INT. J. CONTROL, 1993, VOL. 57, NO. 3, 599–625

Adaptive feedback stabilization in PWM-controlled DC-to-DC power supplies

HEBERTT SIRA-RAMÍREZ†, ROCCO TARANTINO-ALVARADO‡ and ORESTES LLANES-SANTIAGO§

In this article, adaptive discontinuous feedback regulators are proposed for the stabilization of switch mode controlled DC-to-DC power supplies of the buck, boost and buck-boost types. Adaptation is performed under the assumption of an unknown LC 'input' circuit natural oscillating frequency, unknown RC 'output' circuit time constant and unknown input source voltage. An average pulse-width-modulation (PWM) input-current regulation scheme, in continuous conduction mode, which accomplishes indirect stabilization of the average output load voltage, is used as the basis for the adaptive controller design. Adaptation is thus confined to the self-tuning of a (static) duty ratio synthesizer. The actual, discontinuous, adaptive PWM feedback strategy state responses exhibit remarkably good asymptotic convergence properties toward the adaptively controlled average designed responses, thus effectively compensating for the modelling error inherent in the average based feedback controller design. It is shown that only in the buck converter case, is a direct-output voltage-adaptive feedback control scheme feasible. In the boost, and the buck-boost, converter cases, the non-adaptive average output voltage regulation schemes lead to non-minimum phase closed loop situations. The performances of the proposed controllers are evaluated through computer simulations.

1. Introduction

Pulse-width-modulation (PWM) feedback control of switch-mode DC-to-DC power converters has been extensively treated in many conference proceedings (Middlebrook and Cuk, 1981, 1983; yearly IEEE PESC Records), specialized books (see: Kassakian *et al.* 1991, Mohan *et al.* 1989, Severns and Bloom 1985, Csaki *et al.* 1983) and journal publications (see, among many other authors: Sira-Ramirez 1987, 1988, 1989 a, 1990, 1991 a). Most of the available regulation results, and also design issues, deal with the assumption of known parameter values for the several lumped elements constituting the converter circuit. However, this fundamental assumption is sometimes invalid due to imprecise knowledge of the values of the various circuit components and of the input source voltage. Lack of precise knowledge of these quantities arises from inescapable measurement errors, or from unavoidable ageing effects. In general terms, such parametric variations directly influence three key 'characteristic' parameters of the converter—the natural oscillating frequency of the LC 'input' circuit, the time constant of the RC 'output' circuit, and the value of the input

Received 15 December 1991. Revised 15 April 1992.

† Departamento de Sistemas de Control, Universidad de Los Andes, Mérida-Venezuela.
†Address for correspondence: Avenida Los Américas, Edificio Residencias El Roble, Piso 6, Appartamento D-6, Mérida 5101, Venezuela.
‡ Unidad de Telecomunicaciones, Refinería Punta Cardón, Maravén, Punto Fijo-Venezuela.
§ Postgrado en Ingeniería de Control Automático, Universidad de Los Andes, Mérida-Venezuela.

source voltage (see Wood 1974, for further details). These parameters play a crucial role in the normalized state variable representation of the converter circuit and they actually constitute physically measurable parameters. Aside from the obvious physical meaning of such constitutive parameters, their relative values were shown to be highly relevant in many converter controller design issues (Sira-Ramirez and Ilic-Spong 1988, 1989). At the same time, they allow assessment of important qualitative characteristics, such as time scale separation properties of 'input' and 'output' circuit responses for the particular converter under study (see: Sira-Ramirez 1988).

Because of the above reasons, non-adaptive feedback control schemes for DC-to-DC converters, relying on nominal or manufacturer-provided parameter values, may lead to performance degradation due to the mismatch between the adopted controller design parameters and those governing the behaviour of the actual converter. One of the most popular means of feedback regulation in DC-to-DC power converters is through pulse-width-modulation (PWM) control strategies. PWM controller designs are frequently based on average (i.e. infinite switching, or sampling, frequency) models of the controlled converter. In spite of the usually high switching frequencies imposed on the PWM regulator, the actual discontinuously controlled converter behaviour, necessarily restricted to finite switching frequencies, is, therefore, at variance with respect to the continuous predicted average behaviour. Adaptation may then be seen as a most helpful strategy, which accounts for the obvious modelling error intrinsically present in the use of an average-based duty ratio synthesizer for the actual discontinuous closed loop operation of the PWM regulated converter. This article shows that an adaptive feedback strategy leads to a definite performance improvement in the steady-state regulation characteristics of the actual PWM feedback controllers developed on the basis of average models of the various DC-to-DC converters treated here. This fact constitutes a second prevailing reason for resorting to adaptation on the nominally designed controller for DC-to-DC power supplies.

Variations of the individual circuit component values, rather than those in the previously described characteristic parameters, could have been directly taken into account to obtain adaptive feedback regulation schemes for the studied converters. Such a possibility, however, leads to a quite complex, non-standard, adaptive control problem. This is due to the individual nonlinear relationships existing between lumped element values and the linear vector fields defining the converter's state dynamics. On the other hand, the natural oscillating frequency of the LC 'input' circuit, the time constant of the RC 'output' circuit and the normalized input source voltage (referred to, from now on, as ω_0, ω_1, and b, respectively) linearly parametrize the converter's defining vector fields in a natural manner. This convenient feature may then be exploited to propose applications of available adaptive feedback regulation schemes, developed for partially linearizable nonlinear systems in Sastry and Isidori (1989) (see also Sastry and Bodson 1989), for the discontinuous PWM-based regulation of DC-to-DC power supplies.

In this article, we undertake the problem of designing adaptive partially linearizing feedback regulators for DC-to-DC power supplies whose nominal controllers are computed on the basis of average PWM models. Our considerations are specifically restricted to PWM strategies and not to the, also commonly used, Pulse Frequency Modulation control schemes for which the class of (infinite switching) average models here used are not globally valid. Three converter structures are treated in this article: the buck, the boost and the buck-boost converter models. The non-adaptive versions of the proposed controllers are derived from exact partial linearization strategies, carried out on the average PWM converter models. The regulators are thus based on input current stabilization (i.e. indirect output voltage regulation) of the switch-mode controlled DC-to-DC power supply, operating in continuous conduction mode. A direct output voltage control scheme, on the other hand, entitles use of the output voltage error for the regulation of the converter's output load voltage to a pre-specified equilibrium value. However, since partial linearization requires invertibility of the system dynamics, the feasibility of the control scheme, whether adaptive or not, requires a minimum phase condition on the zero dynamics associated with the system. It turns out that only in the buck converter case, is a direct output voltage adaptive feedback control scheme actually feasible. In the boost and buck-boost converter cases, the average non-adaptive output voltage regulation scheme leads to a non-minimum phase situation for the closed loop system thus rendering adaptation impossible. Thus, only indirect average output voltage regulation schemes and their corresponding adaptive versions, realized through input current stabilization strategies, are proposed in this work for the various converter circuits (see also Sira-Ramirez and Lischinsky-Arenas 1991). The performance of the proposed adaptive (discontinuous) PWM controllers are evaluated through extensive computer simulations and compared with the nominal non-adaptive responses.

Section 2 of this article deals with some generalities regarding an adaptive PWM feedback control scheme for standard (infinite frequency) average models of PWM-controlled systems, based on well-established results developed for the adaptive control of linearizable systems. Section 3 deals with the design of indirect output voltage adaptive feedback regulation schemes for the buck, the boost and the buck-boost converters. The non-minimum phase conditions associated with direct output voltage regulation strategies are also presented in this section. Section 4 presents the conclusions and some suggestions for further research in this area.

For a general background on adaptive control of nonlinear systems, the reader is referred to the excellent books by Sastry and Bodson (1989) and Narendra and Annaswammy (1989), to the works of Campion and Bastin (1990), Kanellakopoulos *et al*. (1989, 1991), Teel *et al.* (1991), and the collection of articles in Kokotovic (1991). The background of the effect of infinite frequency average PWM design strategies on the actual discontinuous controlled nonlinear systems, and the various approximation errors and convergence properties of the scheme, may be found in the literature. However, for such basic results, the reader is referred to Sira-Ramirez (1989 b), the appendices in Sira-Ramírez (1991 b, 1991 c) and to Sira-Ramírez *et al.* (1993).

2. An adaptive PWM feedback controller scheme for switch-regulated linearizable systems

In this section, we propose a general adaptive feedback regulation scheme for PWM controlled nonlinear systems with input–output linearizable average models. A slight extension of the standard adaptive control scheme, developed in Sastry and Isidori (1989) for systems of relative degree one, is used in the specification of an adaptive duty ratio synthesizer (i.e. controller) for the idealized, infinite sampling frequency, average model of the nonlinear PWM controlled system.

2.1. *An average model for PWM switch-controlled nonlinear systems*

Consider the n dimensional state representation of a single-input single-output PWM controlled nonlinear system containing a parameter vector θ, with constant but unknown components:

$$\left.\begin{aligned} \dot{x} &= f(x, \theta) + ug(x, \theta) \\ y &= h(x, \theta) \end{aligned}\right\} \tag{2.1}$$

where, without loss of generality, the control input variable u takes values in the discrete set $\{0, 1\}$, possibly representing the values of a switch position function, determining one particular feedback structure among two possible (nonlinear) feedback paths (see Sira-Ramirez and Lischinsky-Arenas 1990). The values of u are specified according to the PWM-controlled law:

$$u = \begin{cases} 1 & \text{for } t_k \leq t < t_k + \mu[x(t_k)]T \\ 0 & \text{for } t_k + \mu[x(t_k)]T < t \leq t_k + T \end{cases} \quad t_k + T = t_{k+1}; \quad k = 0, 1, 2, \ldots \tag{2.2}$$

where the mapping $\mu: R^n \rightarrow R$, represents the duty ratio function, which is usually regarded as a state dependent quantity (i.e. as a truly feedback law) and which specifies, at each sampling instant t_k, the state dependent width $\mu[x(t_k)]T$ of the control input pulse during the upcoming inter-sampling interval of fixed duration T (known also as the duty cycle, or, simply, the sampling period). It is easy to see, from (2.2), that the duty ratio function μ is evidently limited to non-negative values, which do not exceed the upper bound of 1, i.e. $\mu \in [0, 1]$. The scalar function $y = h(x, \theta)$ represents an output error which is to be stabilized to zero.

It has been shown in Sira-Ramirez (1989 b, 1991 b, c) that a piecewise smooth average model of the PWM-controlled system (2.1) can be obtained by assuming an infinitely large sampling frequency $1/T$. This assumption results in a model of (2.1) in which the discrete-valued control input function u is replaced by the limited piecewise smooth (feedback) duty ratio function μ, i.e.:

$$\left.\begin{aligned} \dot{z} &= f(z, \theta) + \mu g(z, \theta) \\ y &= h(z, \theta) \end{aligned}\right\} \tag{2.3}$$

where z denotes the averaged state vector. Such an average model represents a crude smooth approximation to the actual PWM-controlled system behaviour, yet, it has proven to be quite useful in PWM controller design strategies for nonlinear systems in which finite, but relatively large, sampling frequencies are used. In particular, for switch controlled DC-to-DC power supplies, the infinite sampling frequency average model (2.3) has been successfully used in a variety of nonlinear controller design techniques, including exact (static) feedback linearization, extended linearization and nonlinear dynamical feedback control schemes, based on differential algebraic techniques (see Sira-Ramírez, 1987, 1988, 1989 a, 1990, 1991 a, Sira-Ramírez and Lischinsky-Arenas 1991, Sira-Ramírez and Prada-Rizzo 1992).

In Sira-Ramirez (1989 b, 1991 b) and Sira-Ramírez *et al.* (1993), the above mathematical idealization of the infinite frequency switching model has been shown to be equivalent, in various ways, to the geometric averaging theory of Filippov, which is commonly used in obtaining idealized constrained behaviour of systems regulated by variable structure control strategies leading to sliding mode behaviour (Utkin 1978). This averaging technique also enjoys definite relations with standard averaging theory as inferred from the work of Krein *et al.* (1990).

Remark: It should be stressed that, such as it is standard in nonlinear controller design techniques, an average PWM-based controller design for DC-to-DC power supplies is primarily carried out under the assumption of non-saturation conditions for the duty ratio function μ. However, in actual closed loop operation the saturation limits on such a duty ratio function, represented here by the condition $\mu \in [0, 1]$, must be duly imposed. Hence, the average duty ratio synthesizer design must be regarded as being only locally valid around the prescribed equilibrium point. Saturation of the duty ratio function, however, may take place during the transients. For those excursions of the regulated variables which imply a violation of the physically natural bounds for the actual duty ratio function, an open loop condition is obtained from which the system may soon recuperate only if such a saturation condition is briefly held. Indefinite saturation of the duty ratio function leads to an open loop control condition, and the possibility of unstable behaviour. In spite of the local validity of the designed PWM average feedback strategy and the idealization provided by the infinite switching frequency model, the ranges of effectiveness, validity and quality of performance of our proposed adaptive feedback control schemes are, however, quite satisfactory as demonstrated below in the simulation examples presented. □

2.2. *An adaptive PWM feedback control scheme for nonlinear systems*

We assume, in the spirit of Sastry and Isidori (1989), that the parameter-dependent average vector fields $f(z, \theta)g(z, \theta)$ and the nonlinear output function $h(x, \theta)$ exhibit the following *linear* dependence on the components of the parameter vector θ:

$$f(z, \theta) = \sum_{i=1}^{n_1} \theta_i^1 f_i(z); \quad g(z, \theta) = \sum_{j=1}^{n_2} \theta_j^2 g_j(z)$$
$$h(z, \theta) = h_0(z) + \sum_{k=1}^{n_3} \theta_k^3 h_k(Z) \tag{2.4}$$

where $f_i(z)$, $g_j(z)$, $h_0(z)$ and $h_k(Z)$ are smooth functions of their arguments. Z is an n-dimensional constant vector, possibly representing the steady-state value of the average state vector z.

The following result represents a slight extension of that found in Sastry and Isidori (1989).

Theorem 2.1: *Let* $\theta_1 = (\theta_1^1, \ldots, \theta_{n1}^1)$, $\theta_2 = (\theta_1^2, \ldots, \theta_{n2}^2)$, $\theta_3 = (\theta_1^3, \ldots, \theta_{n3}^3)$ and $\theta = (\theta_1, \theta_2, \theta_3)$ *denote, respectively, the parameter vectors associated with the vector fields f, g, the nonlinear output function h, and the composition of the three parameter vectors. Let* ϕ *be the parameter estimation error defined as* $\phi = \theta - \hat{\theta}$, *with* $\hat{\theta}$ *being an estimation of the actual (unknown) value of the composite vector* θ. *Given a relative degree one nonlinear system of the form (2.3), with the vector fields* $f(z, \theta)$ *and* $g(z, \theta)$, *defined as in (2.4), then the feedback control law:*

$$\hat{\mu}(z) = -\frac{1}{\sum_{j=1}^{n_2} \hat{\theta}_j^2 L_{g_j} h_0(z)} \left[\sum_{i=1}^{n_1} \hat{\theta}_i^1 L_{f_i} h_0(z) + \alpha \left(h_0(z) + \sum_{k=1}^{n_3} \hat{\theta}_k^3 h_k(Z) \right) \right] \quad (2.5)$$

where $\hat{\theta}$ *evolves according to the following parameter update law:*

$$\dot{\phi} = -\dot{\hat{\theta}} = -yW = -y \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} = -y \begin{bmatrix} L_{f_1} h_0(z) \\ \cdots \\ L_{f_{n_1}} h_0(z) \\ [L_{g_1} h_0(z)]\hat{\mu}(z) \\ \cdots \\ [L_{g_{n_2}} h_0(z)]\hat{\mu}(z) \\ \alpha h_1(Z) \\ \cdots \\ \alpha h_{n_3}(Z) \end{bmatrix} \quad (2.6)$$

locally yields a bounded asymptotic convergence of the output y to zero, with bounded state variables responses, provided the quantity $L_g h = L_g h_0$ *is bounded away from zero.*

Proof: Suppose system (2.3) is relative degree one. It is desired to impose, in order to obtain an asymptotic approach of the scalar output error to zero, the following asymptotically stable dynamics on the controlled output error: $\dot{y} = -\alpha y$, with $\alpha > 0$. Rewriting this expression as: $L_f h(x, \theta) + u L_g h(x, \theta) = -\alpha h(x, \theta)$ one obtains the following non-adaptive nonlinear controller:

$$u = -\frac{\alpha h(x, \theta) + L_f h(x, \theta)}{L_g h(x, \theta)}.$$

Using, on the above controller expression, the definitions provided in (2.4) and by virtue of the fact that the parameter vector θ is unknown, the above control law is modified, using the 'certainty equivalence' principle, to its adaptive version given in (2.5). Substituting now (2.5) in (2.3) and after some straightforward manipulations, one obtains:

$$\dot{y} = -\alpha y + (\theta - \hat{\theta})^{\mathrm{T}} W = -\alpha y + \phi^{\mathrm{T}} W$$

with W being the 'regressor vector' implicitly defined in (2.6).

Consideration of the following positive definite Lyapunov function candidate: $v(y, \phi) = \frac{1}{2}(y^2 + \phi^2)$ yields, after use of the parameter update law (2.6): $\dot{v}(y, \phi) = -\alpha y^2 \leq 0$. The results follow immediately from the same considerations found in Theorem 3.1 of the work of Sastry and Isidori (1989). □

Remark: The adaptive controller design procedure that we propose in this article consists of, first, specifying a stabilizing adaptive feedback duty ratio synthesizer, for the average PWM model (2.3), using the results of Theorem 2.1 above. Second, the obtained average adaptive controller scheme is then used in the actual discontinuous PWM controlled system (2.1), (2.2) for the generation of the required stabilizing (limited) sampled duty ratio function, as portrayed in the scheme shown in Fig. 1. The presence of the limiter before the PWM actuator, indicates the local validity of the proposed feedback control scheme and the need to avoid control tasks that involve extensive saturations of the duty ratio synthesizer output. This, however, is quite a standard limitation and is present in almost every feedback control scheme, for the regulation of DC-to-DC power supplies, proposed to date. □

The theoretical justification of the proposed adaptive PWM controller rests on the validity of the (infinite sampling frequency) average PWM model (2.3) as a piecewise smooth system that approximates, arbitrarily close, the controlled behaviour of (2.1) as the sampling frequency in the PWM controller (2.2) is made sufficiently large (see Sira-Ramírez 1989 b, 1991 b and Sira-Ramírez *et al.* 1993). Moreover, adaptation is seen actually to improve the regulation characteristics, and performance, of the actual discontinuous PWM controllers (synthesized on the basis of idealized average models) even when nominal parameter values are used, both, for the non-adaptive PWM controller and the regulated converter (see simulation results).

Given system (2.1) as:

$$\left.\begin{aligned} \dot{x} &= f(x, \theta) + ug(x, \theta) \\ y &= h(x, \theta) \end{aligned}\right\} \tag{2.7}$$

we propose the following adaptive PWM feedback controller:

$$u = \begin{cases} 1 & \text{for } t_k \leqslant t < t_k + \hat{\mu}[x(t_k)]T \\ 0 & \text{for } t_k + \hat{\mu}[x(t_k)]T < t \leqslant t_k + T \end{cases} \quad t_k + T = t_{k+1};\ k = 0, 1, 2, \ldots \tag{2.8}$$

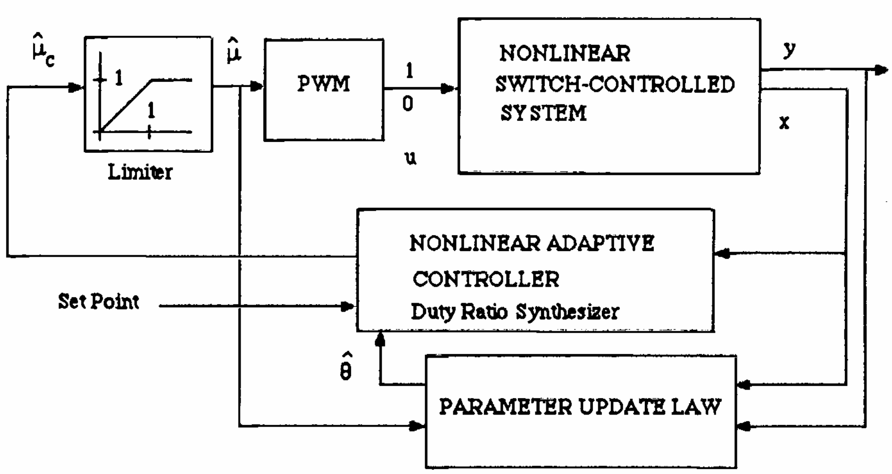

Figure 1. General nonlinear adaptive feedback regulation scheme for nonlinear PWM switched-controlled systems.

with the estimated duty ratio function μ given by:

$$\hat{\mu}(x) = \mathrm{sat}_{[0,1]}\,\hat{\mu}_c(x) := \begin{cases} 1 & \text{if } \hat{\mu}_c(x) \geq 1 \\ \hat{\mu}_c & \text{if } 0 < \hat{\mu}_c(x) < 1 \\ 0 & \text{if } \hat{\mu}_c(x) \leq 0 \end{cases} \tag{2.9}$$

where $\hat{\mu}_c(x)$ is the adaptive computed duty ratio function, directly obtained from the average designed duty ratio synthesizer given in (2.5) as:

$$\hat{\mu}_c(x) = -\frac{1}{\sum_{j=1}^{n_2} \hat{\theta}_j^2 L_{g_j} h_0(x)} \left[\sum_{i=1}^{n_1} \hat{\theta}_i^1 L_{f_i} h_0(x) + \alpha\left(h_0(x) + \sum_{k=1}^{n_3} \hat{\theta}_k^3 h_k(X) \right) \right] \tag{2.10}$$

where the vector X coincides with the steady-state value Z, of the average state vector z.

The parameter update law is given by:

$$\dot{\phi} = -\dot{\hat{\theta}} = -yW = -y \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} = -y \begin{bmatrix} L_{f_1} h_0(z) \\ \cdots \\ L_{f_{n_1}} h_0(x) \\ [L_{g_1} h_0(x)]\hat{\mu}(x) \\ \cdots \\ [L_{g_{n_2}} h_0(x)]\hat{\mu}(x) \\ \alpha h_1(X) \\ \cdots \\ \alpha h_{n_3}(X) \end{bmatrix} \tag{2.11}$$

Remark: Due to the saturation imposed on the computed adaptive duty ratio function, the asymptotic convergence of the average output variable y to zero can only be locally guaranteed in that region of the state-space where the computed duty ratio function does not exhibit such a saturation. Due to the explicit state dependence exhibited by the computed duty ratio function μ_c on the state x of the system, the region of non-saturation may be explicitly estimated by means of its boundaries, defined as $\hat{\mu}_c(x) = 0$ and $\hat{\mu}_c(x) = 1$. □

3. Adaptive stabilization of PWM controlled DC-to-DC power supplies

In this section we use the basic adaptive PWM control scheme, proposed in § 2, to obtain indirect output voltage stabilization for DC-to-DC power supplies of the buck, boost and buck-boost types. Even for the non-adaptive control case, a regulation scheme directly based on output voltage error stabilization is feasible only for the buck converter case. For the boost, and the buck-boost converters, a non-minimum phase condition on the associated zero dynamics precludes the application of the (inversion-based) input–output linearization control techniques and, thus, destroys the possibility of adaptation. Adaptive controllers based on output voltage error stabilization are still feasible if they are derived from other kinds of non-inverting controllers, such as linear, or nonlinear, PI, PD or PID, feedback regulators (see Sira-Ramirez 1990, 1991 a for further details).

We begin our exposition by treating first the boost converter case in a detailed fashion. Next, the buck-boost converter and the buck converter cases are examined. A direct output voltage stabilization for the buck converter case will be treated elsewhere. In spite of the linear nature of the buck converter, we still apply to its indirect output voltage regulation scheme, the same techniques used for the nonlinear cases of the boost and the buck-boost converters. This makes the article consistent and allows some unifying approach to the general problem of designing adaptive feedback regulators for DC-to-DC power supplies.

3.1. *Adaptive indirect output voltage regulation for the PWM-controlled boost converter*

3.1.1. *Generalities about the PWM controlled boost converter.* Consider the following normalized state variable representation of the boost converter shown in Fig. 2 (Wood, 1974, see also Sira-Ramirez 1987):

$$\left.\begin{aligned} \dot{x}_1 &= -(1-u)\omega_0 x_2 + b \\ \dot{x}_2 &= (1-u)\omega_0 x_1 - \omega_1 x_2 \end{aligned}\right\} \tag{3.1}$$

where u represents the switch position function taking values in the discrete set $\{0, 1\}$ which will be specified in accordance with a PWM feedback control law of the form (2.2).

The average PWM model of (3.1), introduced in the previous section, is simply given by:

$$\left.\begin{aligned} \dot{z}_1 &= -(1-\mu)\omega_0 z_2 + b \\ \dot{z}_2 &= (1-\mu)\omega_0 z_1 - \omega_1 z_2 \end{aligned}\right\} \tag{3.2}$$

To facilitate application of the adaptive control results of § 2, we equivalently express (3.2) as:

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \omega_0 \begin{bmatrix} -z_2 \\ z_1 \end{bmatrix} + \omega_1 \begin{bmatrix} 0 \\ -z_2 \end{bmatrix} + b \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \mu\left(\omega_0 \begin{bmatrix} z_2 \\ -z_1 \end{bmatrix}\right) \tag{3.3}$$

and adopt, consistent with § 2, the following notation for the average model (3.3):

$$\dot{z} = f(z, \theta) + \mu g(z, \theta) = \theta_1^1 f_1(z) + \theta_2^1 f_2(z) + \theta_3^1 f_3(z) + \mu[\theta_1^2 g_1(z)]$$

where:

$$\theta_1^1 = \omega_0,\ \theta_2^1 = \omega_1,\ \theta_3^1 = b;\ f_1(z) = \begin{bmatrix} -z_2 \\ z_1 \end{bmatrix},\ f_2(z) = \begin{bmatrix} 0 \\ -z_2 \end{bmatrix},\ f_3(z) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\theta_1^2 = \omega_0;\ g_1(z) = \begin{bmatrix} z_2 \\ -z_1 \end{bmatrix} \tag{3.4}$$

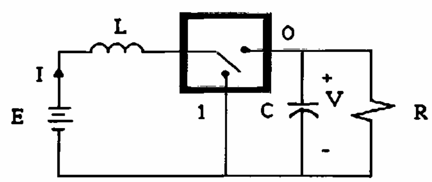

Figure 2. The boost converter.

The average equilibrium point of (3.3), for a constant value $\mu = U$ of the duty ratio, is readily obtained from (3.2) as:

$$z_1 = Z_1(U) = \frac{\omega_1 b}{(1-U)^2\omega_0^2};\ z_2 = Z_2(U) = \frac{b}{(1-U)\omega_0};\ \mu = U \qquad (3.5)$$

It is easily inferred, from (3.5), that:

$$Z_1(U) = \frac{\omega_1}{b} Z_2^2(U) = \frac{\theta_2^1}{\theta_3^1} Z_2^2(U) \qquad (3.6)$$

Let us consider, as the output of the average system (3.2), the average normalized input current error, computed with respect to its average steady-state value:

$$y = h(z) = z_1 - Z_1(U) = z_1 - \frac{\omega_1}{b} Z_2^2(U) = z_1 - \frac{\theta_2^1}{\theta_3^1} Z_2^2(U) \qquad (3.7)$$

Evidently, system (3.2), with output (3.7), is everywhere a relative degree one system, except on the line $z_2 = 0$. Notice that a steady-state condition $z_2 = 0$ is not achievable in practice. As a matter of fact such a condition is reachable only in the unstable mode of operation for the input current, with $\mu = 1$. This behaviour, incidentally, corresponds to a fixed position of the switch at the value $u = 1$. We stress that our results are only locally valid for those controlled state trajectories that do not cross the line $z_2 = 0$.

3.1.2. *Nonadaptive feedback regulation of the average PWM controlled boost converter.* Imposing on the input current state equation the asymptotically stable behaviour:

$$\dot{z}_1 = -\alpha[z_1 - Z_1(U)] = -\alpha\left[z_1 - \frac{\omega_1}{b} Z_2^2(U)\right] \qquad (3.8)$$

with $\alpha > 0$, one immediately obtains, by simple inversion carried out on the first equation of (3.2), the computed duty ratio function μ_c as:

$$\mu_c(c) = \frac{\omega_0 z_2 - \left[b + \alpha\left(z_1 - \frac{\omega_1}{b} Z_2^2(U)\right)\right]}{\omega_0 z_2} \qquad (3.9)$$

The actual duty ratio function $\mu(z)$ is obtained by limiting the values of the computed duty ratio function μ_c to the closed interval $[0, 1]$, in the following manner:

$$\mu(z) = \mathrm{sat}_{[0,1]}\,\mu_c(z) := \begin{cases} 1 & \text{if } \mu_c(z) \geq 1 \\ \mu_c(z) & \text{if } 0 < \mu_c(z) < 1 \\ 0 & \text{if } \mu_c(z) \leq 0 \end{cases} \qquad (3.10)$$

In steady-state stable operation, the computed duty ratio function takes, according to (3.9), the value:

$$\mu_c = U = 1 - \frac{b}{\omega_0 Z_2(U)} \qquad (3.11)$$

which is entirely compatible with (3.5). One further requires that the steady-state value of the duty ratio function be confined to the open interval $(0, 1)$ as demanded by (3.10). One immediately obtains the following (normalized output voltage amplification and positivity) conditions:

$$Z_2(U) > \frac{b}{\omega_0} > 0 \tag{3.12}$$

which guarantees, at least in steady-state operation, the properness of the relative degree definition.

The zero dynamics, associated with the computed duty ratio function μ_c, are obtained by substituting (3.9) into the second state equation in (3.2), and letting $y = z_1 - Z_1(U) = z_1 - (\omega_1/b)Z_2^2(U)$ become identically zero. One then obtains the following nonlinear autonomous dynamics:

$$\dot{z}_2 = -\omega_1 z_2 + \frac{\omega_1 Z_2^2(U)}{z_2} \tag{3.13}$$

which has only one physically meaningful equilibrium point represented by $z_2 = Z_2(U)$.

Moreover, such an equilibrium point is exponentially asymptotically stable, as can be easily verified from a standard argument on the following Lyapunov function candidate (which is strictly positive everywhere except at the equilibrium point, $z_2 = Z_2(U)$, where its value is zero) and its time derivative, evaluated along the solutions of (3.13):

$$V(z) = \tfrac{1}{2}[z_2^2 - Z_2^2(U)]^2 \Rightarrow \dot{V}(z) = -2\omega_1 V(z)$$

In view of the saturation limits (3.10) for the proposed duty ratio synthesizer, and the previous arguments, it can be concluded that the non-adaptive controller (3.9), (3.10) locally exponentially asymptotically stabilizes the average input current behaviour to its equilibrium point while the average normalized output voltage also asymptotically exponentially converges to the corresponding equilibrium value, in accordance with (3.5).

3.1.3. *Adaptive feedback regulation of the PWM controlled boost converter.* An adaptive version of controller (3.9) requires the introduction of a new unknown parameter which bestows on the input current error (3.7) the hypothesized linear parametric dependence of the output, as in (2.4). Indeed we redefine the average input current error (3.7) as:

$$y = h(z) = z_1 - \frac{\theta_2^1}{\theta_3^1} Z_2^2(U) = z_1 - \theta_1^3 Z_2^2(U) \tag{3.14}$$

i.e. in terms of the notation adopted in (2.4) we have: $h_0(z) = z_1$ and $h_1(Z) = -Z_2^2(U)$.

The average adaptive controller, based on (3.9), can then be obtained in terms of the estimated values of the actual parameters, defined in (3.4) and (3.14), as:

$$\hat{\mu}_c(z) = \frac{\hat{\theta}_1^1 z_2 - [\hat{\theta}_3^1 + \alpha(z_1 - \hat{\theta}_1^3 Z_2^2(U))]}{\hat{\theta}_1^2 z_2} \tag{3.15}$$

and

$$\hat{\mu}(z) = \mathrm{sat}_{[0,1]}\,\hat{\mu}_c(z) := \begin{cases} 1 & \text{if } \hat{\mu}_c(z) \geqslant 1 \\ \hat{\mu}_c(z) & \text{if } 0 < \hat{\mu}_c(z) < 1 \\ 0 & \text{if } \hat{\mu}_c(z) \leqslant 0 \end{cases} \tag{3.16}$$

The regressor vector, corresponding to the proposed adaptive scheme of § 2, is given by:

$$w = \begin{bmatrix} L_{f_1} h_0(z) \\ L_{f_2} h_0(z) \\ L_{f_3} h_0(z) \\ \hat{\mu}(z) L_{g_1} h_0(z) \\ \alpha h_1(Z) \end{bmatrix} = \begin{bmatrix} -z_2 \\ 0 \\ 1 \\ z_2 \hat{\mu}(z) \\ -\alpha Z_2^2(U) \end{bmatrix} \tag{3.17}$$

which immediately points to the fact that the proposed adaptive scheme does not allow the estimation of the parameter $\theta_2^1 = \omega_1$, representing the time constant of the RC output circuit. Notice, however, that such a parameter is not present in the control law (3.15) and, hence, lack of knowledge about this parameter is irrelevant for control purposes.

The corresponding parameter adaptation laws for the average PWM controlled system are readily obtained from (2.6) as:

$$\dot{\hat{\theta}}_1^1 = -z_2 y;\ \dot{\hat{\theta}}_2^1 = 0;\ \dot{\hat{\theta}}_3^1 = y;\ \dot{\hat{\theta}}_1^2 = z_2 y \hat{\mu}(z);\ \dot{\hat{\theta}}_1^3 = -\alpha Z_2^2(U) y \tag{3.18}$$

The actual (discontinuous) adaptive PWM controller equations are summarized below:

$$u = \begin{cases} 1 & \text{for } t_k \leqslant t < t_k + \hat{\mu}[x(t_k)]T \\ 0 & \text{for } t_k + \hat{\mu}[x(t_k)]T < t \leqslant t_k + T \end{cases} \quad t_k + T = t_{k+1};\quad k = 0, 1, 2, \ldots \tag{3.19}$$

with,

$$\hat{\mu}(x) = \mathrm{sat}_{[0,1]}\, \hat{\mu}_c(x) := \begin{cases} 1 & \text{if } \hat{\mu}_c(x) \geqslant 1 \\ \hat{\mu}_c & \text{if } 0 < \hat{\mu}_c(x) < 1 \\ 0 & \text{if } \hat{\mu}_c(x) \leqslant 0 \end{cases} \tag{3.20}$$

$$\hat{\mu}_c(x) = \frac{\hat{\theta}_1^1 x_2 - [\hat{\theta}_2^1 + \alpha(x_1 - \hat{\theta}_1^3\, X_2^2(U))]}{\hat{\theta}_1^2 x_2} \tag{3.21}$$

where $X_2(U) = Z_2(U)$.

The parameter update law is obtained from (3.18) as:

$$\dot{\hat{\theta}}_1^1 = -x_2 y;\ \dot{\hat{\theta}}_2^1 = 0;\ \dot{\hat{\theta}}_3^1 = y;\ \dot{\hat{\theta}}_1^2 = x_2 y \hat{\mu}(x);\ \dot{\hat{\theta}}_1^3 = -\alpha X_2^2(U) y \tag{3.22}$$

3.1.4. *Simulation results*. Simulations were carried out on a boost converter with nominal circuit parameter values given by $R = 11{\cdot}2$, $L = 0{\cdot}195$ mH, $C = 2000\ \mu$F and $E = 28{\cdot}0$ V, which correspond with the following nominal values for the characteristic parameters $\omega_0 = 1{\cdot}601281 \times 10^3$, $\omega_1 = 0{\cdot}044642 \times 10^3$ and $b = 2{\cdot}005121 \times 10^3$. A desired normalized steady-state output voltage was set to $Z_2(U) = 2{\cdot}9525$ V, with a corresponding steady state value for the duty ratio of $U = 0{\cdot}6$. The non-adaptive version of the average and the actual PWM controller was first simulated. Figure 3 shows the average normalized input current and the average normalized output voltage trajectories superimposed on the actual (discontinuous) PWM-controlled responses. The sampling frequency was set to 10 kHz while the closed loop eigenvalue for the normalized average input current linearization was chosen as: $\alpha = 2 \times 10^3\ \mathrm{s}^{-1}$. The responses of the actual PWM controlled states are shown to chatter closely

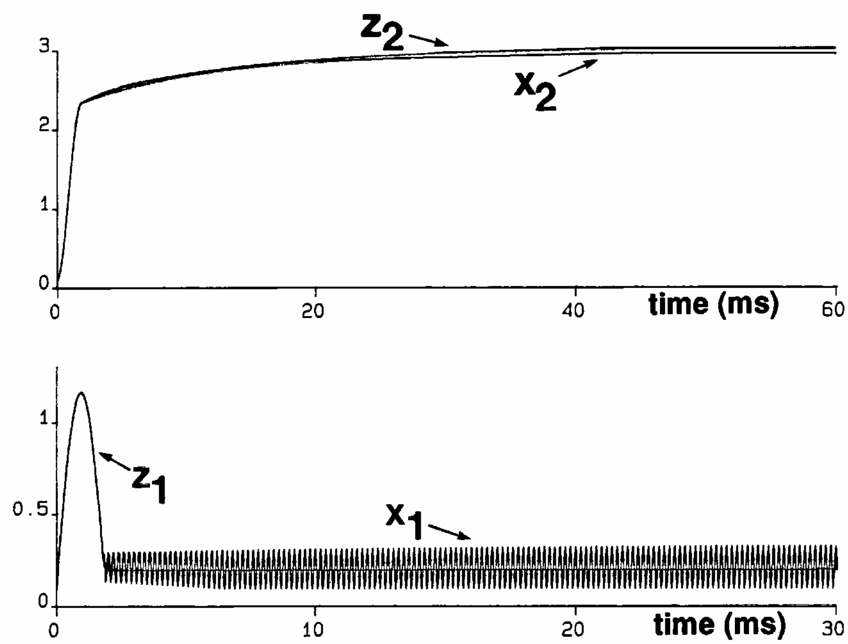

Figure 3. Non-adaptive average normalized input current and average normalized output voltage trajectories superimposed on the actual (discontinuous) PWM-controlled responses for the boost converter.

about the average trajectories with quite close convergence toward the desired equilibrium. The very small discrepancies, and biases, existing between the average and the actual PWM controlled responses are explainable in terms of the fact that a finite sampling frequency is being used, together with the inherent modelling error represented by the use of an average-based duty ratio synthesizer. Figure 4 shows the average and the actual PWM-controlled state trajectories obtained when the adaptive version of the PWM controller is used. The desired equilibrium value is now perfectly achieved for both the average and the actual PWM controlled normalized input current, but without the small discrepancies noticed before in the non-adaptive case. This is, precisely, one of the beneficial effects of the proposed adaptation scheme. Figure 5 depicts the obtained trajectories for the adaptive controller parameters in the actual PWM controlled system. In general, as inferred from the general results of the adaptation method, the steady state values of these parameters may not coincide with the actual (i.e. true) parameter values used for the simulation of the plant.

3.2. *Direct output voltage regulation for the PWM controlled boost converter*

It will now be shown that a (direct) normalized output voltage regulation for the boost converter is infeasible, if one resorts to input–output linearization schemes based on partial inversion of the converter system equations.

Indeed if one adopts, as the output of the boost converter circuit, the output voltage error with respect to its steady state value, i.e.

$$y = z_2 - Z_2(U) \tag{3.23}$$

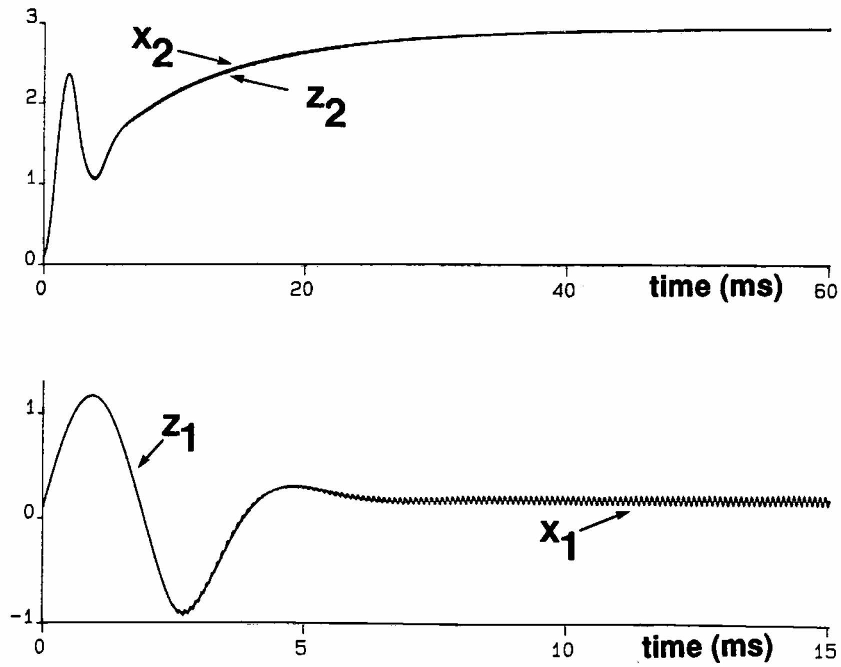

Figure 4. Adaptively controlled average, and actual state trajectories for the boost converter example.

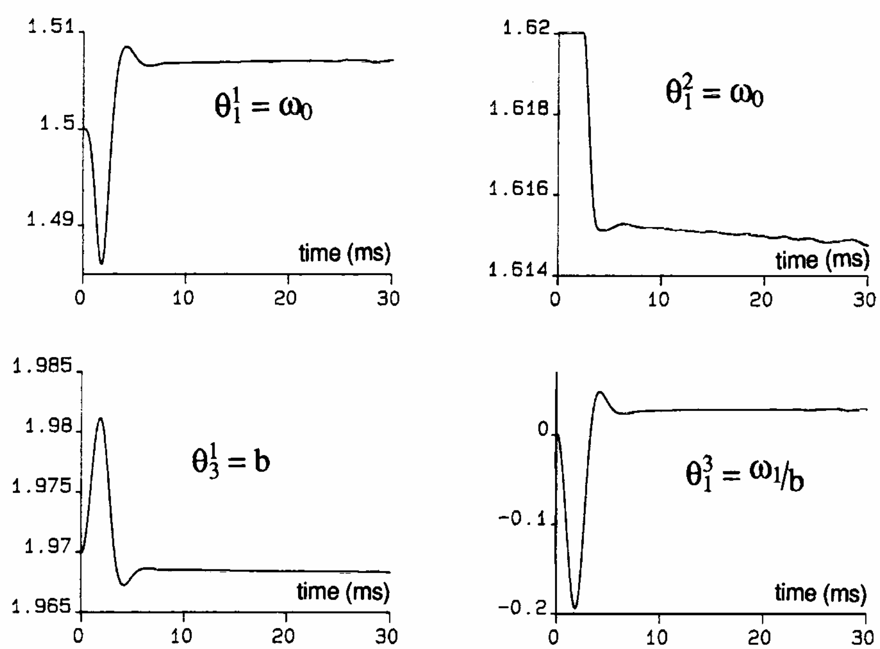

Figure 5. Trajectories for the adaptive controller parameters of the actual PWM-controlled boost converter.

then, it easily follows that the resulting system is everywhere relative degree one, except on the line $z_1 = 0$.

Imposing on the normalized output voltage state equation the asymptotically stable behaviour:

$$\dot{z}_2 = -\alpha \left[z_2 - Z_2(U) \right] \tag{3.24}$$

with $\alpha > 0$, one immediately obtains, by simple inversion carried out on the second state equation of (3.2), the computed duty ratio function μ_c as:

$$\mu_c(z) = \frac{\omega_0 z_1 - \omega_1 z_2 + \alpha \left(z_2 - Z_2(U) \right)}{\omega_0 z_1} \tag{3.25}$$

The associated zero dynamics is obtained by substituting the expression (3.25) into the first state equation in (3.2), and letting $y = z_2 - Z_2(U)$ be zero. One then obtains:

$$\dot{z}_1 = b - \frac{\omega_1 Z_2^2(U)}{z_1} \tag{3.26}$$

which has a unique equilibrium point given by $z_1 = Z_1(U) = \omega_1 Z_2^2(U)/b$. It is easy to verify, by straightforward linearization, that such an equilibrium point is unstable and, thus, the system is non-minimum phase. This fact renders any adaptation scheme based on (3.25) totally infeasible.

3.3. *Adaptive indirect output voltage regulation for the PWM controlled buck-boost converter*

3.3.1. *Generalities about the PWM controlled buck-boost converter.* Consider the following normalized state representation of the buck-boost converter shown in Fig. 6 (Sira-Ramirez 1987):

$$\left.\begin{aligned} \dot{x}_1 &= (1 - u)\omega_0 x_2 + ub \\ \dot{x}_2 &= -(1 - u)\omega_0 x_1 - \omega_1 x_2 \end{aligned}\right\} \tag{3.27}$$

where u takes values in the discrete set $\{0, 1\}$.

The average PWM model of (3.27) is given by:

$$\left.\begin{aligned} \dot{z}_1 &= (1 - \mu)\omega_0 z_2 + \mu b \\ \dot{z}_2 &= -(1 - \mu)\omega_0 z_1 - \omega_1 z_2 \end{aligned}\right\} \tag{3.28}$$

which may also be equivalently expressed as:

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \omega_0 \begin{bmatrix} z_2 \\ -z_1 \end{bmatrix} + \omega_1 \begin{bmatrix} 0 \\ -z_2 \end{bmatrix} + \mu \left(\omega_0 \begin{bmatrix} -z_2 \\ z_1 \end{bmatrix} + b \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) \tag{3.29}$$

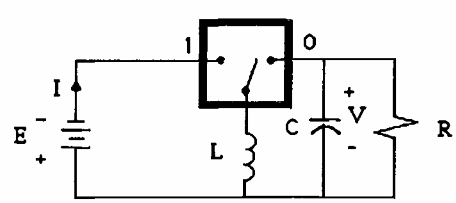

Figure 6. The buck-boost converter.

or, as:

$$\dot{z} = f(z, \theta) + \mu g(z, \theta) = \theta_1^1 f_1(z) + \theta_2^1 f_2(z) + \mu[\theta_1^2 g_1(z) + \theta_2^2 g_2(z)]$$

where:

$$\left.\begin{aligned} &\theta_1^1 = \omega_0,\ \theta_2^1 = \omega_1,\ f_1(z) = \begin{bmatrix} z_2 \\ -z_1 \end{bmatrix},\ f_2(z) = \begin{bmatrix} 0 \\ -z_2 \end{bmatrix} \\ &\theta_1^2 = \omega_0,\ \theta_2^2 = b;\ g_1(z) = \begin{bmatrix} z_2 \\ -z_1 \end{bmatrix},\ g_2(z) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{aligned}\right\} \tag{3.30}$$

The average equilibrium points of (3.28), for constant duty ratio $\mu = U$, are given by:

$$\begin{aligned} z_1 &= Z_1(U) = -\frac{Ub\omega_1}{(1-U)^2\omega_0^2} = \frac{U\theta_2^2\theta_2^1}{(\theta_1^1 - U\theta_1^2)^2}; \\ z_2 &= Z_2(U) = -\frac{Ub}{(1-U)\omega_0} = -\frac{U\theta_2^2}{\theta_1^1 - U\theta_1^2};\ \mu = U \end{aligned} \tag{3.31}$$

From where it follows that:

$$Z_1(U) = \frac{\omega_1}{b} Z_2(U)\,[Z_2(U) - b/\omega_0] = \frac{\theta_2^1}{\theta_3^1} Z_2(U) - \frac{\theta_2^1\theta_1^2}{\theta_1^1\theta_2^2} Z_2^2(U) \tag{3.32}$$

The output of the average system (3.28) is now taken as:

$$\begin{aligned} y = h(z) &= z_1 - Z_1(U) = z_1 - \frac{\omega_1}{b} Z_2(U)\,[Z_2(U) - b/\omega_0] \\ &= z_1 + \frac{\theta_2^1}{\theta_3^1} Z_2(U) - \frac{\theta_2^1\theta_1^2}{\theta_1^1\theta_2^2} Z_2^2(U) \end{aligned} \tag{3.33}$$

Evidently, system (3.28), with output (3.33), is everywhere a relative degree one system, except on the line $\omega_0 z_2 = b$. Our results are only locally valid for those controlled state trajectories that do not cross the above line.

3.3.2. *Non-adaptive feedback regulation of the average PWM controlled buck-boost converter.* Imposing on the input current state equation an asymptotically stable behaviour:

$$\dot{z}_1 = -\alpha\,[z_1 - Z_1(U)] = -\alpha\left[z_1 - \frac{\omega_1}{b}\cdot Z_2(U)\,(Z_2(U) - b/\omega_0)\right] \tag{3.34}$$

with $\alpha > 0$, one immediately obtains, by simple inversion carried out on the first equation of (3.2), the computed duty ratio function μ_c as:

$$\begin{aligned} \mu_c(z) &= \frac{\omega_0 z_2 + \alpha\,(z_1 - Z_1(U))}{\omega_0 z_2 - b} \\ &= \frac{\theta_1^1 z_2 + \alpha[z_1 + (\theta_2^1/\theta_3^1)Z_2(U) - (\theta_2^1\theta_1^2/\theta_1^1\theta_2^2)Z_2^2(U)]}{\theta_1^2 z_2 - \theta_2^2} \end{aligned} \tag{3.35}$$

The actual duty ratio function $\mu(z)$ is simply obtained as before by:

$$\mu(z) = \mathrm{sat}_{[0,1]}\,\mu_c(z) := \begin{cases} 1 & \text{if } \mu_c(z) \geq 1 \\ \mu_c(z) & \text{if } 0 < \mu_c(z) < 1 \\ 0 & \text{if } \mu_c(z) \leq 0 \end{cases} \tag{3.36}$$

In steady-state stable operation, the computed duty ratio function takes, according to (3.9), the value:

$$\mu_c = U = \frac{\omega_0 Z_2(U)}{\omega_0 Z_2(U) - b} \tag{3.37}$$

which is compatible with (3.31). Demanding that the steady-state value of the duty ratio function be confined to the interval (0, 1), one obtains the following condition:

$$\omega_0 Z_2(U) > 0 > -b \Leftrightarrow \omega_0 Z_2(U) + b > 0 \tag{3.38}$$

which guarantees, at least in steady-state operation, the properness of the relative degree definition.

The zero dynamics is obtained in this case as:

$$\dot{z}_2 = -\omega_1 z_2 + \frac{b\omega_1 Z_1(U)}{\omega_0 z_2 - b} \tag{3.39}$$

which has only one physically meaningful equilibrium point represented by the positive solution $z_2 = Z_2(U)$ of:

$$z_2^2 - \frac{b}{\omega_0} z_2 - b\frac{Z_1(U)}{\omega_1} = 0$$

Moreover, such an equilibrium point is locally asymptotically stable, as can be easily verified from a standard argument on the following Lyapunov function candidate and its time derivative along the solutions of the zero dynamics differential equation (3.39):

$$V(z) = \tfrac{1}{2}\left[z_2^2 - \frac{b}{\omega_0} z_2 - \frac{bZ_1(U)}{\omega_1}\right]^2 \Rightarrow \dot{V}(z) = -2\omega_1 \left[\frac{z_2 - b/(2\omega_0)}{z_2 - b/\omega_0}\right] V(z)$$

Notice that for $z_2 > b/(2\omega_0)$ the function:

$$\left[\frac{z_2 - b/(2\omega_0)}{z_2 - b/\omega_0}\right]$$

is strictly positive and bounded above by 1; hence, for initial conditions specified around the assumed positive equilibrium point for z_2, the trajectories of the zero dynamics asymptotically converge towards such an equilibrium point.

3.3.3. *Adaptive feedback regulation of the PWM controlled buck-boost converter.* An adaptive version of the controller (3.35), (3.36) again requires the introduction of new unknown parameters which make the input current error satisfy the hypothesized linear parametric dependence of the output. We thus redefine the average input current error (3.33) as:

$$y = h(z) = z_1 - Z_1(U) = z_1 + \theta_1^3 Z_2(U) - \theta_3^2 Z_2^2(U) \tag{3.40}$$

with:

$$\theta_1^3 = \frac{\theta_2^1}{\theta_3^1}; \quad \theta_2^3 = \frac{\theta/{}_2^1\theta_1^2}{\theta_1^1\theta_2^2} \tag{3.41}$$

i.e. in terms of the notation adopted in (2.4) we have: $h_0(z) = z_1$ and $h_1(Z) = Z_2(U)$, $h_2(Z) = -Z_2^2(U)$.

The average adaptive controller, based on (3.35), can then be obtained in terms of the estimated values of the actual parameters, defined in (3.30) and (3.41), as:

$$\hat{\mu}_c(z) = \frac{\hat{\theta}_1^1 z_2 + \alpha[z_1 + \hat{\theta}_1^3 Z_2(U) - \hat{\theta}_2^3 Z_2^2(U)]}{\hat{\theta}_1^2 z_2 - \theta_2^2} \tag{3.42}$$

and

$$\hat{\mu}(z) = \text{sat}_{[0,1]}\,\hat{\mu}_c(z) := \begin{cases} 1 & \text{if } \hat{\mu}_c(z) \geqslant 1 \\ \hat{\mu}_c(z) & \text{if } 0 < \hat{\mu}_c(z) < 1 \\ 0 & \text{if } \hat{\mu}_c(z) \leqslant 0 \end{cases} \tag{3.43}$$

The regressor vector, corresponding to the proposed adaptive scheme of § 2, is given by:

$$w = \begin{bmatrix} L_{f_1} h_0(z) \\ L_{f_2} h_0(z) \\ \hat{\mu}(z) L_{g_1} h_0(z) \\ \hat{\mu}(z) L_{g_2} h_0(z) \\ \alpha h_1(Z) \\ \alpha h_2(Z) \end{bmatrix} = \begin{bmatrix} z_2 \\ 0 \\ -z_2\hat{\mu}(z) \\ \hat{\mu}(z) \\ \alpha Z_2(U) \\ -\alpha Z_2^2(U) \end{bmatrix} \tag{3.44}$$

The corresponding parameter adaptation laws for the average PWM controlled system are readily obtained from (2.6) as:

$$\begin{gathered} \dot{\hat{\theta}}_1^1 = z_2 y;\ \dot{\hat{\theta}}_2^1 = 0;\ \dot{\hat{\theta}}_1^2 = -\, z_2 y \hat{\mu}(z);\ \dot{\hat{\theta}}_2^2 = \hat{\mu}(z) y; \\ \dot{\hat{\theta}}_1^3 = \alpha Z_2(U) y;\ \dot{\hat{\theta}}_2^3 = -\alpha Z_2^2(U) y \end{gathered} \tag{3.45}$$

The actual (discontinuous) adaptive PWM controller equations are summarized below:

$$u = \begin{cases} 1 & \text{for } t_k \leqslant t < t_k + \hat{\mu}[x(t_k)]T \\ 0 & \text{for } t_k + \hat{\mu}[x(t_k)]T < t \leqslant t_k + T \end{cases} \quad t_k + T = t_{k+1};\quad k = 0, 1, 2, \ldots \tag{3.46}$$

with,

$$\hat{\mu}(x) = \text{sat}_{[0,1]}\,\hat{\mu}_c(x) := \begin{cases} 1 & \text{if } \hat{\mu}_c(x) \geqslant 1 \\ \hat{\mu}_c & \text{if } 0 < \hat{\mu}_c(x) < 1 \\ 0 & \text{if } \hat{\mu}_c(x) \leqslant 0 \end{cases} \tag{3.47}$$

$$\hat{\mu}_c(x) = \frac{\hat{\theta}_1^1 x_2 + \alpha[x_1 + \hat{\theta}_1^3 X_2(U) - \hat{\theta}_2^3 X_2^2(U)]}{\hat{\theta}_1^2 x_2 - \theta_2^2} \tag{3.48}$$

where $X_2(U) = Z_2(U)$.

$$\begin{gathered} \dot{\hat{\theta}}_1^1 = x_2 y;\ \dot{\hat{\theta}}_2^1 = 0;\ \dot{\hat{\theta}}_1^2 = -x_2 y \hat{\mu}(x);\ \dot{\hat{\theta}}_2^2 = \hat{\mu}(x) y \\ \dot{\hat{\theta}}_1^3 = \alpha X_2(U) y;\ \dot{\hat{\theta}}_2^3 = -\alpha X_2^2(U) y \end{gathered} \tag{3.49}$$

3.3.4. *Simulation results.* Simulations were carried out on a buck-boost converter with nominal circuit parameter values given by $R = 1{\cdot}12\ \Omega$, $L = 0{\cdot}195$ mH, $C = 2000\ \mu$F and $E = 28$ V, which correspond to the following nominal values for the characteristic parameters $\omega_0 = 1{\cdot}601281 \times 10^3$, $\omega_1 = 0{\cdot}44642 \times 10^3$ and $b = -2{\cdot}005121 \times 10^3$. A desired normalized steady-state output voltage was set to $Z_2(U) = 1{\cdot}8783$ V, with a corresponding steady-state value for the duty ratio of $U = 0{\cdot}6$. The non-adaptive version of the average and the actual PWM controller was first simulated. Figure 7 shows the non-adaptively controlled average normalized input current and the corresponding average normalized output voltage trajectories superimposed on the actual (discontinuous) PWM controlled responses. The sampling frequency was set to 10 kHz while the closed loop eigenvalue for the normalized average input current linearization was chosen as: $\alpha = 2 \times 10^3\ s^{-1}$. The actual PWM-controlled responses are shown to chatter closely about the average trajectories which exhibit asymptotic convergence towards the desired (average) equilibrium. As before, a small bias may be noticed in the chattering response of the average input inductor current with respect to the corresponding average steady-state response. This is due to the finite sampling frequency being used and the 'modelling error' effect of the infinite frequency average-based controller. Figure 8 shows the average and the actual state trajectories obtained when the adaptive version of the PWM controller is used. Again, the desired equilibrium value is achieved for both the average and the actual adaptive PWM controlled normalized output voltage. No noticeable discrepancies, or steady state biases, exist between the average and the actual PWM-controlled state responses. Figure 9 depicts the obtained trajectories for the adaptive controller parameters in the actual PWM controlled converter.

3.4. *Direct output voltage regulation for the PWM controlled buck–boost converter*

It is now shown that a normalized output voltage regulation for the buck-boost converter is infeasible if one resorts to input–output linearization schemes based on partial inversion of the converter system equations.

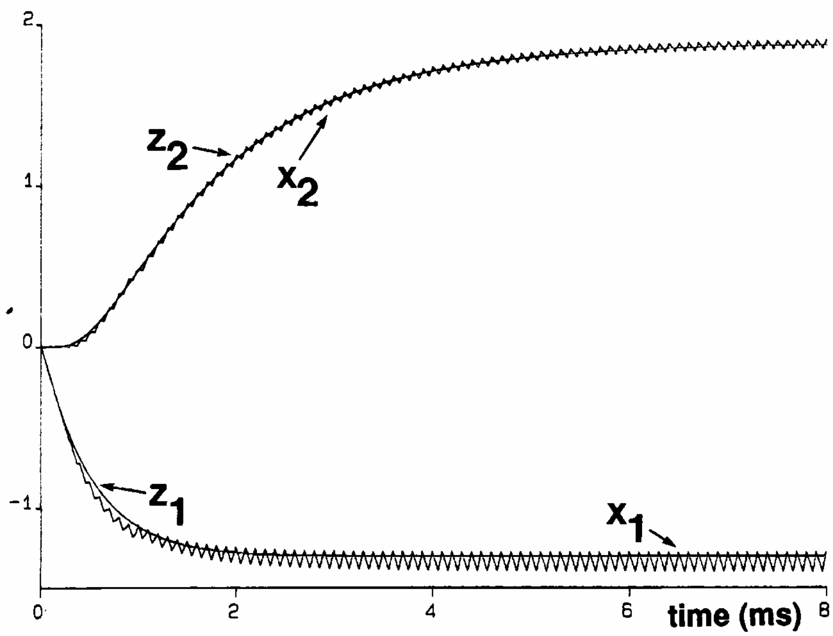

Figure 7. Non-adaptive average normalized input current and average normalized output voltage trajectories superimposed on the actual (discontinuous) PWM-controlled responses for the buck-boost converter.

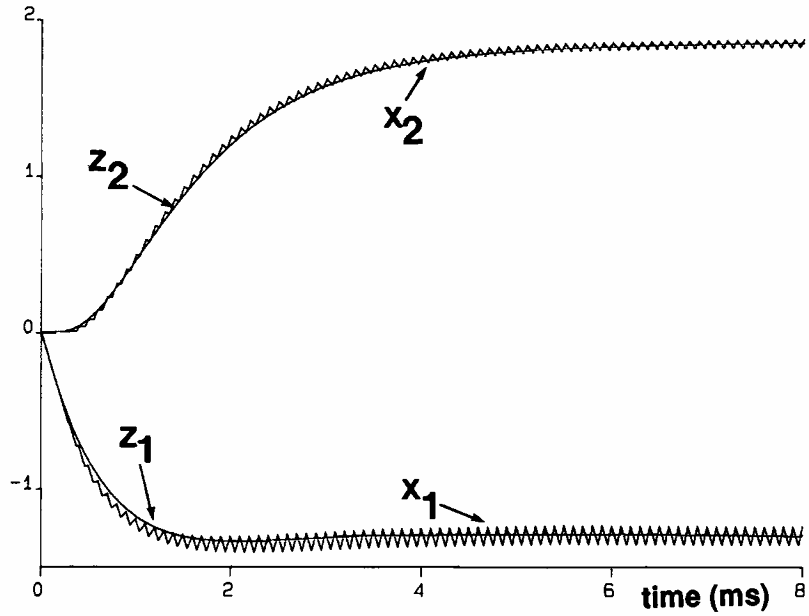

Figure 8. Adaptively controlled average, and actual state trajectories for the buck–boost converter example.

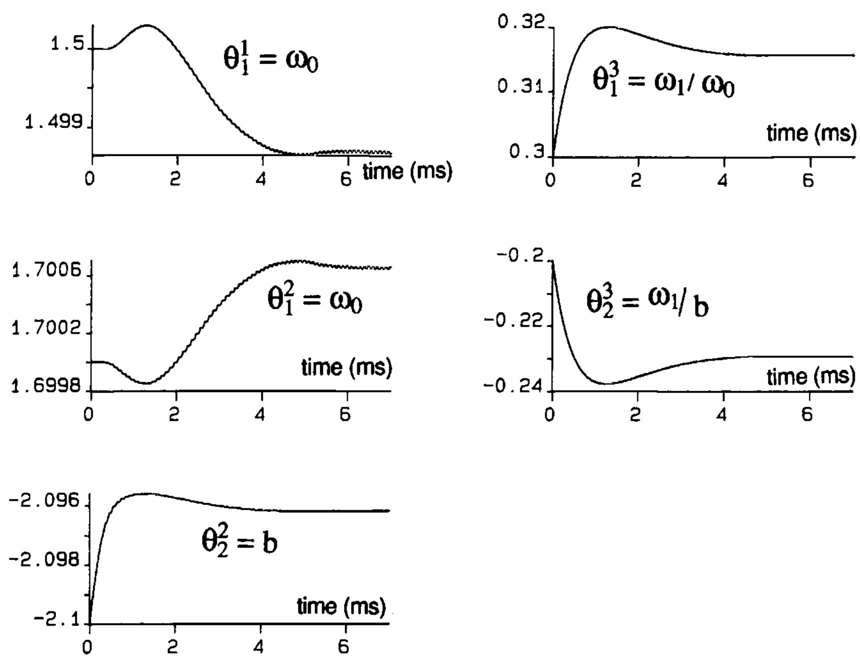

Figure 9. Trajectories for the adaptive controller parameters of actual PWM-controlled buck–boost converter.

Indeed if one adopts, as the output of the buck-boost converter circuit, the output voltage error with respect to its steady state value, i.e.

$$y = z_2 - Z_2(U) \tag{3.50}$$

A desired output voltage dynamics of the form:

$$\dot{z}_2 = -\alpha\,[z_2 - Z_2(U)] \tag{3.51}$$

with $\alpha > 0$, yields the following computed duty ratio function μ_c as:

$$\mu_c(z) = \frac{\omega_0 z_1 + \omega_1 z_2 - \alpha(z_2 - Z_2(U))}{\omega_0 z_1} \tag{3.52}$$

The associated zero dynamics is now:

$$\dot{z}_1 = b - \frac{\omega_1 Z_2(U)[Z_2(U) - b/\omega_0]}{z_1} \tag{3.53}$$

which has a unique equilibrium point given by $z_1 = Z_1(U) = \omega_1/b\; Z_2(U)[Z_2(U) - b/\omega_0]$. It can be verified that such an equilibrium point is unstable and, thus, the system is non-minimum phase. This fact renders any adaptation scheme based on (3.52) totally unfeasible.

3.5. *Adaptive indirect output voltage regulation for the PWM controlled buck converter*

We now summarize the equations and steps leading to an adaptive indirect output voltage regulation for a PWM controlled buck converter (see Fig. 10).

Normalized state representation of the buck converter (Wood 1974, Sira-Ramirez 1987):

$$\begin{aligned}\dot{x}_1 &= -\omega_0 x_2 + ub\\ \dot{x}_2 &= \omega_0 x_1 - \omega_1 x_2\end{aligned} \tag{3.54}$$

Average PWM model

$$\begin{aligned}\dot{z}_1 &= -\omega_0 z_2 + \mu b\\ \dot{z}_2 &= \omega_0 z_1 - \omega_1 z_2\end{aligned} \tag{3.55}$$

$$\begin{bmatrix}\dot{z}_1\\ \dot{z}_2\end{bmatrix} = \omega_0\begin{bmatrix}-z_2\\ z_1\end{bmatrix} + \omega_1\begin{bmatrix}0\\ -z_2\end{bmatrix} + \mu\left(b\begin{bmatrix}1\\ 0\end{bmatrix}\right) \tag{3.56}$$

or:

$$\dot{z} = f(z) + \mu g(z) = \theta_1^1 f_1(z) + \theta_2^1 f_2(z) + \mu[\theta_1^2 g_1(z)]$$

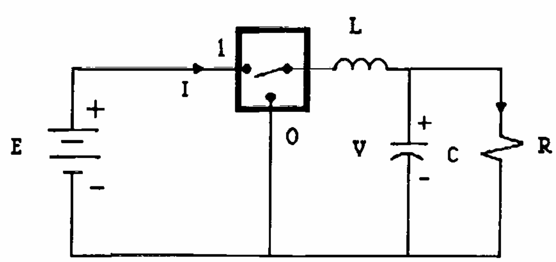

Figure 10. The buck converter.

with:

$$\theta_1^1 = \omega_0,\ \theta_2^1 = \omega_1;\ f_1(z) = \begin{bmatrix} -z_2 \\ z_1 \end{bmatrix},\ f_2(z) = \begin{bmatrix} 0 \\ -z_2 \end{bmatrix} \qquad (3.57)$$
$$\theta_1^2 = b;\ g_1(z) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Average equilibrium point

$$z_1 = Z_1(U) = \frac{U\omega_1 b}{\omega_0^2} = \frac{U\theta_2^1\theta_1^2}{(\theta_1^1)^2} \qquad (3.58)$$
$$z_2 = Z_2(U) = \frac{U\theta_1^2}{\theta_1^1} = ;\quad \mu = U$$

Output function for the average system

$$y = h(z) = z_1 - Z_1(U) = z_1 - \frac{\omega_1}{\omega_0} Z_2(U) = Z_1 - \frac{\theta_2^1}{\theta_1^1} Z_2(U) \qquad (3.59)$$

Output equation with linear parametrization for the purposes of adaptation

$$y = h(z) = z_1 - \frac{\theta_2^1}{\theta_1^1} Z_2(U) = z_1 - \theta_1^3 Z_2(U)$$

Desired input current dynamics

$$\dot{z}_1 = -\alpha\,[z_1 - Z_1(U)] = -\alpha\left[z_1 \frac{\omega_1}{\omega_0} Z_2(U)\right] = -\alpha\left[z_1 \frac{\theta_2^1}{\theta_1^1} Z_2(U)\right] \qquad (3.60)$$

with $\alpha > 0$.

The computed duty ratio function

$$\mu_c(z) = \frac{\omega_0 z_2 - \alpha\,[z_1 - (\omega_1/\omega_0) Z_2(U)]}{b} \qquad (3.61)$$
$$= \frac{\theta_1^1 z_2 - \alpha\,[z_1 - (\theta_2^1/\theta_1^1) Z_2(U)]}{\theta_1^2}$$

Actual duty ratio function

$$\mu = \mathrm{sat}_{[0,1]}\,\mu_c := \begin{cases} 1 & \text{if } \mu_c \geqslant 1 \\ \mu_c & \text{if } 0 < \mu_c < 1 \\ 0 & \text{if } \mu_c \leqslant 0 \end{cases} \qquad (3.62)$$

Steady-state computed duty ratio function

$$\mu_c = U = \frac{\omega_0}{b} Z_2(U) \qquad (3.63)$$

Normalized output voltage attenuation and positivity conditions arising from $0 < \mu = \mu_c < 1$

$$0 < Z_2(U) < \frac{b}{\omega_0} \qquad (3.64)$$

Zero dynamics (exponentially asymptotically stable)

$$\dot{z}_2 = -\,\omega_1\left[z_2 - \frac{\omega_0}{\omega_1} Z_1(U)\right] = -\omega_1[z_2 - Z_2(U)] \qquad (3.65)$$

Adaptive average controller

$$\hat{\mu}_c(z) = \frac{\hat{\theta}_1^1 z_2 - \alpha [z_1 - \hat{\theta}_1^3 Z_2(U)]}{\hat{\theta}_1^2} \tag{3.66}$$

$$\hat{\mu} = \mathrm{sat}_{[0,1]}\, \hat{\mu}_c := \begin{cases} 1 & \text{if } \hat{\mu}_c \geq 1 \\ \hat{\mu}_c & \text{if } 0 < \hat{\mu}_c < 1 \\ 0 & \text{if } \hat{\mu}_c \leq 0 \end{cases} \tag{3.67}$$

The regressor vector

$$w = \begin{bmatrix} L_{f_1} h \\ L_{f_2} h \\ \hat{\mu} L_{g_1} h \\ \alpha h_1(Z) \end{bmatrix} = \begin{bmatrix} -z_2 \\ 0 \\ \hat{\mu} \\ -\alpha Z_2(U) \end{bmatrix} \tag{3.68}$$

The parameter adaptation laws for average PWM-controlled system

$$\dot{\hat{\theta}}_1^1 = -y\, z_2;\ \dot{\hat{\theta}}_2^1 = 0;\ \dot{\hat{\theta}}_1^2 = y\hat{\mu};\ \dot{\hat{\theta}}_1^3 = -\alpha y Z_2(U) \tag{3.69}$$

Discontinuous adaptive PWM controller

$$u = \begin{cases} 1 & \text{for } t_k \leq t < t_k + \hat{\mu}[x(t_k)]T \\ 0 & \text{for } t_k + \hat{\mu}[x(t_k)]T < t \leq t_k + T \end{cases} \quad t_k + T = t_{k+1};\quad k = 0, 1, 2, \ldots \tag{3.70}$$

$$\hat{\mu}(x) = \mathrm{sat}_{[0,1]}\, \hat{\mu}_c(x) := \begin{cases} 1 & \text{if } \hat{\mu}_c(x) \geq 1 \\ \hat{\mu}_c & \text{if } 0 < \hat{\mu}_c(x) < 1 \\ 0 & \text{if } \hat{\mu}_c(x) \leq 0 \end{cases} \tag{3.71}$$

$$\hat{\mu}_c(x) = \frac{\hat{\theta}_1^1 x_2 - \alpha [x_1 - \hat{\theta}_1^3 X_2(U)]}{\hat{\theta}_1^2} \tag{3.72}$$

with: $X_1(U) = Z_1(U)$.

Parameter update laws

$$\dot{\hat{\theta}}_1^1 = -x_2 y;\ \dot{\hat{\theta}}_2^1 = 0;\ \dot{\hat{\theta}}_1^2 = y\hat{\mu}(x);\ \dot{\hat{\theta}}_1^3 = -\alpha y Z_2(U) \tag{3.73}$$

Simulation results

Simulations were carried on a buck converter with nominal circuit parameter values given by $R = 0{\cdot}81\ \Omega$, $L = 0{\cdot}20$ mH, $C = 500{\cdot}0\ \mu$F and $E = 63{\cdot}28$ V, which correspond to the following nominal values for the characteristic parameters $\omega_0 = 3{\cdot}1623 \times 10^3$, $\omega_1 = 2{\cdot}4691 \times 10^3$ and $b = 4{\cdot}4746 \times 10^3$. A desired normalized steady-state output voltage was set to $Z_2(U) = 0{\cdot}7794$ V, with a corresponding steady-state value for the duty ratio of $U = 0{\cdot}5$. Figure 11 shows the average normalized input current and the average normalized output voltage trajectories superimposed in the actual (discontinuous) non-adaptive PWM controlled responses. The sampling frequency used was set to 20 kHz and the closed loop eigenvalue for the normalized average input current linearization was chosen as $\alpha = 3 \times 10^3\ \mathrm{s}^{-1}$. Convergence toward the desired equilibrium normalized input current is obtained for the average PWM controlled responses while the actual (i.e. discontinuous) non-adaptive PWM controlled responses exhibit very small discrepancies with respect to the average response. Figure 12 shows the average and the actual state trajectories obtained when the adaptive version of the PWM controller is used. Again, the desired equilibrium value is

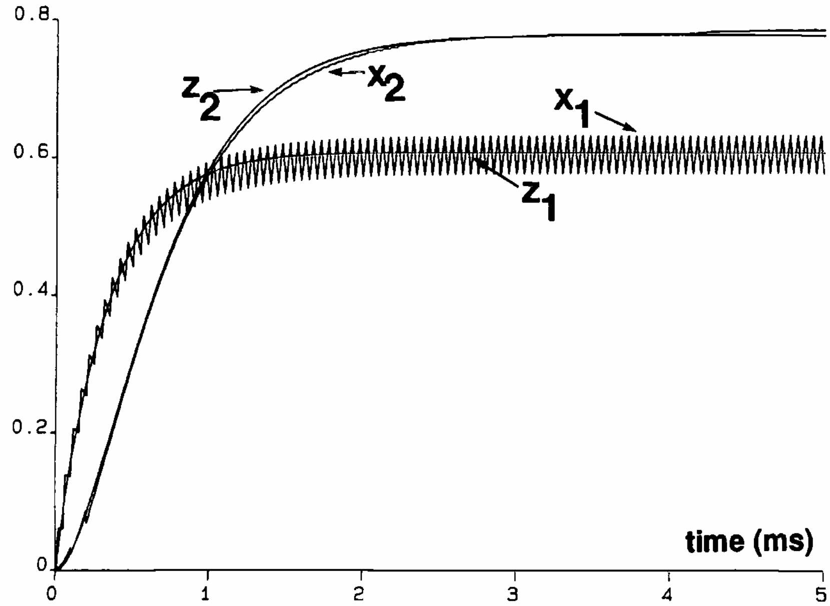

Figure 11. Non-adaptive average normalized input current and average normalized output voltage trajectories superimposed on the actual (discontinuous) PWM-controlled responses for the buck converter.

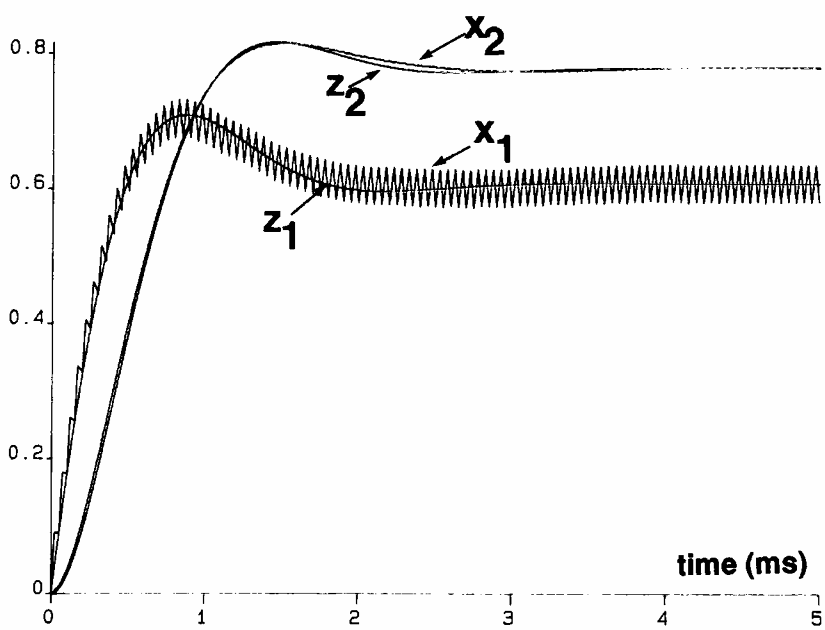

Figure 12. Adaptively controlled average, and actual state trajectories for the buck converter example.

perfectly achieved without noticeable discrepancies for both the average and the actual normalized input current and output voltages. Figure 13 depicts the obtained trajectories for the adaptation parameters in the actual PWM-controlled converter.

3.6. *Remarks on the direct output voltage regulation for the PWM controlled buck converter*

It is easy to see that the average model of the buck converter constitutes a linear time-invariant system controlled by the (hard limited) duty ratio function. This linear system is, both, controllable and observable from the output voltage error. Moreover, from an input–output viewpoint, the average model is relative degree two. Direct output voltage error stabilization is thus feasible, without the minimum phase problems present in the boost and buck–boost cases.

A non-adaptive controller which imposes a desired second-order linear dynamics on the output voltage error is readily computed from the observability canonical form of the average system. The adaptation scheme, developed in § 2, no longer applies to this case and a more involved adaptive feedback regulator must be computed using the results of Sastry and Isidori (1989) for such a class of systems. One of the key issues that must be dealt with, for the resulting adaptive regulator, is the necessary over parametrization (see Campion and Bastin 1990), due to the several products and powers of the unknown system parameters. Such an overparametrization results from the need to obtain parametric linearity in the adaptively controlled second-order dynamics. This fact makes the resulting adaptive controller quite complex and of dubious practical interest since 28 differential equations must be considered for the adaptation system.

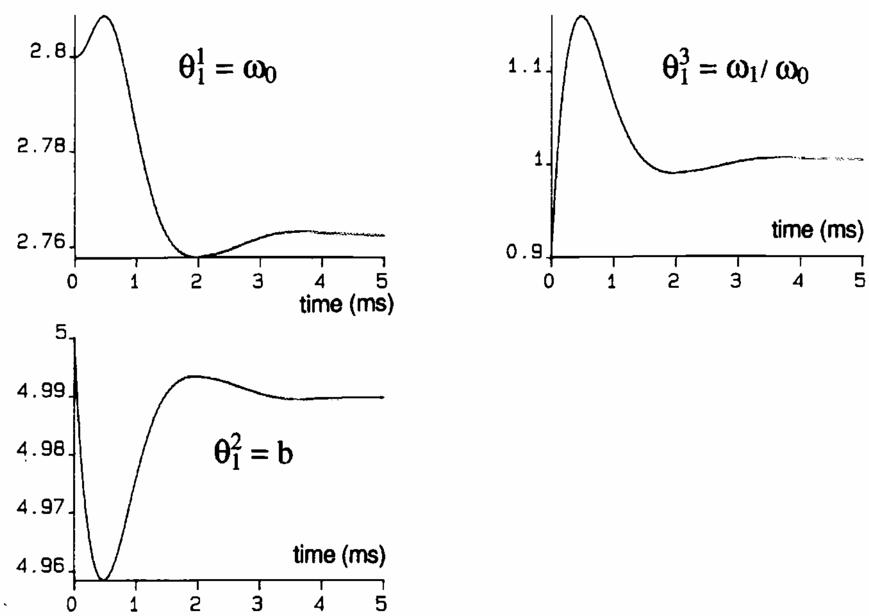

Figure 13. Trajectories for the adaptive controller parameters of the actual PWM-controlled buck converter.

4. Conclusions and suggestions for further work

In this article, a general adaptive feedback regulation scheme for PWM controlled DC-to-DC power converters has been proposed. The treated cases included the buck, the boost and the buck-boost converters. Using modern nonlinear feedback controller design techniques, based on partial linearization and system inversion, a non-adaptive regulator is developed for the average PWM system which indirectly stabilizes the output load voltage, to a desirable constant equilibrium point, through an input current stabilization strategy. In order to avoid non-minimum phase problems, the proposed adaptive controllers necessarily process input current error information, computed in terms of the required steady state value of the output voltage, rather than directly operating on the basis of the output voltage error information. The non-adaptive average-model based design strategy is shown to yield feedback controlled state responses which represent a rather good approximation to the actual discontinuous PWM controlled responses, with very small steady-state errors. The actual discontinuous adaptive PWM controlled responses are generally shown to yield improved feedback controlled state responses which are in almost perfect steady-state agreement with the (ideal) adaptive average PWM-controlled responses. Thus, adaptation is seen to compensate effectively not only for unmodelled parameter variations but also for the errors inherent in the actual use of an average-based feedback controller design.

Some other non-adaptive feedback regulation schemes for PWM controlled DC-to-DC power supplies, such as those based on either linear or nonlinear PD, PI and PID actions may also be made adaptive using techniques similar to those developed in this preliminary study (see Sira-Ramirez 1990, 1991 a). Adaptive dynamical feedback regulation, based on dynamical duty ratio synthesizers, can be developed for DC-to-DC converters along similar, but certainly more involved, lines (see Sira-Ramirez and Lischinsky-Arenas 1991 and Sira-Ramírez *et al*. 1992).

Acknowledgment

This work was supported by the Consejo de Desarrollo Científico, Humanístico y Tecnológico of the Universidad de Los Andes under Research Grants I-362-91 and I-358-91.

References

Campion, G., and Bastin, G., 1990, Indirect adaptive state feedback control of linearly parametrized non-linear systems. *International Journal of Adaptive Control Signal Processing*, **4**, 345–358.

Csaki, F., Gansky, K., Ipsitz, I., and Marti, S., 1983, *Power Electronics* (Budapest, Hungary: Akademia Kiado).

IEEE Power Electronics Specialists Conference Records.

Kanellakopoulos, I., Kokotovic, P. V., and Marino, R., 1989, Robustness of adaptive nonlinear control under and extended matching condition. In *Proceedings of the IFAC Symposium Nonlinear Control of Systems Design* (NOLCOS), Capri, Italy, pp. 192–197.

Kanellakopoulos, I., Kokotovic, P. V., and Morse, S., 1991, Systematic design of adaptive controllers for feedback linearizable systems. *IEEE Transactions on Automatic Control* **36**, 1241–1253.

KASSAKIAN, J. G., SCHLECHT, M. F., and VERGHESE, G. C., 1991, *Principles of Power Electronics* (Reading, Mass: Addison-Wesley).

KOKOTOVIC, P. V., 1991, *Foundations of Adaptive Control*. Lecture Notes in Control and Information Sciences (Berlin: Springer-Verlag).

KREIN, P. T., BENTSMAN, J., BASS, R. M., and LESIEUTRE, B. L., 1990, On the use of averaging for the analysis of power electronics systems. *IEEE Transactions on Power Electronics*, **5**, 182–190.

MIDDLEBROOK, R. D., and CÚK, S., 1981, *Advances in Switchedmode Power Converter Circuits* (Pasadena, Calif.: Tesla); 1983, *Advances in Switchedmode Power Converter Circuits* (Pasadena, Calif.: Tesla).

MOHAN, N., UNDELAND, T. M., and ROBBINS, W. P., 1989, *Power Electronics, Converters, Applications and Design* (New York: Wiley).

NARENDRA, K., and ANNASWAMMY, A., 1989, *Stable Adaptive Systems* (Englewood Cliffs, NJ: Prentice Hall).

SASTRY, S. and BODSON, M., 1989, *Adaptive Control* (Englewood Cliffs, NJ: Prentice Hall).

SASTRY, S., and ISIDORI, A., 1989, Adaptive control of linearizable systems. *IEEE Transactions on Automatic Control*, **34**, 1123–1131.

SEVERNS, R. P. and BLOOM, G., 1985, *Modern DC-to-DC Switchmode Power Converter Circuits* (New York: Van Nostrand-Reinhold).

SIRA-RAMÍREZ, H., 1987, Sliding motions in bilinear switched networks. *IEEE Transactions on Circuits and Systems*, **34**, 919–933; 1988, Sliding motions on slow manifolds of systems with fast actuators. *International Journal of Systems Science* **47**, 875–887; 1989 a, Switched control of bilinear converters via pseudolinearization. *IEEE Transaction on Circuits and Systems*, **36**, 858–865; 1989 b, A geometric approach to pulse-width-modulated control in nonlinear dynamical systems. *IEEE Transactions on Automatic Control*, **34**, 184–187; 1990, *International Journal of Control*, Design of P–I controllers for dc-to-dc power supplies via extended linearization, **51**, 601–620; 1991 a Nonlinear P–I controller design for switch-mode dc-to-dc power converters. *IEEE Transactions on Circuits and Systems*, **38**, 410–417. 1991 b, Nonlinear dynamically feedback controlled descent on a non atmosphere-free planet: a differential algebraic approach. *Control: Theory and Advanced Technologies* **7**, 301–320; 1991 c, Nonlinear dynamical feedback strategies in aerospace systems control: a differential algebraic approach. *Proceedings of First European Control Conference*, Grenoble, France, Vol. 3, pp. 2038–2043.

SIRA-RAMÍREZ, H., and ILIC-SPONG, M., 1988, A geometric approach to the feedback control of switchmode dc-to-dc power supplies. *IEEE Transactions on Circuits and Systems*, **35**, 1291–1298; 1989, Exact linearization in switch mode dc-to-dc power converters. *International Journal of Control*, **50**, 511–524.

SIRA-RAMÍREZ, H., and LISCHINSKY-ARENAS, P., 1990, Dynamical discontinuous feedback control of nonlinear systems. *IEEE Transactions on Automatic Control* **35**, 1373–1378; 1991, The differential algebraic approach in nonlinear dynamical compensator design for dc-to-dc power converters. *International Journal of Control*, **54**, 111–134.

SIRA-RAMÍREZ, H., LISCHINSKY-ARENAS, P., and LLANES-SANTIAGO, O., 1993, Dynamical compensator design in nonlinear aerospace systems. *IEEE Transactions on Aerospace and Electronic Systems* (to be published).

SIRA-RAMÍREZ, H., and PRADA-RIZZO, M. T., 1992, Nonlinear feedback regulator design for the Cúk converter. *IEEE Transactions on Automatic Control* **37**, 1173–1180.

SIRA-RAMÍREZ, H., ZRIBI, M., and AHMAD, S., 1992, Adaptive dynamical feedback regulation strategies for linearizable uncertain systems. *International Journal of Control*, to be published.

TEEL, A., KADIYALA, R., KOKOTOVIC, P. V., and SASTRY, S., 1991, Indirect techniques for adaptive input-output linearization of non-linear systems. *International Journal of Control*, **53**, 193–222.

UTKIN, V. I., 1978, *Sliding Modes and Their Applications in Variable Structure Systems* (Moscow: MIR).

WOOD, J. R., 1974, Power conversion in electrical networks. NASA Report No. CR-120830, Harvard University Division of Engineering and Applied Physics, Cambridge, Massachusetts.