Sliding Mode Control of dc-to-dc Power Converters via Extended Linearization

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Abstract—**The method of Extended Linearization is proposed for the systematic solution of sliding mode controller design in dc-to-dc Power Converters of the Boost and the Buck-Boost type. A nonlinear sliding surface with suitable stabilizing properties is synthesized on the basis of the extension of a linear sliding design carried out for the parametrized average linear incremental model of the converter. The obtained feedback strategies lead to asymptotically stable sliding modes with remarkable self-scheduling properties. Simulation examples are presented for illustrative purposes.**

I. Introduction

In this article, a new method is proposed for the synthesis of stabilizing sliding mode controllers (Utkin [1]) in bilinear switch-regulated dc-to-dc Power Supplies. The method of *Extended Linearization*, developed by W. Rugh and his co-workers (Rugh [2], Baumann and Rugh [3], and Wang and Rugh [4]) is used for the specification of the nonlinear sliding surface. In general terms, the design technique, primarily, entitles resorting to parametrized approximate linearization, about a general constant equilibrium point, of a suitably defined average system model. Using linear sliding-mode design results [1], a traditional stabilizing sliding hyperplane design is carried out on the basis of the family of parametrized linear systems. A most convenient framework for this purpose consists in placing the average incremental (i.e., linearized) model in standard controllable canonical form by means of an invertible linear state coordinates transformation. The linear design is led by imposing, on the resulting ideal sliding dynamics, a set of stable closed loop eigenvalues, chosen independently of the constant operating point. The core of the proposed method consists in specifying a suitable "extension" of the sliding hyperplane design which results in a *nonlinear* switching manifold. The designed surface, which is tangent to the prescribed hyperplane, contains the equilibrium point and it is parametrizable in terms of the nominal operating conditions. A conceptual advantage of this procedure is that the resulting ideal sliding dynamics can also be made locally linear (modulo a suitable local diffeomorphic state coordinate transformation directly derivable from the linearized system model). Nonlinear switching manifolds, which are only required to be tangent to the prescribed linear hyperplane, may be, generally speaking, nonuniquely defined. A direct integration procedure, carried out on the synthesized incremental sliding hyperplane, is then proposed as a systematic procedure for the synthesis of nonlinear sliding surfaces and their associated discontinuous feedback controllers. The nonlinear sliding mode switching logic is directly synthesized on the basis of the obtained nonlinear sliding surface coordinate function. The region of existence of a stabilizing sliding regime is easily assessed from knowledge of the parametrized equivalent control.

The proposed sliding mode controller exhibits a most important property, aside from those already mentioned, related to adaptability to sudden changes in the nominal operating conditions. Thus, if a desirable, or accidental, change of the nominal operating conditions of the converter takes place, the proposed discontinuous control scheme automatically creates a sliding regime which stabilizes the converters trajectories to the new equilibrium point, located on a new corresponding sliding surface. This last property results in no need for a "scheduling" process of the sliding manifold and of the switching "gains". Similar features are known to be "characteristic" of standard extension schemes commonly used in nonlinear controller design techniques based on the Extended Linearization approach [2]–[4]. Sliding regimes, based on Extended Linearization, have been also recently proposed by the authors for a variety of aerospace control problems (see Sira-Ramírez and Rios-Bolívar [5], [6]).

Section 2 of this article presents a general procedure for synthesizing stable nonlinear sliding manifolds, for a large class of switched controlled systems, via Extended Linearization. Section 3 is devoted to apply the proposed controller design procedure to the Boost Converter and the Buck-Boost Converter models. The performances of the obtained sliding mode controllers are assessed by means of computer simulations. Section 4 summarizes the conclusions and presents some suggestions for further work.

II. An Extended Linearization Synthesis Procedure for Sliding Mode Controllers in Nonlinear Switching Systems

A. Problem Formulation

Consider the n-dimensional switched controlled dynamical nonlinear system:

$$\dot{x} = f(x) + ug(x) + \eta \tag{1}$$

where $f(\cdot)$ and $g(\cdot)$ are smooth vector fields defined on an open set of $\mathcal{R}^n$. and η is a constant vector. The control input function u takes values on the binary discrete set $\{0, 1\}$. This general formulation corresponds to the typical situation in bilinear switched controlled circuits as well as in the most

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common category of switch-mode controlled dc-to-dc power converters (see Sira-Ramirez [7])

Associated to (1), and under the assumption of *fast switchings*, we define an *Average Model* by formally replacing the discontinuous control function u in (1) by a continuous piecewise smooth function μ (see also Sira-Ramírez, [8])

$$\dot{z} = f(z) + \mu g(z) + \eta \tag{2}$$

where the state vector is now denoted by z, just to differentiate it from the actual state x.

One of the main difficulties in attempting to use *any*linearization method in the controller specification for a switching system, of the form (1), lies in the fact that (1) *cannot* be linearized due the discrete nature of u and the high frequency control discontinuities associated with the operation of such class of systems. However, in two important discontinuous feedback control schemes represented by: 1) sliding mode controlled systems, and 2) Pulse Width Modulation (PWM) control strategies, an *infinite switching frequency average model* of system (1) may be obtained, precisely, in the form of (2). In both cases the, so called, *ideal sliding dynamics*, or the *average PWM model*, are equally obtained by substitution of the discontinuous control function u by the piecewise smooth *equivalent control*, or by the piecewise smooth *duty ratio function*. In any of the two above cases, the average control function μ takes values in the closed interval $[0,1]$. Notice that a linearization procedure is then entirely feasible, possibly in a local fashion, on average models of the form (2). This reasoning justifies our use of the extended linearization technique in switching systems (see also Sira-Ramirez [9])

The average controlled system (2) is assumed to have a continuous family of constant state equilibrium points, $Z(U)$, corresponding to average constant inputs, $\mu = U$, which are neither 0 nor 1, i.e., $0 < U < 1$. The equilibrium points satisfy:

$$f(Z(U)) + Ug(Z(U)) + \eta = 0.$$

The pair of linearized maps, given by:

$$\left[\frac{\partial f}{\partial z}(Z(U)) + U\frac{\partial g}{\partial z}(Z(U)), g(Z(U))\right]$$

is assumed to be *controllable*.

It is desired to locally maintain, in a stable fashion, the trajectories of the nonlinear system (1) in the vicinity of the constant nominal average equilibrium trajectory, $x = X(U) := Z(U)$, by means of a sliding motion, suitably induced on a manifold S which contains such an equilibrium point $x = X(U)$. In other words, it is required to synthesize

1) A nonlinear sliding surface S, parametrized by the nominal average control input U, of the form:

$$S = \{x \in \mathcal{R}^n | s(x,U) = 0\} \tag{3}$$

such that $s(X(U),U) = 0$, and

2) An associated variable structure control law:

$$u(x,U) = \begin{cases} 1 & \text{for } s(x,U) > 0 \\ 0 & \text{for } s(x,U) < 0 \end{cases} \tag{4}$$

which automatically forces every small state deviation, from the nominal operating conditions, to zero, via the local creation of a stable sliding regime, taking place on S, and leading the state trajectory to $X(U)$. This stabilization is to be accomplished, of course, modulo small chattering around the prescribed equilibrium point.

In order to specify such a sliding manifold we propose to resort to the method of *Extended Linearization* ([2]–[4]) as indicated in the following paragraphs.

B. A Nonlinear Sliding Mode Controller Design Based on Extended Linearization

1) Linearize the average dynamical system (2) about each point in the family of average constant operating trajectories, $[Z(U), U]$, obtaining the following parametrized family of linear systems:

$$\dot{z}_\delta = A(U)z_\delta + b(U)\mu_\delta \tag{5}$$

where, for fixed U, the input and state perturbation variables are defined, respectively, as: $\mu_\delta = \mu(t) - U$, $z_\delta(t) = z(t) - Z(U)$, while the $n \times n$ matrix $A(U)$ and the n-vector $b(U)$ are defined as:

$$A(U) := \frac{\partial f}{\partial z}(Z(U)) + U\frac{\partial f}{\partial z}(Z(U)) \;\; ; \quad b(U) := g(Z(U)) \tag{6}$$

Since the pair $[A(U), b(U)]$ is assumed to be controllable, a similarity transformation exists of the form:

$$\zeta_\delta = P(U)z_\delta = [p_1(U), p_2(U), \ldots, p_n(U)]\, z_\delta \tag{7}$$

such that (5) may be represented as a *controllable canonical realization*. The nonsingular matrix $P(U)$ is obtained from the well known expression:

$$P^{-1}(U) = [b(U), A(U)b(U), \ldots, A^{n-1}(U)b(U)]\, M(U)$$

$$M(U) = \begin{bmatrix} \alpha_1(U) & \alpha_2(U) & \cdots & 1 \\ \alpha_2(U) & \alpha_3(U) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{n-1}(U) & 1 & \cdots & \vdots \\ 1 & 0 & \cdots & 0 \end{bmatrix} \tag{8}$$

where:

$$\det[\lambda I - A(U)] = \lambda^n + \alpha_{n-1}(U)\lambda^{n-1} + \alpha_{n-2}(U)\lambda^{n-2} + \ldots + \alpha_0(U).$$

2) Obtain the transformed system in *controllable canonical* form as:

$$\begin{aligned} \dot{\zeta}_{1\delta} &= \zeta_{2\delta} \\ \dot{\zeta}_{2\delta} &= \zeta_{3\delta} \\ &\vdots \\ \dot{\zeta}_{(n-1)\delta} &= \zeta_{n\delta} \\ \dot{\zeta}_{n\delta} &= -\alpha_{n-1}(U)\zeta_{n\delta} - \alpha_{n-2}(U) \cdot \zeta_{(n-1)\delta} - \cdots - \alpha_0(U)\zeta_{1\delta} + \mu_\delta \end{aligned} \tag{9}$$

3) Use as a sliding surface the linear manifold:

$$\Sigma_\delta = \left\{ \zeta_\delta \in \mathcal{R}^n | \sigma_\delta(\zeta_\delta) = \sum_{i=1}^{n} c_i \zeta_{i\delta} = 0 \;\; ; \;\; c_n = 1 \right\} \tag{10}$$

and choose the coefficients c_i, independently of the operating point $[Z(U), U]$, such that the roots of the *characteristic polynomial*:

$$\sum_{i=1}^{n} c_i \lambda^{i-1} = 0 \tag{11}$$

for the (reduced) linear ideal sliding dynamics are specified at convenient locations in the open left half of the complex plane. i.e., so that the autonomous ideal sliding mode dynamical system:

$$\begin{aligned} \dot{\zeta}_{1\delta} &= \zeta_{2\delta} \\ \dot{\zeta}_{2\delta} &= \zeta_{3\delta} \\ &\vdots \\ \dot{\zeta}_{(n-1)\delta} &= -c_{n-1}\zeta_{(n-1)\delta} - c_{n-2}\zeta_{(n-2)\delta} - \cdots - c_1\zeta_{1\delta} \end{aligned} \tag{12}$$

is asymptotically stable toward the origin of transformed coordinates.

4) Obtain, on the basis of the previously described design steps, the parametrized sliding hyperplane specification in terms of the average perturbed state coordinates z_δ, as follows:

$$S_\delta = \{ z_\delta \in \mathcal{R}^n | s_\delta(z_\delta, U) := \sigma_\delta(P(U) z_\delta) = c^T P(U) z_\delta = 0 \} \tag{13}$$

5) Obtain a nonlinear sliding manifold S, characterized by the *parametrized surface coordinate function* $s(z, U) = 0$ such that its corresponding linearization about the operating point $[Z(U), U]$, yields back the sliding hyperplane (13). In other words, find a nonlinear switching surface, in average state coordinates z, which is tangent to the sliding hyperplane (13) at the equilibrium point. This sliding manifold can be immediately expressed in actual state coordinates x as $s(x, U) = 0$

- **Sliding Manifold**. We must, thus, find a nonlinear sliding surface coordinate function $s(x, U)$, parametrized by the constant operating point U, such that the following relations are satisfied:

$$\frac{\partial s(x, U)}{\partial x} \Big|_{x=X(U)} = c^T P(U) = \left[c^T p_1(U), c^T p_2(U), \ldots, c^T p_n(U) \right] \tag{14}$$

or, componentwise:

$$\frac{\partial s(x, U)}{\partial x_i} \Big|_{x=X(U)} = c^T p_i(U) \;\; ; \;\; i = 1, 2, \ldots, n \tag{15}$$

with the additional (boundary) condition: $s(X(U), U) = 0$, with $X(U) = Z(U)$.

Remark In general, there are infinitely many parametrized sliding surface coordinate functions, $s(x, U)$, which satisfy relations (14) and the boundary condition. Such a lack of uniqueness of solution may not be totally inconvenient. However, the following direct integration procedure, inspired by the results in [2], allows one to obtain a nonlinear sliding manifold in a systematic manner:

a) Assume, without loss of generality, that the first component $X_1(U)$ of the vector $X(U)$ is invertible, i.e., let there exist a unique solution, $X_1^{-1}(x_1)$, for U in the equation $x_1 = X_1(U)$.

b) It can be verified, after partial differentiation with respect to the components of the vector x and substitution of the equilibrium point, that the following manifold is one possible solution for the required parametrized nonlinear sliding manifold:

$$S = \left\{ x \in \mathcal{R}^n \;|\; s(x, U) = \int_U^{X_1^{-1}(x_1)} c^T P(\nu) \frac{\mathrm{d}X(\nu)}{\mathrm{d}\nu} \mathrm{d}\nu + \sum_{j=2}^{n} c^T p_j(X_1^{-1}(x_1)) \left[x_j - X_j (X_1^{-1}(x_1)) \right] = 0 \right\} \tag{16}$$

□

- **Equivalent Control** Once the nonlinear sliding surface coordinate function $s(x, U)$ is known, computation of the equivalent control follows by imposing the well known (ideal) invariance conditions, which make of the switching manifold a local integral manifold of the constrained system (ideally) smoothly controlled by the equivalent control policy:

$$s(x, U) = 0 \;\; ; \;\; \frac{\mathrm{d}}{\mathrm{d}t} s(x, U) = 0 \tag{17}$$

- **Sliding Mode Switching Logic**. A nonlinear sliding mode switching strategy is usually synthesized such that the sliding mode existence conditions ([1]) are satisfied, at least, in a local fashion. Such well known conditions are given by:

$$\lim_{s \to 0^+} \frac{\mathrm{d}s(x, U)}{\mathrm{d}t} < 0 \;\; ; \;\; \lim_{s \to 0^-} \frac{\mathrm{d}s(x, U)}{\mathrm{d}t} > 0 \tag{18}$$

It has been shown that, for nonlinear systems which are linear in the scalar control input, a necessary and sufficient conditions for the local existence of a sliding mode is that the equivalent control locally exhibits values which are *intermediate* between the extreme numerical values representing the switch position values (i.e., $0 < u_{EQ}(x, U) < 1$). The region of existence of such a sliding regime coincides, precisely, with the region where such an *intermediacy* condition is satisfied by the equivalent control. One may, therefore, synthesize the nonlinear sliding mode switching logic from knowledge of the sliding manifold coordinate function, $s(x, U)$, as follows:

$$u(x, U) = \frac{1}{2}[1 + \text{sign } s(x, U)] \tag{19}$$

In more general cases, where there is no special input structure to the system, the above switching logic, or any one satisfying the equivalent control intermediacy condition, may still locally create a sliding regime provided the system exhibits a *control foliation property* (See Sira-Ramirez [10]). For the class of examples that we will be presenting in the next section, a switching control law of the form (19) suffices.

Notice that due to the discrete nature of the control input set, the equivalent control function is necessarily limited to the closed interval $[0, 1]$. Thus, the region of existence of a sliding mode, on the switching manifold, may not be global in the state space of the system. A necessary and sufficient condition for assessing the region of existence of sliding regimes in the above class of variable structure systems was given by Sira-Ramirez in [11]. Such a region of existence is simply defined, for the class of switch-regulated systems given by (1), by:

$$\{x \in \mathcal{R}^n \mid 0 \; < \; u_{EQ}(x, U) \; < \; 1 \;\} \tag{20}$$

Let $L_\phi s(x, U)$ denote the the *directional*, or *Lie derivative*, of the sliding manifold coordinate function $s(x, U)$ along the smooth vector field $\phi(x)$. It is easy to show that for a system of the form (1), with sliding surface given by $S = \{x \in \mathcal{R}^n \mid s(x, U) = 0\}$, the equivalent control, readily obtained from the condition $\dot{s}(x, U) = 0$, is given by

$$u_{EQ}(x, U) = -\frac{L_{f+\eta}s(x, U)}{L_g s(x, U)} \tag{21}$$

The region of existance (20) is therefore given by (see [11]):

$$\left\{x \in \mathcal{R}^n \mid 0 < u_{EQ}(x, U) = -\frac{L_{f+\eta}s(x, U)}{L_g s(x, U)} \; < 1\right\} \tag{22}$$

which, in local coordinates, is simply expressed as:

$$\left\{x \in \mathcal{R}^n \mid 0 < -\frac{[\partial s(x, U)/\partial x](f(x) + \eta)}{[\partial s(x, U)/\partial x]g(x)} < 1\right\} \tag{23}$$

In all of the examples treated in the next section such a region of existence is explicitly computed. In the presented simulations, the behavior of the discontinuously controlled trajectories, around the prescribed switching manifold, are clearly shown, both, outside and inside such a region.

III. Sliding Mode Controller Design for Bilinear Switched-controlled Converters

In this section we use the extended linearization-based sliding mode control synthesis procedure, developed in Section 2, for the specification of discontinuous feedback regulation schemes in typical bilinear switch-mode controlled dc-to-dc power converters.

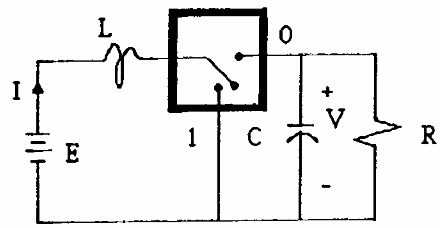

Fig. 1. The Boost converter.

A. Boost Converter

Consider the Boost converter model shown in Fig. 1. This converter is described by the following bilinear system of controlled differential equations:

$$\begin{aligned}\dot{x}_1 &= -w_0 x_2 + u w_0 x_2 + b\\ \dot{x}_2 &= w_0 x_1 - w_1 x_2 - u w_0 x_1\end{aligned} \tag{24}$$

where, $x_1 = I\sqrt{L}$, $x_2 = V\sqrt{C}$ represent *normalized input current* and *normalized output voltage* variables, respectively. The quantity $b = E/\sqrt{L}$ is the *normalized external input voltage* and, $w_0 = 1/\sqrt{LC}$ and $w_1 = 1/RC$ are, respectively, the LC (input) circuit natural oscillating frequency and the RC (output) circuit time constant. The variable u denotes the switch position function, acting as a control input, and taking values in the discrete set $\{0, 1\}$. System (24) is of the same form as (1), with $\eta = [b \quad 0]'$. We now summarize, according to the theory presented in the previous section, the formulae leading to a nonlinear sliding mode controller design for the average model of (24) using extended linearization.

Average Boost converter model

$$\begin{aligned}\dot{z}_1 &= -w_0 z_2 + \mu w_0 z_2 + b\\ \dot{z}_2 &= w_0 z_1 - w_1 z_2 - \mu w_0 z_1\end{aligned} \tag{25}$$

Constant parametrized operating equilibrium points

$$\mu = U \;\; ; \;\; Z_1(U) = \frac{b w_1}{w_0^2(1 - U)^2} \;\; ; \;\; Z_2(U) = \frac{b}{w_0(1 - U)} \tag{26}$$

Parametrized family of linearized systems about the constant operating points

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} z_{1\delta} \\ z_{2\delta} \end{bmatrix} = \begin{bmatrix} 0 & -w_0(1 - U) \\ w_0(1 - U) & -w_1 \end{bmatrix} \begin{bmatrix} z_{1\delta} \\ z_{2\delta} \end{bmatrix} + \begin{bmatrix} \frac{b}{1-U} \\ -\frac{b w_1}{w_0(1-U)^2} \end{bmatrix} \mu_\delta \tag{27}$$

with:

$$z_{i\delta}(t) = z_i(t) - Z_i(U); i = 1, 2 \;\; ; \;\; \mu_\delta(t) = \mu(t) - U.$$

Transformation of linearized family of systems to controllable canonical form

$$\begin{bmatrix} \zeta_{1\delta} \\ \zeta_{2\delta} \end{bmatrix} = \frac{w_0^2(1 - U)^2}{b^2(2w_1^2 + w_0^2(1 - U)^2)} \begin{bmatrix} \frac{b w_1}{w_0^2(1-U)^2} & \frac{b}{w_0(1-U)} \\ b & -\frac{2b w_1}{w_0(1-U)} \end{bmatrix} \begin{bmatrix} z_{1\delta} \\ z_{2\delta} \end{bmatrix} \tag{28}$$

Prametrized family of linearizations in controllable canonical form

$$\dot{\zeta}_{1\delta} = \zeta_{2\delta}$$
$$\dot{\zeta}_{2\delta} = -w_0^2(1-U)^2\zeta_{1\delta} - w_1\zeta_{2\delta} + \mu_\delta \quad (29)$$

Linear sliding surface and ideal sliding dynamics in transformed state coordinates

$$\sigma_\delta(\zeta_\delta) = \zeta_{2\delta} + c_1\zeta_{1\delta} \quad (30)$$

$$\dot{\zeta}_{1\delta} = -c_1\zeta_{1\delta} \;\; ; \;\; c_1 > 0 \quad (31)$$

Linear sliding surface in original (average) state coordinates

$$s_\delta(z_\delta) = \left[b + \frac{c_1 b w_1}{w_0^2(1-U)^2}\right] z_{1\delta} + \left[\frac{c_1 b - 2bw_1}{w_0(1-U)}\right] z_{2\delta} = 0 \quad (32)$$

Nonlinear sliding surface, equivalent control and sliding mode controller

$$s(x,U) = b[x_1 - Z_1(U)] + \frac{1}{2}c_1[x_1^2 - Z_1^2(U)] + \frac{c_1 - 2w_1}{2}[x_2^2 - Z_2^2(U)] = 0 \quad (33)$$

$$u_{EQ}(x,U) = 1 - \frac{b(b+c_1x_1) - w_1(c_1-2w_1)x_2^2}{w_0(b+c_1x_1) - w_0(c_1-2w_1)x_1x_2} \quad (34)$$

$$u = \frac{1}{2}[1 + \text{sign}\, s(x,U)] \quad (35)$$

The region of existence of a sliding mode on the switching manifold, according to (20), is given by the zone bounded between the following two curves in the $x_1 - x_2$ coordinate plane (see Fig. 3).

$$x \in \mathcal{R}^2 | u_{EQ}(x,U) = 0\} = (x_1,x_2) \mid \frac{b(b+c_1x_1) - w_1(c_1-2w_1)x_2^2}{w_0(b+c_1x_1) - w_0(c_1-2w_1)x_1x_2} = 1\} \quad (36)$$

$$\{x \in \mathcal{R}^2 | u_{EQ}(x,U) = 1\} = \{(x_1,x_2) \mid b(b+c_1x_1) - w_1(c_1-2w_1)x_2^2 = 0\} \quad (37)$$

A local diffeomorphic state coordinate transformation, which can be inferred from the linearized transformation (28) takes the average ideal sliding dynamics into an autonomous stable linear system.

$$\xi_1 = \frac{1}{2}(z_1^2 + z_2^2) \;\; ; \;\; \xi_2 = bz_1 - w_1z_2^2 \quad (38)$$

This transformation coincides with the diffeomorphism achieving exact linearization found in Sira-Ramirez and Ilic [12] and, not surprisingly, it is the same found by *Pseudo-linearization* techniques (see Sira-Ramirez [13]). The interpretation of (38), in terms of total average energy and average consumed power, can be found in [12].

In the new coordinates (38), the ideal sliding dynamics is given by:

$$\dot{\xi}_1 = -c_1\left(\xi_1 - \frac{X_1^2 + X_2^2}{2}\right)$$
$$\dot{\xi}_2 = -c_1\left(\xi_2 - \frac{bX_1 - w_1X_2^2}{2}\right) = -c_1\xi_2 \quad (39)$$

which is evidently linear, as claimed from the outset.

B. A Simulation Example

A Boost converter circuit with parameter values: $R = 30\Omega$, $C = 20\mu$F, $L = 20$mH and $E = 15$V was considered for sliding mode controller design based on nonlinear switching manifolds computed via extended linearization. The constant operating value of μ was chosen to be $U = 0.1619$ while the corresponding desirable normalized constant output voltage turned out to be $Z_2(0.1619) = 0.08$. The equilibrium value of the average normalized input current is $z_1(0.1619) = 0.1007$.

Fig. 2 shows several state trajectories corresponding to different initial conditions set on the ideal Boost converter model feedback regulated by means of a sliding mode controller of the form (3.10)-(3.12). The average controlled state variables, z_1 and z_2, are shown to converge towards the desirable equilibrium point, $(0.1007, 0.0800)$. The region of existence of a sliding regime is bounded by the curves $u_{EQ}(x,U) = u_{EQ}(x, 0.1619) = 0$ (actually shown in the figure) and $u_{EQ}(x,U) = u_{EQ}(x, 0.1619) = 1$ (not shown in the figure). Outside the region of existence of a sliding regime, the trajectories are clearly seen *not* to create a sliding regime on the manifold, $s(x,U) = s(x, 0.1619) = 0$.

Fig. 3 clearly shows the extent of the region of existence of a sliding motion by depicting the boundary lines (36) given by $\{x \in \mathcal{R}^2 \mid u_{EQ}(x,U) = 0\}$ and the boundary line (37), given by $\{x \in \mathcal{R}^2 \mid u_{EQ}(x,U) = 1\}$. The first boundary line corresponds to an hyperbola which exhibits intersection with the sliding manifold, $s(x,U) = 0$, only on one of its branches located in the first quadrant, $x_1 > 0$, $x_2 > 0$. The boundary line (37) corresponds to a parabola confined to the second and third quadrants. The intersection of the second branch of the hyperbola with the parabola clearly corresponds to a nonphysical situation whereby the equivalent control adopts, both, the value 1 and 0. The region of existence of a sliding regime is thus located *above* the first branch of the hyperbola located in the first quadrant and, theoretically speaking, it extends towards the infinity. This fact precisely corresponds with the (ideal) output voltage "amplification" capabilities of the Boost converter. Such a magnification even extends towards infinite values, as the equilibrium value of the duty ratio, $\mu = U$, approaches 1 (see (26)).

Fig. 4 shows the effect of a sudden step change in the desired average equilibrium value of the converter output voltage $x_2 = Z_2(U)$ from 0.0800 to 0.2000. This change causes a corresponding change in the operating equilibrium point of the duty ratio, μ, as well as the consideration of a *new* sliding surface S_2. The equilibrium value of the duty ratio is seen to change from the value $\mu = U_1 = 0.1619$ to a new value of $\mu = U_2 = 0.6646$.

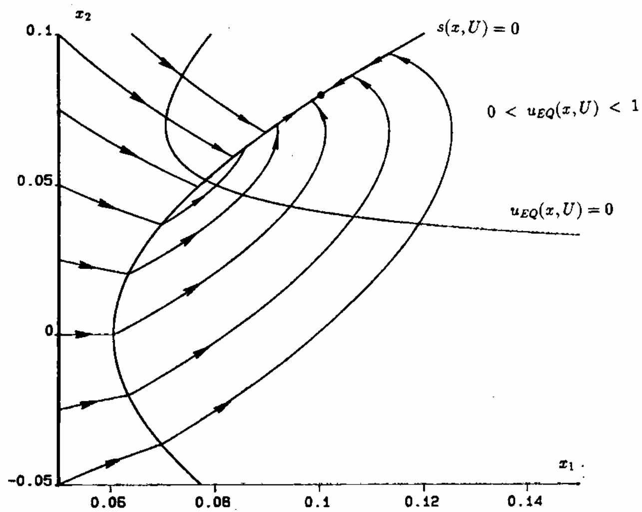

Fig. 2. Sliding mode controlled trajectories for the Boost converter.

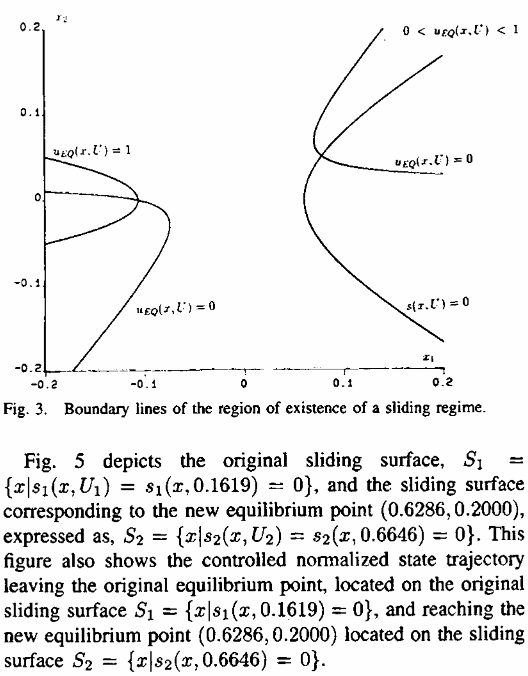

Fig. 3. Boundary lines of the region of existence of a sliding regime.

Fig. 5 depicts the original sliding surface, $S_1 = \{x|s_1(x, U_1) = s_1(x, 0.1619) = 0\}$, and the sliding surface corresponding to the new equilibrium point (0.6286, 0.2000), expressed as, $S_2 = \{x|s_2(x, U_2) = s_2(x, 0.6646) = 0\}$. This figure also shows the controlled normalized state trajectory leaving the original equilibrium point, located on the original sliding surface $S_1 = \{x|s_1(x, 0.1619) = 0\}$, and reaching the new equilibrium point (0.6286, 0.2000) located on the sliding surface $S_2 = \{x|s_2(x, 0.6646) = 0\}$.

C. Buck-Boost Converter

Consider the Buck-Boost converter model (see Fig. 6). This device is described by the following bilinear state equation model:

$$\begin{aligned} \dot{x}_1 &= w_0 x_2 - u w_0 x_2 + u b \\ \dot{x}_2 &= -w_0 x_1 - w_1 x_2 + u w_0 x_1 \end{aligned} \tag{40}$$

where, $x_1 = I\sqrt{L}$, $x_2 = V\sqrt{C}$ represent normalized input current and output voltage variables respectively, $b = E/\sqrt{L}$ is the normalized external input voltage and it is here assumed to be a negative quantity (i.e., reversed polarity) while, $w_0 = 1/\sqrt{LC}$ and $w_1 = 1/RC$ are, respectively, the LC (input) circuit natural oscillating frequency and the RC (output) circuit time constant. The switch position function, acting as a control input, is denoted by u and takes values in the discrete set $\{0, 1\}$. System (3.17) is of the same form as (2.1), with $\eta = 0$ and $g = [-w_0 x_2 + b \;\; w_0 x_1]'$. We now summarize the formulae leading to a nonlinear sliding mode controller design for the Buck-Boost model (40).

Average Buck-Boost converter model

$$\begin{aligned} \dot{z}_1 &= w_0 z_2 - \mu w_0 z_2 + \mu b \\ \dot{z}_2 &= -w_0 z_1 - w_1 z_2 + \mu w_0 z_1 \end{aligned} \tag{41}$$

Constant equilibrium points

$$\mu = U \;\; ; \;\; Z_1(U) = \frac{bUw_1}{w_0^2(1-U)^2};$$
$$Z_2(U) = -\frac{bU}{w_0(1-U)} \tag{42}$$

Parametrized family of linearized systems about the constant operating points

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} z_{1\delta} \\ z_{2\delta} \end{bmatrix} = \begin{bmatrix} 0 & w_0(1-U) \\ -w_0(1-U) & -w_1 \end{bmatrix} \begin{bmatrix} z_{1\delta} \\ z_{2\delta} \end{bmatrix} + \begin{bmatrix} \frac{-b}{1-U} \\ \frac{-bw_1}{w_0(1-U)^2} \end{bmatrix} \mu_\delta \tag{43}$$

with:

$$z_{i\delta}(t) = z_i(t) - Zi(U) \;\; ; \;\; i = 1, 2. \;\; ; \;\; \mu_\delta(t) = \mu(t) - U.$$

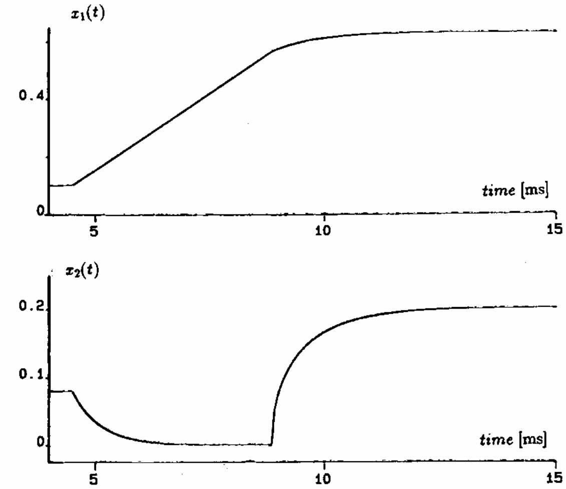

Fig. 4. State variables responses due to a sudden change in the operating point (Boost converter).

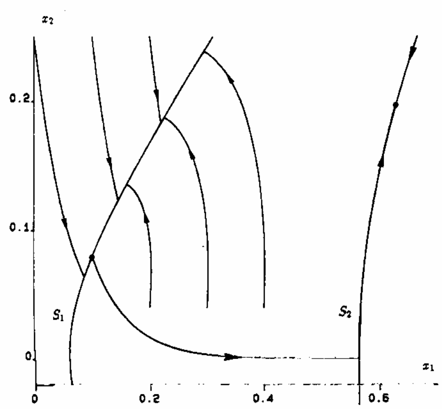

Fig. 5. Effect of a sudden change in the operating point in the state space (Boost converter).

Transformation of linearized family of systems to controllable canonical form

$$\begin{bmatrix} \zeta_{1\delta} \\ \zeta_{2\delta} \end{bmatrix} = \frac{w_0^2(1-U)^3}{b^2(w_1^2 U(1+U) + w_0^2(1-U)^2)} \cdot \begin{bmatrix} \frac{bw_1U}{w_0^2(1-U)^2} & -\frac{b}{w_0(1-U)} \\ b & \frac{bw_1(1+U)}{w_0(1-U)} \end{bmatrix} \begin{bmatrix} z_{1\delta} \\ z_{2\delta} \end{bmatrix} \tag{44}$$

Prametrized family of linearizations in controllable canonical form

$$\begin{aligned} \dot{\zeta}_{1\delta} &= \zeta_{2\delta} \\ \dot{\zeta}_{2\delta} &= -w_0^2(1-U)^2\zeta_{1\delta} - w_1\zeta_{2\delta} + \mu_\delta \end{aligned} \tag{45}$$

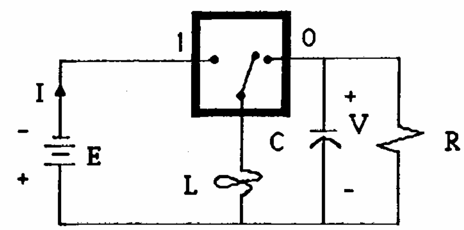

Fig. 6. The Buck-Boost converter.

Linear sliding surface and ideal sliding dynamics in transformed state coordinates

$$\sigma_\delta(\zeta_\delta) = \zeta_{2\delta} + c_1\zeta_{1\delta} \tag{46}$$

$$\dot{\zeta}_{1\delta} = -c_1\zeta_{1\delta} \;\; ; \;\; c_1 > 0 \tag{47}$$

Linear sliding surface in original (average) state coordinates

$$s_\delta(z_\delta) = \left[b + \frac{c_1 b w_1 U}{w_0^2(1-U)^2}\right] z_{1\delta} + \left[\frac{bw_1(1+U) - c_1 b}{w_0(1-U)}\right] z_{2\delta} = 0 \tag{48}$$

Nonlinear sliding surface, equivalent control and sliding mode controller

$$\begin{aligned} s(x,U) = b[x_1 - Z_1(U)] & \\ + \frac{c_1}{2}[x_1^2 - Z_1^2(U)] &+ \frac{c_1 - 2w_1}{2}[x_2^2 - Z_2^2(U)] \\ - \frac{b}{w_0}(c_1 - w_1)&[x_2 - Z_2(U)] = 0 \end{aligned} \tag{49}$$

$$u_{EQ}(x,U) = 1 - \frac{bw_0(b+c_1x_1) - w_1w_0(c_1-2w_1)x_2^2 + bw_1(c_1-w_1)x_2}{w_0b(b+w_1x_1) - w_0^2(b+2w_1x_1)x_2} \tag{50}$$

$$u = \frac{1}{2}[1 + \text{sign}\ s(x,U)] \tag{51}$$

Bounding curves for the region of existence of a sliding mode

See (52) and (53) at the bottom of this page.

Linearizing local diffeomorphic state coordinate transformation for average ideal sliding dynamics

$$\xi_1 = \frac{1}{2}\left[z_1^2 + \left(z_2 - \frac{b}{w_0}\right)^2\right] \ ; \ \xi_2 = bz_1 - w_1z_2\left(z_2 - \frac{b}{w_0}\right) \tag{54}$$

Ideal sliding dynamics in transformed coordinates

$$\dot{\xi}_1 = -c_1\left[\xi_1 - \frac{X_1^2 + \left(X_2 - \frac{b}{w_0}\right)^2}{2}\right]$$
$$\dot{\xi}_2 = -c_1\xi_2 \tag{55}$$

D. A Simulation Example

A Buck-Boost converter circuit, with the same parameter values as in the previous example, was considered for nonlinear sliding mode controller design. The constant operating value of μ was chosen to be $U = 0.6508$. The corresponding desirable average normalized constant output voltage turned out to be $Z_2(0.6508) = -0.125$ while the corresponding average input current was $Z_1(0.6508) = 0.3774$. Fig. 7 shows several state trajectories corresponding to different initial conditions set on the buck-boost converter model controlled by the sliding mode based regulator of the form (49)-(51). The average controlled state variables, z_1 and z_2, are shown to converge towards the desirable equilibrium point represented by $Z_1(U) = 0.3773$ and $Z_2(U) = -0.125$. The region of existence of a sliding regime is bounded by the curves $u_{EQ}(x,U) = u_{EQ}(x, 0.6508) = 0$ (actually shown in the figure) and $u_{EQ}(x,U) = u_{EQ}(x, 0.6508) = 1$ (not shown in the figure). Outside the region of existence of a sliding regime, the trajectories are clearly seen not to create a sliding regime on the manifold $s(x,U) = s(x, 0.6508) = 0$.

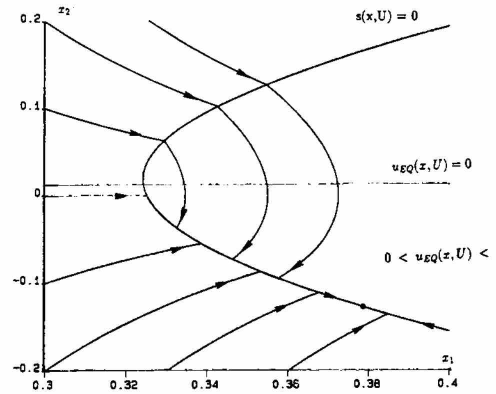

Fig. 7. Sliding mode controlled trajectories for the Buck-Boost converter.

Fig. 8 clearly shows the extent of the region of existence of a sliding motion by depicting the boundary line (52), given by: $\{x \in \mathcal{R}^2 \mid u_{EQ}(x,U) = 0\}$, and the boundary line (53), given by $\{x \in \mathcal{R}^2 \mid u_{EQ}(x,U) = 1\}$. The first boundary line corresponds to an hyperbola which exhibits intersection with the sliding manifold, $s(x,U) = 0$, only on one of its branches. Roughly speaking with the one located on the fourth quadrant, $x_1 > 0$ $x_2 < 0$. The boundary line (53) corresponds to a parabola confined to the second and third quadrants. As before, the intersection of the second branch of the hyperbola with the parabola corresponds to a nonphysical situation. The region of existence of a sliding regime is thus located *below* the first branch of the hyperbola, located in the fourth quadrant, and, theoretically speaking, it extends towards the infinity. This fact precisely corresponds with the (ideal) "negative amplification" capabilities of the Buck-Boost converter. Such magnification ideally reaches infinite values as the equilibrium value of the duty ratio, $\mu = U$, approaches 1 (see (42)).

Fig. 9 shows the effect of a sudden step change in the average equilibrium value of the converter output voltage, $x_2 = Z_2(U)$, from -0.1250 to -0.0500. This change causes a corresponding change in the operating equilibrium value of the duty ratio μ, as well as the consideration of a new sliding surface S_2, containing the new equilibrium point $(0.0920, -0.0500)$. The equilibrium value of the duty ratio is seen to change from the value $\mu = U_1 = 0.6508$, to a new value of $\mu = U_2 = 0.4271$.

$$\{x \in \mathcal{R}^2 | u_{EQ}(x,U) = 0\} = \left\{(x_1,x_2) \middle| \frac{bw_0(b+c_1x_1) - w_1w_0(c_1-2w_1)x_2^2 + bw_1(c_1-w_1)x_2}{w_0b(b+w_1x_1) - w_0^2(b+2w_1x_1)x_2} = 1\right\} \tag{52}$$

$$\{x \in \mathcal{R}^2 | u_{EQ}(x,U) = 1\} = \{(x_1,x_2) \mid bw_0(b+c_1x_1) - w_1w_0(c_1-2w_1)x_2^2 + bw_1(c_1-w_1)x_2 = 0\} \tag{53}$$

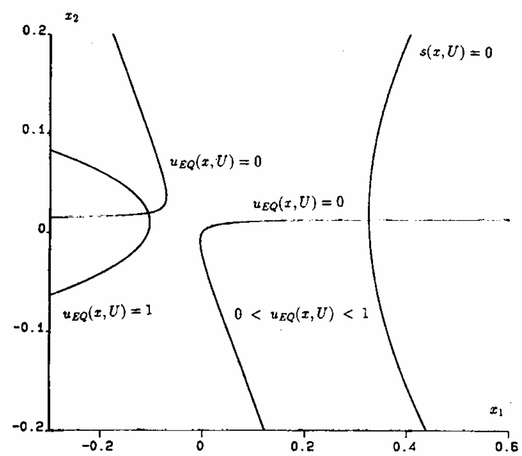

Fig. 8. Boundary lines of the region of existence of a sliding regime.

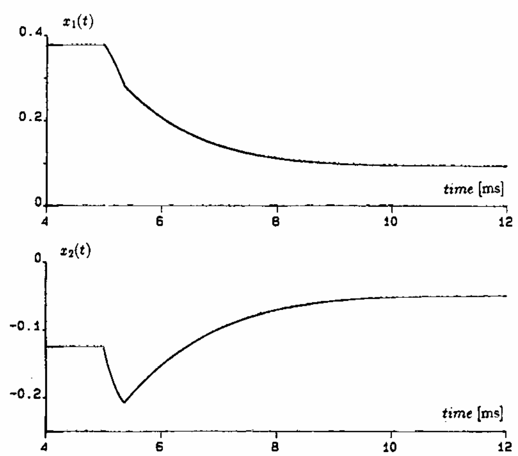

Fig. 9. State variables responses due to a sudden change in the operating point (Buck-Boost converter).

Fig. 10 depicts the original sliding surface $S_1 = \{x|s_1(x,U_1) = s_1(x,0.6508) = 0\}$ and the sliding surface corresponding to the new equilibrium point $S_2 = \{x|s_2(x,U_2) = s_2(x,0.4271) = 0\}$. The figure also shows the controlled normalized state trajectory leaving the original equilibrium point, $(0.3774, -0.125)$, located on the original sliding surface S_1, and reaching the new equilibrium point $(0.0920, -0.0500)$, located on the sliding surface S_2.

IV. Conclusions and Suggestions for Further Work

A systematic approach has been proposed for the synthesis of families of nonlinear sliding surfaces, parametrized by constant equilibrium points, defining sliding-mode regulators for dc-to-dc power converters. The method entitles the use

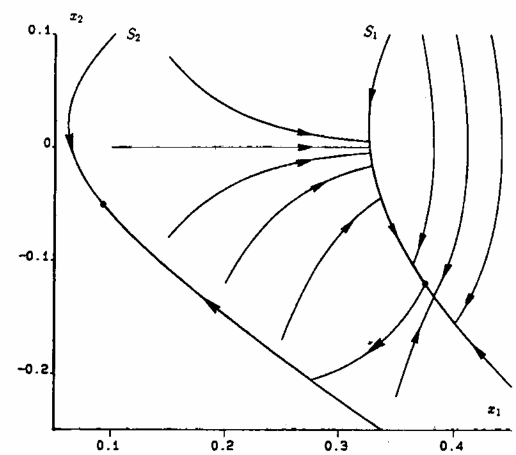

Fig. 10. Effect of a sudden change in the operating point in the state space (Buck-Boost converter).

of the extended linearization technique for the specification of the nonlinear switching manifold. On the basis of the proposed parametrized nonlinear manifold, one specifies—in a standard fashion—the associated equivalent control, the required switching strategy and the sliding mode existence region. One of the main advantages of the proposed regulator design scheme resides in the "self-scheduling" properties of the synthesized controller.

The proposed design scheme exhibits the following features:

1) The approach benefits from an extensive list of well known theoretical contributions for design of linear sliding modes, including efficient computer packages already developed for such design tasks.
2) The possibilities of nontrivial applications can be greatly enhanced, and carried out, by means of existing algebraic manipulation systems.
3) The method naturally enjoys rather useful self-scheduling properties when nominal operating conditions are abruptly changed. This is particularly important in the field of control of mechanical manipulators, aerospace systems and other practical nonlinear control application areas.
4) The method developed in this article also constitutes an alternative approach, for approximate linearization of nonlinear systems, to the method developed by Bartolini and Zolezzi in [14].

As a topic for future work, practical implementation of the switching regulators can be attempted on a real converter. Also, automation of the design process via computational algebra packages, such as MACSYMA, REDUCE, or MAPLE, is strongly encouraged.

Chattering alleviation is a topic of general prevailing interest in discontinuous feedback control of dynamical systems (see Fliess and Messager [15], Zhou and Fisher [16] and others). A topic that deserves some attention is represented by the need of devising chattering-free dc-to-dc power conversion schemes.

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