CONTROL-THEORY AND ADVANCED TECHNOLOGY
Vol. 9, No. 4, pp. 935–945, December, 1993

C91049sR

Short Paper

PULSE-FREQUENCY-MODULATION CONTROL OF NONLINEAR SYSTEMS*

H. SIRA-RAMÍREZ[1]

Abstract. The design of stabilizing Pulse-Frequency-Modulated (PFM) controllers is addressed, in all generality, for the case of nonlinear single-input single-output analytic systems. A dynamical PFM control strategy is developed, in full detail, on the basis of an elementary scalar PFM controlled system result. An Illustrative design example is provided.

Key Words—Pulse-Frequency-Modulation control, nonlinear systems.

1. Introduction

Pulse-Frequency-Modulated (PFM) feedback control strategies have been relatively little studied in the second half of this century. The main reference in this area is constituted by the work of Skoog and Blankenship (1970) where many early references can be found. Early works are all centered around the case of linear time-invariant systems. To our knowledge, no further extensions of these works, to the nonlinear case, were pursued later on.

In this article, we present a general design method for synthesizing dynamical PFM feedback control laws stabilizing to a constant equilibrium point any *minimum phase* single-input single-output nonlinear system. By means of dynamical discontinuous feedback, we effectively circumvent the chattering problem associated with the bang-bang type of discontinuities associated with PFM control inputs. The dynamical PFM controller design is accomplished by first proposing a static PFM controller on the corresponding Normal Canonical Form of a generalized version of the *extended system* (Nijmeijer and Van der Schaft, 1990). In contradistinction to the extended system, which uses only one integrator before the input, the *Generalized Extended System* is obtained by joining to the original system input a string of integrators of length equal to the dimension of the *zero dynamics* of the original input-output system.

Section 2 presents a fundamental result on the PFM control of an elementary scalar dynamical system. A full PFM controller design procedure, for higher order nonlinear plants, is based on this elementary result. Section 2 also introduces the general version of the extended system and discusses some of its properties. The Normal Canonical Form of the proposed generalized extended system is intimately related to Fliess' *Generalized Observability Canonical Form* (see Fliess, 1988; Conte et al., 1988). Section 3 develops a dynamical

* Received by the editors May 9, 1991 and in revised form December 21, 1992.
This work was supported by the Consejo de Desarrollo Científico, Humanístico y Tecnológico of the Universidad de Los Andes under Research Grant 1-358-91.
[1] Departamento Sistemas de Control, Universidad de Los Andes, Mérida, Venezuela.

PFM control design scheme for nonlinear minimum-phase systems. Section 4 presents an application example of chemical process control. It should be emphasized that the control of chemical processes represents an area in which discontinuous feedback control policies have not been traditionally used. The conclusions are collected in Sec. 5.

2. Some Fundamental Results

2.1 PFM control of a scalar system Consider the scalar PFM controlled dynamical system, in which the constants ρ_1, ρ_2, ρ_3 and W, are all strictly positive quantities.

$$\left.\begin{aligned}
\dot{s} &= -Wv,\\
v = \mathrm{PFM}_{\tau,T}(s) &= \begin{cases} \operatorname{sgn} s(t_k) & \text{for} \quad t_k \le t < t_k + \tau[s(t_k)]T[s(t_k)] \\ 0 & \text{for} \quad t_k + \tau[s(t_k)]T[s(t_k)] \le t < t_k + T[s(t_k)] \end{cases}\\
\tau[s(t)] &= \begin{cases} 1 & \text{for} \quad |s(t)| > \dfrac{1}{\rho_1} \\ \rho_1 |s(t)| & \text{for} \quad |s(t)| \le \dfrac{1}{\rho_1} \end{cases}\\
T[s(t)] &= \begin{cases} T_{\max} & \text{for} \quad |s(t)| \ge \dfrac{1}{\rho_2} \\ T_{\min} + \dfrac{\rho_2\rho_3}{\rho_3 - \rho_2}[T_{\max} - T_{\min}]\left(s(t) - \dfrac{1}{\rho_3}\right) & \text{for} \quad \dfrac{1}{\rho_3} < |s(t)| < \dfrac{1}{\rho_2} \\ T_{\min} & \text{for} \quad |s(t)| \le \dfrac{1}{\rho_3} \end{cases}\\
& k = 0, 1, 2, \cdots; \quad t_{k+1} = t_k + T[s(t_k)]
\end{aligned}\right\}, \tag{2.1}$$

where it is assumed that $\rho_2 < \rho_1 < \rho_3$. The t_k's represent *irregularly* spaced sampling instants, determined by the sampled values of the *duty cycle function*, denoted here by $T[s(t_k)]$. The duty cycle function, $T[s(t)]$, takes values on the closed interval $[T_{\min}, T_{\max}]$, and it varies linearly with respect to $s(t)$ in the region $|s| < 1/\rho_2$. The duty cycle, or sampling period, saturates to $T_{\max}$ for large values of s, and remains fixed at the constant lower bound $T_{\min}$ for small values of s. At each sampling instant, t_k, the value of the width of the sign-modulated, unit amplitude, control pulse is determined by the sampled value of the *duty ratio function*, represented by $\tau[s(t_k)]$. In general, the duty cycle and the duty ratio functions may be quite independent of each other. The function "sgn" stands for the *signum* function

$$\operatorname{sgn}(s) = \begin{cases} +1 & \text{if} \quad s > 0, \\ 0 & \text{if} \quad s = 0, \\ -1 & \text{if} \quad s < 0. \end{cases}$$

The condition $\rho_2 < \rho_1 < \rho_3$ indicates that the pulse width, τ, is saturated to the value of the duty cycle, T, as long as the value of the duty cycle is itself saturated to $T_{\max}$ (see Fig. 1). When the state, s, of the scalar system is decreased, in absolute value, below the boundary value $1/\rho_2$, the duty cycle, T, starts also decreasing, in a linear fashion with respect to s, while the pulse width temporarily continues to be saturated to the same values adopted by T. If the state s further decreases and reaches the interval $[-1/\rho_1, 1/\rho_1]$ (notice that $1/\rho_1$ is intermediate between $1/\rho_3$ and $1/\rho_2$), the pulse width also starts decreasing linearly with respect to s. When the state s is finally confined to the band $[-1/\rho_3, 1/\rho_3]$, the duty cycle (sampling period) reaches its minimum value $T_{\min}$. In this region, the duty ratio still continues to linearly decrease towards zero, even if the duty cycle is already saturated to its minimum value $T_{\min}$.

The following proposition establishes a sufficient condition for the asymptotic stability to zero of the PFM controlled system (2.1).

Proposition 2.1. The PFM controlled system (2.1) is asymptotically stable to $s = 0$, if

$$0 < \rho_3 W T_{\max} < 2. \tag{2.2}$$

Proof. Due to the piecewise constant nature of the control inputs and the linearity of the continuous system, it suffices to study the stability of the discretized version of (2.1) at the sampling instants. An exact discretization of the PFM controlled system (2.1) thus yields

$$s(t_k + T) = s(t_k) - W\mathrm{sgn}[s(t_k)]\tau[s(t_k)]T[s(t_k)]. \tag{2.3}$$

Suppose the initial condition $s(t_0)$ is chosen deep into the region $|s| > 1/\rho_2$. The evolution of the sampled values of $s(t)$ obey, according to (2.1)

$$\left.\begin{aligned} s(t_k + T) &= s(t_k) - WT_{\max} \quad \text{for} \quad s(t_k) > 0 \\ s(t_k + T) &= s(t_k) + WT_{\max} \quad \text{for} \quad s(t_k) < 0 \end{aligned}\right\}. \tag{2.4}$$

Hence, given an arbitrary initial condition $s(t_0)$ for s, it is obvious from (2.4) that the condition: $0 < \rho_3 W T_{\max} < 2$ is sufficient to ensure, that the value of $s(t_k)$ will be eventually found within the bounded region $|s| < 1/\rho_2$. This is due to the fact that the controlled increments taken by $s(t_k)$, in the considered

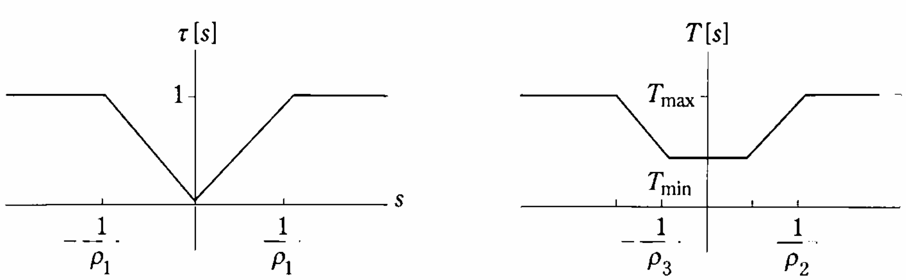

Fig. 1. Duty ratio and duty cycle functions for PFM actuator.

region $1/\rho_3 < | s | > 1/\rho_2$, are of width $WT_{\max}$ and, therefore, the condition: $WT_{\max} < 2/\rho_3$ also guarantees that $WT_{\max} < 2/\rho_2$. It follows that $s(t_k)$ cannot "jump" over the band $| s | < 1/\rho_2$, and hence, $s(t_k)$ will land on this region for sufficiently large k. Two possibilities arise then: either $s(t_k)$ is found in the "band" $1/\rho_3 < | s(t_k) | < 1/\rho_2$, or $s(t_k)$ satisfies $| s(t_k) | < 1/\rho_3$. Suppose first that: $1/\rho_3 < | s(t_k) | < 1/\rho_2$, for some sufficiently large k. In this region, the value of $| s(t_k) |$ can only further decrease, as it is easily seen from (2.1). Indeed, the increments: $\Delta s(t_k) = s(t_{k+1}) - s(t_k)$, that the variable s may take in the region $1/\rho_3 < | s | < 1/\rho_2$, satisfy: $WT_{\min} < | \Delta s(t_k) | < WT_{\max}$. Since, by assumption, $WT_{\max} < 2/\rho_3$, then one has: $WT_{\min} < | \Delta s(t_k) | < WT_{\max} < 2/\rho_3 < 2 | s(t_k) |$. It follows that $| s(t_k) |$ decreases, and that the controlled evolution of $s(t_k)$ will reach the region: $| s(t_k) | < 1/\rho_3$. In this last region, the sampled values of s, in accordance to (2.1), evolve, satisfying

$$\begin{aligned} s(t_k + T) &= s(t_k) - \rho_1 WT_{\min} \operatorname{sgn}[s(t_k)] \, | s(t_k) | \\ &= (1 - \rho_1 WT_{\min}) s(t_k), \end{aligned} \tag{2.5}$$

which is asymptotically stable to zero, if and only if: $0 < \rho_1 WT_{\min} < 2$. This last condition is evidently equivalent to $WT_{\min} < 2/\rho_1$. Notice, however, that from the assumptions about the parameters in (2.1): $WT_{\min} < WT_{\max} < 2/\rho_3 < 2/\rho_1$, i.e., the proposed condition (2.2) implies the asymptotical stability requirement for (2.5). The result follows.

2.2 The generalized extended system Consider the analytical n-dimensional state variable representation of a relative degree r single-input single-output system

$$\left.\begin{aligned} \dot{x} &= F(x, u) \\ y &= h(x) \end{aligned}\right\}. \tag{2.6}$$

One defines a *Generalized Extended System* of (2.6), as the $2n - r$ dimensional system obtained by placing a chain of $n - r$ integrators before the original system input u, and feeding the resulting system by an external auxiliary input signal v, i.e.,

$$\left.\begin{aligned} \dot{x} &= F(x, x_{n+1}) \\ \dot{x}_{n+1} &= x_{n+2} \\ \dot{x}_{n+2} &= x_{n+3} \\ &\cdots \\ \dot{x}_{2n-r-1} &= x_{2n-r} \\ \dot{x}_{2n-r} &= v \\ y &= h(x) \end{aligned}\right\}. \tag{2.7}$$

The *extended system*, as defined in Nijmeijer and Van der Schaft (1990), only requires placing a single integrator before the input u, regardless of the value of the relative degree of the system.

The following state coordinate transformation takes the $2n - r$ dimensional system (2.7) into Normal Canonical form (see Isidori, 1989):

$$(\hat{\xi}, \eta) = \hat{\Phi}(x, x_{n+1}, x_{n+2}, \cdots, x_{2n-r}) =: \hat{\Phi}(\hat{x})$$
$$= \begin{bmatrix} h(x) \\ \dot{h}(x) \\ \cdots \\ h^{(r)}(x, x_{n+1}) \\ \cdots \\ h^{(n-1)}(x, x_{n+1}, x_{n+2}, \cdots, x_{2n-r}) \\ \phi_{n+1}(x) \\ \cdots \\ \phi_{2n-r}(x) \end{bmatrix}, \tag{2.8}$$

where

$$\hat{\xi} := (\xi_1, \cdots, \xi_r, \xi_{r+1}, \cdots, \xi_n) = (\xi, \xi_{r+1}, \cdots, \xi_n),$$
$$\eta := (\phi_{n+1}, \cdots, \phi_{2n-r}).$$

The components of η are set to be independent of the first r coordinate components comprising the vector ξ. It should be obvious that if the η's, in (2.8), are chosen in this manner, they are also independent of the new set of extra state coordinates $\xi_{r+1}, \cdots, \xi_n$, and the transformation (2.8) is full rank.

The normal canonical form of system (2.7) is, therefore, given by

$$\left.\begin{aligned} \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= \xi_3 \\ &\cdots \\ \dot{\xi}_{n-1} &= \xi_n \\ \dot{\xi}_n &= c(\hat{\xi}, \eta, v) \\ \dot{\eta} &= q(\hat{\xi}, \eta) \\ y &= \xi_1 \end{aligned}\right\}. \tag{2.9}$$

In order to be able to solve, even if in a local sense, for the auxiliary control input v on any relation involving the function $c(\hat{\xi}, \eta, v)$, it is assumed that $\partial c / \partial v \neq 0$ in the region of interest. This is equivalent to avoiding the regions where *impasse* points may exist for the traditional definition of the state of a dynamical controller, derived on the basis of (2.9) (see Fliess and Hasler, 1990, for related details).

3. Dynamical PFM Control of Nonlinear Systems

Let $p(\lambda)$ be an $(n-1)$th order Hurwitz polynomial with constant coefficients

$$p(\lambda) = \lambda^{n-1} + a_{n-1}\lambda^{n-2} + \cdots + a_2\lambda + a_1. \tag{3.1}$$

Consider now the following auxiliary output function of the system (2.6):

$$s(\xi) = \xi_n + a_{n-1}\xi_{n-1} + \cdots + a_2\xi_2 + a_1\xi_1. \tag{3.2}$$

If the condition $s = 0$ is achieved by means of suitable controls, the restricted motions of the generalized extended system (2.7) satisfy the following asymptotically stable linear time-invariant dynamics:

$$\left.\begin{aligned} \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= \xi_3 \\ &\cdots \\ \dot{\xi}_{n-1} &= -a_{n-1}\xi_{n-1} - \cdots - a_2\xi_2 - a_1\xi_1 \end{aligned}\right\}. \tag{3.3}$$

The following proposition is a direct consequence of the preceding considerations and of Proposition 2.1.

Proposition 3.1. A minimum phase nonlinear system of the form (2.6) is locally asymptotically stable to an equilibrium point $(u, x, y) = (U, X(U), 0)$ if the control action u is specified as a dynamical PFM control policy given, with slight abuse of notation, by the solution of the following implicit, time-varying, nonlinear discontinuous differential equation:

$$\begin{aligned} &c(\Phi(x), u, \dot{u}, \cdots, u^{(n-r)}) \\ &= -\sum_{i=1}^{r} a_{i-1} h^{(i-1)}(x) - \sum_{i=r+1}^{n} a_{i-1} h^{(i-1)}(x, u, \dot{u}, \cdots, u^{(i-r-1)}) \\ &\quad - W\mathrm{PFM}_{\tau,T}\left[\sum_{i=1}^{r} a_i h^{(i-1)}(x) + \sum_{i=r+1}^{n} a_i h^{(i-1)}(x, u, \dot{u}, \cdots, u^{(i-r-1)})\right], \end{aligned} \tag{3.4}$$

where $a_0 = 0$, and $a_n = 1$.

Proof. Imposing on the auxiliary output function $s(\xi)$, given in (3.2), the asymptotically stable discontinuous PFM controlled dynamics defined by (2.1), one immediately obtains an implicit PFM static controller for v, in terms of the transformed state variables. In original state and input coordinates, the controller adopts the form (3.4).

Notice that one cannot, in general, assume that a global state variable representation exists for the dynamics of the implicit controller given by (3.4). As it is now known from the ***differential algebraic*** approach to system analysis, state variable representations are only locally possible, in general (see the outstanding work of Fliess (1990), and the references therein).

4. An Application Example

4.1 Example (A Dynamical PFM Control Approach for Regulating the Concentration in an Exothermic Continuously Stirred Tank Reactor).

Consider the following nonlinear dynamical controlled model of an exothermic reaction occurring inside a CSTR (see Parrish and Bosilow, 1986), where the control objective is to regulate the outlet concentration through manipulation of the water jacket temperature:

$$\left.\begin{aligned} \dot{x}_1 &= \frac{F}{V}(c_0 - x_1) - a x_1 \exp\left(-\frac{b}{x_2}\right) \\ \dot{x}_2 &= \frac{F}{V}(T_0 - x_2) + \frac{aL}{c_p} x_1 \exp\left(-\frac{b}{x_2}\right) - \frac{h}{Vc_p}(x_2 - u) \\ y &= x_2 - T \end{aligned}\right\}, \tag{4.1}$$

where x_1 represents the product concentration. The state variable x_2 represents the reactor temperature. The control variable u is the water jacket temperature. F is the reactor throughput in lb/hr, c_0 is the inlet flow concentration in lb/lb, T_0 is the inlet flow temperature measured in deg.R, c_p is the material heat capacity in BTU/lb.R while V and L are, respectively, the reactor holdup (in lb.) and the heat of the reaction (in BTU/lb.). The constant h is the heat transfer parameter (in BTU/hr.R), b is the activation constant (in deg.R) and a is the pre-exponential factor in hr^{-1}. A constant temperature T is to be stably maintained to control indirectly the product concentration x_1 to its constant equilibrium value X_1.

A stable constant equilibrium point for this system is then given by

$$\left.\begin{aligned} x_2 &= T; \quad x_1 = X_1(T) = \frac{c_0}{1 + V/Fa \exp(-b/T)} \\ u &= U(T) = T - \frac{c_p F}{h}(T_0 - T) - \frac{aLV}{h} \cdot \frac{c_0 \exp(-b/T)}{1 + V/Fa \exp(-b/T)} \end{aligned}\right\}. \tag{4.2}$$

We next summarize the design procedure leading to a dynamical stabilizing PFM controller for system (4.2), based on the extended model. As it is easily verified, the relative degree of the system (4.1) is equal to one, and hence, the dimension of the zero dynamics is also one.

Extended system model of CSTR.

$$\left.\begin{aligned} \dot{x}_1 &= \frac{F}{V}(c_0 - x_1) - a x_1 \exp\left(-\frac{b}{x_2}\right) \\ \dot{x}_2 &= \frac{F}{V}(T_0 - x_1) + \frac{aL}{c_p} x_1 \exp\left(-\frac{b}{x_2}\right) - \frac{h}{Vc_p}(x_2 - x_3) \\ \dot{x}_3 &= v \\ y &= x_2 - T \end{aligned}\right\}. \tag{4.3}$$

State coordinate transformation to normal canonical form for the extended system.

$$\left.\begin{aligned} \xi_1 &= x_2 - T \\ \xi_2 &= \frac{F}{V}(T_0 - x_2) + \frac{aL}{c_p} x_1 \exp\left(-\frac{b}{x_2}\right) - \frac{h}{Vc_p}(x_2 - x_3) \\ \eta &= x_3 \end{aligned}\right\}, \tag{4.4}$$

$$\left.\begin{aligned} x_1 &= \frac{c_p}{aL}\cdot\exp\left(\frac{b}{\xi_1+T}\right)\left\{\xi_2 - \frac{F}{V}(T_0 - T - \xi_1) + \frac{h}{Vc_p}(\xi_1 + T - \eta)\right\} \\ x_2 &= \xi_1 + T \\ x_3 &= \eta \end{aligned}\right\}. \quad (4.5)$$

Normal canonical form of the extended system.

$$\left.\begin{aligned} \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= \frac{aL}{c_p}\exp\left(-\frac{b}{\xi_1+T}\right)\left[\frac{F}{V}c_0 - \frac{c_p}{aL}\cdot\exp\left(\frac{b}{\xi_1+T}\right)\frac{F}{V}\left(1+\frac{V}{F}\cdot a\right.\right. \\ &\quad \left.\times\exp\left(-\frac{b}{\xi_1+T}\right) - \frac{bV}{F}\frac{\xi_2}{(\xi_1+T)^2}\right)\left(\xi_2 - \frac{F}{V}(T_0 - T - \xi_1)\right. \\ &\quad \left.\left. + \frac{h}{Vc_p}(\xi_1 + T - \eta)\right)\right] - \left(\frac{F}{V} + \frac{h}{Vc_p}\right)\xi_2 + \frac{h}{Vc_p}\cdot v \\ \dot{\eta} &= v \\ y &= \xi_1 \end{aligned}\right\}. \quad (4.6)$$

Auxiliary output function.

$$s = \xi_2 + a_1\xi_1; \quad a_1 > 0. \quad (4.7)$$

Restricted asymptotically stable motions of the controlled dynamics.

$$\dot{\xi}_1 = -a_1\xi_1. \quad (4.8)$$

Static PFM controller for the extended system.

$$\begin{aligned} v = \frac{Vc_p}{h}\cdot\Bigg\{ & -\frac{aL}{c_p}\exp\left(-\frac{b}{\xi_1+T}\right)\left[\frac{F}{V}c_0 - \frac{c_p}{aL}\exp\left(\frac{b}{\xi_1+T}\right)\frac{F}{V}\right. \\ & \cdot\times\left(1 + \frac{V}{F}\cdot a\exp\left(-\frac{b}{\xi_1+T}\right) - \frac{bV}{F}\cdot\frac{\xi_2}{(\xi_1+T)^2}\right) \\ & \left.\times\left(\xi_2 - \frac{F}{V}(T_0 - T - \xi_1) + \frac{h}{Vc_p}(\xi_1 + T - \eta)\right)\right] \\ & + \left(\frac{F}{V} + \frac{h}{Vc_p} - a_1\right)\xi_2 - W\mathrm{PFM}_{\tau,T}[s(\xi)]\Bigg\}. \end{aligned} \quad (4.9)$$

Asymptotically stable zero dynamics.

$$\begin{aligned} \dot{\eta} = & -\frac{F}{V}\left(1 + \frac{V}{F}\cdot a\exp\left(-\frac{b}{T}\right)\right) \\ & \times\left[\eta - T + \frac{c_pF}{h}(T_0 - T) + \frac{aLV}{h}\frac{c_0\exp(-b/T)}{1 + V/Fa\exp(-b/T)}\right]. \end{aligned} \quad (4.10)$$

Dynamical PFM controller in original state and input coordinates.

$$\left.\begin{aligned}\dot{u} &= \frac{Vc_p}{h}\Bigg\{\left(\frac{F}{V}+\frac{h}{Vc_p}-\frac{abL}{c_p}\exp\left(-\frac{b}{x_2}\right)\frac{x_1}{x_2^2}-a_1\right)\Bigg[\frac{F}{V}(T_0-x_2)\\ &\quad+\frac{aL}{c_p}x_1\exp\left(-\frac{b}{x_2}\right)-\frac{h}{Vc_p}(x_2-u)\Bigg]-\frac{aL}{c_p}\exp\left(-\frac{b}{x_2}\right)\\ &\quad\times\left(\frac{F}{V}(c_0-x_1)-ax_1\exp\left(-\frac{b}{x_2}\right)\right)-W\mathrm{PFM}_{\tau,T}[s(x,u)]\Bigg\}\\ s(x,u) &= \frac{F}{V}T_0-a_1T-\left(\frac{F}{V}+\frac{h}{Vc_p}-a_1\right)x_2\\ &\quad+\frac{aL}{c_p}x_1\exp\left(-\frac{b}{x_2}\right)+\frac{h}{Vc_p}u\end{aligned}\right\}. \quad (4.11)$$

Simulations were performed for a dynamical PFM controller CSTR characterized by the following parameters (Parrish and Bosilow, 1986):

$F = 2000$ [lb/hr]; $c_0 = 0.50$ [lb/lb]; $V = 2400$ [lb.];

$a = 7.08 \times 10^{10}$ [hr^{-1}]; $b = 15080$ [deg.R]; $T_0 = 5320$ [deg.R];

$L = 600$ [BTU/lb.]; $cp = 0.75$ [BTU/lb.R]; $h = 15000$ [BTU/hr.R].

For such values of the parameters, the equilibrium point (4.2) of the system results in

$$x_2 = T = 600 \text{ [deg.R]};\quad u = U(T) = 107.679 \text{ [deg.R]};\quad X_1(T) = 0.246 \text{ [lb/lb]}.$$

The PFM controller parameters were chosen as: $a_1 = 8$, $W = 50$, $T_{\max} = 8 \times 10^{-4}$ [hr], $T_{\min} = 2 \times 10^{-4}$ [hr], $\rho_1 = 15$, $\rho_2 = 10$, $\rho_3 = 40$. Figure 2 portrays the time response of the dynamical PFM controlled state variables x_1 and x_2, the chattering-free (smoothed) continuous control input trajectory $u(t)$ and the evolution of the auxiliary output function $s(x, u)$.

5. Conclusions

A general stabilizing design procedure, based on dynamical PFM feedback control policies, has been presented for minimum phase nonlinear single-input single-output systems. Such a controller is obtained on the basis of zeroing an auxiliary output function, defined in terms of the *normal canonical variables* of the *Generalized Extended* system. The dynamical controller results are based on elementary considerations concerning the asymptotic stabilization of such a scalar auxiliary output function by means of a simple PFM feedback controller of the ON-OFF-ON type. Zeroing of the auxiliary output function induces an asymptotically stable motion of the constrained dynamics, characterized by a linear time-invariant system with eigenvalues placeable at will. The discontinuities, generated by the PFM controller, take place in the state space

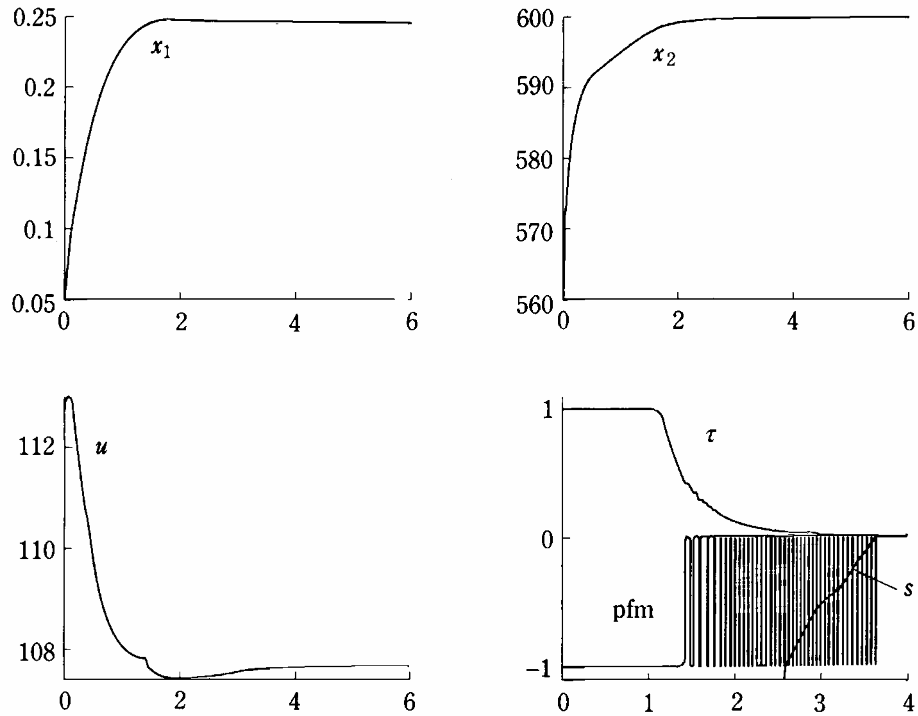

Fig. 2. Dynamical PFM controlled state variables, chattering-free control input trajectory and auxiliary output function for CSTR example.

of the dynamical controller, and not in the state space of the system. The resulting integrated control actions are, thus, *continuous* with substantially reduced (smoothed out) chattering.

Within the proposed approach, no need arises to resort to *average* controlled system considerations, nor high sampling frequency assumptions, in order to establish the salient stability features of the actual closed loop PFM controlled system. Such approximation schemes have been customarily exercised in, both the analysis and the design methods available for discontinuously controlled systems. New applications areas, such as nonlinear chemical process control, nonlinear electromechanical systems control etc., in which PFM control was not traditionally feasible, can now benefit from the inherent robustness and high performance characteristics of this class of discontinuous control strategies.

The obtained results may be extended to nonlinear multi-input systems.

References

Conte, G., C. Moog and A. Perdon (1988). Un théoreme sur la represéntation entréesortie d'un système non linéaire. *Compte Rendu Academie Science, Paris*, **307**, I, 363–366.

Fliess, M. (1988). Nonlinear control theory and differential algebra. Ch. I. Byrnes and A. Kurzhanski (eds.), *Modeling and Adaptive Control*, Lecture Notes in Control and Information Sciences, **105**, Springer-Verlag, N.Y., 134–145.

Fliess, M. (1990). Generalized controller canonical form for linear and nonlinear dynamics. *IEEE Trans. Automatic Control*, **AC-35**, 9, 994–1001.

Fliess, M. and M. Hasler (1990). Questioning the classic state-space description via circuit examples. M.A. Kashoek, J.H. Van Schuppen and A.C.M. Ran (eds.), *Realization and Modelling in System Theory*, Birkhäuser, Boston.

Isidori, A. (1989). *Nonlinear Control Systems*. Springer-Verlag, N.Y.

Nijmeijer, H. and A. Van der Schaft (1990). *Nonlinear Dynamical Control Systems*. Springer-Verlag, N.Y.

Parrish, J. and C. Bosilow (1986). Nonlinear inferential control of reactor effluent concentration from temperature and flow measurements. *Proceedings of the American Control Conference*, Seattle, WA, June, **2**, 1027–1033.

Skoog, R.A. and G. Blankenship (1970). Generalized pulse-modulated feedback systems: Norms, gains, Lipschitz constants and stability. *IEEE Trans. Automatic Control*, **AC-15**, 3, 300–315.

Hebertt Sira-Ramírez obtained the degree of Ingeniero Electricista from the Universidad de Los Andes in Mérida-Venezuela in 1970. He pursued graduate studies from 1972 to 1977 at the Massachusetts Institute of Technology (MIT), Cambridge, MA, where he obtained the degree of Master of Science in Electrical Engineering (MSEE) and the degree of Electrical Engineer (EE) in 1974. He later obtained the degree of Doctor of Philosophy (Ph.D.) in Electrical Engineering in 1977. He is currently a Full Professor in the Control Systems Department of the Systems Engineering School of the Universidad de Los Andes, where he has also held the position of Head of the Control Systems Department from 1978 to 1980 and was elected Vice-President of the University for the term 1980–1984. He was appointed Head of the Graduate School in Control Engineering in 1992. Dr. Sira-Ramírez is a Senior Member of the Institute of Electrical and Electronics Engineers (IEEE), where he serves as a member of the IEEE International Committee and is a Distinguished Lecturer. He is also a member of the International Federation of Automatic Control (IFAC), Sigma Xi, The Planetary Society and of the Venezuelan College of Engineers (CIV).

He has held visiting positions in the Coordinated Science Laboratory of the University of Illinois at Urbana-Champaign, in the Department of Aeronautical and Astronautical Engineering and the Department of Electrical Engineering of the same University in 1986. He has also held brief visiting positions in the School of Electrical Engineering of Purdue University in 1989, 1990 and 1991. He was appointed Associate Research Director at the Laboratoire de Signaux et Systems of the CNRS in Plateau du Moulon, (Paris) France in 1992. Dr. Sira-Ramírez received the Venezuelan College of Engineers Award for Scientific Research in 1987, the Senior Researcher Scholarship Award from the Venezuelan National Council for Scientific and Technological Research (CONICIT) in 1990 and the National Prize for Best Engineering Work (CONICIT) in 1993. Dr. Sira-Ramírez is interested in the theory and applications of discontinuous feedback control strategies for nonlinear dynamic systems.