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Pulse width modulated control of the full bridge buck converter

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Two approaches are examined for the design a pulse width modulated (PWM) feedback control scheme regulating a full bridge buck converter in both stabilization and AC signal tracking tasks. The first approach is based on a direct strategy for the specification of a static PWM feedback control which accomplishes asymptotic stabilization to a preselected constant operating point for the output capacitor voltage. The same approach is also examined for the asymptotic output tracking of AC reference signals. The second approach, or indirect approach, proposes a dynamical feedback regulation scheme based on input inductor current stabilization or tracking using reference signals obtained via partial inversion of the system dynamics. The indirect approach emphasizes the use of Fliess' generalized observability canonical form of the average converter model.

1. Introduction

The full bridge buck converter (FBBC) constitutes a popular means of generating automatically regulated constant output voltages, of either polarity, which represent an arbitrarily specified fraction of the constant input source voltage (see Boudjema *et al.* 1989). The FBBC may also be used in the generation of controlled sinusoidal voltages through the asymptotic tracking of a suitable AC reference signal referred to as the 'template signal'. A typical application of this capability lies in the area of low power emergency inversion. For a detailed account on the principles of operation of this versatile class of DC power converters, the reader is referred to the books by Csaki *et al.* (1983), Mohan *et al.* (1989) and Severns and Bloom (1985), as well as the current literature appearing in specialized journals.

Sliding mode and PWM control schemes were proposed by Boudjema *et al.* (1989) for the efficient handling of AC signal tracking problems in FBBC. For the PWM regulator case, the design was accomplished using static pole assignment techniques through full state feedback defined on the basis of the average PWM converter model.

In this article, two approaches are examined for the specification of a pulse width modulated (PWM) feedback control scheme regulating a full bridge buck converter in both stabilization and AC signal tracking objectives. The first approach is based on a direct, static, output regulation strategy for the specification of a PWM feedback control law which accomplishes asymptotic output voltage stabilization to a preselected constant operating point for the output capacitor voltage. The same approach is used for the asymptotic output tracking of AC reference signals. The second approach is constituted by the use of a dynamical PWM feedback regulator which indirectly accomplishes asymptotic stabilization of the output voltage to a preselected constant operating point via asymptotic stabilization of the input current.

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The same dynamical feedback strategy may be used in indirect asymptotic sinusoidal output voltage tracking for the average PWM converter model. The proposed indirect approach emphasizes the use of a partial (state) inversion strategy in conjunction with Fliess' generalized observability canonical form (GOCF) (see Fliess 1990 a) of the average PWM controlled converter.

Section 2 of this article presents some generalities about the FBBC and the static PWM feedback regulation scheme for the output voltage stabilization and AC signal tracking problems based on the average FBBC model. Section 3 presents the dynamical PWM feedback regulation strategy for both asymptotic stabilization and tracking problems.

2. Static PWM feedback controller for stabilization and tracking in the full bridge buck converter

2.1. *Full bridge buck converter*

Consider the following system of ordinary differential equations describing a FBBC which includes an ideal high frequency isolation transformer (see Fig. 1):

$$\dot{x}_1 = -w_0 x_2 + ub, \quad \dot{x}_2 = w_0 x_1 - w_1 x_2 \tag{2.1}$$

where x_1 is the normalized input inductor current, defined as $x_1 = I_L \sqrt{L}$, x_2 is the normalized output capacitor voltage given by $x_2 = V_0 N \sqrt{C}$, where N is the transformer's winding turn ratio and V_0 is the primary winding voltage drop. The constant w_0 represents the natural oscillating frequency of the LC input circuit, $w_0 = 1/(N\sqrt{LC})$, where the capacitance refers to the primary of the transformer, and w_1 is the inverse time constant associated with the RC output circuit, i.e., $w_1 = 1/RC$. The constant b is the normalized input source voltage, $b = V_s/\sqrt{L}$, assumed to be constant. The variable u acts as the control input taking values in the discrete set $\{-1, 0, 1\}$, according to the positions of the switches. This feature makes the FBBC truly a generalization of the standard two position switch-controlled Buck converter, also known as the 'step-down' converter (see, for instance, Sira-Ramirez 1987). A suitable operation of the switch arrangement, which is capable of realizing such a discrete set of available control inputs, is simply summarized as (see also Boudjema *et al.* 1989) shown in the following table.

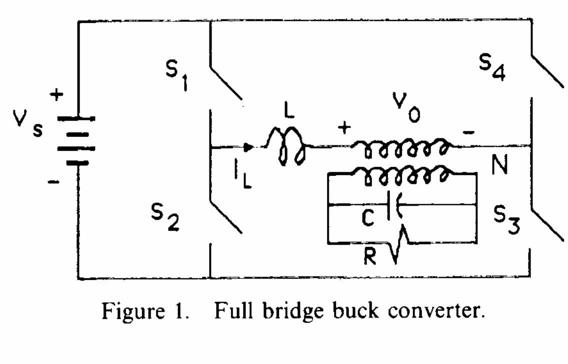

Figure 1. Full bridge buck converter.

	S_1	S_2	S_3	S_4
$u = 1$	ON	OFF	ON	OFF
$u = 0$	OFF	ON	ON	OFF
$u = -1$	OFF	ON	OFF	ON

The FBBC constitutes, therefore, a variable structure system that may be naturally controlled by means of an ON–OFF–ON PWM control policy (see the Appendix in Sira-Ramirez 1991 c) of the following type:

$$u = \begin{cases} \operatorname{sign} \mu[x(t_k)], & \text{for } t_k \le t \le t_k + |\mu[x(t_k)]| T \\ 0, & \text{for } t_k + |\mu[x(t_k)]| T \le t < t_k + T \end{cases} \tag{2.2}$$

where $\mu : R^n \to R$ is the duty ratio function taking values in the closed interval $[-1, 1]$ of the real line, $x(t_k)$ is the sampled value of the state at time t_k and T represents the constant sampling period. The function 'sign' stands for the 'signum' function. The absolute value of the duty ratio function μ specifies, at each sampling instant t_k, the width of the sign modulated control pulse that will be enforced within the upcoming inter-sampling period of length T. The duty ratio function $\mu(x)$ must be regarded, effectively, as a feedback control policy.

The PWM controller design problem carried out in this section, consists of specifying such a bounded feedback control policy $\mu(x)$ for stabilization and tracking purposes. The use of the following, infinite switching frequency, average PWM controlled converter model, for controller design purposes, has been thoroughly justified in several journal publications by Sira-Ramirez and his co-workers (see Sira-Ramirez 1989, 1991 a, Sira-Ramirez and Prada-Rizzo 1992):

$$\dot{z}_1 = -w_0 z_2 + \mu b, \quad \dot{z}_2 = w_0 z_1 - w_1 z_2 \tag{2.3}$$

where μ is now regarded as a smooth function of the average state vector z, taking values in the closed interval $[-1, 1]$.

Given a constant value U of the duty ratio μ, such that $U \in [-1, 1]$, the corresponding average equilibrium state of the FBBC is simply obtained from (3.3) as

$$z_1 = Z_1(U) = \frac{b w_1}{w_0^2} U, \quad z_2 = Z_2(U) = \frac{b}{w_0} U \tag{2.4}$$

A steady state relationship can be derived from (2.4) for the average equilibrium values of the normalized input circuit current z_1 and the normalized output capacitor voltage z_2 which is independent of the constant value U of the duty ratio function. Such a relation is readily obtained as

$$Z_1 = \frac{w_1}{w_0} Z_2 \tag{2.5}$$

Hence, regulation of the average output capacitor voltage z_2 towards constant equilibrium values, specified by (2.4), can be indirectly accomplished through regulation of the average input inductor current z_1 toward its equilibrium value Z_1. This simple observation also allows for the possibility of specifying a dynamical feedback regulator instead of a static one (Boudjema *et al.* 1989). For tracking problems related to the automatic generation of regulated *AC* signals, such an indirect control policy requires partial inversion of the FBBC dynamical equations.

2.2. *Stabilization of the FBBC through static PWM control*

Consider the average model (2.3) of the FBBC with output equation

$$\dot{z}_1 = -w_0 z_2 + \mu b, \quad \dot{z}_2 = w_0 z_1 - w_1 z_2, \quad y = z_2 - Z_2 \tag{2.6}$$

The following invertible duty-ratio dependent (i.e. control-dependent) state coordinate transformation

$$\left.\begin{aligned} \xi_1 &= z_2 - Z_2, \quad \xi_2 = w_0 z_1 - w_1 z_2, \\ z_1 &= \frac{1}{w_0}\xi_2 + \frac{w_1}{w_0}(\xi_1 + Z_2), \quad z_2 = \xi_1 + Z_2 \end{aligned}\right\} \tag{2.7}$$

takes the average model (2.6) into Fliess' GOCF (Fliess 1990 a)

$$\left.\begin{aligned} \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= -w_0^2\xi_1 - w_1\xi_2 - w_0^2 Z_2 + b w_0 \mu \\ y &= \xi_1 \end{aligned}\right\} \tag{2.8}$$

Notice that if a feedback control is specified which asymptotically stabilizes the transformed coordinates ξ_1 and ξ_2 to zero (i.e. the system output y and its time derivative dy/dt are stabilized to zero), then according to the definitions in (2.7), one obtains asymptotic stabilization of the original average state coordinates z_1 and z_2 to their equilibrium values Z_1 and Z_2, respectively.

An unrestricted dynamical feedback controller specifying the required (computed) stabilizing duty ratio function μ is immediately obtained from (2.8) by simply equating the last differential equation in (2.8) to a suitable linear combination of the transformed variables. This operation is to bestow on the closed-loop transformed system convenient, stable, pole locations characterized by, say, a damping ratio ζ and a natural undamped frequency ω_n:

$$\begin{aligned} \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= -w_0^2\xi_1 - w_1\xi_2 - w_0^2 Z_2 + b w_0 \mu \\ &= -2\zeta\omega_n\xi_2 - \omega_n^2\xi_1 \\ y &= \xi_1 \end{aligned}$$

that is

$$\mu = \left(\frac{w_1 - 2\zeta\omega_n}{b w_0}\right)\xi_2 + \left(\frac{w_0^2 - \omega_n^2}{b w_0}\right)\xi_1 + \frac{w_0}{b} Z_2 \tag{2.9}$$

We denote the proposed static control law in (2.9) as $\hat{\mu}$, and regard it as the computed duty ratio function.

It is quite straightforward to interpret the static duty ratio synthesizer (2.9), expressed in transformed coordinates, as a classical proportional-derivative controller including a set-point feedforward term. Figure 2 depicts such an interpretation of the proposed controller (see also Sira-Ramirez (1991 b) and Sira-Ramirez and Lischinsky-Arenas (1993)).

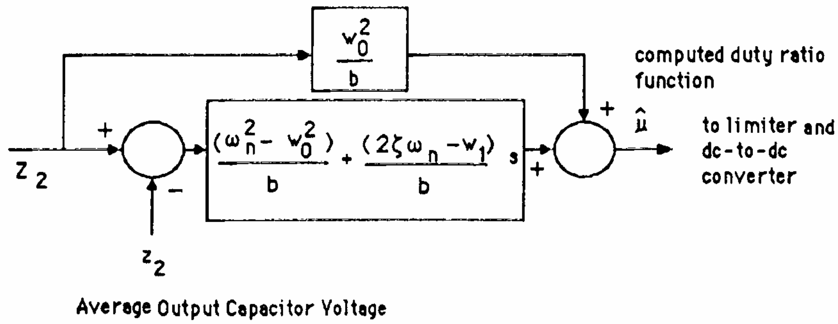

Figure 2. Interpretation of duty ratio synthesizer as a classical P-D compensator for a static PWM controller full bridge buck converter.

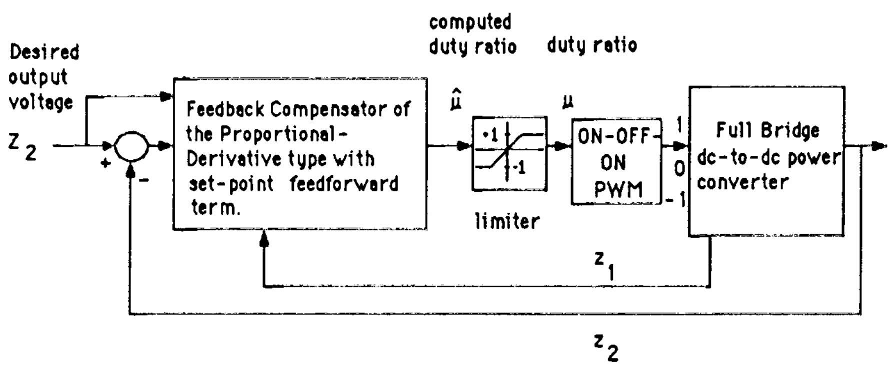

Figure 3. Static PWM feedback compensator structure for direct output stabilization and tracking tasks in a full bridge buck converter.

In original average coordinates, one obtains the following dynamical feedback controller:

$$\hat{\mu} = -\left[\frac{\omega_n^2 - w_0^2 + (w_1 - 2\zeta\omega_n)w_1}{bw_0}\right]z_2 - \frac{(2\zeta\omega_n - w_1)}{b}z_1 + \frac{\omega_n^2}{bw_0}Z_2 \tag{2.10}$$

The actual duty ratio function μ is simply obtained by bounding the solutions of (2.10) within the physically meaningful interval $[-1, 1]$, that is

$$\mu = \begin{cases} +1, & \text{if } \hat{\mu} > 1 \\ \hat{\mu}, & \text{if } -1 \leq \hat{\mu} \leq +1 \\ -1, & \text{if } \hat{\mu} < -1 \end{cases} \tag{2.11}$$

Figure 3 depicts the static PWM controller scheme accomplishing asymptotic stabilization of the output capacitor voltage to a desirable average equilibrium point for the FBBC.

Simulation example

Simulations were carried out on a FBBC with parameter values $R = 1.5\,\Omega$, $C = 2700\,\mu\text{F}$, $L = 40\,\mu\text{H}$, $V_s = 30$ V and a transformer winding turn ratio $N = 10$ (i.e.

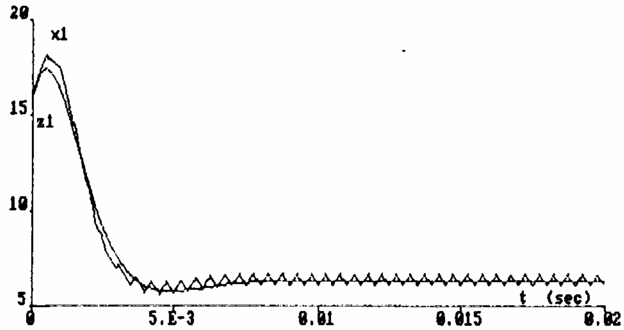

Figure 4. Average and actual input inductor current response for static PWM stabilization of a full bridge buck converter.

Figure 5. Average and actual output capacitor voltage response for static PWM stabilization of a full bridge buck converter.

$w_0 = 304{\cdot}29$, $w_1 = 246{\cdot}91$, $b = 4{\cdot}743 \times 10^4$). The chosen damping ratio and natural oscillating frequency of the closed-loop system stable complex poles were set, respectively, set at $\zeta = 0{\cdot}7$ and $w_n = 1000$ rad/s. The required equilibrium value for the normalized output voltage was specified as 7·794, which corresponds to an actual output voltage of 15 V. The corresponding equilibrium value for the normalized input current set point is 6·324 (i.e. $I_L = 1$ A). These equilibrium values correspond to a constant steady state value of the duty ratio function, $\mu = U = 0{\cdot}5$. A sampling frequency of 2 kHz was used for the proposed PWM controller. Figures 4 and 5 depict the transient response of the average and the discontinuously PWM controlled input current and output voltage of the FBBC, respectively. Figure 6 represents the evolution of the statically synthesized duty ratio function μ and the corresponding discontinuous PWM control actions u.

2.3. *Sinusoidal signal generation of the FBBC through static PWM feedback control*

In this section, we propose a static PWM feedback control for the adequate tracking of sinusoidal template signals by the output capacitor voltage in a FBBC. The natural limitations imposed on the synthesized duty ratio function imply some template amplitude-frequency tradeoffs that can be explicitly computed in terms of the system parameters.

Suppose the average normalized output capacitor voltage z_2 is to track a

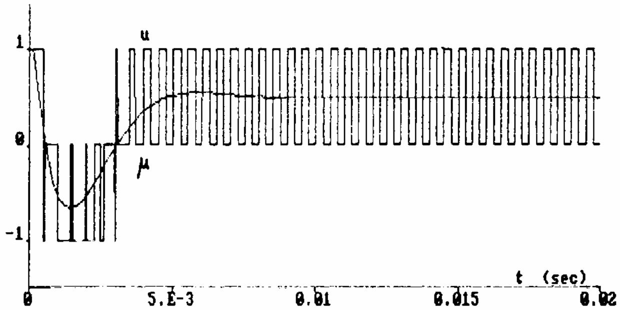

Figure 6. Duty ratio function and control input responses in static PWM stabilization of a full bridge buck converter.

(template) sinusoidal signal of the form

$$z_{2d}(t) = A \sin(\omega t) \tag{2.12}$$

The following invertible time-varying, input-dependent state coordinate transformation,

$$\left.\begin{aligned} &\xi_1(t) = z_2 - A \sin \omega t, \quad \xi_2(t) = w_0 z_1 - w_1 z_2 - A\omega \cos \omega t \\ &z_1(t) = \frac{\xi_2}{w_0} + \frac{w_1}{w_0}\xi_1 + \frac{A}{w_0}(w_0^2 + \omega^2)^{1/2} \sin\left(\omega t + \operatorname{tg}^{-1}\frac{\omega}{w_1}\right) \\ &z_2 = \xi_1 + A \sin \omega t \end{aligned}\right\} \tag{2.13}$$

places the average FBBC model into Fliess' GOCF,

$$\left.\begin{aligned} \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= -w_0^2\xi_1 - w_1\xi_2 + bw_0\mu \\ &\quad - A[(\omega w_1)^2 + (\omega^2 - w_0^2)^2]^{1/2} \sin\left[\omega t + \tan^{-1}\left(\frac{\omega w_1}{w_0^2 - \omega^2}\right)\right] \\ y &= \xi_1 \end{aligned}\right\} \tag{2.14}$$

Asymptotic stabilization of the output ξ_1 to zero implies perfect, direct, tracking of the output voltage reference signal $z_{2d} = A \sin \omega t$, by the average output capacitor voltage z_2.

A static duty ratio synthesizer is readily obtained by equating the last differential equation in (2.14) to an appropriate linear expression in the transformed coordinates which ensures proper stable pole location in the complex plane. Proceeding as in the stabilization case of § 2.2, one obtains the following expression for the computed duty ratio function μ in transformed state coordinates:

$$\begin{aligned} \hat{\mu} &= \left(\frac{w_0^2 - \omega_n^2}{bw_0}\right)\xi_1 + \left(\frac{w_1 - 2\zeta\omega_n}{bw_0}\right)\xi_2 \\ &\quad + \frac{A}{bw_0}[(\omega w_1)^2 + (\omega^2 - w_0^2)^2]^{1/2} \sin\left[\omega t + \tan^{-1}\left(\frac{\omega w_1}{w_0^2 - \omega^2}\right)\right] \end{aligned} \tag{2.15}$$

In original coordinates, the controller is given by

$$\hat{\mu} = \left(\frac{w_1 - 2\zeta\omega_n}{b}\right)z_1 + \left[\frac{w_0^2 - w_n^2 - w_1(w_1 - 2\zeta\omega_n)}{bw_0}\right]z_2$$
$$+ \frac{A}{bw_0}[(2\zeta\omega_n)^2 + (\omega_n^2 - \omega^2)^2]^{1/2}\sin\left(\omega t + \tan^{-1}\frac{2\zeta\omega_n\omega}{w_n^2 - \omega^2}\right) \quad (2.16)$$

The actual duty ratio function μ is obtained via appropriate bounding of the computed duty ratio function (2.16), as expressed in (2.11).

After stabilization of ξ_1 and ξ_2 to zero, the corresponding steady state value of the synthesized duty ratio function $\hat{\mu}$ is obtained from (2.15) as

$$\hat{\mu} = \frac{A}{w_0 b}[(\omega^2 - w_0^2)^2 + (\omega w_1)^2]^{1/2}\sin\omega t + \tan^{-1}\left(\frac{\omega w_1}{\omega_0^2 - \omega^2}\right)\Bigg] \quad (2.17)$$

It should be clear that perfect tracking of the desired output voltage is achieved whenever the computed duty ratio function $\hat{\mu}$ does not saturate beyond the bounding values prescribed by the interval $[-1, 1]$, i.e. when the actual and the computed duty ratio functions coincide. In order to guarantee that, under steady state operating conditions, the amplitude of the synthesized duty ratio function $\hat{\mu}$ does not violate the bounding limits of the interval $[-1, 1]$, one imposes on the amplitude of the steady state sinusoidal wave given in (2.17), the following frequency-dependent magnitude restriction:

$$\frac{A}{w_0 b}[(\omega w_1)^2 + (\omega^2 - w_0^2)^2]^{1/2} < 1 \Leftrightarrow A < \frac{w_0 b}{[(\omega w_1)^2 + (\omega^2 - w_0^2)^2]^{1/2}} \quad (2.18)$$

The above 'tradeoff' relation only involves the FBBC parameters w_0, w_1, b and the defining parameters of the desired (template) reference sinusoidal output voltage amplitude A, and frequency ω.

Simulation example

Simulations were carried out on a FBBC with the same parameter values as in the previous simulation example, except for the fact that, here, V_s was taken as 300 V. The chosen damping ratio and natural oscillating frequency of the closed-loop system stable complex poles were set at $\zeta = 0{\cdot}7$ and $\omega_n = 300$ rad/s, respectively. The required normalized sinusoidal output voltage was specified as $z_{2d} = 120\sin(314t)$. Figures 7 and 8 depict the transient response of the average and the discontinuously PWM controlled input current and output voltage of the FBBC, respectively, in comparison with the corresponding AC reference signals. Figure 9 represents the evolution of the synthesized duty ratio feedback function μ, the steady state duty ratio signal and the corresponding discontinuous PWM control actions u.

3. Dynamical PWM feedback controller for stabilization and tracking in the full bridge buck converter

3.1. *Stabilization of the FBBC through dynamical PWM control*

Consider the average model (2.3) of the FBBC with input inductor current error as the output equation:

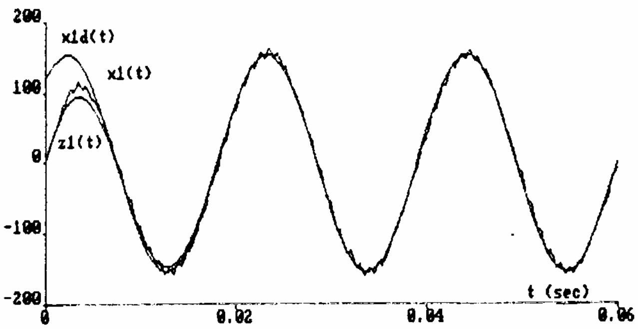

Figure 7. Average and actual input inductor current response for static PWM AC tracking task in a full bridge buck converter.

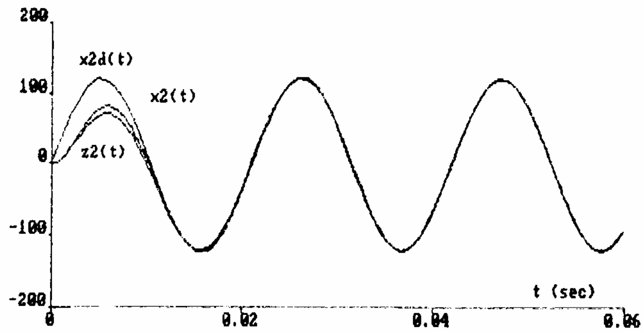

Figure 8. Average and actual output capacitor voltage response for static PWM AC tracking task in a full bridge buck converter.

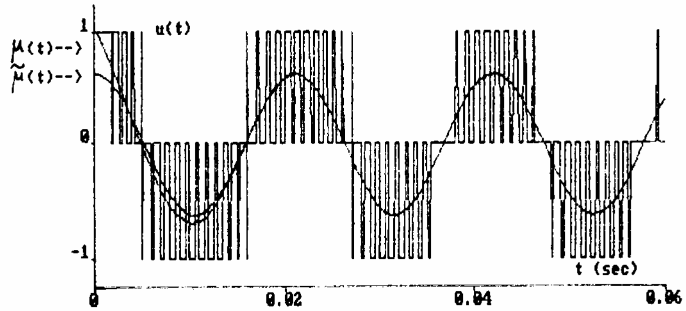

Figure 9. Duty ratio function and control input responses for static PWM AC tracking task in a full bridge buck converter.

$$\left.\begin{aligned} \dot{z}_2 &= -w_0 z_2 + \mu b \\ \dot{z}_2 &= w_0 z_1 - w_1 z_2 \\ y = z_1 - Z_1 &= z_1 - \frac{w_1}{w_0} Z_2 \end{aligned}\right\} \qquad (3.1)$$

The following invertible duty-ratio dependent state coordinate transformation,

$$\left.\begin{aligned} \xi_1 &= z_1 - \frac{w_1}{w_0} Z_2, \quad \xi_2 = -w_0 z_2 + \mu b \\ z_1 &= \xi_1 + \frac{w_1}{w_0} Z_2, \quad z_2 = \frac{-\xi_2 + \mu b}{w_0} \end{aligned}\right\} \tag{3.2}$$

takes the average model (3.1) into Fliess' GOCF (Fliess 1990):

$$\left.\begin{aligned} \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= -w_0^2 \xi_1 - w_0 w_1 Z_2 - w_1 \xi_2 + b w_1 \mu + \dot{\mu} b \\ y &= \xi_1 \end{aligned}\right\} \tag{3.3}$$

Notice that if a feedback control is specified that asymptotically stabilizes the transformed coordinates ξ_1 and ξ_2 to zero then, according to (3.2) one obtains asymptotic stabilization of the original average state coordinates z_1 and z_2 to their equilibrium values Z_1 and Z_2, respectively.

The zero dynamics, associated with the steady state equilibrium condition, $\xi_1 = 0$, $\xi_2 = 0$, is obtained by equating the last differential equation in (3.8) to zero (see Fliess 1990 b),

$$\dot{\mu} = -w_1\left(\mu - \frac{w_0}{b} Z_2\right) = -w_1(\mu - U) \tag{3.4}$$

which evidently implies an asymptotically stable solution towards $\mu = U$, i.e. the achieved equilibrium corresponds to a minimum phase behaviour.

An unrestricted dynamical feedback controller specifying the required (computed) stabilizing duty ratio function μ may be obtained from (3.3):

$$\begin{aligned} \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= -w_0^2 \xi_1 - w_0 w_1 Z_1 - w_1 \xi_2 + b w_1 \mu + \dot{\mu} b \\ &= -2\zeta\omega_n \xi_2 - \omega_n^2 \xi_1 \\ y &= \xi_1 \end{aligned}$$

that is

$$\dot{\mu} = -w_1 \mu + \left(\frac{w_1 - 2\zeta\omega_n}{b}\right)\xi_2 + \left(\frac{w_0^2 - \omega_n^2}{b}\right)\xi_1 + \frac{w_0 w_1}{b} Z_2 \tag{3.5}$$

As before, we denote the solution of (3.5) as $\hat{\mu}$, and regard it as the computed duty ratio function.

The dynamical duty ratio synthesizer (3.5), expressed in transformed coordinates, is readily interpreted as a classical proportional-derivative controller, followed by a low pass filter, with cut off frequency w_1, and a set-point feedforward term. Figure 10 depicts such an interpretation of the proposed controller (see also Sira-Ramirez (1991 b) and Sira-Ramirez and Lischinsky-Arenas (1993)).

In original average coordinates, one obtains the following dynamical feedback

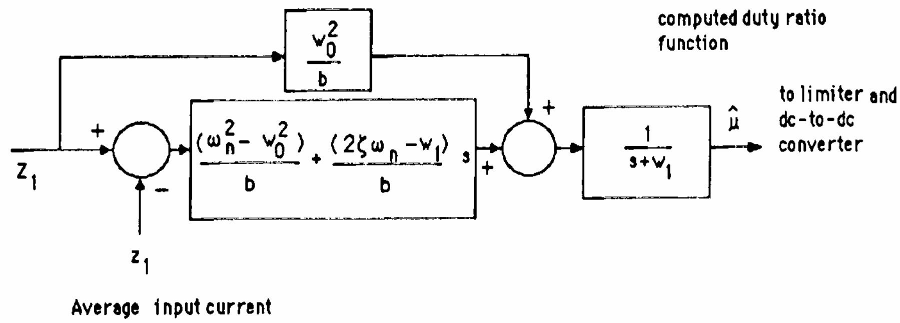

Figure 10. Classical P-D compensator interpretation of average dynamical duty ratio synthesizer in a PWM controlled full bridge buck converter.

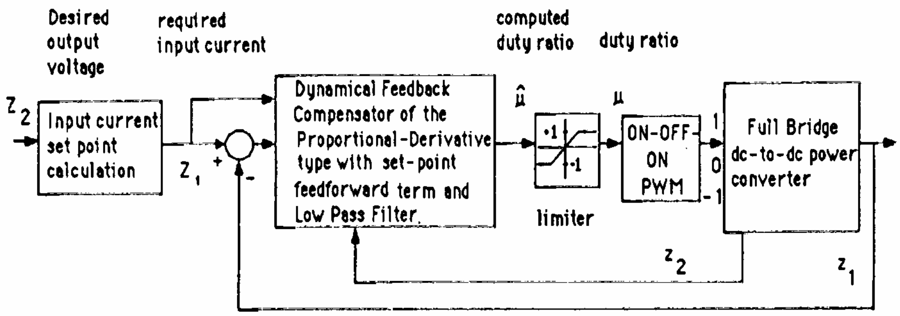

Figure 11. Dynamical PWM feedback compensator structure for indirect output stabilization and tracking tasks in a full bridge buck converter.

controller:

$$\frac{d}{dt}\hat{\mu} = -2\zeta\omega_n\hat{\mu} + \left(\frac{w_0^2 - \omega_n^2}{b}\right)z_1 + \frac{(2\zeta\omega_n - w_1)}{b}w_0 z_2 + \frac{\omega_n^2 w_1}{b w_0}Z_2 \tag{3.6}$$

The actual duty ratio function μ is simply obtained by bounding the solutions of (3.6) within the physically meaningful interval $[-1, 1]$, as was previously done in (2.11).

Figure 11 depicts the dynamical PWM controller scheme accomplishing asymptotic stabilization to a desirable average equilibrium point for the FBBC.

Simulation example

Simulations were carried on a FBBC with the same parameter values, the same designed pole locations and sampling frequency used in the example in § 2. Figures 12 and 13 depict the transient response of the average and the discontinuously PWM controlled input current and output voltage of the FBBC, respectively. Figure 14 represents the evolution of the dynamically synthesized duty ratio function μ and the corresponding discontinuous PWM control actions u.

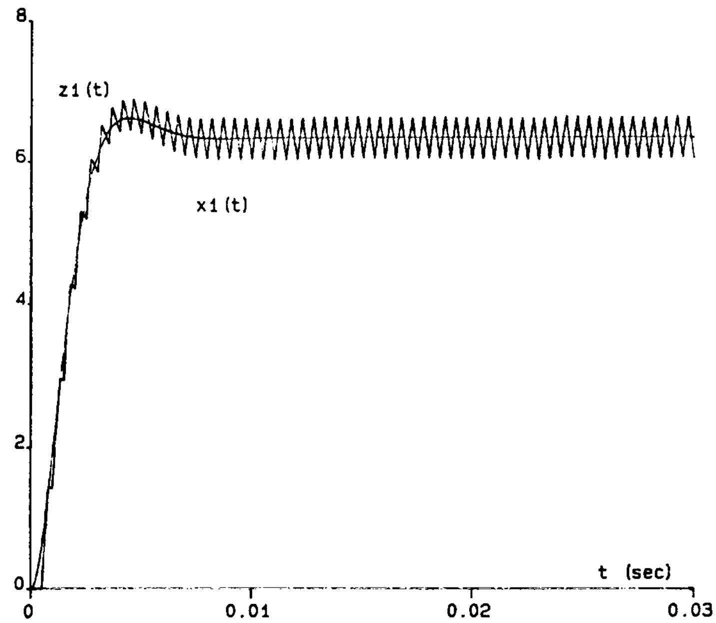

Figure 12. Average and actual input inductor current response for a dynamical PWM stabilization of a full bridge buck converter.

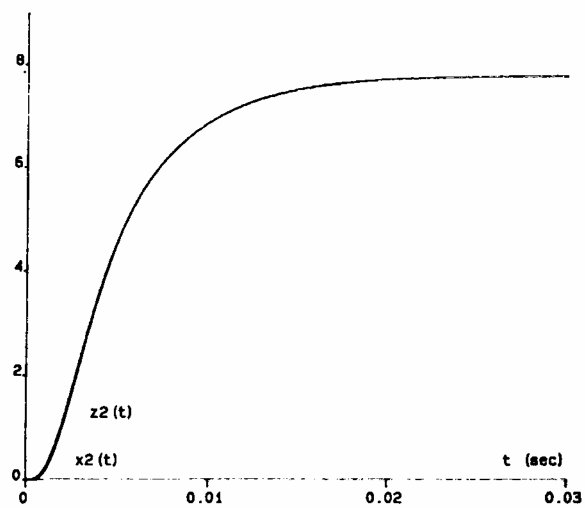

Figure 13. Average and actual output capacitor voltage response for a dynamical PWM stabilization of a full bridge buck converter.

3.3. *Sinusoidal signal generation of the FBBC through dynamial feedback PWM control*

In this section, we propose the use of partial (state) inversion and dynamical PWM feedback control for the adequate tracking of sinusoidal template signals by the output capacitor voltage in a FBBC. Partial inversion is referred to the indirect tracking policy by which the average input inductor current is made to follow a

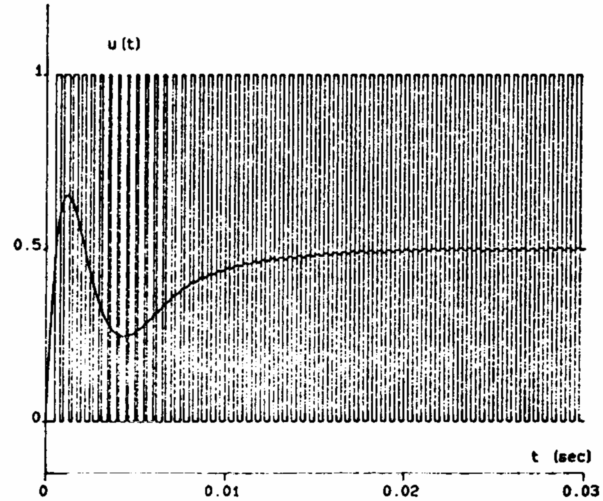

Figure 14. Duty ratio function and control input responses in dynamical PWM stabilization of a full bridge buck converter.

periodical reference signal which, in turn, corresponds to a perfect sinusoidal voltage at the output capacitor terminals in the FBBC. The natural limitations imposed on the synthesized duty ratio function imply some template amplitude-frequency tradeoffs which can be explicitly computed in terms of the system parameters.

Suppose the average normalized output capacitor voltage z_2 is to track a (template) sinusoidal signal of the form

$$z_{2d}(t) = A \sin(\omega t) \tag{3.7}$$

Inversion of the average system dynamics (3.1) leads to the corresponding required reference signal for the average input inductor current z_1,

$$z_{1d}(t) = \frac{A}{w_0}(\omega^2 + w_1^2)^{1/2} \sin\left(\omega t + \tan^{-1}\frac{\omega}{w_1}\right) \tag{3.8}$$

The following invertible time-varying input-dependent state coordinate transformation,

$$\left.\begin{aligned} \xi_1(t) &= z_1 - \frac{A}{w_0}(\omega^2 + w_1^2)^{1/2} \sin\left(\omega t + \tan^{-1}\frac{\omega}{w_1}\right) \\ \xi_2(t) &= -w_0 z_2 + \mu b - \frac{A\omega}{w_0}(\omega^2 + w_1^2)^{1/2} \cos\left(\omega t + \tan^{-1}\frac{\omega}{w_1}\right) \end{aligned}\right\} \tag{3.9}$$

places the average FBBC model into Fliess' GOCF,

$$\left.\begin{aligned} \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= -w_0^2\xi_1 - w_1\xi_2 + \mu b w_1 + \dot{\mu} b - \frac{A}{w_0}[(\omega^2 + w_1^2)[(\omega w_1)^2 + (\omega^2 - w_0^2)^2]^{1/2} \\ &\qquad \times \sin\left[\omega t + \tan^{-1}\left(\frac{\omega}{w_1}\right) + \tan^{-1}\left(\frac{\omega w_1}{w_0^2 - \omega^2}\right)\right] \\ y &= \xi_1 \end{aligned}\right\} \tag{3.10}$$

Asymptotic stabilization of the output ξ_1 to zero implies perfect tracking of the input current reference signal z_{1d} by the average input inductor current z_1 and, hence, indirect perfect tracking of the desired AC signal, $z_{2d} = A \sin \omega t$, by the average output capacitor voltage z_2.

The zero dynamics associated with system (3.16) is simply given by

$$\dot{\mu} = -w_1 \mu + \frac{A}{w_0 b} \{(\omega^2 + w_1^2)[(\omega w_1)^2 + (\omega^2 - w_0^2)^2]\}^{1/2} \times \sin\left[\omega t + \tan^{-1}\left(\frac{\omega}{w_1}\right) + \tan^{-1}\left(\frac{\omega w_1}{w_0^2 - \omega^2}\right)\right] \quad (3.11)$$

After some tedious but straightforward manipulations, one finds the steady state solution of the forced linear differential equation (3.11) as

$$\tilde{\mu}(t) = \frac{A}{w_0 b} [(\omega w_1)^2 + (\omega^2 - w_0^2)^2]^{1/2} \sin\left[\omega t + \tan^{-1}\left(\frac{\omega w_1}{w_0^2 - \omega^2}\right)\right] \quad (3.12)$$

A dynamical duty ratio synthesizer is readily obtained by equating the last differential equation in (3.10) to an appropriate linear expression in the transformed coordinates which ensures proper stable pole location in the complex plane. Proceeding as in the stabilization case of § 3.1, one obtains the following time-varying differential equation for the computed duty ratio function $\hat{\mu}$ in original average state coordinates:

$$\frac{d}{dt}\hat{\mu} = -2\zeta\omega_n \hat{\mu} + \left(\frac{w_0^2 - \omega_n^2}{b}\right) z_1 + \left(\frac{2\zeta\omega_n - w_1}{b}\right) w_0 z_2 + \frac{A}{w_0 b}[(\omega^2 + w_1^2)[(\omega_n^2 - w_0^2)^2 + (2\zeta\omega_n\omega)^2]]^{1/2} \times \sin\left[\omega t + \tan^{-1}\left(\frac{\omega}{w_1}\right) + \tan^{-1}\left(\frac{2\zeta\omega_n\omega}{\omega_n^2 - w_0^2}\right)\right] \quad (3.13)$$

The actual duty ratio function μ is obtained via appropriate bounding of the solution of (3.13) as expressed in (2.11).

In order to guarantee that, under steady state operating conditions, the amplitude of the synthesized duty ratio function μ does not violate the bounding limits of the interval $[-1, 1]$, one imposes on the amplitude of the steady state sinusoidal wave given in (3.12), the following frequency-dependent magnitude restriction:

$$\frac{A}{w_0 b}[(\omega w_1)^2 + (\omega^2 - w_0^2)^2]^{1/2} < 1 \Leftrightarrow A < \frac{w_0 b}{[(\omega w_1)^2 + (\omega^2 - w_0^2)^2]^{1/2}} \quad (3.14)$$

The above 'tradeoff' relation is the same relation previously obtained for the static controller case, as expected.

Simulation example

Simulations were carried out on a FBBC with the same parameter values as in the previous simulation example, except for the fact that, here, V_s was taken as 300 V. The chosen damping ratio and natural oscillating frequency of the closed-loop system stable complex poles were set, respectively, at $\zeta = 0{\cdot}7$ and $\omega_n = 300$ rad/s. The

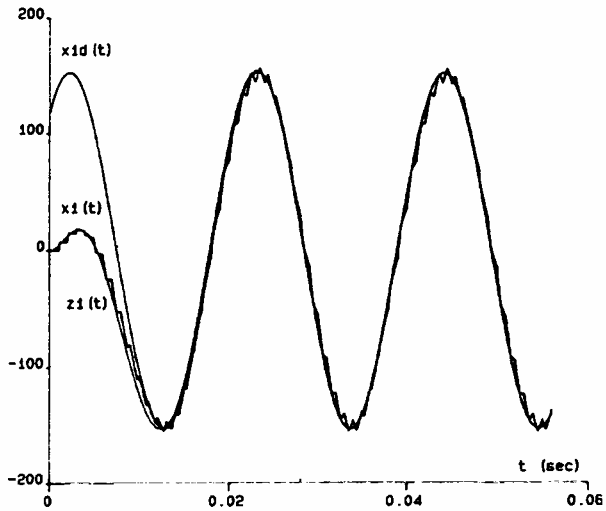

Figure 15. Average and actual input inductor current response for dynamical PWM AC tracking task in a full bridge buck converter.

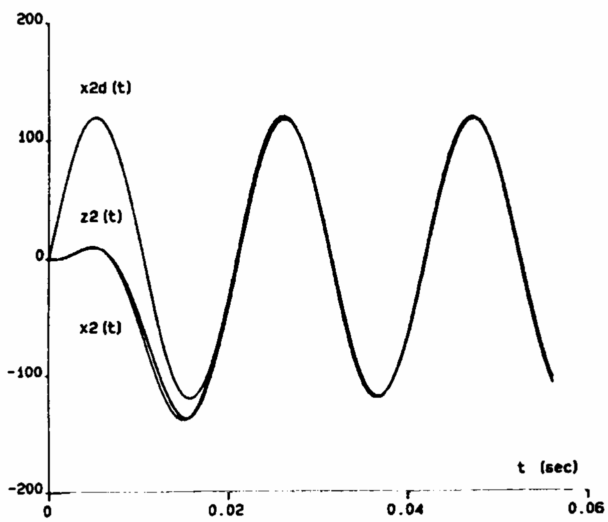

Figure 16. Average and actual output capacitor voltage response for dynamical PWM AC tracking task in a full bridge buck converter.

required normalized sinusoidal output voltage was specified as $z_{2d} = 120 \sin (314t)$. Figures 15 and 16 depict, the transient response of the average and the discontinuously PWM controlled input current and output voltage of the FBBC, respectively, in comparison with the corresponding AC reference signals. Figure 17 represents the evolution of the synthesized duty ratio feedback function μ, the steady state duty ratio signal and the corresponding discontinuous PWM control actions u.

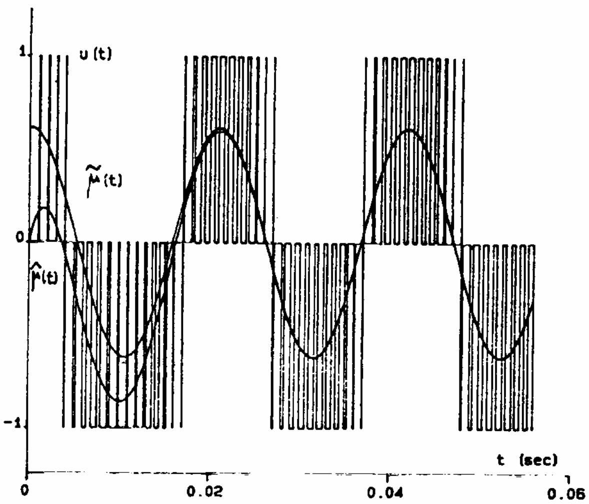

Figure 17. Duty ratio function and control input responses for dynamical PWM AC tracking task in a full bridge buck converter.

4. Conclusions

In this article, both static and dynamical PWM control schemes have been presented for stabilization and tracking problems in DC-to-DC power converters of the full bridge buck type. The static PWM feedback approach results in a P-D controller. This approach has been presented by several authors from slightly different viewpoints (see Boudjema *et al.* 1989 and the references therein). The dynamic approach is believed to be new, although it may be effectively interpreted as a classical P-D controller and low-pass filter arrangement. Our viewpoint, however, considers a unifying design resource represented by Fliess' generalized observability canonical form of the average PWM converter model for direct and indirect output voltage regulation strategies. Different average GOCFs are obtained by considering as the system's output variable either the output capacitor voltage or the input inductor current. In the first case, Fliess' GOCF coincides with the traditional controller canonical form and it leads to a traditional static state feedback regulation scheme interpretable as a P-D classical controller. The indirect feedback control synthesis approach is based on the specification of a dynamical, unrestricted, smooth feedback control law determining the required duty ratio function for the regulation of the infinite frequency average model of the PWM controlled converter. Such a feedback scheme corresponds to a dynamical pole placement approach which imposes a desirable stable dynamics on Fliess' generalized observability canonical form of the error dynamics associated with the average stabilization or tracking problem defined on the bridge converter. Aside from the high response quality of the controlled variables, the proposed dynamical approach also allows for an explicit determination of the stabilization and tracking limitations exhibited by the treated class of 'step down' converters. In the tracking task, where sinusoidal reference signal tracking is demanded from the output capacitor voltage of the converter, the amplitude-frequency tradeoffs, inherent in the physically meaningful limitations of the duty ratio function, were explicitly computed.

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