The Differential Algebraic Approach in Nonlinear Dynamical Feedback Controlled Landing Maneuvers

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Abstract—**A differential algebraic approach is proposed for the synthesis of a dynamical feedback controller regulating a spacecraft smooth descent toward the surface of a planet which exhibits nonnegligible atmospheric resistance. An exact linearization-based controller is synthesized using Fliess' generalized observability canonical form of the controller system. The smooth controlled trajectory is regulated by means of assumed amplitude modulated thrusting capabilities of the spacecraft. The robustness of the regulator is tested in the presence of significant unmodelled spatial changes in the coefficient of atmospheric resistance. A simulation example is provided.**

I. Introduction

The problem of soft landing on a planet has been studied mainly from an optimal control viewpoint. Starting from Miele's solution [1] in 1960, using calculus of variations, the problem gained interest from both theoretical and practical viewpoints. A minimum-time approach was proposed by Meditch [2] in the early 1960's. In his studies, Meditch showed that the minimum-time and the minimum-fuel landing problems are completely equivalent. The optimal control approach was also used by Flemming and Rishel to illustrate Pontryaguin's minimum principle in their book [3]. Cantoni and Finzi [4] also contributed significantly to the problem by further modifying Meditch's solution. Recently, a sliding mode control approach was proposed by Sira-Ramirez in [5], with various practical alternatives for the final touchdown stage. In [5], a suitable sliding manifold is synthesized which induces an exponentially stable behavior in the ideal sliding trajectories associated to average height and average vertical speed variables.

In a series of outstanding recent articles, Fliess [6]–[9] has

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introduced a new and general approach based on *differential algebra* for the study of linear and nonlinear lumped, or distributed controlled dynamical systems. A number of long standing problems in automatic control theory, such as feedback decoupling, invertibility, model matching, and realization, have been conceptually clarified and generalized by Fliess in a powerful and most elegant manner.Crucially based on the extension to *differential fields* of the *theorem of the primitive element* [10], any controlled dynamical system, described by a set of forced ordinary differential equations, was shown to possess a *generalized controller canonical form* (GCCF) depending on the input and a finite number of its time derivatives. In [9] a *generalized observability canonical form* (GOCF) was also introduced which is closely related to previous results. Such canonical form is obtainable by means of output-based state coordinate transformations which are, in general, control-dependent and, possibly, including a finite number of the control input time derivatives. As a direct consequence of this result, the problem of *output feedback linearization* of a controlled dynamical system is always trivially solvable, in a local manner, using nonlinear, possibly time-varying, dynamical feedback. The linearizing dynamical compensator is clearly suggested by the canonical form itself. However, for systems with constant operating points, the asymptotic stability of the linearized closed-loop dynamics, around such an equilibrium point, crucially depends on the *minimum-phase* character of the nonlinear GOCF about such an equilibrium.

In this note, a nonoptimal dynamical feedback solution is proposed for the problem of soft controlled landing on the surface of a nonatmosphere-free planet.The proposed scheme is based on the specification of a nonlinear amplitude modulated (AM) dynamical feedback controller, obtained on the basis of Fliess' generalized observability canonical form [9] of the dynamical model describing the smooth controlled descent over the planet's surface.

It should be pointed out that the controlled soft descent problem can also be treated from the perspective of *discontinuous* dynamical feedback regulation using the differential algebraic approach. Indeed, our AM solution constitutes the basis for a dynamical feedback controller of the On-Off pulse width modulated (PWM) type, should a sampled "bang-bang" policy need be imposed on the main thruster firings. Within this more realistic setting, our obtained amplitude modulated dynamical controller represents a truly feedback strategy for the specification of the required *duty ratio* function associated to the PWM option. Arbitrarily close approximations to the smoothly regulated responses obtained here are still possible, provided a sufficiently high sampling frequency is imposed on the actuator. The details of this extension will be fully reported in a forthcoming article [11].

Section II presents the main results of this article in relation to the soft landing maneuver via a dynamical feedback linearizing controller of the smoothly regulated system. Simulations are presented that illustrate the performance of the proposed controller under the assumption of a constant atmospheric resistance coefficient. The controller is also evaluated in the presence of significant unmodelled changes in such a coefficient. Section III is devoted to the conclusions. The Appendix presents Fliess' derivation of the GCCF and the GOCF and their associated exact dynamical feedback linearization result.

II. A Dynamical Feedback Solution for Soft Controlled Landing of a Spacecraft

A. The Dynamical Model of a Soft Controlled Landing Including Atmospheric Resistence

Consider the nonlinear dynamical model describing the vertical descent, including the spacecraft mass behavior, of a thrust controlled vehicle, with amplitude modulation capabilities in the braking force, attempting a regulated landing on the surface of a planet of gravity acceleration g and nonnegligible atmospheric resistance force opposing the vertical downwards motion (see [12, p. 4])

$$\frac{dz_1}{dt} = z_2$$
$$\frac{dz_2}{dt} = g - \left(\frac{\gamma}{z_3}\right) z_2^2 - \left(\frac{\sigma\alpha}{z_3}\right) u \tag{2.1}$$
$$\frac{dz_3}{dt} = -\alpha u$$

where z_1 is the position (height) on the vertical axis, chosen here to be positively oriented downwards (i.e., $z_1 < 0$, for actual positive height), z_2 is the downwards velocity (see Fig. 1) and z_3 represents the combined mass of the vehicle and the residual fuel. The function u is a continuous piecewise smooth control function with values in the closed set $[0, 1]$ regulating, in a smooth manner, the constant rate of ejection per unit time α and effectively acting as a control parameter. The constant σ represents the relative ejection velocity of the gases in the thruster. Thus, $\sigma\alpha$ is the maximum thrust of the braking engine, while γ is a positive quantity representing the atmospheric resistance coefficient.

The control signal u is assumed to be synthesized on the basis of an *amplitude modulated* feedback strategy, as $u = \mu(x(t))$, synthesized from knowledge of the measured state vector $x(t)$. As a function of the state x, the control function μ is continuous piecewise smooth function constrained within the bounds $0 < \mu(x(t)) < 1$. The feedback synthesis problem is then defined as the problem of specifying a suitable feedback control law μ.

A soft landing on the surface $z_1 = 0$ may be seen as a particular case of a controlled descent toward a sustained hovering about certain prespecified height $z_1 = K$. Usually, the landing maneuver entitles a regulated descent toward a small height (typically 1 m, or so, i.e., $K = -1$) on which a short hovering takes place before the main thruster is safely shut off. The final touchdown stage is actually a free fall toward the surface from the small hovering height. Taking the output function of the system as $y = h(x) = z_1 - K$, the problem of sustained hovering is translated into the problem of zeroing the output y that one can associate to the nonlinear system (2.1).

Remark 1: It is evident from the dynamical system equations (2.1) that the maximum value of the downwards velocity z_2 takes place only under free fall (i.e., uncontrolled) conditions ($\mu = 0$, $z_3 = \text{constant} = M$). This maximum velocity value is precisely given by $(gM/\gamma)^{1/2}$. In such a case the downwards spacecraft acceleration is zero. A braking maneuver toward a sustained hovering, starting from free fall conditions, entitles a negative controlled acceleration until reaching zero downwards velocity at the prespecified hovering height $z_1 = K$. At this point, the controlled acceleration should also become zero. It follows that, during the controlled descent, the downwards acceleration is always bounded above by zero.

B. Nonlinear Dynamic Feedback Controller Design for the Soft Controlled Landing of an Amplitude Modulated Thrusted Spacecraft

We proceed to specify the *generalized observability canonical form* (see [9]) of system (2.1) which allows us to derive a nonlinear dynamical feedback controller for the smooth descent maneuver.

It is easy to verify that $q_1 = z_1 - K$ is a *differential primitive element* that allows one to also write the AM model (2.1) in a GOCF of the form (A.5) with $\nu = 1$. Thus, a control-dependent state coordinate transformation of the AM controlled system (2.1)

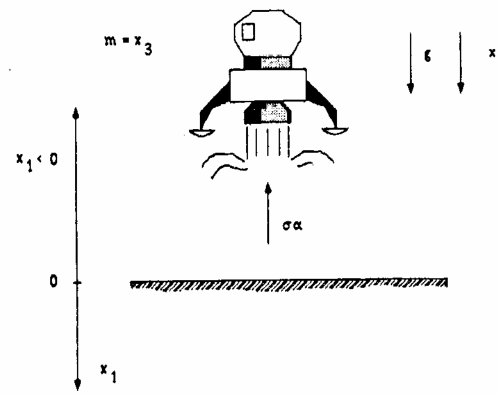

Fig. 1. Vertically controlled descent on the surface of a planet.

given by:

$$q_1 = z_1 - K, \quad q_2 = z_2, \quad q_3 = g - \frac{\gamma z_2^2 + \sigma\alpha\mu}{z_3}$$
$$z_1 = q_1 + K, \quad z_2 = q_2, \quad z_3 = \frac{\gamma q_2^2 + \sigma\alpha\mu}{g - q_3} \quad (2.2)$$

yields the following transformed system in GOCF:

$$\dot{q}_1 = q_2$$
$$\dot{q}_2 = q_3$$
$$\dot{q}_3 = -(g - q_3)\left[\frac{2\gamma q_2 g + \sigma\alpha\dot{\mu}}{\gamma q_2^2 + \sigma\alpha\mu}\right] + (2\gamma q_2 - \alpha\mu)\left[\frac{(g - q_3)^2}{\gamma q_2^2 + \sigma\alpha\mu}\right]. \quad (2.3)$$

Exact feedback linearization to *Brunovsky's observer canonical form* is then accomplished by the following input derivative-dependent control space coordinate transformation:

$$v = -(g - q_3)\left[\frac{2\gamma q_2 g + \sigma\alpha\dot{\mu}}{\gamma q_2^2 + \sigma\alpha\mu}\right] + (2\gamma q_2 - \alpha\mu)\left[\frac{(g - q_3)^2}{\gamma q_2^2 + \sigma\alpha\mu}\right]. \quad (2.4)$$

Such a control space dynamical feedback transformation evidently yields

$$\dot{q}_1 = q_2$$
$$\dot{q}_2 = q_3$$
$$\dot{q}_3 = v$$
$$y = q_1. \quad (2.5)$$

The exactly linearized system (2.5) is now easily stabilized around the origin of transformed coordinates by a standard linear state-feedback controller of the following form:

$$v = -\alpha_1 q_1 - \alpha_2 q_2 - \alpha_3 q_3. \quad (2.6)$$

In other words, by suitably choosing the constant coefficients α_1, α_2, and α_3, the dynamical feedback controller synthesizing the *computed feedback control law* (henceforth denoted by $\hat{\mu}$) accomplishes, within nonsaturating conditions for the actuator's control values, any desirable exponential rate of decay on the height, vertical velocity, and vertical acceleration variables. Such a dynamical feedback controller, yielding the computed feedback control law $\hat{\mu}$, is immediately obtained from (2.5) and (2.6) as

$$\frac{d}{dt}\hat{\mu} = \frac{\gamma q_2^2 + \sigma\alpha\hat{\mu}}{\sigma\alpha(g - q_3)}\left\{\alpha_1 q_1 + \alpha_2 q_2 + \alpha_3 q_3 + (2\gamma q_2 - \alpha\hat{\mu})\left[\frac{(g - q_3)^2}{\gamma q_2^2 + \sigma\alpha\hat{\mu}}\right]\right\} - \frac{2\gamma q_2 g}{\sigma\alpha}. \quad (2.7)$$

Notice that no singularity is implied by the presence of the factor $(g - q_3)^{-1}$ in (2.7) due to the established negativity of the vertical acceleration q_3 during the descent maneuver. In original coordinates, the dynamical feedback controller is given by:

$$\frac{d}{dt}\hat{\mu} = \frac{z_3}{\sigma\alpha}\left[\alpha_1(z_1 - K) + \alpha_2 z_2 + \alpha_3\left(g - \frac{\gamma z_2^2 + \sigma\alpha\hat{\mu}}{z_3}\right) + (2\gamma z_2 - \alpha\hat{\mu})\left(\frac{\gamma q_2^2 + \sigma\alpha\hat{\mu}}{z_3^2}\right)\right] - \frac{2\gamma z_2 g}{\sigma\alpha}. \quad (2.8)$$

The actual feedback control function μ is obtained by properly limiting between 0 and 1 the values of the computed feedback control law $\hat{\mu}$, obtained as a solution of the nonlinear time-varying differential equation (2.8), i.e.,

$$\mu = \begin{cases} 1 & \text{if } \hat{\mu} > 1 \\ \hat{\mu} & \text{if } 0 < \hat{\mu} < 1 \\ 0 & \text{if } \hat{\mu} < 0. \end{cases} \quad (2.9)$$

A block diagram depicting the complete nonlinear feedback scheme for the dynamically controlled vertical descent is shown in Fig. 2.

C. Stability Considerations About a Sustained Hovering Condition

A hovering condition on $y = 0$ implies a zero equilibrium point for the position, vertical velocity and vertical acceleration in (2.3). As it can be easily seen from the last state equation in (2.3), the hovering condition: $q_1 = q_2 = q_3 = 0$, entitles an exponentially stable autonomous trajectory for the feedback control law μ, governed by

$$\frac{d\mu}{dt} = -\left(\frac{g}{\sigma}\right)\mu. \quad (2.10)$$

From (2.2), it follows that, under such hovering condition, the total mass behavior is governed by

$$z_3 = \left(\frac{\sigma\alpha}{g}\right)\mu. \quad (2.11)$$

This reveals the consequences of sustaining a hovering condition in an indefinite manner. Since the total spacecraft mass z_3 asymptotically converges to zero, the equilibrium point is not physically meaningful. As a matter of fact, since the fuel mass is depleted in finite time, the control model (2.1) becomes unrealistic after the fuel mass has been exhausted.

We may also establish the stability characteristics of the hovering condition by resorting to considerations about the *normal canonical form* and the associated *zero dynamics* of the AM controlled system (2.1) (see [13] and [5]).

The system (2.1) is already in *normal canonical form* and it exhibits *relative degree* [13] equal to 2, i.e., the output y must be differentiated twice, with respect to time, before the input function μ appears explicitly on the derivative. However, the stability charac-

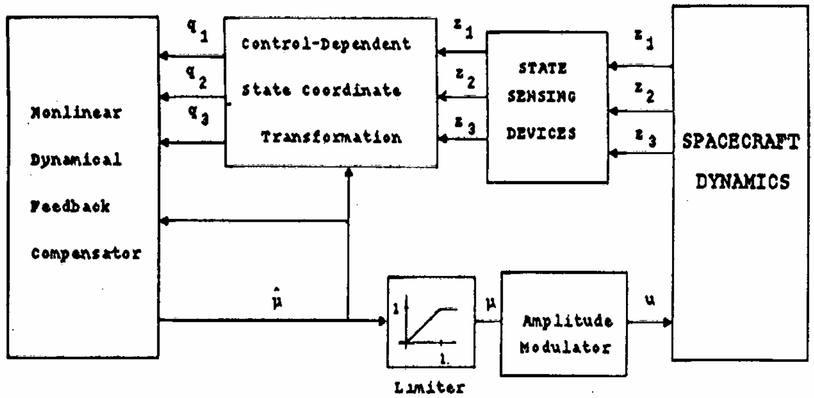

Fig. 2. Nonlinear dynamical feedback control scheme for regulation of the spacecraft landing.

teristics of the associated *zero dynamics* can be more clearly portrayed by a suitable state coordinate transformation which eliminates the control influence from the last state equation in (2.1).

Let $M := z_3(0)$ denote the total initial mass of the spacecraft at the beginning of the landing maneuver, and let $\xi_1 = z_1 - K$, $\xi_2 = z_2$ and $\eta = z_2 - \sigma \ln(z_3/M)$ define a local diffeomorphic transformation of the dynamical system state coordinates. The transformed system is now written as

$$\frac{d\xi_1}{dt} = \xi_2$$

$$\frac{d\xi_2}{dt} = g - \left[\gamma\xi_2^2 + \sigma\alpha\mu\right] M^{-1} \exp\left(-\frac{\xi_2 - \eta}{\sigma}\right) \quad (2.12)$$

$$\frac{d\eta}{dt} = g - \gamma\xi_2^2 M^{-1} \exp\left(-\frac{\xi_2 - \eta}{\sigma}\right)$$

$$y = \xi_1$$

which shows that the zero dynamics, corresponding to the evolution of the η coordinate when ξ_1 and ξ_2 are both identically zero, is governed by the autonomous differential equation

$$\frac{d\eta}{dt} = g \quad (2.13)$$

which evolves according to $\eta(t) = \eta(0) + gt = z_2(0) + gt$, i.e., the η coordinate has the interpretation of a virtual free fall vertical velocity which grows without bound. By virtue of the relation between z_3 and η, in the state coordinates transformation, the total spacecraft mass behavior z_3 is again seen to be exponentially decreasing to a nonphysically meaningful equilibrium value located at zero, while the spacecraft ideally remains in the hovering condition: $\xi_1 = 0$, $\xi_2 = 0$ (i.e., $z_1 = K$, $z_2 = 0$). This can also be seen from (2.1) and the fact that even though neither the control action nor the mass z_3 of the spacecraft remain constant on such a hovering condition, the ratio μ/z_3 does remain constant and equals the value $g/\sigma\alpha$. It follows that, under such hovering conditions, the total spacecraft mass z_3 (residual fuel mass plus spacecraft "dead" mass) obeys the same linear differential equation obeyed by the feedback control law μ

$$\frac{dz_3}{dt} = -\left(\frac{g}{\sigma}\right) z_3. \quad (2.14)$$

The total spacecraft mass z_3 thus asymptotically decreases to zero. In spite of this fact, the controlled descent toward the surface can still be practically performed at the expense of sustained fuel mass expenditure within an allowable safety limit in the hovering condition. The final free fall descent, from the hovering position, via switching off of the main engine, must be performed so as to guarantee enough residual fuel for the ascending maneuver, if any, later on (see [5]).

D. A Simulation Example

Simulations were performed for controlled landing model discussed above, with the following constant parameters

$$\sigma = 200\ [\text{m/s}] \qquad \alpha = 50\ [\text{kg/s}]$$

$$g = 3.72\ [\text{m/s}^2] \qquad \gamma = 1\ [\text{kg/m}].$$

On a planet with the given physical constants, the free fall limit velocity is 51.03 [m/s]. The three poles of the exactly linearized closed-loop system were located at $-1.2\ s^{-1}$. Fig. 3 shows the evolution of the controlled state variables x_1 and x_2 (height and vertical velocity). Fig. 4 depicts the behavior of the spacecraft mass under the designed control policy. Fig. 5 represents the time evolution of the feedback control μ during the controlled descent maneuver. Initial states were chosen, from a free fall condition, at

$$x_1(0) = -500\ [\text{m}], \quad x_2 = 51.03\ [\text{m/s}], \quad x_3(0) = 700\ [\text{kg}]$$

In order to evaluate the controller performance in the presence of unmodeled time-varying perturbations in the coefficient of atmospheric resistance, the value of γ in the dynamical system model was assumed to be a function of the height coordinate z_1 of the form $\gamma = \gamma_0 + \gamma_\delta(z_1)$, as shown in Fig. 6, with $\gamma_0 = 1$ [kg/m] taken as the nominal value of the coefficient γ to be used only in the dynamical controller equations. Figs. 7, 8, and 9 depict the behavior of the state and control input variables during the perturbed descending maneuver.

III. Conclusions

A nonoptimal dynamical feedback control scheme of the amplitude modulation-type has been presented for the soft landing of a vertically controlled vehicle on the surface of a planet provided with an atmosphere. An exact dynamical feedback linearization using Fliess' generalized observability canonical form was shown to allow an exponentially controlled descent trajectory for the height, vertical velocity, and vertical acceleration variables. The nonlinear system describing the landing dynamics was shown, by means of a suitable state coordinate transformation, to have a critical case zero dynam-

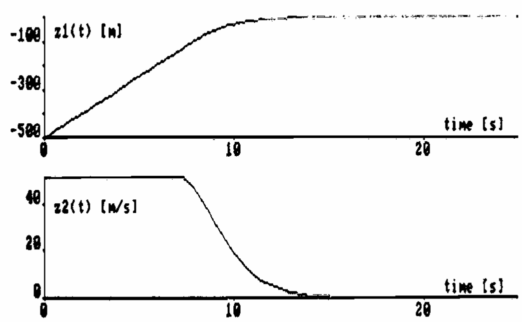

Fig. 3. Amplitude modulated controlled trajectories for dynamically feedback controlled position and vertical velocity state variables.

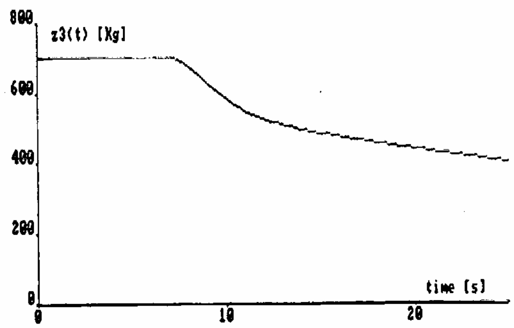

Fig. 4. Controlled behavior of combined spacecraft and residual fuel mass.

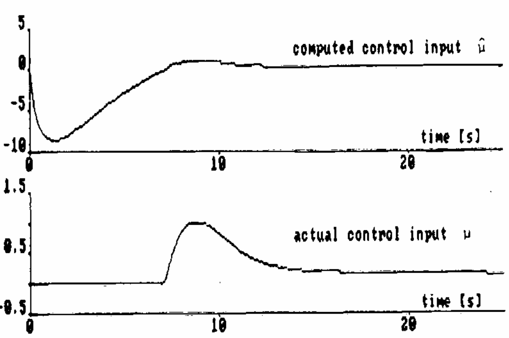

Fig. 5. Computed and actual control functions for soft landing maneuver.

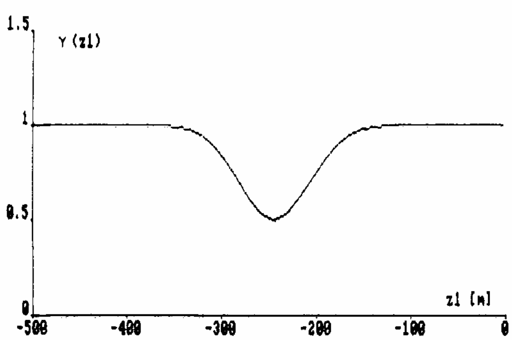

Fig. 6. Spatial variation of atmospheric resistance coefficient.

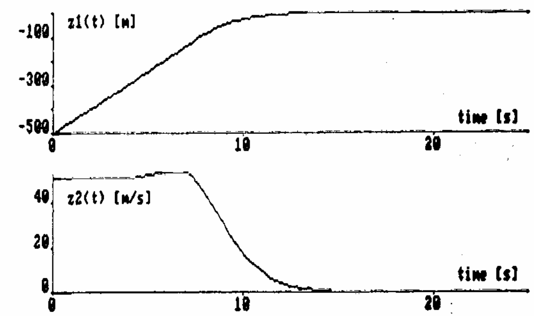

Fig. 7. Amplitude modulated controlled position and vertical velocity state variable trajectories for perturbed landing maneuver.

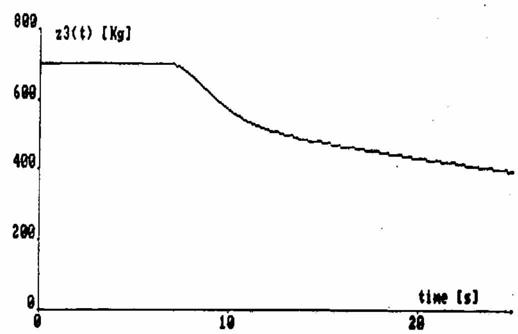

Fig. 8. Controlled behavior of combined spacecraft and residual fuel mass for perturbed landing maneuver.

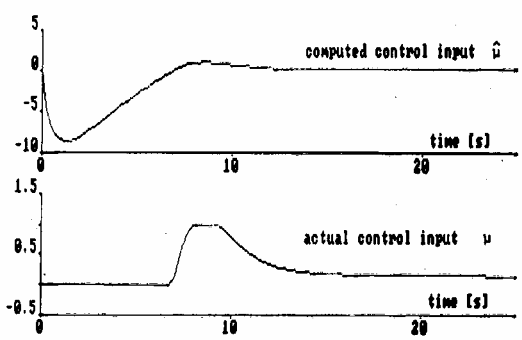

Fig. 9. Computed and actual control functions for perturbed landing maneuver.

ics—characterized by an eigenvalue at the origin. Indeed, an imposed asymptotically stable behavior in the controlled state vector not only implies infinite-time reachability of the proposed hovering height but it also entitles total residual fuel mass exhaustion and a nonphysically meaningful asymptotic equilibrium point for the total controlled spacecraft mass. In order to handle this situation, the control policy must be necessarily combined with either a free fall or, alternatively, a time-optimal strategy for the final touchdown stage of the landing maneuver from a small achieved hovering height (see also [5]).

The results of this note can also be used, or appropriately interpreted, within an *average-based* feedback design strategy in dynamical PWM controlled options for the soft landing maneuver problem. In contradistinction to the amplitude modulated option, presented in this article, the possibility of a discontinuous feedback policy is appropriate, and entirely feasible, whenever the thruster regulating actions must be restricted to take values on a binary set representing the On-Off available inputs for the main thruster braking force (see also [11]).

Appendix

The Differential Algebraic Approach to Systems Dynamics

In this Appendix, Fliess' derivation of the GCCF and GOCF for nonlinear dynamical systems is presented. The results are directly taken from Fliess [6], [9]. They are presented here for the sake of self-containment. Background on *differential algebra* can be found in Kolchin's book [10]

Fliess' Generalized Controller Canonical Form for Nonlinear Systems and Exact Dynamical Feedback Linearization [6]

Let u be a differential scalar indeterminate and let k be a *differential field*, with derivation denoted by d/dt. A *dynamics* is defined as a *finitely generated differentially algebraic extension* $K/k\langle u\rangle$ of the differential field $k\langle u\rangle$.The *input* u is said to be *independent* if u is a *differential transcendence basis* of K/k. Suppose $x = (x_1, x_2, \cdots, x_n)$ is a *nondifferential transcendence basis* of $K/k\langle u\rangle$. It follows that the derivatives dx_i/dt $(i = 1, \cdots, n)$ are $k\langle u\rangle$-algebraically dependent on the components of x. Thus, there exists exactly n polynomial differential equations of the form

$$P_i\left(dx_i/dt, x, u, du/dt, \cdots, d^{\upsilon}u/dt^{\upsilon}\right) = 0; \qquad i = 1, \cdots, n \tag{A.1}$$

implicitly describing the controlled dynamics. Under the assumption that such equations can be locally solved in *normal form*, i.e., as

$$dx_i/dt = p_i\left(x, u, du/dt, \cdots, d^{\upsilon}u/dt^{\upsilon}\right); \qquad i = 1, \cdots, n \tag{A.2}$$

one obtains a nonredundant description of the dynamics. Such is not the case if one uses a generator system of $K/k\langle u\rangle$ which strictly contains a transcendence basis. Any other transcendence basis, say $z = (z_1, z_2, \cdots, z_n)$ also qualifies as a "state" and similar expressions can be obtained for the given dynamics. The components of x are $k\langle u\rangle$-algebraically dependent upon the components of z and vice versa. Such transformations, from one state to another, involve equations dependent upon the control input u and its derivatives.

According to the *theorem of the differential primitive element* [10], there exists an element $x \in K$ such that $K = k\langle u, x\rangle$. The (nondifferential) transcendence degree n of $K/k\langle u\rangle$ is the smallest integer such that $x^{(n)}$ is $k\langle u\rangle$-algebraically dependent on $x, dx/dt, \cdots, d^{(n-1)}x/dt^{(n-1)}$. We let $x = q_1 = x_1$, $q_2 = dx/dt, \cdots, q_n = d^{(n-1)}x/dt^{(n-1)}$. It follows that $q = (q_1, \cdots, q_n)$ is also a transcendence basis of $K/k\langle u\rangle$. One, hence, obtains a nonlinear generalization of the controller canonical form

$$\begin{aligned} \frac{d}{dt}q_1 &= q_2 \\ \frac{d}{dt}q_2 &= q_3 \\ &\vdots \\ \frac{d}{dt}q_{n-1} &= q_n \\ C\left(\dot{q}_n, q, u, \dot{u}, \cdots, u^{(\upsilon)}\right) &= 0 \end{aligned} \tag{A.3}$$

where C is a polynomial with coefficients in k. If one can locally solve for the time derivative of q_n in the last equation, one obtains and explicit system of first-order differential equations, known as the *generalized controller canonical form* (GCCF):

$$\begin{aligned} \frac{d}{dt}q_1 &= q_2 \\ \frac{d}{dt}q_2 &= q_3 \\ &\vdots \\ \frac{d}{dt}q_{n-1} &= q_n \\ \frac{d}{dt}q_n &= c\left(q, u, \dot{u}, \ddot{u}, \cdots, u^{(\upsilon)}\right). \end{aligned} \tag{A.4}$$

Consider the scalar output y and let n also be the smallest integer such that $y^{(n)}$ is algebraically dependent on $y, y^{(1)}, \cdots, y^{(n-1)}, u, u^{(1)}, \cdots$, i.e., $y^{(n)} = c(y, y^{(1)}, \cdots, y^{(n-1)}, u, u^{(1)}, \cdots,)$. Let now $q_i = y^{(i-1)}$, $i = 1, \cdots, n$. Then, one can write a local state space representation in the special form of a generalization of the observability canonical form. Such a canonical form is the *generalized observability canonical form* (See [9])

$$\begin{aligned} \frac{d}{dt}q_1 &= q_2 \\ \frac{d}{dt}q_2 &= q_3 \\ &\vdots \\ \frac{d}{dt}q_{n-1} &= q_n \\ \frac{d}{dt}q_n &= c\left(q, u, \dot{u}, \ddot{u}, \cdots, u^{(\upsilon)}\right) \\ y &= q_1. \end{aligned} \tag{A.5}$$

Exact dynamic output, or state feedback linearization is simply achieved by equating the expression in the last differential equations in (A.4) and (A.5), respectively, to a (stable) linear equation in the components of q, possibly including an external reference input signal υ, as follows:

$$c\left(q, u, \dot{u}, \ddot{u}, \cdots, u^{(\upsilon)}\right) = -\alpha_1 q_1 - \alpha_2 q_2 - \cdots - \alpha_n q_n + k\upsilon. \tag{A.6}$$

The last equation implicitly defines a dynamical nonlinear state feedback law which accomplishes an exact output linearization of the nonredundant dynamics. The obtained linear system has prespecified asymptotic stability properties chosen by means of the α's.

It is evident that the nonlinear dynamical feedback linearization scheme presented above is based on exact cancellation of the nonlinear plant dynamics by means of the proposed controller. One intuitively expects that this cancellation may lead to internal instabilities in some special cases. For instance, by means of a straightforward linearization around a constant equilibrium point, if any, of the combined COCF and the proposed dynamical feedback controller, one can easily demonstrate that the dynamical linearizing controller is asymptotically stable if and only if the linearized transfer function of the given plant is *minimum phase*.

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