_3 = mgL \geq U$, implies that no equilibrium point exists for the system. This should also be clear from physical considerations.

Linearization about the above equilibrium point yields the following *minimum phase* scalar transfer function [42] relating the Laplace transform of the incremental displacement, $\delta q_m(t) = x_1(t) - x_1(U)$, and the Laplace transform of the incremental torque variable, $\delta\tau(t) = \tau(t) - U$, is shown in (28b) (shown at the bottom of the page).

It is easy to see that, provided $mgL \geq U$, the numerator polynomial of (28b) is a Hurwitz polynomial. The zero dynamics of the system is then minimum phase. The minimum phase character of the zero dynamics depends only on the constant B and it is independent of the value of the equilibrium angular position. We say then that the zero dynamics if *globally* minimum phase. It follows also from (28b) that the assumption $B \neq 0$ is crucial for the flexible joint system to be minimum phase (for related developments see [31], and [43]).

$$\frac{\delta q_m(s)}{\delta\tau(s)} = \frac{Ds^2 + Bs + K_s + \sqrt{(mgL)^2 - U^2}}{[D_ms^2 + B_ms + K_s]\left[Ds^2 + Bs + K_s + \sqrt{(mgL)^2 - U^2}\right] - K_s^2} \tag{28b}$$

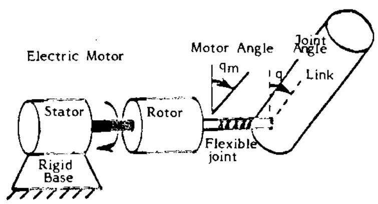

Fig. 1. Single-link flexible joint robot.

Remark:[1] Notice that a purely oscillatory zero dynamics is obtained if the assumption $B \neq 0$is not verified. The theory developed in this paper is not applicable to those cases in which $B = 0$. In practice most mechanical manipulators will have $B \neq 0$, as bearings and other mechanical components have nonzero friction and viscous damping.

The following input-dependent state coordinate transformation, which allows one to obtain a GOCF in the form of (2), represents a direct particularization of the multivariable case results found in Conte *et al.* [14] to our single-input system:

$$\begin{aligned} \eta_1 &= x_1 \\ \eta_2 &= x_2 \\ \eta_3 &= -a_5 x_2 + a_1 x_3 + a_1 u \\ \eta_4 &= a_5^2 x_2 - a_1 a_5 x_3 + a_1 \rho^{-1} x_4 - a_1 a_5 u + a_1 \dot{u} \end{aligned} \tag{29}$$

and its corresponding inverse:

$$\begin{aligned} x_1 &= \eta_1 \\ x_2 &= \eta_2 \\ x_3 &= \frac{1}{a_1}(\eta_3 + a_5 \eta_2 - a_1 u) \\ x_4 &= \frac{\rho}{a_1}(\eta_4 + a_5 \eta_3 = a_1 \dot{u}). \end{aligned} \tag{30}$$

The Jacobian matrix of the state coordinate transformation (29) is given by

$$J = \frac{\partial \eta}{\partial x} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -a_5 & a_1 & 0 \\ 0 & a_5^2 & -a_5 a_1 & \rho^{-1} a_1 \end{bmatrix} \tag{31}$$

which is clearly nonsingular for finite ρ.

The GOCF for the flexible joint manipulator system is then obtained as

$$\begin{aligned} \dot{\eta}_1 &= \eta_2 \\ \dot{\eta}_2 &= \eta_3 \\ \dot{\eta}_3 &= \eta_4 \\ \dot{\eta}_4 &= -\frac{a_1 a_2 a_3}{\rho^2} \sin\left[\eta_1 + \frac{\rho^2}{a_1}(a_5 \eta_2 - a_1 u + \eta_3)\right] \\ &\quad - \frac{1}{\rho^2}(a_1 a_7 + a_4 a_5)\eta_2 \\ &\quad - \left(\frac{a_4}{\rho^2} + a_5 a_6\right)\eta_3 - (a_5 + a_6)\eta_4 \\ &\quad + \frac{a_1}{\rho^2}(a_4 - a_1)u + a_1 a_6 \dot{u} + a_1 \ddot{u} \\ y &= \eta_1 \end{aligned} \tag{32}$$

[1] This remark was kindly provided by a reviewer.

from which an input–output representation of the manipulator system is readily obtained as

$$\begin{aligned} & y^{(4)} + (a_5 + a_6) y^{(3)} + \left(\frac{a_4}{\rho^2} + a_5 a_6\right) y^{(2)} + \\ & \quad \frac{1}{\rho^2}(a_1 a_7 + a_4 a_5) y^{(1)} + \frac{a_1 a_2 a_3}{\rho^2} \cdot \\ & \quad \sin\left[y + \frac{\rho^2}{a_1}\left(a_5 y^{(1)} - a_1 u + y^{(2)}\right)\right] - \\ & \quad \frac{a_1}{\rho^2}(a_4 - a_1) u - a_1 a_6 \dot{u} - a_1 \ddot{u} = 0 \end{aligned} \tag{33}$$

B. Dynamically Controlled Output Tracking Tasks for Flexible Joint Manipulators

In this section we shall present both the smooth output tracking regulator for the flexible joint robot and the Variable Structure Control scheme presented in the previous section. In both cases we shall also illustrate the stabilization properties of the regulator towards prespecified constant equilibrium points.

1) Reference Output Tracking via Exact Error Dynamics Linearization: Let $y_L(t)$ be a desired reference trajectory for the link angular position $z = q$. One can compute, from the dynamical equations (26), the corresponding desired reference trajectory $y_R(t)$ for the motor position as

$$y_R(t) = \rho^2\left(D y_L^{(2)}(t) + B y_L^{(1)}(t) + mgL \sin y_L\right) + y_L(t). \tag{34}$$

From expression (34), it is straightforward to compute the corresponding required time derivatives $y_R^{(1)}(t)$, $y_R^{(2)}(t)$, $y_R^{(3)}(t)$, constituting the vector $\xi_R(t)$, defined in (13), as well as the fourth time derivative $y_R^{(4)}(t)$, which will be used in the dynamical controller (10).

Defining the tracking error as $e = y - y_R(t)$, one obtains, based on the results of Section II and expression (11), the system of differential equations describing the tracking error dynamics as

$$\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= e_3 \\ \dot{e}_3 &= e_4 \\ \dot{e}_4 &= -\frac{a_1 a_2 a_3}{\rho^2} \sin\Bigg(e_1 + y_R + \frac{\rho^2}{a_1}\Big[a_5\Big(e_2 + y_R^{(1)}\Big) \\ &\qquad - a_1 u + e_3 + y_R^{(2)}\Big]\Bigg) \\ &\quad - \frac{1}{\rho^2}(a_1 a_7 + a_4 a_5)\Big(e_2 + y_R^{(1)}\Big) \\ &\quad - \left(\frac{a_4}{\rho^2} + a_5 a_6\right)\Big(e_3 + y_R^{(2)}\Big) \\ &\quad - (a_5 + a_6)\Big(e_4 + y_R^{(3)}\Big) \\ &\quad + \frac{a_1}{\rho^2}(a_4 - a_1) u + a_1 a_5 \dot{u} + a_i \ddot{u} \\ e &= e_1. \end{aligned} \tag{35}$$

Exact linearization of the tracking error dynamics (35) can now be accomplished by equating the last differential equation in (35) to a linear time invariant expression in the error coordinates. This is equivalent to having the close loop error dynamics obey $e^{(4)} = v$, with:

$$v = -\gamma_4 e_4 - \gamma_3 e_3 - \gamma_2 e_2 - \gamma_1 e_1 \tag{36}$$

An asymptotically stable error response is thus easily designed by an elementary pole placement strategy on a fourth order controllable system expressed in Brunovsky canonical form.

Using original state coordinates one can write the nonlinear time-varying dynamical regulator equation as

$$\begin{aligned}\ddot{u} + (\gamma_4 - a_5)\dot{u} + \left(a_5^2 - a_5\gamma_4 + \gamma_3 - \frac{a_1}{\rho^2}\right)u = \\ -\frac{\gamma_1}{a_1}x_1 + \left(\frac{a_7}{\rho^2} - \frac{\gamma_2}{a_1} + \frac{a_5\gamma_3}{a_1} + \frac{a_5^3}{a_1} - \frac{a_5^2\gamma_4}{a_1}\right)x_2 \\ + \left(\frac{a_4}{\rho^2} - \gamma_3 - a_5^2 + a_5\gamma_4\right)x_3 \\ + \left(\frac{a_5}{\rho} + \frac{a_6}{\rho} - \frac{\gamma_4}{\rho}\right)x_4 \\ + \frac{a_2 a_3}{\rho^2}\sin\left(x_1 + \rho^2 x_3\right) \\ + y_R^{(4)} + \gamma_4 y_R^{(3)} + \gamma_3 y_R^{(2)} + \gamma_2 y_R^{(1)} + \gamma_1 y_R. \qquad (37a)\end{aligned}$$

Remark: For any $\rho \neq 0$, the obtained dynamical feedback controller for the flexible joint manipulator does not exhibit any singularities whatsoever. It should be pointed out that, in general, the GOCF approach for synthesis of dynamical controller does suffer from difficulties related to *impasse points, nonminimum phase regions,* and other singularities. The impasse points arise from the impossibility of explicitly solving for the highest derivative of the control in the linearizing equation. The worst case probably happens when the equilibrium point lies on a hypersurface of impasse points. A second difficulty is usually represented by having the equilibrium point in a *nonminimum phase region,* or else when the controlled trajectories visit such instability regions. The usual remedy for these situations has been extensively explored by Fliess and his co-workers from the perspective of discontinuous control actions [31], [41].

2) Simulation Results for the Stabilization and Tracking Tasks for Flexible Joint Manipulator by Means of a Smooth Linearizing Dynamical Controller: Simulations were performed for a flexible joint robot with the following parameters:

$$\begin{aligned} a_1 &= 3.333\ \left(\mathrm{m^2Kg}\right)^{-1} \\ a_2 &= 1.0\ \left(\mathrm{m^2Kg}\right)^{-1} \\ a_3 &= 5.0\ \mathrm{N.m} \\ a_4 &= 4.33\ \left(\mathrm{m^2Kg}\right)^{-1} \\ a_5 &= 0.333\ \mathrm{s^{-1}} \\ a_6 &= 0.1\ \mathrm{s^{-1}} \\ a_7 &= -0.233\ \mathrm{s^{-1}} \\ K_s &= 100\ \mathrm{N/m\ rad^{-1}}. \end{aligned}$$

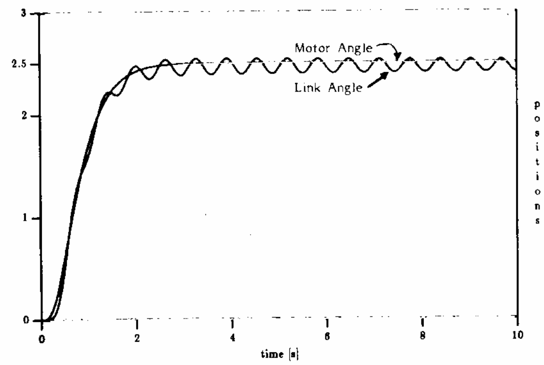

Fig. 2. Stabilization with exactly linearizing dynamical controller. Motor and link responses.

The first task for the proposed smooth dynamical controller was that of a stabilization problem by dynamical exact linearization around a constant equilibrium point. The desired angular position of the link was set to $z = q = 1.5$ rad. The corresponding desired angular position for the motor was then computed to be $y_R = 2.529$ rad and used as a reference value for q_m. The constant steady state torque was $U = 2.992$ Nm and the closed loop poles were chosen at: -10, -10, -3, -3 $\left[s^{-1}\right]$. All initial conditions, for the system and the dynamical controller, were chosen to be zero.

The link angular position response and the motor position response are both shown in Fig. 2. The response of the link exhibits a slow decaying oscillation about the required constant equilibrium trajectory with maximum deviation of about 4.1% about the desired value. The corresponding motor position response is absolutely smooth and it goes to the desired reference value approximately within 2 s. The dynamically generated control input torque is shown in Fig. 3. Notice that inversion of the plant to compute the desired motor trajectory cannot be accomplished for the unit step desired link angular position due to the impulsive character of the required time derivatives of the step reference trajectory. The absence of impulsive controls in our simulations, together with the low stiffness that we have used, account for this relatively poor performance. In the next section we show that this response is substantially improved by the use of the variable structure control strategy.

Simulations were carried out for the tracking problem using a desired sinusoidal angular position response for the link. The desired reference output $y_L(t)$ was chosen as

$$y_L(t) = \frac{\pi}{3}\sin(\pi t).$$

The corresponding reference trajectory $y_R(t)$ for the motor angular position, and the required time derivatives $y_R^{(1)}(t)$, $y_R^{(2)}(t)$, $y_R^{(3)}(t)$, $y_R^{(4)}(t)$, were then computed using $y_L(t)$ and equation (34). The closed loop poles for the linearized error dynamics were set at the same values as in the simulations used in the previous stabilization example.

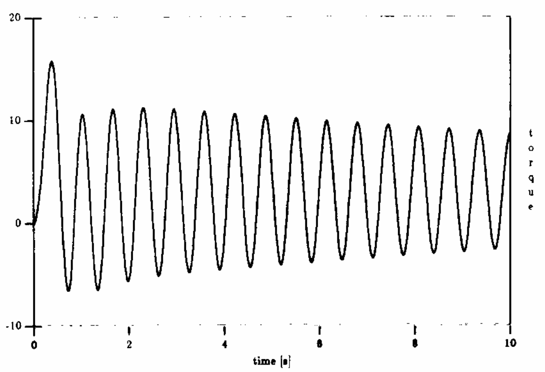

Fig. 3. Dynamically generated exactly linearizing input torque for the stabilization task.

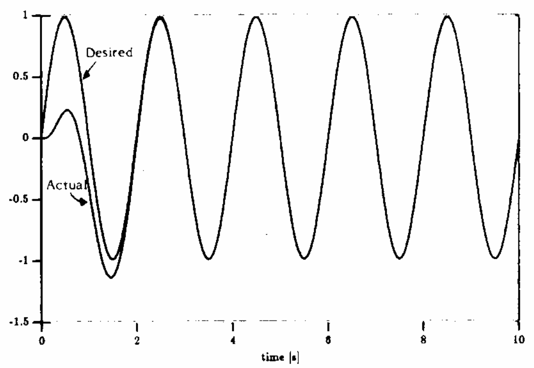

Fig. 5. Desired and actual motor angular position for sinusoidal trajectory tracking with error dynamics exactly linearizing controller.

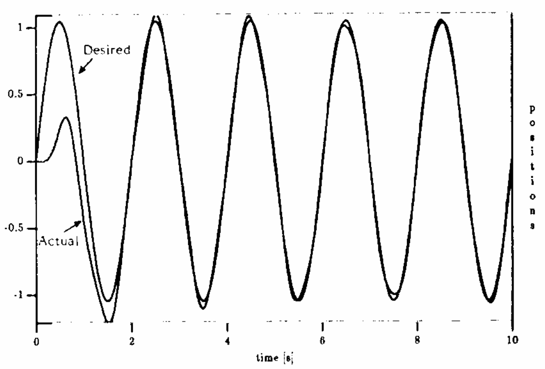

Fig. 4. Desired and actual link's angular position for sinusoidal trajectory tracking with error dynamics exactly linearizing controller.

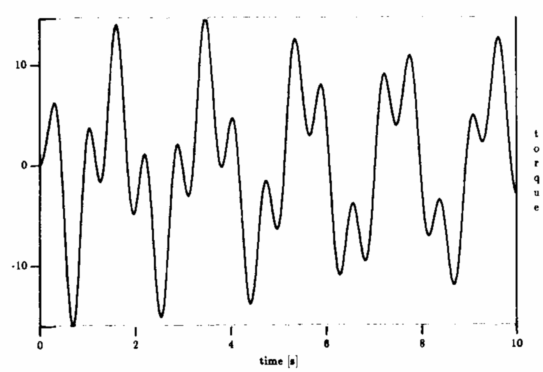

Fig. 6. Dynamically generated exactly linearizing input torque for the sinusoid tracking task.

Fig. 4 shows the desired sinusoidal link angular trajectory $y_L(t)$ and the actual dynamically controlled response for the link angular position $z(t)$. The agreement is remarkably close with slowly decaying oscillating error. Fig. 5 depicts the desired (computed) motor angular position and the resulting dynamically controlled response for the motor position $x_1(t)$. The tracking error asymptotically converges to zero in approximately 3 s. Fig. 6 represents the dynamically generated input torque for the exact error dynamics linearization controller. Notice that the performance of the tracking controller is superior than that obtained in the output stabilization case. This is achieved due to the computability of the required time derivatives of the desired output reference, something that we did not have for the stabilization about a constant (step) output reference due to the impulsive character of such derivatives.

3) Discontinuous Output Tracking Controller Via Variable Structure Error Dynamics Linearization: According to Section II-C a dynamical variable structure controller is synthesized by first forming an auxiliary output equation ω, as prescribed by (20) of the form:

$$\omega(t) = e_4(t) + m_2 e_3(t) + m_1 e_2(t) + m_0 e_1(t). \tag{37b}$$

In original coordinates such an auxiliary output function is simply given by

$$\begin{aligned}\omega = {} & a_1\dot{u} + a_1(m_2 - a_5)u + \left(a_5^2 - a_5 m_2 + m_1\right)x_2 \\ & + a_1(m_2 - a_5)x_3 + \frac{a_1}{\rho}x_4 + m_0 x_1 \\ & - m_0 y_R(t) - m_1 y_R^{(1)}(t) - m_2 y_R^{(2)} - y_R^{(3)}\end{aligned} \tag{38}$$

one obtains, after imposing on ω the discontinuous dynamics given in (17), a time-varying differential equation for the dynamical controller, synthesizing the control input torque u, in terms of the desired reference input signal $y_R(t)$, its time derivatives $y_R^{(i)}(t)$ $(i = 1, 2, 3, 4)$, and the original state variables of the manipulator:

$$\begin{aligned}& \ddot{u} + (m_2 - a_5)\dot{u} + \left(-\frac{a_1}{\rho^2} + a_5^2 - a_5 m_2 + m_1\right)u \\ & = \left(\frac{a_7}{\rho^2} - \frac{m_0}{a_1} + \frac{m_1 a_5}{a_1} + \frac{a_5^3}{a_1} - \frac{a_5^2 m_2}{a_1}\right)x_2 \\ & \quad + \left(\frac{a_4}{\rho^2} - m_1 - a_5^2 + a_5 m_2\right)x_3 + \left(\frac{a_5}{\rho} + \frac{a_6}{\rho} - \frac{m_2}{\rho}\right)x_4 \\ & \quad + \frac{a_2 a_3}{\rho^2}\sin(x_1 + \rho^2 x_3) - \frac{\mu}{a_1}[\omega(t) + \Omega \operatorname{sgn} \omega(t)] \\ & \quad + \frac{1}{a_1}\left(m_2 y_R^{(3)} + m_1 y_R^{(2)} + m_0 y_R^{(1)} + y_R^{(4)}\right).\end{aligned} \tag{39}$$

Since the flexible joint manipulator is relative degree 2 with respect to the chosen output x_1, the linearization of the GOCF by means of a variable structure controller strategy involves a second order dynamical system for synthesizing the input torque u. This checks with the form of equation (39). Notice, then that the controller output u, given by the solution of (39), is a *continuous* signal with continuous first order time derivative and discontinuous second order time derivative. The smoothness of both the output response x_1 and the input torque u, should be evident from (39).

4) Simulation Results for the Stabilization and Tracking Tasks by Means of a Dynamical Variable Structure Control: Simulations were performed for a flexible joint manipulator with the same parameters previously described in the simulation example of Section III-B-2). In this case the dynamical variable structure controller, given in equation (39), was used. The parameters corresponding to this discontinuous controller were chosen as

$$\mu = 1.0\ \mathrm{s}^{-1}; \qquad \Omega = 5.0; \qquad m_0 = 210;$$
$$m_1 = 107; \qquad m_2 = 18.$$

The chosen values for the coefficients m_0, m_1, m_2, correspond to poles located at -5, -6, and $-7\,[\mathrm{s}^{-1}]$. A stabilization task, identical to that described in the simulation example of Section III-B-2), was used to evaluate the performance of the dynamical variable structure feedback policy. Fig. 7 shows the link and motor angular position responses, while Fig. 8 represents the dynamically generated control input torque. Fig. 9 represents the time response of the auxiliary output function ω exhibiting the induced sliding regime. Link oscillations were successfully eliminated and the required constant desired equilibrium values were exactly achieved by the motor and the link responses. The steady state difference between the link and motor positions, due to the joint flexibility, precisely coincides with the value of the computed steady state difference between these coordinates, which equals 0.029 rad. Notice that the performance of the dynamical variable structure controller scheme is vastly superior to that obtained by the exact error dynamics linearization approach of Section III-B-2) in spite of the fact that impulses generated by the output reference step function time derivatives were not used in the controller design. Furthermore, the dynamically synthesized input torque *does not exhibit a chattering behavior,* traditionally associated with variable structure control schemes. Even though an underlying high frequency switching strategy is being used to generate the control input, the flexibility of the joint does not induce high frequency oscillations on the output response.

Simulations were carried out for the tracking problem using a desired sinusoidal angular position response for the link identical to that used in Section III-B-2). The closed loop poles for the ideal linearized error dynamics were set at the same locations as in the stabilization experiment described above.

Fig. 10 shows the desired sinusoidal link angular trajectory $y_L(t)$ and the actual dynamically sliding mode controlled response for the link angular position $z(t)$. The agreement between the desired and the actual link position trajectories is practically perfect after 3 s. Fig. 11 depicts the desired (computed) motor angular position and the resulting dynamically controlled response for the motor position $x_1(t)$. The tracking

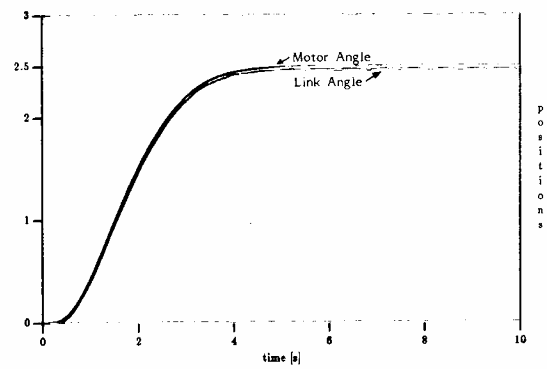

Fig. 7. Stabilization with variable structure dynamical controller. Motor and link responses.

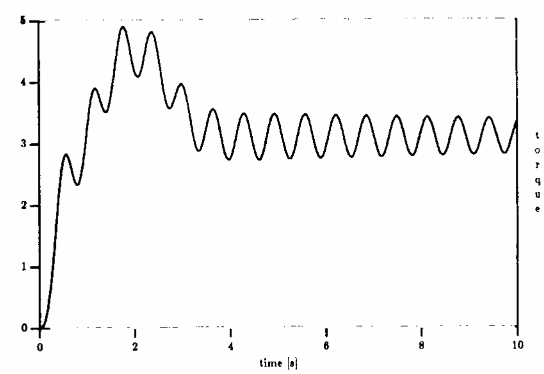

Fig. 8. Sliding mode dynamically generated input torque for the stabilization task.

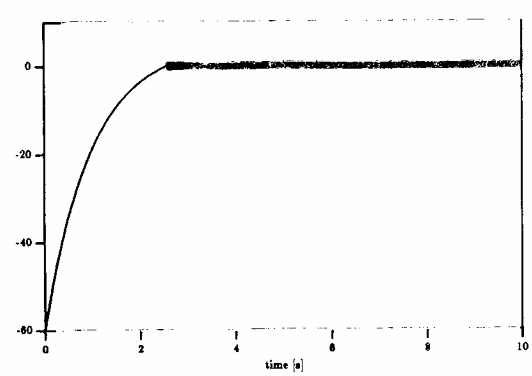

Fig. 9. Time response of auxiliary output function undergoing sliding regime.

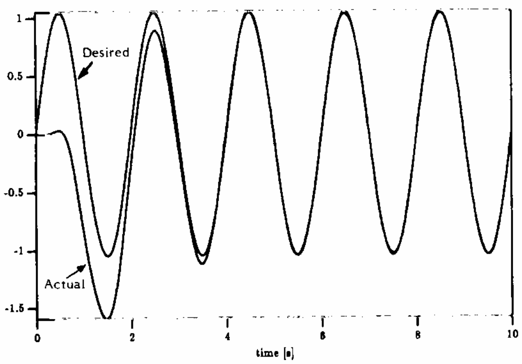

Fig. 10. Desired and actual link's angular position for sinusoidal trajectory tracking with dynamical variable structure controller.

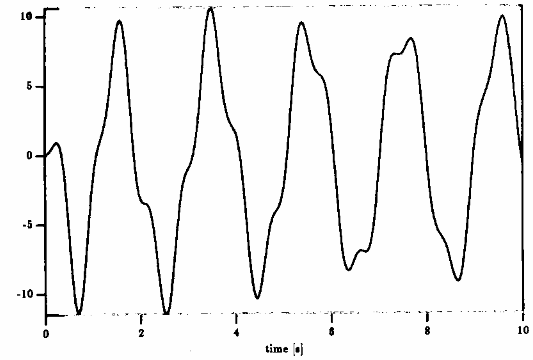

Fig. 12. Sliding mode dynamically generated input torque for the sinusoid tracking task.

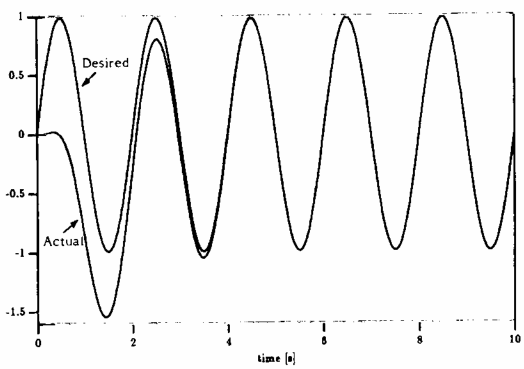

Fig. 11. Desired and actual motor angular position for sinusoidal trajectory tracking with dynamical variable structure controller.

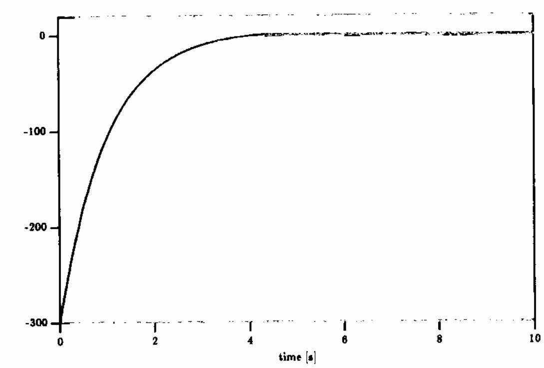

Fig. 13. Time response of auxiliary output function undergoing sliding regime.

error asymptotically converges to zero in approximately 3.5 s. Fig. 12 represents the dynamically generated input torque for the variable structure controller exhibiting a smooth profile without any noticeable chattering whatsoever. Fig. 13 represents the time response of the auxiliary output function ω exhibiting the induced sliding regime.

IV. Conclusion

Dynamical controllers accomplishing stabilization and asymptotic reference output tracking are readily obtainable for nonlinear systems described in Fliess' generalized observability canonical form. Such a canonical form naturally leads to a dynamical linearizing controller that asymptotically zeroes the output tracking error with prescribed transients. The approach also allows for the design of highly efficient dynamical variable structure control schemes on the basis of finite time nulling of an auxiliary output function. This auxiliary function is defined in terms of the tracking error and a finite number of its time derivatives expressible in terms of the original state variables. The resulting ideal sliding dynamics induces an asymptotic stabilization of the output tracking error function with arbitrarily assigned eigenvalues. Aside from the well-known robustness properties, which are implicit in every sliding mode control scheme, the obtained discontinuous controller design exhibits several interesting advantages. These advantages are related to the possibilities of obtaining a high level of smoothness in the output error response, as well as a substantially reduced chattering in the control input signals (i.e., effective chattering reduction for, both, the input and the output signals is entirely feasible, without resorting to the well known high gain saturating amplifier alternative).

In the method proposed here the discontinuities, associated with the underlying sliding mode control actions, take place on the dynamical controller's state space and not on the systems inputs or state variable trajectories. This feature, aside from generating sufficiently smooth inputs, greatly facilitates the implementation aspects of the sliding mode controller. The required electronic hardware may even be synthesized with solid state analog devices, capable of nonlinear algebraic operations, and power transistor switches. If digital implementation is preferred, some limitations may be found due to the significant amount of computation required to implement, on line, the proposed dynamical controller. Research is proceeding along these avenues with due consideration of the neural network alternative.

An application was presented in this paper, for the stabilization and tracking problems for flexible joint robotic manipulators. Encouraging simulation results were obtained, dealing with the smooth control of angular position stabilization and sinusoidal signal tracking in a single-link elastic joint manipulator.

An important issue that was not considered in this preliminary study refers to the inescapable fact of imprecisely known parameters in the system dynamics. Adaptation schemes for the GOCF is an important open research topic that deserves to be looked from the perspective of differential algebra. As additional topics for further research, the proposed dynamical controller could be implemented in an actual laboratory manipulator using nonlinear analog electronics. The technique presented could be used in the control of a different class of mechanical systems as well. Interesting difficulties may be encountered for instance in the application of the described approach for multivariable flexible joint manipulators and some other mechanical systems such as helicopters and pneumatic motors.

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