A Dynamical Variable Structure Control Strategy in Asymptotic Output Tracking Problems

Hebertt Sira-Ramírez

Abstract—**In this article a dynamical discontinuous feedback strategy of the sliding mode type is presented for asymptotic output tracking problems in nonlinear dynamical systems. Fliess' generalized observability canonical form (GOCF) is used in the derivation of the dynamical variable structure feedback controller. For a large class of nonlinear systems, a truly effective smoothing of the sliding mode controlled responses is possible, while substantially reducing the chattering for the control input. For this reason, the technique is especially suitable for the control of some mechanical and electromechanical systems. An example, including simulations, is provided.**

I. Introduction

Asymptotic output tracking problems in nonlinear dynamical systems have been extensively studied in the control systems literature. Contributions from a *differential geometric* viewpoint are summarized in Isidori's outstanding book [1], where clear connections are established with the concept of the *inverse system*, and the *zero dynamics* (see also Nijmeijer and Van der Schaft [2]). Within the same setting, an adaptive control approach to the output tracking problem has also been explored by Isidori and Sastry [3].

Recently, *differential algebra* has been proposed by Prof. M. Fliess for the study of nonlinear controlled systems (see Fliess [4] and [5]). Among many other deep contributions, Fliess' remarkable studies have found, for instance, that the concept of *state* only has a *local* validity. *Implicit ordinary differential equations* account for a more general and enlightening setting from which a unified and far reaching treatment is possible for basic control theoretic concepts. Within this new viewpoint, naturally emerging *generalized canonical forms* for linear and nonlinear controlled systems are introduced which explicitly exhibit time derivatives of the control input functions on the state and output equations [5]. Only in the case of linear systems, elimination of these input derivatives from the state equations is possible via control-dependent state coordinate transformations. Kalman's original formulation is thus, completely recovered (see [5] and Diop [6]).

Section II of this article presents the asymptotic output tracking problem from the perspective of *dynamical variable structure feedback control*. The proposed control scheme is based on Fliess' *generalized observability canonical form* (GOCF) [4]. The approach represents a viable feedback alternative exhibiting attractive features such as robustness and, more importantly, certain degree of input–output smoothness, dependent upon the relative degree of the system. The proposed "chattering-free" features, nontypical of sliding mode controlled behavior, make the approach especially suitable for controlling some electromechanical devices (see Sira-Ramirez *et al.* [7]). In Section III, we present, along with computer simulations, an application example that illustrates the advantages of the proposed discontinuous

Manuscript received January 31, 1991; revised August 20, 1991. Paper recommended by Associate Editor, A. Arapostathis. This work was supported by the Consejo de Desarrollo Científico, Humanístico y Tecnológico of the Universidad de Los Andes under Grant I-325-90.

The author is with the Departamento Sistemas de Control, Universidad de los Andes, Mérida, Venezuela.

IEEE Log Number 9204977.

0018-9286/93$03.00

dynamical controller for a DC-motor angular velocity tracking task. The concluding remarks and proposals for further work are collected in Section IV.

A different approach to dynamically generated sliding regimes has been presented by Fliess and Messeger in [8]. In a simple—but important—example, dynamical sliding regimes were shown to asymptotically stabilize a nonlinear second-order system which is not stabilizable by any smooth feedback strategy. Output stabilization problems treated via dynamical discontinuous control have recently been presented by Sira-Ramírez in [9]–[11] for some aerospace control problems.

II. Asymptotic Output Tracking Via Dynamical Variable Structure Feedback Control

The following proposition is quite basic in the developments presented in this section:

Proposition: Let μ and W represent strictly positive quantities and let "sgn" stand for the *signum* function. Then, the scalar discontinuous system:

$$\dot{w} = -\mu(w + W \operatorname{sgn} w) \tag{2.1}$$

globally exhibits a sliding regime on $w = 0$. Furthermore, any trajectory starting on the initial value $w = w(0)$, at time $t = 0$, reaches the condition $w = 0$ in finite time T, given by:

$$T = \mu^{-1} \ln\left(1 + \frac{|w(0)|}{W}\right).$$

Proof: Immediate upon checking that globally: $w\,dw/dt < 0$ for $w \neq 0$, which is a well-known condition for sliding mode existence (see Utkin [12]). The second part follows easily from the linearity of the two intervening system "structures." ■

Let α be a strictly positive integer. Consider a nonlinear dynamical system expressed in Fliess' GOCF [4]:

$$\begin{aligned} \dot{\eta}_1 &= \eta_2 \\ \dot{\eta}_2 &= \eta_3 \\ &\cdots \\ \dot{\eta}_{n-1} &= \eta_n \\ \dot{\eta}_n &= c(\eta, u, \dot{u}, \cdots, u^{(\alpha)}) \\ y &= \eta_1. \end{aligned} \tag{2.2}$$

Under rather mild conditions, any analytic nonlinear system, given in the traditional Kalman state variable representation, $dx/dt = F(x, u)$, $y = h(x)$, can be transformed to Fliess' GOCF by means of a suitable input-dependent state coordinate transformation (see Fliess [13], Conte *et al.* [14], and also [2, ch. 4] and references therein). Notice that the integer α in (2.2) is intimately related to the *relative degree* r of the system [1] by the relation: $\alpha = n - r$. Hence, α coincides with the dimension of the *zero dynamics*.

Tracking problems in systems which are exactly linearizable by static state feedback, i.e., those in which $\alpha = 0$, can be similarly treated by the techniques presented here simply by considering the "extended system" proposed by Nijmeijer and Van der Schaft in [2].

Let $y_R(t)$ be a prescribed reference output function, assumed to be sufficiently smooth. The *asymptotic output tracking problem* consists in specifying a dynamical controller, possibly described by an implicit time-varying scalar ordinary differential equation with discontinuous right-hand side, which accepts as input functions: 1) the output reference signal $y_R(t)$, together with a finite number of its time derivatives $y_R^{(i)}(t)$ $(i = 1, \cdots, n)$ and 2) the *generalized phase variable coordinates* η_i $(i = 1, \cdots, n)$, of the given system, and is capable of producing—as a solution output signal—a scalar control input function u, which locally forces the system output $y = \eta_1$ to asymptotically converge toward the desired output reference signal $y_R(t)$.

Define a tracking error function $e(t)$ as the difference between the actual system output $y(t)$ and the output reference signal $y_R(t)$:

$$e(t) = y(t) - y_R(t). \tag{2.3}$$

We then have:

$$e^{(i)}(t) = \eta_{i+1} - y_R^{(i)}(t); \quad 0 \le i \le n-1 \tag{2.4}$$

$$e^{(n)}(t) = \dot{\eta}_n - y_R^{(n)}(t) = c(\eta, u, \dot{u}, \cdots, u^{(\alpha)}) - y_R^{(n)}. \tag{2.5}$$

Defining $e_i = e^{(i-1)}$ $(i = 1, 2, \cdots, n)$, as components of an error vector e, we may also express the tracking error system (2.4) and (2.5) in GOCF as:

$$\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= e_3 \\ &\cdots \\ \dot{e}_{n-1} &= e_n \\ \dot{e}_n &= c(\xi_R(t) + e, u, \dot{u}, \cdots, u^{(\alpha)}) - y_R^{(n)}(t) \\ e &= e_1 \end{aligned} \tag{2.6}$$

with

$$\xi_R(t) = \operatorname{col}(y_R(t), y_R^{(1)}(t), \cdots, y_R^{(n-1)}(t))$$
$$e = \operatorname{col}(e_1, e_2, \cdots, e_n). \tag{2.7}$$

Suppose that the asymptotic equilibrium point of the controlled tracking error system (2.6), for some suitable control input strategy, is given by $e_1 = e_2 = \cdots = e_n = 0$. Hence, under such an equilibrium condition, i.e., under perfect tracking, the system exhibits the following "remaining dynamics" or "*inverse dynamics*"

$$c(\xi_R(t), u, \dot{u}, \cdots, u^{(\alpha)}) = y_R^{(n)}(t). \tag{2.8}$$

For the particular case in which the tracking signal $y_R(t)$ is identically zero, or a given constant, i.e., for the case of an output nulling, or stabilization, task, the expression (2.8) constitutes the so called *zero dynamics* (see Fliess [15] and also Isidori *et al.* [16]). In such a case, the nature of the local stability of (2.8) around an equilibrium point determines the feasibility of the stabilizing controller scheme. The system is said to be *minimum phase*, around such an equilibrium point, if the zero dynamics is asymptotically stable to the particular equilibrium point. It will be a *nonminimum phase* if the zero dynamics is unstable [1]. Critical stability cases are usually treated via *center manifold theory* (see Appendix B of Isidori's book [1]), or other techniques such as Lyapunov's stability theory.

The stability features of (2.8) for reference signals $y_R(t)$ which are bounded with bounded derivatives, $y_R^{(i)}(t)$ $(i = 1, 2, \cdots, n)$ also determine, to a large extent, the physical realizability of any tracking control strategy which asymptotically achieves the perfect tracking condition $e = 0$. We assume that the solution u of (2.8) is defined for all times, and is bounded for all bounded input functions $y_R(t)$ which also exhibit bounded derivatives (see [3]).

Let the set of real coefficients $\{m_0, \cdots, m_{n-2}\}$ be such that the following polynomial, in the complex variable "s," is Hurwitz:

$$s^{n-1} + m_{n-2}s^{n-2} + \cdots + m_1 s + m_0. \tag{2.9}$$

Consider now an auxiliary scalar output variable w, defined in terms of the output tracking error coordinates e_i $(i = 1, \cdots, n)$ as:

$$w = \sum_{i=1}^{n} m_{i-1}e^{(i-1)} = \sum_{i=1}^{n} m_{i-1}e_i; \quad \text{with } m_{n-1} = 1. \tag{2.10}$$

If we impose on the evolution of the auxiliary output variable w, the discontinuous dynamics considered in (2.1), one obtains, from (2.6) and (2.10):

$$\begin{aligned}\dot{w} &= \dot{e}_n + \sum_{i=1}^{n-1} m_{i-1}e_{i+1}\\ &= -\mu\left[\sum_{i=1}^{n} m_{i-1}e_i + W \operatorname{sgn}\left(\sum_{i=1}^{n} m_{i-1}e_i\right)\right].\end{aligned} \tag{2.11}$$

Using (2.5) one obtains the following dynamical feedback controller in terms of an *implicit ordinary differential equation* with discontinuous right-hand side:

$$\begin{aligned}c(\xi_R + e, u, \dot{u}, \cdots, u^{(\alpha)}) &= y_R^{(n)} - \sum_{i=1}^{n-1} m_{i-1}e_{i+1}\\ &\quad - \mu\left[\sum_{i=1}^{n} m_{i-1}e_i + W \operatorname{sgn}\left(\sum_{i=1}^{n} m_{i-1}e_i\right)\right].\end{aligned} \tag{2.12}$$

On each one of the regions $w > 0$, and $w < 0$, a different feedback control "structure" is generated by (2.12) and the corresponding implicit differential equation is to be independently solved for the controller u, on the basis of knowledge of the error vector e and the vector of functions $\xi_R(t)$. Under the additional assumption that, locally, $\partial c/\partial u^{(\alpha)}$ is *nonzero* in (2.12), then no singularities, of the *impasse* points type (Fliess and Hasler [17]) need be considered. Moreover, by virtue of the implicit function theorem, controller equation (2.12) is then locally equivalent to an *explicit system of first-order discontinuous differential equations* which can be solved *on line* with no further difficulties than those involved in, say, a dynamical sliding mode observer system acting in a closed-loop scheme.

Since w was shown to exhibit a sliding regime on the discontinuity surface $w = 0$, *Filippov's continuation method* (see Filippov [18]), or, alternatively, the *method of the equivalent control* [12], must be used for defining the idealized solutions of (2.12) on the switching manifold $w = 0$. According to the *method of the equivalent control*, the discontinuous motions on the sliding surface $w = 0$ can be described, in an idealized fashion, by the following *invariance conditions*: $w = 0$ and $dw/dt = 0$. The condition: $dw/dt = 0$, allows, in turn, the definition of a *virtual* control action, known as the *equivalent control*, which would be responsible for locally smoothly maintaining the evolution of the tracking error system state variables e_i on the manifold $w = 0$, should the motions precisely started on such a manifold. The resulting autonomous dynamics for the controlled output tracking error, ideally constrained to the switching manifold and "regulated" by the equivalent control, is generally known as the *ideal sliding dynamics*. It follows from (2.6), (2.10), and the invariance conditions, that such an ideal sliding dynamics is nonredundantly given by:

$$\begin{aligned}\dot{e}_1 &= e_2\\ \dot{e}_2 &= e_3\\ &\cdots\\ \dot{e}_{n-1} &= -\sum_{i=1}^{n-1} m_{i-1}e_i\end{aligned} \tag{2.13}$$

which exhibits an asymptotically stable motion toward the origin of the error vector coordinates, with eigenvalues uniquely specified by the prescribed set of constant coefficients $\{m_0, \cdots, m_{n-2}\}$. In particular, the output tracking error function $e_1 = \eta_1 - y_R(t)$ asymptotically converges to zero, as desired. Using the condition $dw/dt = 0$, on (2.11), with $w = 0$, it follows that the equivalent control, denoted by u_{EQ}, is defined as the solution of the implicit time-varying differential equation:

$$\begin{aligned}c\left(\xi_R(t) + e, u_{EQ}, \dot{u}_{EQ}, \cdots, u_{EQ}^{(\alpha)}\right) &= y_R^{(n)}(t) - \sum_{i=1}^{n-1} [m_{i-2} - m_{n-2}m_{i-1}]e_i\end{aligned} \tag{2.14}$$

with $m_{-1} = 0$.

In view of the stability property imposed on (2.13) and by virtue of (2.10), when $w = 0$, the tracking error vector e asymptotically converges to zero. The equivalent control is then given by:

$$c\left(\xi_R(t), u_{EQ}, \dot{u}_{EQ}, \cdots, u_{EQ}^{(\alpha)}\right) = y_R^{(n)}(t) \tag{2.15}$$

i.e., once more we find that the stability properties of the "inverse dynamics" play a crucial role in the solution of the tracking problem. In this case, it bears a definite influence in the rightful definition and existence of the equivalent control function under perfect tracking conditions.

Remark: Two important advantages can be readily established about the dynamical variable structure controller represented by (2.12). The first one is the fact that the output tracking error function $e(t)$ asymptotically approaches zero with substantially reduced or smoothed out "chattering." Notice that n integrators stand between the tracking error variable $e_1 = e(t)$ and the discontinuous control actions regulating the behavior of the auxiliary output variable w to zero. Therefore, with respect to a *static* variable structure controller alternative, based on Isidori's *normal canonical form* approach (see Sira-Ramírez [19] and [20]), $n - r$ additional integrations contribute to further smooth out the controlled tracking error signal $e(t)$. Secondly, and this is possibly the most important advantage of the approach, a traditional explicit *canonical phase variable* representation for the dynamical controller (2.12) indicates that the control input u is the outcome of α integrations performed on a nonlinear function of the discontinuous actions that lead the auxiliary output w to zero. This implies substantially smoothed control inputs which do not result in a "bang-bang" behavior for the actuator. ■

III. An Application Example

The following nonlinear dynamical model of a field controlled DC-motor is taken from Rugh [21, page 98].

$$\begin{aligned}\dot{x}_1 &= -\frac{R_a}{L_a}x_1 - \frac{K}{L_a}x_2 u + \frac{V_a}{L_a}\\ \dot{x}_2 &= -\frac{B}{J}x_2 + \frac{K}{J}x_1 u.\end{aligned} \tag{3.1}$$

Where x_1 represents the armature circuit current, x_2 is the angular velocity of the rotating axis. V_a is a fixed voltage applied to the armature circuit, while u is the field winding input voltage, acting as the control variable. The constants R_a, L_a, and K represent, respectively, the resistance, the inductance in the armature circuit and the torque constant. The parameters J and B are the load's moment of inertia and the associated viscous damping coefficient.

Suppose $y_R(t)$ is a known, desired, bounded reference trajectory for the angular velocity x_2 considered as the output function. One can obtain a GOCF for the dynamics of the tracking error $e = x_2 - y_R(t)$, by defining a time-varying input dependent state coordinate transformation of the form:

$$e_1 = x_2 - y_R(t) \qquad x_2 = e_1 + y_R(t)$$

$$e_2 = -\frac{B}{J}x_2 + \frac{K}{J}x_1 u - \dot{y}_R(t)$$

$$x_1 = \frac{J}{Ku}\left[e_2 + \frac{B}{J}(e_1 + y_R(t)) + \dot{y}_R(t)\right] \quad (3.2)$$

yielding

$$\dot{e}_1 = e_2$$

$$\dot{e}_2 = -\frac{R_a B}{L_a J}(e_1 + y_R(t)) - \left(\frac{B}{J} + \frac{R_a}{L_a}\right)(e_2 + \dot{y}_R(t)) + \frac{KV_a}{L_a J}u - \frac{K^2}{L_a J}(e_1 + y_R(t))u^2 + \frac{\dot{u}}{u}\left[e_2 + \frac{B}{J}(e_1 + y_R(t)) + \dot{y}_R(t)\right] - \ddot{y}_R(t)$$

$$e = e_1. \quad (3.3)$$

Notice that $u = 0$ corresponds to a singularity of the transformation (3.2) and, hence, stabilization or tracking tasks that imply polarity reversals in the field winding input voltage must be treated by different techniques which imply inducing appropriate "jumps," or discontinuities, in the input variable u or in some of its time derivatives (see Fliess *et al.* [22] and Abu el Ata and Fliess [23]).

Since the problem of smoothly transferring the constant operating angular velocity, $y_R(t) = \Omega$, to a new constant reference value, $y_R(t) = \Omega^*$, eventually entitles the need for a controlled stable steady-state operation, we first study the stability features associated to the zero dynamics of system (3.3) when $y_R(t)$ is a nonzero constant of value, say, Ω.

The *zero dynamics*, as defined in Section II, is easily obtained from (3.3) as follows:

$$-\frac{R_a}{L_a}u + \frac{KV_a}{\Omega L_a B}u^2 - \frac{K^2}{L_a B}u^3 + \dot{u} = 0. \quad (3.4)$$

The constant equilibrium points $u = U$ of (3.4) are obtained from the solutions of the following third order algebraic equation:

$$-R_a B u + \frac{KV_a}{\Omega}u^2 - K^2 u^3 = 0. \quad (3.5)$$

One of the possible solutions of (3.5) corresponds to the singular equilibrium solution $u = 0$, which is, hence, discarded. The two other solutions of (3.5) are given by:

$$U = \frac{V_a K}{2\Omega}\left(1 \pm \sqrt{1 - 4\frac{R_a B\Omega^2}{V_a^2}}\right). \quad (3.6)$$

If the discriminant $D := V_a^2 - 4R_a B\Omega^2$ is negative, then there is no real solution to the stabilization problem. If, on the other hand, D is zero, or positive, then there are two real, positive, roots for u in (3.5). The stability properties of these equilibria may be directly determined via approximate linearization of (3.4).

Linearization of the zero dynamics (3.4) around the equilibrium point $u = U$ yields:

$$\dot{u}_\delta + \frac{1}{BL_a}(R_a B - K^2 U^2)u_\delta = 0 \quad (3.7)$$

where $u_\delta := u - U$ represents the incremental field circuit input voltage. The linearized zero dynamics (3.7) is evidently asymptotically stable to zero, provided the constant equilibrium input voltage U satisfies the condition:

$$R_a B > K^2 U^2 \quad (3.8)$$

which identifies, together with the condition: $V_a^2 \geq 4R_a B\Omega^2$, the minimum phase region in the input–output space for the given system (3.1). The operating equilibrium points, Ω and Ω^*, associated to the smooth angular velocity transfer maneuver defined via a suitably proposed tracking problem, must then be tested via the corresponding values U and U^* obtained from (3.6) against condition (3.8). This will assess the stability of the corresponding zero dynamics on the involved equilibria.

Consider the auxiliary output variable w, written now in terms of the tracking error velocity and acceleration error variables, as it was defined in (2.10):

$$w = e_2 + m_0 e_1. \quad (3.9)$$

Imposing on w the discontinuous dynamics given in (2.1), a time-varying differential equation for the dynamical controller is obtained which synthesizes the control signal u, in terms of the reference input signal $y_R(t)$, its time derivative $dy_R(t)/dt$ and the tracking error variables e_1 and e_2. Writing, however, the variable structure dynamical feedback controller in terms of the original state coordinates x_1 and x_2 of the controlled system (3.1), one obtains

$$\dot{u} = \frac{J}{Kx_1}\Bigg[\left(-\frac{B^2}{J^2} + \frac{B}{J}(\mu + m_0) - \mu m_0\right)x_2 + \frac{K}{J}\left(\frac{B}{J} + \frac{R_a}{L_a} - \mu - m_0\right)x_1 u - \frac{V_a K}{JL_a}u + \frac{K^2}{JL_a}x_2 u^2 + \mu m_0 y_R(t) + (\mu + m_0)\dot{y}_R(t) + \ddot{y}_R(t) - \mu W \operatorname{sign}\left(\left(-\frac{B}{J} + m_0\right)x_2 + \frac{K}{J}x_1 u - m_0 y_R(t) - \dot{y}_R(t)\right)\Bigg]. \quad (3.10)$$

Simulation Results

Simulations of a tracking task were performed for a DC-motor with the following parameter values:

$$R_a = 7\ \text{Ohm};\quad L_a = 120\ \text{mH};\quad V_a = 5\ V;$$

$$B = 6.04 \times 10^{-6}\ N\text{-}m\text{-}s/\text{rad};\quad J = 1.06 \times 10^{-6}\ N\text{-}m\text{-}s^2/\text{rad};$$

$$K = 1.41 \times 10^{-2}\ N\text{-}m/A.$$

A desired output reference trajectory $y_R(t)$ was considered which allowed for a smooth transition from a nominal (equilibrium) angular velocity Ω, to a new chosen operating angular velocity Ω^*. Such reference function was set to be:

$$y_R(t) = \begin{cases} \Omega \text{ for } 0 < t < t_1 \\ \Omega^* + (\Omega - \Omega^*) \exp(-kt^2) \text{ for } t > t_1; \quad k > 0. \end{cases} \tag{3.11}$$

Fig. 1 portrays the time response of the dynamical sliding mode controlled angular velocity. The dynamical variable structure controller smoothly leads the angular velocity from $\Omega = 300$ rad/s to a new operating value $\Omega^* = 200$ rad/s. The parameters of the induced dynamics (2.1) were set as: $\mu = 100$, $W = 10$, $m_0 = 20$. For all practical purposes, perfect tracking was achieved in this simulation since the obtained plots for $y_R(t)$ and $x_2(t)$ are indistinguishable. It may be verified that according to the chosen values of the parameters, the initial and final angular velocities are located on the *minimum phase* region of the system. Time t_1 and the constant k in (3.7) were set, respectively, as $t_1 = 0.5$ s and $k = 3$. Fig. 2 portrays the time response of the armature circuit current for the transition maneuver, while Fig. 3 shows the corresponding control input voltage trajectory exhibiting almost no chattering.

IV. Conclusion

Dynamical variable structure controllers accomplishing asymptotic reference output tracking are readily obtainable for nonlinear systems described in Fliess' *local generalized observability canonical form*. Such a canonical form naturally leads to a dynamical sliding mode controller which zeros, in finite time, an auxiliary output function defined in terms of the tracking error time derivatives. The resulting ideal sliding dynamics induces an asymptotic stabilization of the output tracking error function with eigenvalues totally prescribed at will. The obtained discontinuous controller design exhibits two main advantages, aside from the well-known robustness properties, which are implicit in every sliding mode control scheme. These advantages are related to the possibilities of obtaining a degree of smoothness in the output error response, as well as reduced chattering in the synthesized control input signals, in strict accordance with the relative degree of the given nonlinear dynamical system (i.e., effective chattering reduction for, both, the input and the output signals is entirely feasible, without resorting to the well-known high-gain amplifier alternative. See Slotine and Li [24]). The approach, however, requires full state feedback and it entitles dealing with the complexity of nonlinear time-varying implicit dynamical controllers, which may not be globally defined. Some of the associated difficulties include the presence of *impasse* points, or the operation of the controller in a *region of nonminimum phase* characteristics. In such pathological cases, the usual remedy indicates the use of discontinuities in the control signal. This prescription has been shown to produce the required desirable results, without disturbing side effects (see [22] and [23]).

It should be stressed that using time varying, input-dependent, sliding surfaces, the discontinuities associated to the proposed dynamical sliding mode control strategy take place in the *state space of the dynamical controller* and not in the state space of the system itself. Since the vast majority of controllers are nowadays synthesized by means of *electronics hardware*, or software, the fast switchings requirement is much easier to handle. This is to be compared with the demands of the traditional (static) sliding mode control techniques whereby the controlled system vari-

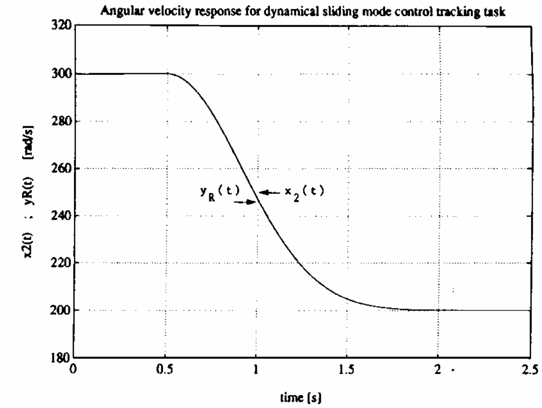

Fig. 1. Angular velocity response for dynamical sliding mode controlled tracking task.

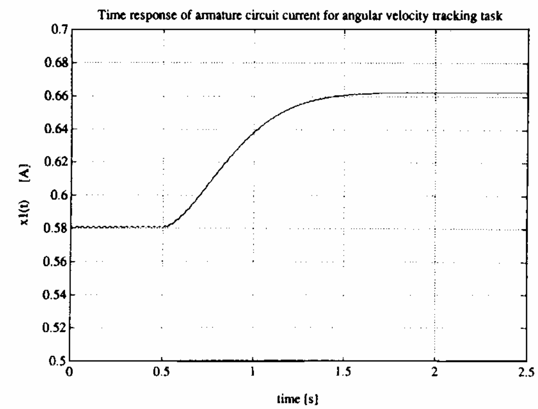

Fig. 2. Time response of armature circuit current for angular velocity tracking task.

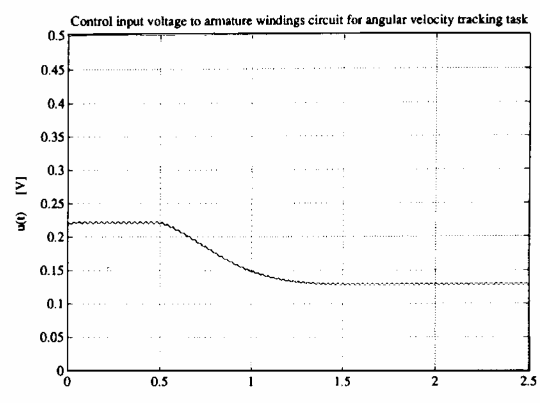

Fig. 3. Control input voltage to field windings circuit for angular velocity tracking task.

ables as well as actuator inputs and outputs directly undergo the (usually disastrous) effects of the generated bang-bang type of discontinuities. This fact makes possible the application of sliding mode control techniques to areas where they were not traditionally feasible, such as, chemical process control, bilogical systems control, and the regulation of mechanical and electromechanical systems (see also [7]).

In this article a nonlinear DC-motor example, dealing with smooth controlled transitions of nominal angular velocities to new constant operating values was presented along with encouraging simulation results. As topics for further research, the dynamical variable structure feedback controller here proposed could be implemented in an actual DC-motor using nonlinear analog electronics. Also, a robust controller that effectively handles the uncertainty of system parameters could be developed. Profitable connections could also be established with the work of Charlet *et al.* [25].

ACKNOWLEDGMENTS

The author is sincerely grateful to Profs. Michel Fliess and Claude Moog for kindly sending him a good number of their valuable contributions.

REFERENCES

[1] A. Isidori, *Nonlinear Control Systems*, 2nd ed. Berlin: Springer-Verlag, 1989.

[2] H. Nijmeijer and A. J. Van der Schaft, *Nonlinear Dynamical Control Systems*. Berlin: Springer-Verlag, 1990.

[3] S. Sastry and A. Isidori, "Adaptive control of linearizable systems," *IEEE Trans. Automat. Contr.*, vol. 34, no. 11, pp. 1123–1131. Nov. 1989.

[4] M. Fliess, "Nonlinear control theory and differential algebra," in *Modeling and Adaptive Control*, ch. I. Byrnes and A. Khurzhansky, Eds. (Lecture Notes in Control and Information Sciences, Vol. 105). New York: Springer-Verlag, 1989.

[5] ——, "Generalized controller canonical forms for linear and nonlinear dynamics," *IEEE Trans. Automat. Contr.*, vol. 35, no. 9, pp. 994–1001, Sept. 1990.

[6] S. Diop, "Elimination in control theory," *Math. Contr., Signals Syst.*, vol. 4, no. 1, pp. 17–32, 1991.

[7] H. Sira-Ramírez, S. Ahmad, and M. Zribi, "Dynamical feedback control of robotic manipulators with joint flexibility," Tech. Rep. TR-EE-70-90, School Electric. Eng., Purdue Univ., West Lafayette, IN, Dec. 1990.

[8] M. Fliess and F. Messager, "Vers une stabilisation non linéaire discontinue," in *Analysis and Optimization of Systems*, A. Bensoussan and J. L. Lions, Eds. (Lecture Notes in Control and Information Sciences, Vol. 144), pp. 778–787. New York: Springer-Verlag, 1990.

[9] H. Sira-Ramírez, "Dynamical feedback strategies in aerospace systems control: A differential algebraic approach," *European Control Conference* (EEC-91), vol. 3, pp. 2238–2243, Grenoble, France, July 2–5, 1991.

[10] ——, "Asymptotic output stabilization for nonlinear systems via dynamical variable structure control," *Dyn. Contr.*, vol. 2, no. 1, pp. 45–58, Feb. 1992, to appear.

[11] ——, "Nonlinear dynamically feedback controlled descent on a nonatmosphere-free planet: A differential algebraic approach," *Contr. Theory Adv. Tech.*, vol. 7, no. 2, pp. 301–320, June 1991.

[12] V. I. Utkin, *Sliding Modes and Their Applications in Variable Structure Systems*. Moscow: MIR, 1978.

[13] M. Fliess, "Nonlinear control theory and differential algebra," in *Modeling and Adaptive Control*, ch. I. Byrnes and A. Khurzhansky, Eds. (Lecture Notes in Control and Information Sciences, Vol. 105). New York: Springer-Verlag, 1989.

[14] G. Conte, C. H. Moog, and A. Perdon, "Un théorème sur la représentation entrée-sortie d'un système non linéaire," *C.R. Acad. Sci. Paris*, 307, Serie I, pp. 363–366, 1988.

[15] M. Fliess, "What the Kalman state representation is good for," in *Proc. 29th IEEE Conf. Decision Contr.*, vol. 3, Honolulu, HI, Dec. 1990, pp. 1282–1287.

[16] A. Isidori, P. V. Kokotovic, S. S. Sastry, and C. I. Byrnes, "The analysis of singularly perturbed zero dynamics," in *Analysis and Optimization of Systems*, A. Bensoussan and J. L. Lions, Eds. (Lecture Notes in Control and Information Sciences, Vol. 144) pp. 861–870. New York: Springer-Verlag, 1990.

[17] M. Fliess and M. Hasler, "Questioning the classic state-space description via circuit examples," in *Mathematical Theory of Networks and Systems* (MTNS-89), M. A. Kaashoek, A. C. M. Ram, and J. H. van Schuppen, Eds., *Progress in Systems and Control.* Boston, MA: Birkhauser, 1990.

[18] A. F. Filippov, *Differential Equations with Discontinuous Right-Hand Sides*. The Netherlands: Kluwer, 1988.

[19] H. Sira-Ramírez, "Structure at infinity, zero dynamics and normal forms of systems undergoing sliding motions," *Int. J. Syst. Sci.*, vol. 21, no. 4, pp. 665–674. Apr. 1990.

[20] ——, "Sliding regimes in general nonlinear systems: A relative degree approach," *Int. J. Contr.*, vol. 50, no. 4, pp. 1487–1506, Oct. 1989.

[21] W. Rugh, *Nonlinear System Theory—The Volterra/Wiener Approach*. Baltimore, MD: The Johns Hopkins Univ. Press, 1981.

[22] M. Fliess, P. Chantre, S. Abu el Ata, and A. Coíc, "Discontinuous predictive control, inversion and singularities; Application to a heat exchanger," in *Analysis and Optimization of Systems*, A. Bensoussan and J. L. Lions, Eds. in (Lecture Notes in Control and Information Sciences, Vol. 144), pp. 851–860. New York: Springer-Verlag, 1990.

[23] S. Abu el Ata-Doss and M. Fliess, "Nonlinear predictive control by inversion," in *Proc. IFAC Symp. Nonlinear Contr. Syst. Design*, Capri, June 1989.

[24] J. J. E. Slotine and W. Li, *Applied Nonlinear Control*. Englewood Cliffs, NJ: Prentice-Hall, 1991.

[25] B. Charlet, J. Lévine, and R. Marino, "Sufficient conditions for dynamic state feedback linearization," *SIAM J. Contr. Optimiz.*, vol. 29, no. 1, pp. 38–57, 1991.