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Asymptotic Output Stabilization for Nonlinear Systems via Dynamical Variable-Structure Feedback Control

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Abstract. In this article, the problem of asymptotic output stabilization in nonlinear controlled systems is approached from the perspective of dynamical sliding-mode control. The proposed controller is based on Fliess's Generalized Observability Canonical Form, recently derived from the differential algebraic approach to system dynamics.

1. Introduction

In this article, a new perspective is explored for dynamical feedback controllers based on sliding regimes (Utkin [1]). The proposed approach is based on Fliess's Generalized Observability Canonical Form (GOCF), recently derived as a consequence of a series of outstanding applications of differential algebra (Kolchin [2]; Kaplansky [3]) to systems dynamics (see the seminal works of Fliess [4–6]).

Sliding-mode controllers and other classes of discontinuous feedback controllers, such as pulse width modulation (PWM) and pulse frequency modulation (PFM), for linear or nonlinear systems have been traditionally designed within the restrictive character of being memoryless or static (see Sira-Ramirez [7]–[9]). Recently, however, Fliess and Messager [10] initiated an interesting line of thought by considering the possibilities of dynamical variable-structure controllers leading to sliding regimes. Also, PWM controllers based on dynamical feedback synthesis of the associated duty ratio function were proposed by Sira-Ramirez and Lischinsky Arenas [11]. Previously, a series of applications of these ideas had been developed by the author for regulation problems in dc-to-dc power converters and some aerospace control tasks (see Sira-Ramirez [12], 13, 14]). Given the close relationship between sliding regimes and PWM controlled responses, established in [15], one is naturally led to investigate the possibilities of designing dynamical feedback regulators based on sliding modes, within an approach that is independent of the known existing relationship. Such a task is undertaken in this article, and the properties of these dynamical discontinuous controllers are studied and applied to an illustrative example.

Recently, M. Fliess has made remarkable contributions to the understanding of dynamical controlled systems from the perspective of differential algebra. His results have conceptually clarified a large number of traditional problems in linear and nonlinear controlled systems, such as realization, invertibility, model matching, and feedback decoupling, among others. A Generalized Controller Canonical Form and a Generalized Observability Canonical Form were shown to exist for a large class of nonlinear controlled systems. These canonical forms render the exact linearization problem into a trivial one, if nonlinear time-varying dynamical feedback is allowed in the control scheme (see [4]).

In section 2 of this article, a dynamical sliding-mode controller is proposed for the output-stabilization problem in nonlinear systems. The controller is synthesized on the basis of Fliess's GOCF by using an auxiliary "dummy" output function whose time derivative depends nonlinearly on the generalized phase variables and on a finite number of the control input derivatives. A sliding regime is then imposed on the zero-level set of this auxiliary output function, which results in a linear asymptotically stable behavior to zero, with arbitrarily prescribed stable eigenvalues, for the original output variable and the rest of the associated generalized phase variables. Section 3 presents an illustrative example accompanied by computer simulations, while section 4 contains the conclusions and suggestions for further work in this area.

2. A dynamical sliding-mode controller for output stabilization

2.1. Fliess's Generalized Observability Canonical Form

Consider the n-dimensional single-input single-output nonlinear system in state-space form:

$$\begin{aligned} \dot{x} &= f(x, u) \\ y &= h(x), \end{aligned} \tag{1}$$

where f is a nonlinear polynomial function of the state and input variables. It may be argued that this represents no great loss of generality, since the function f might have arisen from an elimination procedure carried out on a lower-dimensional nonlinear-state-space description of the controlled system, which included differential equations whose transcendental coefficients satisfied, in turn, algebraic differential equations (see Fliess [4]; Rubel and Singer [16]). We refer to system (1) as the pair (f, h).

It has been shown by Fliess [5] that in case the output function y is a differential primitive element for the system dynamics, there exists a set of coordinates (which we call here generalized phase coordinates) obtainable by nonlinear input-dependent state coordinate transformations—generally depending also upon a finite number of the input time derivates—in which system (1) is locally expressed in the following GOCF:

$$\begin{aligned} \dot{\eta}_1 &= \eta_2 \\ \dot{\eta}_2 &= \eta_3 \\ &\vdots \\ \dot{\eta}_{n-1} &= \eta_n \\ \dot{\eta}_n &= c(\eta, u, u^{(1)}, \ldots, u^{(\alpha)}) \\ y &= \eta_1 \end{aligned} \tag{2}$$

where α is assumed to be a positive integer. The results in this article also apply to the case in which $\alpha = 0$ (exactly input-output linearizable system), but only through the GOCF of the associated *extended system* (see Nijmeijer and van der Schaft [17]). Details are left for the reader.

If we let $\mathbf{b}^T = [0, \ldots, 0, 1]$, $\mathbf{c} = [1, 0, \ldots, 0]^T$, and $\mathbf{A}$ an $n \times n$ matrix in companion form, with zeros in the last row, then GOCF (2) is briefly expressed as $d\eta/dt = \mathbf{A}\eta + \mathbf{b}c(\eta, u, \ldots, u^{(\alpha)})$; $y = \mathbf{c}^T\eta$.

2.2. *A dynamical sliding-mode controller for output stabilization*

The following proposition is quite basic in our forthcoming developments:

Propostion 1. The on-dimensional discontinuous system

$$\dot{\omega} = -\lambda(\omega + \Omega \operatorname{sgn} \omega) \tag{3}$$

globally exhibits a sliding regime on $\omega = 0$. Here, λ and Ω are stricly positive quantities and "sgn" stands for the signum function, defined as

$$\begin{aligned} \operatorname{sgn} \omega &= +1 \text{ if } \omega > 0 \\ \operatorname{sgn} \omega &= 0 \text{ if } \omega = 0 \\ \operatorname{sgn} \omega &= -1 \text{ if } \omega < 0. \end{aligned} \tag{4}$$

Furthermore, any trajectory starting on the value $\omega = \omega(0)$, at time $t = 0$, reaches the conditions $\omega = 0$ in finite time T, given by $T = \lambda^{-1}\ln[1 + |\omega(0)|/\Omega]$.

Proof. The proof is immediate upon checking that, globally, $\omega d\omega/dt < 0$ for $\omega \neq 0$, which is a well-known condition for sliding-mode existence [1]. The second part follows from the linearity of the two intervening system "structures." ■

Let the set of real coefficients $\{m_0, \ldots, m_{n-2}\}$ be such that the following polynomial is Hurwitz:

$$s^{n-1} + m_{n-2}s^{n-2} + \ldots + m_1 s + m_0. \tag{5}$$

Consider the auxiliary output variable:

$$\omega = \sum_{i=1}^{n} m_{i-1}\, y^{(i-1)}, \text{ with } m_{n-1} = 1, \tag{6}$$

which, in terms of the generalized phase-coordinates vector η, is also simply expressed as

$$\omega = \sum_{i=1}^{n} m_{i-1}\, \eta_i =: \mathbf{m}^T\eta. \tag{7}$$

From equations (2) and (7) and the fact that $\mathbf{m}^T\mathbf{b} = 1$, it follows easily that:

$$\dot{\omega} = \mathbf{m}^T\mathbf{A}\eta + c(\eta, u, u^{(1)}, \ldots, u^{(\alpha)}). \tag{8}$$

A dynamical variable structure feedback controller is obtained if we impose on the evolution of the auxiliary output variable ω, the discontinuous dynamics considered in equation (3). From equations (3), (7), and (8), one obtains:

$$\mathbf{c}(\eta, u, u^{(1)}, \ldots, u^{(\alpha)}) = -\mathbf{m}^T [\lambda I + \mathbf{A}]\, \eta - \lambda\Omega \operatorname{sgn} \mathbf{m}^T\eta, \tag{9}$$

which is to be viewed as an implicit differential equation with discontinuous right-hand side. On each one of the regions $\omega = \mathbf{m}^T\eta > 0$, and $\omega = \mathbf{m}^T\eta < 0$, a different "structure" is valid and the implicit differential equation is to be solved for the controller u, on the basis of knowledge of η. Since ω was shown to exhibit a sliding regime on the discontinuity surface $\omega = 0$, Filippov's continuation method (see Filippov [18]) or, equivalently, the method of the equivalent control [1] is to be used for defining the idealized solutions of equation (9) on the switching manifold $\omega = 0$.

According to the method of the equivalent control, the discontinuous motions on the sliding surface $\omega = 0$ can be described, in an idealized fashion, by the invariance conditions $\omega = 0$ and $d\omega/dt = 0$. These conditions allow, in turn, the definition of a virtual control action, known as the *equivalent control*, which would be responsible for locally smoothly maintaining the evolution of the state variables on the manifold $\omega = 0$, should the motions start on this manifold. The resulting autonomous dynamics, ideally constrained to the switching manifold and "controlled" by the equivalent control, is known as the *ideal sliding dynamics*. It follows from equations (2) and (6) that such an ideal sliding dynamics is given by

$$\begin{aligned} \dot{\eta}_1 &= \eta_2 \\ \dot{\eta}_2 &= \eta_3 \\ &\vdots \\ \dot{\eta}_{n-1} &= -m_{n-2}\,\eta_{n-1} - m_{n-3}\,\eta_{n-2} - \cdots - m_0\,\eta_1 \end{aligned} \tag{10}$$

which represents an asymptotically stable motion toward the origin of generalized phase coordinates with eigenvalues uniquely specified by the prescribed set of coefficients $\{m_0, \ldots, m_{n-2}\}$. In particular, the output function $y = h(x) = \eta_1$ asymptotically converges to zero. From the invariance conditions, the equivalent control, denoted by u_{EQ}, is defined as the solution of the implicit differential equation:

$$\mathbf{c}(\eta, u, u_{EQ}^{(1)}, \ldots, u_{EQ}^{(\alpha)}) = -\mathbf{m}^T\mathbf{A}\eta;\ \text{with } \mathbf{m}^T\eta = 0 \tag{11}$$

Remark 1. Of course, on does not really need to prescribe dynamical variable-structure controllers to asymptotically stabilize the output function to zero. As a matter of fact, using normal canonical forms (see Isidori [19]), a static sliding-mode controller can always achieve such a stabilization with the aid of an auxiliary output function that involves only r output time derivatives (r being the relative degree [19] of the system (f, h)). These output derivatives, in turn, can always be asymptotically synthesized with the aid of postprocessors (see [19]; also Sira-Ramírez [9 or 20]). Furthermore, since the generalized phase variables in equation (9) are locally obtained from input-dependent state coordinate transformations—involving also a finite number of input time derivatives—it is clear that full original state feedback is required, without the benefit of being able to use postprocessors in the synthesis of the controller equations and of the auxiliary output function. However, two important advantages can be readily established about the dynamical variable-structure controller represented by equation (9). The first is the fact that the output function $y = h(x)$ asymptotically approaches zero with substantially reduced or smoothed-out "chattering." Notice that at least n integrators stand between the output variable y and the regulated chattering behavior of the auxiliary output variable ω. Therefore, with respect to the static variable-structure controller alternative based on normal canonical forms, α additional integrations of the input contribute to further smooth out the controlled output signal. Secondly, and this is possibly the most important advantage, a canonical phase variable representation for the dynamical controller indicates that the control input u is the outcome of at least α integrations performed on a nonlinear function of the discontinuous actions that lead the auxiliary output ω to zero. This means substantially smoothed control inputs that do not result in a "bang-bang" behavior for the actuator, something that cannot be avoided in the static-controller alternative. ■

3. An application example

3.1. Smoothed control landing on the surface of a non-atmosphere-free planet

The following example has been previously treated in Sira-Ramírez [20], using a memoryless sliding-mode feedback controller, and it was also treated in Sira-Ramírez [21] in the context of a dynamical feedback ON-OFF PWM controller scheme.

Consider the nonlinear dynamical model describing the vertical descent, including the spacecraft mass behavior, of a thrust-controlled vehicle attempting a regulated landing on the surface of a planet of gravity acceleration g and nonnegligible atmospheric-resistance force opposing the vertical downwards motion.

$$
\begin{aligned}
\frac{dx_1}{dt} &= x_2 \\
\frac{dx_2}{dt} &= g - \left(\frac{\gamma}{x_3}\right) x_2^2 - \left(\frac{\sigma\alpha}{x_3}\right) u, \\
\frac{dx_3}{dt} &= -\alpha u
\end{aligned}
\tag{12}
$$

where x_1 is the position (height) on the vertical axis, chosen here to be positively oriented downwards (i.e., $x_1 < 0$, for actual positive height), x_2 is the downwards velocity, and x_3 represents the combined mass of the vehicle and the residual fuel. The function u is a control function taking values in the closed interval [0, 1], regulating, in an amplitude-modulated manner, the constant rate of ejection per unit time α and effectively acting as a control parameter. The constant σ represents the relative ejection velocity of the gases in the thruster. Thus, $\sigma\alpha$ is the maximum thrust of the braking engine, while γ is a positive quantity representing the atmospheric-resistance coefficient.

A soft landing on the surface $x_1 = 0$ may be seen as a particular case of a controlled descent toward a sustained hovering at a certain prespecified height $x_1 = K$. Usually, the landing maneuver entails a regulated descent toward a small height (typically 1 m or so, i.e., $K = -1$) at which a short hovering takes place before the main thruster is safely shut off. The final touchdown stage is actually a free fall toward the surface from the small hovering height. Taking the output function of the system as $y = h(x) = x_1 - K$, the problem of sustained hovering is translated into the problem of zeroing the output y that one can associate with the nonlinear system (12).

Remark 2. It is evident from the dynamical system equations (12) that the maximum value of the downwards velocity x_2 takes place only under free fall (i.e., uncontrolled) conditions ($u = 0$, $x_3 =$ constant $= M$). This maximum velocity value is precisely given by $(gM/\gamma)^{1/2}$. In such a case, the downwards spacecraft acceleration is zero. A braking maneuver toward a sustained hovering, starting from free-fall conditions, entitles a negative controlled acceleration until zero downwards velocity is reached at the prespecified hovering height $x_1 = K$. At this point, the controlled acceleration should also become zero. It follows that, during the controlled descent, the downwards acceleration is always bounded above by zero. ■

We proceed to specify the GOCF of system (12), which allows us to derive a nonlinear dynamical variable-structure feedback controller for achieving a slow descent maneuver.

It is easy to verify that $\eta_1 = x_1 - K$ is a differential primitive element that allows one to write the model (12) in the GOCF. Thus, the control-dependent state coordinate transformation of the controlled system (12), given by

$$\begin{aligned} \eta_1 &= x_1 - K,\ \eta_2 = x_2,\ \eta_3 = g - \frac{\gamma x_2^2 + \sigma\alpha u}{x_3} \\ x_1 &= \eta_1 + K,\ x_2 = \eta_2,\ x_3 = \frac{\gamma\eta_2^2 + \sigma\alpha u}{g - \eta_3}, \end{aligned} \tag{13}$$

yields the following transformed system in GOCF:

$$\begin{aligned} \dot{\eta}_1 &= \eta_2 \\ \dot{\eta}_2 &= \eta_3 \\ \dot{\eta}_3 &= -(g - \eta_3)\left[\frac{2\eta\gamma_2 g + \sigma\alpha\dot{u}}{\gamma\eta_2^2 + \sigma\alpha u}\right] + (2\gamma\eta_2 - \alpha u)\left[\frac{(g - \eta_3)^2}{\gamma\eta_2^2 + \sigma\alpha u}\right]. \\ y &= \eta_1 \end{aligned} \tag{14}$$

The auxiliary output function ω is given, in this case, by

$$\omega = \eta_3 + m_1\eta_2 + m_0\eta_1, \tag{15}$$

with m_1 and m_0 representing design constants. A dynamical variable-structure feedback controller may then be synthesized by enforcing the dynamics described by equation (9) with the equality

$$- (g - \eta_3) \left[\frac{2\eta\gamma_2 g + \sigma\alpha\dot{u}}{\gamma\eta_2^2 + \sigma\alpha u} \right] + (2\gamma\eta_2 - \alpha u) \left[\frac{(g - \eta_3)^2}{\gamma\eta_2^2 + \sigma\alpha u} \right] + m_1\eta_3 + m_0\eta_2$$
$$= - \lambda[\eta_3 + m_1\eta_2 + m_0\eta_1 + \Omega \operatorname{sgn} (\eta_3 + m_1\eta_2 + m_0\eta_1)], \tag{16}$$

with λ and Ω being positive quantities. Equation (16) thus leads to synthesizing the computed control function (henceforth denoted by μ) as

$$\frac{d}{dt} \mu = \frac{\gamma\eta_2^2 + \sigma\alpha u}{\sigma\alpha(g - \eta_3)} \Bigg\{ (m_1 + \lambda)\eta_3 + (m_0 + \lambda m_1)\eta_2 + \lambda m_0 \eta_1$$
$$+ \lambda\Omega \operatorname{sgn} [\eta_3 + m_1\eta_2 + m_0\eta_1] + (2\gamma\eta_2 - \alpha\mu) \left[\frac{(g - \eta_3)^2}{\gamma\eta_2^2 + \sigma\alpha v} \right] \Bigg\} - \frac{2\gamma\eta_2 g}{\sigma\alpha}. \tag{17}$$

The actual control input function u is obtained by properly limiting between 0 and 1 the values of the computed control function μ, obtained as a solution of the nonlinear time-varying differential equation (17); i.e., the actual control input u is given by

$$u = \begin{cases} 1 & \text{if } \mu > 1 \\ \mu & \text{if } 0 < \mu < 1. \\ 0 & \text{if } \mu < 0 \end{cases} \tag{18}$$

Notice that no singularity is implied by the presence of the factor $(g - \eta_3)^{-1}$ in equation (17) due to the established negativity of the vertical acceleration η_3 during the descent maneuver. In original state coordinates, the dynamical variable-structure feedback controller is given by

$$\frac{d}{dt} \mu = \frac{x_3}{\sigma\alpha} \cdot \Bigg[\lambda m_0(x_1 - K) + (m_0 + \lambda m_1)x_2 + (m_1 + \lambda) \left(g - \frac{\gamma x_2^2 + \sigma\alpha v}{x_3} \right)$$
$$+ \lambda\Omega \operatorname{sgn} \left[\left(g - \frac{\gamma x_2^2 + \sigma\alpha\mu}{x_3} \right) + m_1 x_2 + m_0(x_1 - K) \right] \tag{19}$$
$$+ (2\gamma x_2 - \alpha\mu) \left(\frac{\gamma x_2^2 + \sigma\alpha\mu}{x_3^2} \right) \Bigg] - \frac{2\gamma x_2 g}{\sigma\alpha}.$$

Remark 3. The sliding regime imposed on the auxiliary output function ω, and the reaching time computed in proposition 1, are entirely valid, in terms of the system variables, as long as the actual control input function u, administered to the system, coincides with the computed control input function μ, obtained as a solution of equation (19). Notice that if physical considerations demand the existence of boundaries in the control input space, beyond which there is no actual availability of control actions—as in the case of the control input saturation constraints (equation (18))—then, generally speaking, the control input functions u and μ do not coincide in those regions of saturation and, as a consequence, the sliding mode is not locally sustained on $\omega = 0$. The saturation effects may also affect the value of the reaching time considered in proposition 1. These facts are illustrated in the computer simulations of the controlled system presented below. ■

Under sliding-mode behavior on the switching manifold, the invariance conditions $\omega = 0$ and $d\omega/dt = 0$ result in the following ideal sliding dynamics:

$$\begin{aligned} \dot{\eta}_1 &= \eta_2 \\ \dot{\eta}_2 &= -m_1\eta_2 - m_0\eta_1 \\ y &= \eta_1 \end{aligned} \tag{20}$$

which can be made exponentially asymptotically stable to $y = 0$ by appropriately choosing of the (positive) coefficients m_0 and m_1. In other words, the dynamical variable-structure feedback controller accomplishes, within nonsaturating conditions for the actuator's limiting values, any desirable exponential rate of decay on the height error η_1 and vertical velocity η_2. The equivalent control function is obtained as the solution of

$$\frac{d}{dt}\mu_{EQ} = \frac{\gamma\eta_2^2 + \sigma\alpha u_{EQ}}{\sigma\alpha(g - \eta_3)}\left\{ -m_1\eta_3 - m_0\eta_2 + (2\gamma\eta_2 - \alpha u_{EQ})\left[\frac{(g - \eta_3)^2}{\gamma\eta_2^2 + \sigma\alpha u_{EQ}}\right]\right\} - \frac{2\gamma\eta_2 g}{\sigma\alpha} \tag{21}$$

with the restriction

$$\eta_3 = -m_1\eta_2 - m_0\eta_1 \tag{22}$$

A sliding regime exists in those regions of the sliding manifold where $0 < u_{EQ} < 1$ (see [8]).

Notice that under ideal sliding conditions, both η_1 and η_2 asymptotically approach zero. Hence, by virtue of equation (15), η_3 also decreases to zero. The hovering condition is then characterized by the zero dynamics condition: $\eta_1 = \eta_2 = \eta_3 = 0$. It follows from equation (21) that the equivalent control is governed by

$$\dot{u}_{EQ} = -\frac{g}{\sigma} u_{EQ}. \tag{23}$$

By virtue of equation (13), the total mass is governed by

$$x_3 = \frac{c\sigma\omega}{g} u_{EQ}. \tag{24}$$

It is easy to see from equations (23) and (24) that the hovering conditions cannot be indefinitely sustained, since this would imply that the total mass of the spacecraft asymptotically converges to zero. This cannot physically happen, due to earlier total depletion of the fuel mass. The mathematical model therefore becomes unrealistic after the fuel mass has been totally consumed. Under these circumstances, the equilibrium point reached would not be physically meaningful. This fact coincides with the zero dynamics analysis developed in [21] for a related but different control strategy, based on PWM control of the descending spacecraft.

3.2. *A simulated example*

Simulations were performed with the following constant parameters:

$\sigma = 200$ [m/s] $\quad\quad \alpha = 50$ (Kg/s)

$g = 3.71$ [m/s^2] $\quad\quad \gamma = 1$ [kG/m].

$K = -1$[m]

The poles of the ideal sliding dynamics were both located at $-1.0\ s^{-1}$ ($m_1 = 2$, $m_0 = 1$), while Ω and λ were set to 1 m/s and $5s^{-1}$, respectively. On a planet with the given physical constants, the free-fall limit velocity is 51.03 [m/s]. Figures 1 through 3 show the evolution of the controlled state variables x_1, x_2 (height and vertical velocity) and the total mass x_3. Figure 4 represents the time evolution of the actual (i.e., limited) control inut u during the controlled descent maneuver. Figure 5 depicts the actual, limited, control input u in comparison with the computed control input μ. This figures clearly shows two regions of saturation for the actual input u. Saturation of the control input u occurs roughly in the time intervals ranging from 0 to 7.64 seconds and from 8.22 to 11.04 seconds. Consequently the computed reaching time, given by proposition 1, does not apply due to the existence of the lower-bound saturation interval ($u = 0$) at the beginning of the maneuver, where μ is actually negataive. One also obtains, during the upper-bound saturation condition ($u = 1$, $\mu > 1$), a temporary loss of the sliding-mode conditions on $\omega = 0$. This is due to the fact that on those intervals of nonsaturation, the computed control input μ and the actual control input u entirely coincide. Initial states were chosen, from a free-fall condition, at

$$x_1(0) = -500\ [\text{m}],\ x_2 = 51.03\ [\text{m/s}],\ x_3(0) = 700\ [\text{Kg}].$$

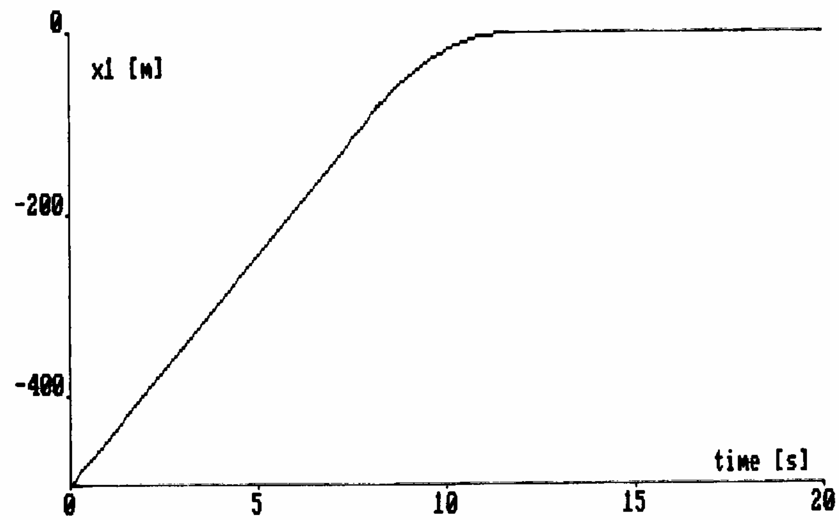

Figure 1. Dynamical variable-structure controlled position for soft landing maneuver.

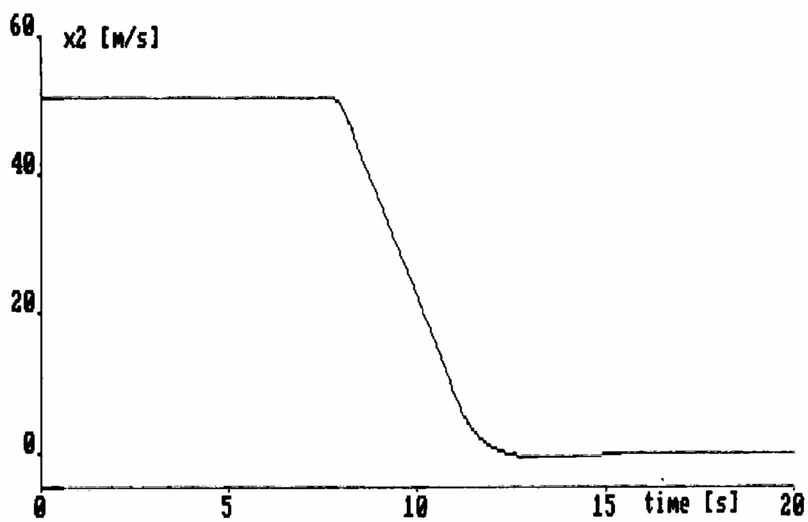

Figure 2. Dynamical variable-structure controlled veritical velocity for soft landing maneuver.

The given initial conditions correspond to an initial value of the auxiliary output function ω given by $\omega(0) = -396.94$. Figure 7 shows the time evolution of the auxiliary output function $\omega(t)$. If no restrictions had been imposed on u, the reaching time T of the sliding condition, $\omega(0) = 0$, according to proposition 1, would have been $T = 1.197$ seconds.

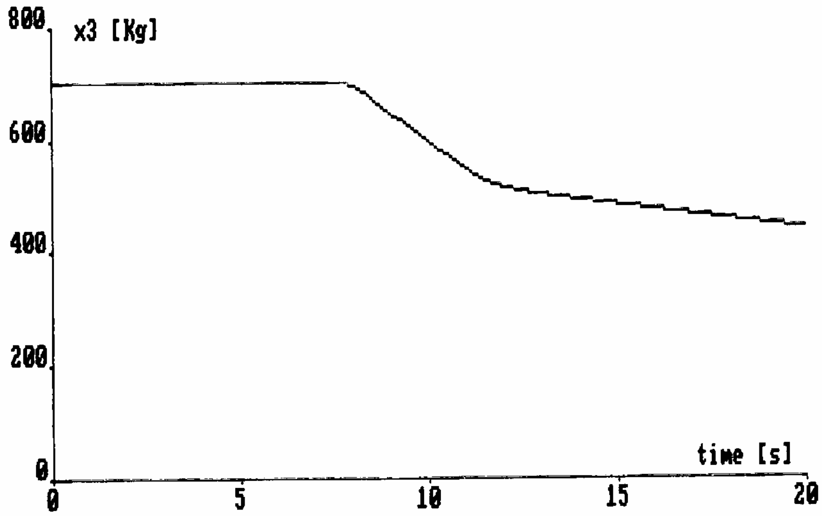

Figure 3. Controlled behavior of total spacecraft mass.

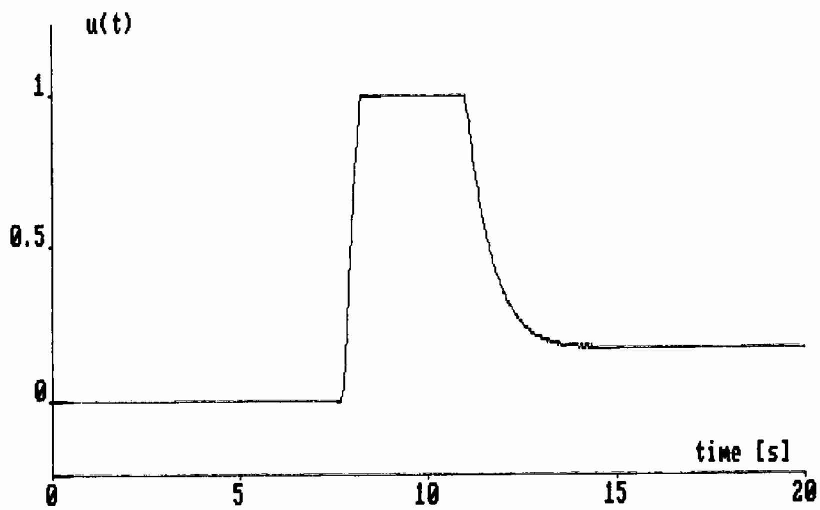

Figure 4. Actual amplitude-modulated control input for sliding-mode soft controlled landing.

4. Conclusions

Dynamical variable-structure controllers accomplishing asymptotic output stabilization are readily obtainable for the class of nonlinear system where the scalar output function qualifies as a differential primitive element for the system dynamics. For this class of systems, Fliess's GOCF naturally leads to a dynamical sliding-mode controller that zeroes, in finite

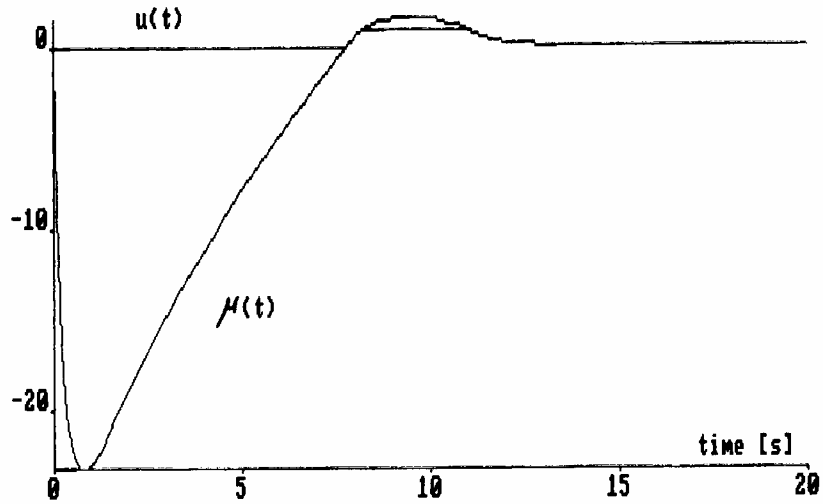

Figure 5. Time evolution of actual bounded control input function u and computed control input function μ.

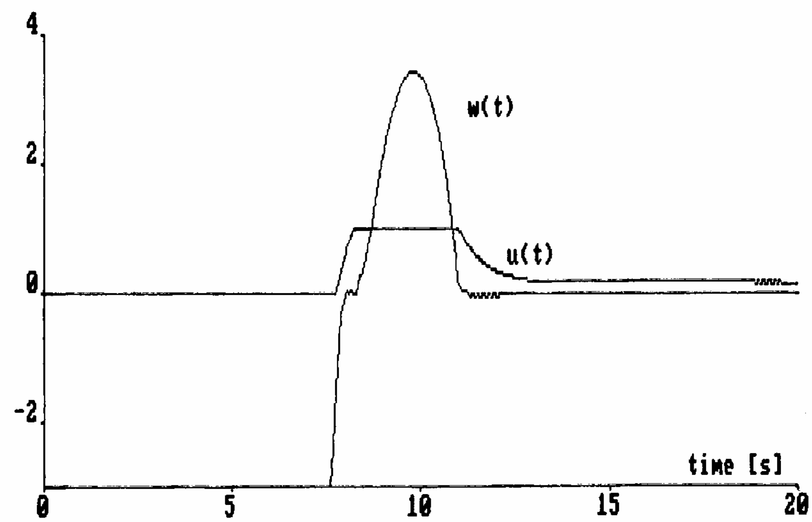

Figure 6. Auxiliary output function exhibiting sliding regime in regions of nonsaturated controller output.

time, an auxiliary output function defined in terms of the output and a finite number of its time derivatives. The resulting ideal sliding dynamics induces an asymptotic stabilization of the original outut function with eigenvalues totally prescribed at will. The obtained discontinuous controller design exhibits two main advantages, aside from the well-known robustness properties implicit in every sliding-mode control scheme. These advantages are related to the possibilities of obtaining a degree of smoothness in output response as well

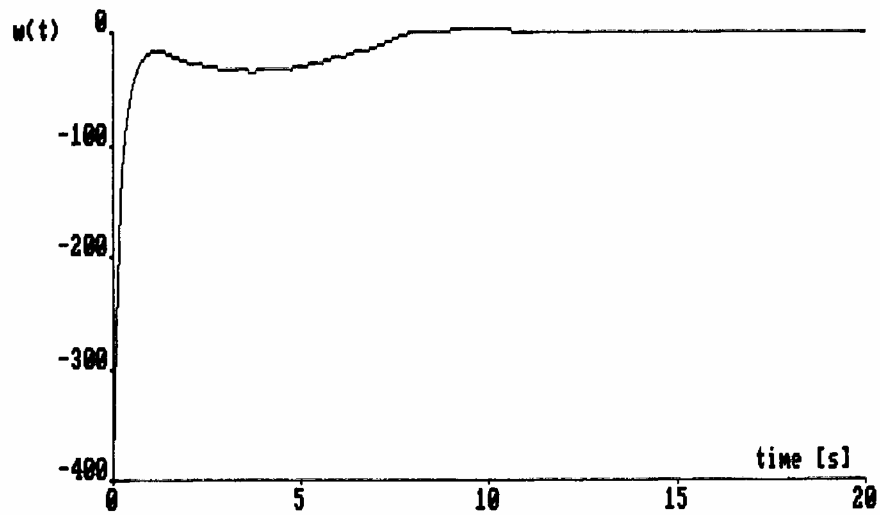

Figure 7. Time response of auxiliary output function $\omega(t)$ due to bounded control actions.

as in the control input signals in strict accordance with the relative degree of the given nonlinear dynamical system (i.e., effective chattering reduction for both the input and the output signals). The approach, however, requires full state feedback and the complexity of nonlinear time-varying implicit dynamical controllers, which may be not globally defined. An illustrative physical example considering a soft controlled landing maneuver was presented. In further research, the results should be extended to the multi-input case.

Notes

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