Dynamical Discontinuous Feedback Strategies in the Regulation of Nonlinear Chemical Processes

Hebertt Sira-Ramírez and Orestes Llanes-Santiago

Abstract— **In this article, a unified approach is proposed for the design of dynamical discontinuous feedback controllers leading to the chattering-free stabilization of nonlinear single input single output systems describing chemical processes. The adopted framework is that of a Generalized State Representation form of the given nonlinear plant. Use is made of the associated Generalized Observability Canonical Form of such representation. Unification of discontinuous feedback policies is achieved by zeroing of an input-dependent auxiliary output function using simple discontinuous feedback control paradigms of various kinds. The zeroing of such scalar stabilizing function induces asymptotically stable controlled dynamics on the given nonlinear minimum-phase plant. Pulse-Frequency-Modulation, Pulse-Width-Modulation and Sampled Sliding Mode control strategies are considered from this unified viewpoint. Examples are provided including simulations.**

Index Terms— **Pulse-Frequency-Modulation, Pulse-Width-Modulation ; Sampled Sliding Mode Control, Chemical Processes**

I. Introduction

RECENTLY, RESULTS FROM THE differential algebraic approach to control theory,pioneered by Prof. M. Fliess [1], [2] have greatly improved the applicability of discontinuous feedback strategies, especially those of the sliding mode (SM) type, leading to asymptotic stabilization, and tracking, in nonlinear systems (see Sira-Ramírez [3] and Sira-Ramírez *et al* [4] for applications to mechanical and electromechanical systems). Some of the traditional disadvantages of sliding and sampled sliding mode control policies are fundamentally related to the "bang-bang" character of the input signals and the associated "chattering" of output and state variables response signals (Utkin [5]). These difficulties are circumvented via dynamical sliding mode controllers while retaining the outstanding robustness, and simplicity, of this class of feedback control schemes.

In this article, Fliess's Generalized Observability Canonical Form (GOCF)is shown to naturally allow for dynamical feedback controller design based on Pulse-Frequency Modulation (PFM) strategies, Pulse-Width-Modulation (PWM) policies and Sampled Sliding Modes (SSM). The obtained control input signals are substantially smoothed with respect to their corresponding static alternative and, hence, chattering-free discontinuously controlled responses are generated. The obtained PFM and PWM controller designs do not resort to traditional approximation schemes, based on (infinite frequency) average models of the discontinuously controlled systems (Sira-Ramírez [6]). These features are particularly important in chemical process control tasks in which discontinuities, or "jumps", cannot be simply allowed on the actuator behaviour, and where fast vibrations of the regulated variables are usually not tolerated, due to their negative effect on the quality specifications of the final product. A sufficiently "smooth" control policy is, therefore, usually desirable while a need, definitely, exists for certain degree of robustness, and precision, of the proposed feedback control scheme.

The synthesis of several dynamical discontinuous regulators, here proposed,is based on Fliess's GOCF for nonlinear single-input single-output systems ([1]). In section II of this article, we briefly address the dynamical Sliding Mode (SM) control solution of the output stabilization problem and present the PFM, PWM and SSM controller design schemes. In section III, we present some illustrative examples, along with encouraging simulations. The first example, taken from Kravaris and Palanki [7], is concerned with the regulation of total concentration control in an isothermal Continuously Stirred Tank Reactor (CSTR). In the second example, a discontinuous feedback control regulator is designed for the stabilization of the output concentration of a certain chemical agent, in a Double Effect Evaporator (DEE) system, extensively used in the Food and Paper industries among others. In both examples, simulations are provided which depict the advantageous features of dynamical discontinuous controls. Concluding remarks are collected at the end of the article.

II. Dynamical Discontinous Feedback Control of Nonlinear Systems

The results of this section may be extended to tracking problems([3], [4]) and to multivariable cases (see Sira-Ramírez and Llanes-Santiago [8]).

Fliess's Generalized Observability Canonical Form

Let $u^{(i)}$ stand for the i-th time derivative of theinput function $u(t)$. We denote $u^{(0)}$ simply by u, while the first time derivative of $u(t)$ is indistinctively represented by $u^{(1)}$ or $\dot{u}$.

This research was supported by the Consejo de Desarrollo Científico, Humanístico and Tecnológico of the Universidad de Los Andes under Research Grant I-358-91.

Hebertt Sira-Ramírez: Departamento Sistemas de Control Universidad de Los Andes Mérida 5101, Venezuela.

Orestes Llanes-Santiago: Postgrado de Ingeniería de Control Automático Facultad de Ingeniería Universidad de Los Andes Mérida 5101, Venezuela.

Manuscript received January 6, 1993; revised manuscript received June 21, 1993.

This paper was recommended by the Associate Editor E. King.

IEEE Log number 9215861.

1063-6536/94$04.00

It has been shown in [1] (see also Conte *et al* [9])that a nonlinear, single-input single-output n-dimensional analytic system of the form:

$$\dot{x} = f(x,u) \qquad y = h(x) \tag{1}$$

can be locally transformed, via an input-dependent state coordinate transformation of the form

$$z = \Phi(x, u, \dot{u}, \ldots, u^{(\alpha-1)}) \tag{2}$$

with $z \in R^n$, into an n-dimensional system of the form

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ &\cdots \\ \dot{z}_n &= c(z, u, \dot{u}, \ldots, u^{(\alpha)}) \\ y &= z_1 \end{aligned} \tag{3}$$

provided the following *observability* matrix of the system

$$\begin{bmatrix} \frac{\partial h(x)}{\partial x} \\ \frac{\partial h^{(1)}(x)}{\partial x} \\ \cdots \\ \frac{\partial h^{(n-1)}(x,u,\ldots,u^{(\alpha-1)})}{\partial x} \end{bmatrix} \tag{4}$$

is full rank n, where $h^{(i)}$ stands for the *total* i-th time derivative of the output function $h(x(t))$. We also used h for $h^{(0)}$ and $h^{(1)}$ for the first time derivative of $h(x(t))$. In (3), the integer α is assumed to be a strictly positive integer, i.e, the system is not necessarily *relative degree n*, and therefore it is not exactly linearizable by means of *static* state feedback (see Isidori [10]). Extension of the results presented here to systems of this class is found in Sira-Ramírez and Llanes-Santiago [11].

It should be remarked, however, that, in general, (3) is not,necessarily, n-dimensional. Our assumption, thus, corresponds to one of a *minimal realization* on (1).

The state coordinate transformation (2) is evidently given by thelocal diffeomorphism:

$$z = \Phi(x, u, \dot{u}, \ldots, u^{(\alpha-1)}) = \begin{bmatrix} h(x) \\ h^{(1)}(x) \\ \cdots \\ h^{(n-1)}(x, u, \dot{u}, \ldots, u^{(\alpha-1)}) \end{bmatrix} \tag{5}$$

Suppose $u = U$, $x = X(U)$ describe a constant equilibrium point for the original system (1), such that $h(X(U))$ is zero. Then, $z = 0$ is an equilibrium point for (3). The autonomous dynamics described by:

$$c(\mathbf{0}, u, \dot{u}, \ldots, u^{(\alpha)}) = 0 \tag{6}$$

is the *zero dynamics* (see Fliess [12]). The stability nature of an equilibrium point $u = U$ of (6) determines the *minimum* or *nonminimum phase* character of the system (1). The equilibrium point for (1) entitles a constant input signal u given by, $u = U$, while the corresponding equilibrium value of the state vector, x, is denoted by $x(U) = X(U)$ and the resulting output signal y is assumed to have a constant equilibrium value equals to zero, i.e. $y(U) = Y(U) = 0$. We briefly denote the constant equilibrium point for system (1) $(X(U), U, Y(U))$ as $(X(U), U, 0)$.

A GOCF Approach to Dynamical Discontinuous FeedbackController Design for Nonlinear Systems

Consider the following auxiliary output function $s : R^n \rightarrow R$,defined in terms of the transformed variable z,

$$s(z) = \left(\sum_{i=1}^{n-1} \gamma_i \, z_i\right) + z_n \tag{7}$$

such that the following corresponding polynomial in the complex variable λ is Hurwitz:

$$\left(\sum_{i=1}^{n-1} \gamma_i \, \lambda^{i-1}\right) + \lambda^{n-1} \tag{8}$$

Suppose that the system is locally minimum phase around $(X(U), U, 0)$. It is easy to see that if (7) is forcefully constrained to zero (whether in finite time, or in an asymptotically stable fashion) by means of appropriate control actions (possibly of discontinuous nature), the resulting controlled dynamics locally evolves in accordance with:

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ &\cdots \\ \dot{z}_{n-1} &= -\sum_{i=1}^{n-1} \gamma_i \, z_i \\ y &= z_1 \end{aligned} \tag{9}$$

which is asymptotically stable to zero.

Two of the dynamical discontinuous feedback controller design schemes,here proposed, rely on inducing on (3) an asymptotically stable linear time invariant controlled dynamics such as (9), with eigenvalues placeable at will. This is done by driving the proposed auxiliary output function $s(z)$ to zero. As it will be shown, SM controllers can always accomplish such a task in *finite time*, PFM and PWM controllers, on the other hand, can only accomplish this task in an asymptotically stable fashion, while SSM control can only do it approximately.

Dynamical Sliding Mode Control of Nonlinear Systems

proposition 2.1 Let W be a strictly positive scalar quantity, and let "sign"stand for the *signum* function. The one-dimensional discontinuous system:

$$\frac{ds}{dt} = \dot{s} = -W \text{ sign } s \tag{10}$$

globally exhibits a sliding regime on $s = 0$. Furthermore, anytrajectory starting on the value $s = s(0)$, at time 0, reaches,the condition $s = 0$ in finite time T, given by $T = W^{-1} \, |s(0)|$.

Proof: Immediate upon checking that globally : $s ds/dt < 0$, which is a well known condition for sliding mode existence [5]. The second part follows trivially from the fact that

$$|s(t)| = -W\,t + |s(0)| \quad \text{for} \quad 0 \le t < W^{-1}\,|s(0)|$$

Proposition 2.2 A minimum phase nonlinear system of the form (1) is locallyasymptotically stabilizable to the equilibrium point $(X(U), U, 0)$ if the control action u is specified as a dynamical SM control policy given by the solution of the following implicit, time-varying, nonlinear discontinuous differential equation:

$$c(z, u, \dot{u}, \ldots, u^{\alpha}) = -\sum_{i=1}^{n} \gamma_i\, z_i - W \,\mathrm{sign}\left[\left(\sum_{i=1}^{n-1} \gamma_i\, z_i\right) + z_n\right] \quad (11)$$

where $\gamma_0 = 0$.

Proof: Immediate upon imposing on the auxiliary output functions(z), defined in (7), the dynamics defined by (10).

Due to the implicit character of the differential equation definingu in (11), singularities, known as *impasse points*, may locally arise in those regions where $u^{(\alpha)}$ cannot be explicitly solved for. We, therefore, assume, according to the *Implicit Function Theorem* (see [10]) that in (11) the quantity $\partial c/\partial u^{(\alpha)}$ is locally nonzero and, hence, a unique solution exists for the highest control input derivative $u^{(\alpha)}$ from (11). Under such circumstances, no such singularities need be considered. For more details on ways to avoid such singularities through suitable discontinuities, the reader is referred to Abu el Ata-Dos *et al* [13].

The controller (11) is easily represented in terms of theoriginal state space coordinates x by using the input-dependent state coordinate transformation (5).

Dynamical PFM Control of Nonlinear Systems Consider the scalar PFM controlled dynamical system, in which the constantsr_1, r_2, r_3 and W, are all strictly positive quantities.

$$\dot{s} = -W\,\nu \quad (12)$$

$$\nu = \mathrm{PFM}_{\tau,T}(s(t)) = \begin{cases} \mathrm{sign}\; s(t_k) & \text{for } t_k \le t < t_k + \tau[s(t_k)]\,T[s(t_k)] \\ 0 & \text{for } t_k + \tau[s(t_k)]\,T[s(t_k)] \le t < t_k + T[s(t_k)] \end{cases}$$

$$\tau[s(t)] = \begin{cases} 1 & \text{for } |s(t)| > \frac{1}{r_1} \\ r_1\,|s(t)| & \text{for } |s(t)| \le \frac{1}{r_1} \end{cases}$$

$$T[s(t)] = \begin{cases} T_{max} & \text{for } |s(t)| \ge \frac{1}{r_2} \\ T_{min} + \frac{r_2 r_3}{r_3 - r_2} \cdot [T_{max} - T_{min}]\left(s(t) - \frac{1}{r_3}\right) & \text{for } \frac{1}{r_3} < |s(t)| < \frac{1}{r_2} \\ T_{min} & \text{for } |s(t)| \le \frac{1}{r_3} \end{cases}$$

$$k = 0, 1, 2, \ldots \;;\; t_{k+1} = t_k + T[s(t_k)]$$

where it is assumed that $r_2 < r_1 < r_3$. The t_k's represent *irregularly* spaced sampling instants, determined by the sampled values of the *duty cycle* function, denoted by $T[s(t_k)]$. The duty cycle function , $T[s(t_k)]$, takes values on the closed interval $[T_{min}, T_{max}]$ and it varies linearly with respect to $s(t)$ in the region $1/r_3 <: |s| < 1/r_2$. The duty cycle, or sampling period, saturates to T_{max} for large values of s, and remains fixed at the constant lower bound T_{min} for small values of s (see fig. 1). At each sampling instant, t_k, the width of the sign-modulated, unit amplitude, control pulse is determined by the value of the *duty ratio* function, represented by $\tau[s(t_k)]$. The duty ratio function is such that the width of the pulse saturates to 100% of the sampling interval when the absolute value of the variable s exceeds some threshold value given by $1/r_1$. Below such a threshold value, the width of the pulses decrease towards zero linearly with respect to $|s|$ (see fig. 2). The duty cycle and the duty ratio functions may be quite independent of each other. The function "sign", in (12), stands, again, for the *signum function*.

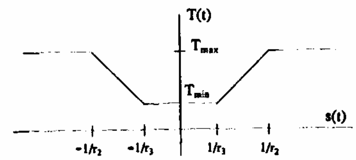


Fig. 1. Duty cycle function for scalar PFM controlled system.

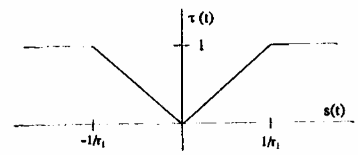


Fig. 2. Duty ratio function for scalar PFM controlled system.

The following proposition establishes a sufficient condition for theasymptotic stability to zero of the PFM controlled system (12).

Proposition 2.3 The PFM controlled system (12) is asymptotically stable to$s = 0$, if

$$0 < r_3\, W\, T_{max} < 2 \quad (13)$$

Proof: Due to the piecewise nature of the control inputs and thelinearity of the continuous system, it suffices to study the stability of the discretized version of (12) at the sampling instants. An exact discretization of the PFM controlled system (12) yields:

$$s(t_k + T) = s(t_k) - W\,\mathrm{sign}\,[s(t_k)]\,\tau[s(t_k)]\,T[s(t_k)] \quad (14)$$

The stability of (13) follows easily from Lyapunov type of arguments as applied to each one of the possible regions involved in (12). Condition (12) is easily seen to be sufficient to guarantee, both, the uniform decrease of $s(t_k)$ and the absence of limit cycles in the underlying sampled system. For more details of the proof of this proposition the reader is referred to Sira-Ramírez [14] and Sira-Ramírez and Llanes-Santiago [15].

Proposition 2.4 A minimum phase nonlinear system of the form (1) is locallyasymptotically stabilizable to the equilibrium point $(X(U), U, 0)$ if the control action u is specified as a dynamical PFM control policy given by the solution of the following implicit, time-varying, nonlinear discontinuous differential equation :

$$c(z,u,\dot{u},\dots,u^{(\alpha)}) = -\sum_{i=1}^{n}\gamma_{i-1}\, z_i - W\ \mathrm{PFM}_{\tau,T}\left[\left(\sum_{i=1}^{n-1}\gamma_i\, z_i\right)+z_n\right] \tag{15}$$

where $\gamma_0 = 0$.

Proof: Immediate upon imposing on the auxiliary output functions(z) in (7) the asymptotically stable discontinuous PFM dynamics defined in (12).

Dynamical PWM Control of Nonlinear Systems Consider the scalar PWM controlled system, in which $r > 0$ and$W > 0$:

$$\begin{aligned}
\dot{s} &= -W\,\nu \qquad (16)\\
\nu &= \mathrm{PWM}_\tau(s(t))\\
&= \begin{cases}\mathrm{sign}\ s(t_k) & \text{for } t_k \le t < t_k + \tau[s(t_k)]\,T\\ 0 & \text{for } t_k + \tau[s(t_k)]\,T \le t < t_k + T\end{cases}\\
\tau[s(t)] &= \begin{cases} 1 & \text{for } |s(t)| > \frac{1}{r}\\ r\,|s(t)| & \text{for } |s(t)| \le \frac{1}{r}\end{cases}\\
k &= 0,1,2,\dots\ ;\ t_{k+1} = t_k + T
\end{aligned}$$

It is easy to see that (15) is just a particular case of the PFMcontrolled system (12) in which the duty cycle function $T[s(t_k)]$ is now taken as a constant of value T for all values of $s(t_k)$. The following result follows immediately form this fact.

Proposition 2.5 The PWM controlled system (15) is asymptotically stable to$s = 0$ if, and only if, :

$$0 < rWT < 2 \tag{17}$$

Proof: Sufficiency is clear form the preceeding proposition. Necessity followsfrom the fact that for convergence of the trajectories of system (15) to zero, and in order to avoid a possible limit cycle, there must exist an instant of time t_k such that, independently of the initial condition, $s(t_k)$ lies in the region $|s(t_k)| \le 1/r$. In this region, the PWM controlled dynamics adopts the form $s(t_{k+1}) = (1 - rWT)s(t_k)$. The result follows (see also Sira-Ramírez [16]).

Proposition 2.6 A minimum phase nonlinear system of the form (1) is locallyasymptotically stabilizable to the equilibrium point $(X(U), U, 0)$ if the control action u is specified as a dynamical PWM control policy given by the solution of the following implicit, time-varying, nonlinear discontinuous differential equation

$$c(z,u,\dot{u},\dots,u^{(\alpha)}) = -\sum_{i=1}^{n}\gamma_{i-1}\, z_i - W\ \mathrm{PWM}_{\tau}\left[\left(\sum_{i=1}^{n-1}\gamma_i\, z_i\right)+z_n\right] \tag{18}$$

where $\gamma_0 = 0$.

Proof: Immediate upon imposing on the auxiliary output function $s(z)$ in(7) the asymptotically stable discontinuous dynamics defined in (15).

Dynamical SSM Control of Nonlinear Systems

Proposition 2.7 Consider the following one-dimensional SSM controlled system:

$$\begin{aligned}
\dot{s} &= -W\,\nu \qquad (19)\\
\nu &= \mathrm{SSM}(s(t)) = \mathrm{sign}\ s(t_k) \quad \text{for } t_k \le t < t_k + T\\
k &= 0,1,2,\dots\ ;\ t_{k+1} = t_k + T
\end{aligned}$$

Then, given an $\epsilon > 0$, there exists a sampling interval $T(\epsilon) = \epsilon/W$ for which the trajectories of (17) satisfy the condition

$$|s(t)| \le 2\epsilon\ \ \forall\ t > W^{-1}\,|s(0)| = \frac{T(\epsilon)}{\epsilon}\,|s(0)|$$

Proof: The proof is immediate from the exact discretization of (17):

$$s(t_k + T) = s(t_k) - W\,T\ \mathrm{sign}\,[s(t_k)]$$

hence,

$$|s(t_k + T) - s(t_k)| = WT$$

The first part follows by letting $WT = \epsilon$. The second part is immediate from the linearity of the system and the fact that for all $t \ge 0$, $|\mathrm{d}s/\mathrm{d}t| = W$.

Chattering of s around the value $s = 0$, can be made ofarbitrarily small amplitude according to the width of the sampling interval $T(\epsilon)$. As $T(\epsilon) \to 0$, the response to a SSM strategy asymptotically converges to the response of a SM policy.

Proposition 2.8 A minimum phase nonlinear system of the form (1) is locallystabilizable around the equilibrium point $(X(U), U, 0)$, modulo a small chattering, if the control action u is specified as a dynamical SSM control policy given by the solution of the following implicit, time-varying, nonlinear discontinuous differential equation:

$$c(z,u,\dot{u},\dots,u^{(\alpha)}) = -\sum_{i=1}^{n}\gamma_{i-1}\, z_i - W\ \mathrm{SSM}\left[\left(\sum_{i=1}^{n-1}\gamma_i\, z_i\right)+z_n\right] \tag{20}$$

Proof: Immediate upon imposing on the auxiliary output function $s(z)$in (7) the discontinuous dynamics defined by (17).

A SSM control policy may also be viewed as a particularcase of a PWM control policy in which the pulse width $\tau[s(t_k)]\ T$ is saturated to the constant value of the sampling interval T (i.e. the duty ratio, $\tau[s(t_k)]$, is equal to 1 for all values of $s(t_k)$).

III. Some Application Examples

Dynamical PFM, PWM and SSM Control for Regulation ofTotal Concentration in a Continuously Stirred Tank Reactor

Consider the following simple nonlinear dynamical model of a controlledCSTR in which an isothermal, liquid-phase, multi-component chemical reaction takes place (see [7]):

$$\begin{aligned} \dot{x}_1 &= -(1+D_{a1})x_1 + u \\ \dot{x}_2 &= D_{a1}x_1 - x_2 - D_{a2}x_2^2 \\ y &= x_1 + x_2 - C \end{aligned} \tag{21}$$

where x_1 represents the normalized (dimensionless) concentration C_P/C_{P0} of a certain species P in the reactor, with $C = C_{P0}$ being the desired total concentration of the species P and Q measured in mol.m^{-3}. The state variable x_2 represents the normalized concentration $C_Q/CP0$ of specied Q. The control variable u is defined as the ratio of the per-unit volumetric molar feed rate of species P, denoted by N_{PF}, and the desired concentration C_{P0}, i.e. $u = N_{PF}/(FC_{P0})$ where F is the volumetric feed rate in $\text{m}^3\text{sec}^{-1}$. The constants D_{a1} and D_{a2} are respectively defined as k_1V/F and k_2VC_{P0}/F with V being the volume of the reactor in m^3, and k_1 and k_2 are first order rate constants expressed in sec^{-1}.

It is assumed that the species Q is highly acidic while the reactantspecies R is neutral. In order to avoid corrosion problems in the downstream equipment, it is desired to regulate the total concentration error y towards zero, so that the total concentration value $x_1 + x_2$ converges to a prescribed set-point value, specified by the constant C. It is also assumed that the control variable u is naturally bounded in the closed interval $[0, U_{max}]$ reflecting the bounded (physical) limits of the molar feed rate of the species P. The numerical value of U_{max} is a given process parameter whose knowledge is considered to be critical, even if not violated by the resulting numerical values of the computed control input strategy.

It is easy to verify that for the given system (19), the rankof the following 2 by 2 matrix:

$$S = \begin{bmatrix} \frac{\partial y}{\partial x} \\ \frac{\partial \dot{y}}{\partial x} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & -(1+2D_{a2}x_2) \end{bmatrix} \tag{22}$$

is everywhere equals to 2, except on the line $x_2 = 0$, which is devoid of practical significance.

A stable constant equilibrium point for this system is given by:

$$\begin{aligned} u &= U \\ x_1(U) &= X_1(U) = \frac{U}{(1+D_{a1})} \\ x_2(U) &= X_2(U) = \frac{1}{2D_{a2}}\left[-1 + \sqrt{1 + \frac{4D_{a1}D_{a2}U}{(1+D_{a1})}}\right] \end{aligned} \tag{23}$$

The following input-dependent state coordinate transformation:

$$\begin{aligned} z_1 &= y = x_1 + x_2 - C \\ z_2 &= \dot{y} = -x_1 - x_2 - D_{a2}x_2^2 + u \end{aligned} \tag{24}$$

allows one to obtain a GOCF for the system in the form given by (3). The inverse of this transformation is obtained by solving (22) with respect to x_1 and x_2. One obtains:

$$\begin{aligned} x_1 &= z_1 + C - \sqrt{\frac{u - (z_1 + z_2 + C)}{D_{a2}}} \\ x_2 &= \sqrt{\frac{u - (z_1 + z_2 + C)}{D_{a2}}} \end{aligned} \tag{25}$$

Note that from (22), it follows that the quantity inside the square root in (23) equals, precisely, the quantity $D_{a2}x_2^2$ and, hence, it is never smaller than zero.

Using (22) and (23) one obtains $\dot{z}_2$ in terms of thetransformed coordinates. The transformed system equations are then given by:

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= -2(1+D_{a1})(z_1 + C) - (3 + 2D_{a1})z_2 \\ &\quad - 2D_{a1}D_{a2}(z_1 + C)\sqrt{\frac{u - (z_1 + z_2 + C)}{D_{a2}}} \\ &\quad + 2D_{a2}^2\sqrt{\left(\frac{u - (z_1 + z_2 + C)}{D_{a2}}\right)^3} + 2(1+D_{a1})u + \dot{u} \\ y &= z_1 \end{aligned} \tag{26}$$

which is in GOCF.

The zero dynamics associated to the output nulling in (24) isgiven, according to (5), by:

$$\begin{aligned} &\dot{u} + 2(1+D_{a1})(u - C) \\ &\quad - 2D_{a1}D_{a2}C\sqrt{\frac{u - C}{D_{a2}}} + 2Da2^2\sqrt{\left(\frac{u - C}{D_{a2}}\right)^3} \\ &= 0 \end{aligned} \tag{27}$$

The equilibrium points of the zero dynamics are obtained from the solutions of the following algebraic equation, obtained directly from (25) by simply letting $\dot{u} = 0$:

$$\begin{aligned} &2(1+D_{a1})(u - C) - 2D_{a1}D_{a2}C\sqrt{\frac{u - C}{D_{a2}}} \\ &\quad + 2Da2^2\sqrt{\left(\frac{u - C}{D_{a2}}\right)^3} = 0 \end{aligned}$$

Clearly $u = C$ is an equilibrium point for (25). It may be shown that such an equilibrium point corresponds to an unstable, i.e. nonminimum phase, equilibrium point. On the other hand, it may be verified, after tedious but straightforward algebraic manipulations, that the constant equilibrium point $u = U$, $y(U) = Y(U) = 0$, corresponding to $X_1(U) + X_2(U) = C$, as computed from (21), is an asymptotically stable equilibrium point. The system is, hence, minimum phase around this equilibrium point.

Consider the following auxiliary output function, with $\gamma_1 > 0$:

$$s = z_2 + \gamma_1 z_1 \tag{28}$$

Note that if s is zeroed, by means of a discontinuous control strategy, then, it follows, from the first of (24) and (26), that the time response of the controlled output function $y = z_1$ is ideally governed by the asymptotically stable linear time-invariant dynamics:

$$\dot{z}_1 = -\gamma_1 z_1 \tag{29}$$

Dynamical PFM Controller Design Imposing on s the asymptotically stable discontinuous dynamics (12) one obtains, after reverting to original state coordinates x_1 and x_2, the following expression for the dynamical PFM controller:

$$\begin{aligned}\dot{u} = &-(1-\gamma_1)(u+x_1+x_2)\\ &+2D_{a1}D_{a2}x_1x_2-(3-\gamma_1)D_{a2}x_2^2\\ &-2D_{a2}^2x_2^3-W\,\mathrm{PFM}_{\tau,T}\,[-(x_1+x_2)\\ &-D_{a2}x_2^2+u+\gamma_1(x_1+x_2-C)]\end{aligned} \tag{30}$$

Dynamical PWM Controller Design Following the same procedure outlined for obtaining the dynamical PFM controller, one obtains a dynamical PWM controller. The resulting expression has, precisely, the same form as that of (28) except for the fact that a PWM control function is used

$$\begin{aligned}\dot{u} = &-(1-\gamma_1)(u+x_1+x_2)\\ &+2D_{a1}D_{a2}x_1x_2-(3-\gamma_1)D_{a2}x_2^2\\ &-2D_{a2}^2x_2^3-W\,\mathrm{PWM}_{\tau}\,[-(x_1+x_2)\\ &-D_{a2}x_2^2+u+\gamma_1(x_1+x_2-C)]\end{aligned} \tag{31}$$

.Dynamical SSM Controller Design Imposing on s the asymptotically stable discontinuous dynamics (17) one readily obtains the following stabilizing dynamical sampled sliding mode controller in original coordinates:

$$\begin{aligned}\dot{u} = &-(1-\gamma_1)(u+x_1+x_2)\\ &+2D_{a1}D_{a2}x_1x_2-(3-\gamma_1)D_{a2}x_2^2\\ &-2D_{a2}^2x_2^3-W\,\mathrm{SSM}\,[-(x_1+x_2)\\ &-D_{a2}x_2^2+u+\gamma_1(x_1+x_2-C)]\end{aligned} \tag{32}$$

Simulation Results Simulations were performed for a reactor characterized by the followingparameters:

$$D_{a1} = 1.0 \ ; \ D_{a2} = 1.0$$

The simulated control task considered the problem of stabilizing the output y to zero. This task is equivalent to having the total concentration variable converging to the prespecified constant reference value $C = 3$. In other words, the total normalized concentration $x_1 + x_2$ in the system (19) is to be driven to C by asymptotically driving the concentration error $y = x_1 + x_2 - C$ to zero.

Fig. 3 portrays the time response of the dynamical PFM-controlled output y, the smoothed input signal u and the corresponding controlled state trajectories. As before, these variables are seen to converge to their respective equilibrium values. Fig. 4, shows the evolution of the auxiliary output function s, with defining parameter $\gamma_1 = 1.0$, and the time response of the duty cycle function $T[s(t)]$, as well as the duty ratio function $\tau[s(t)]$.

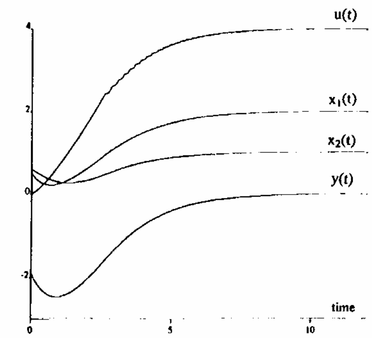


Fig. 3. Output, states and input variables trajectories ofdynamical PFM controlled CSTR.

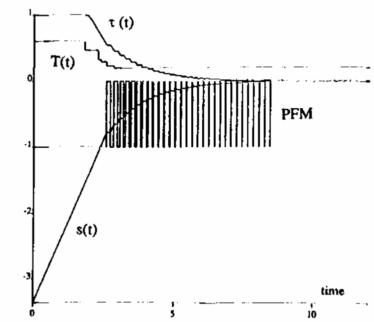


Fig. 4. Evolution of auxiliary output function, duty cycle andduty ratio functions for dynamical PFM controlled CSTR.

The dynamical PFM controller parameters were set, in accordance with condition (12) to

$$W = 1 \ ; \ r_1 = 1 \ ; \ r_2 = 0.5 \ ;$$
$$r_3 = 1.5 \ ; \ T_{max} = 0.6 \ ; T_{min} = 0.2$$

Fig. 5 depicts the time responses of the output, statesand (smoothed) input variables for the dynamical PWM controlled system. Fig. 6 shows the evolution of the auxiliary output function s, with $\gamma_1 = 1.0$, and the time response of the duty ratio function $\tau[s(t)]$. The dynamical PWM controller parameters were set, in accordance with (15), to $W = 1, r = 1$, and sampling period $T = 0.5$.

Fig. 7 portrays the time responses of dynamical SSM-controlled output y, the input signal u, exhibiting a small chattering, as generated by the dynamical SSM controller (30), and the corresponding controlled state trajectories x_1 and x_2. These variables are seen to converge in an asymptotically stable fashion towards their equilibrium values: $y = Y(U) = 0$, $u = U = 4$, $x_1 = X_1(4) = 2$ and $x_2 = X_2(4) = 1$. Fig. 8 shows the evolution of the sliding surface coordinate function s. The parameter γ_1 was set to 1.0.

The dynamical variable structure controller parameters in (11)were set to be $W = 1$, $T = 0.2$.

In order to test the performance of the proposed dynamical PWM controllerwe induced a temporary plant parameter per-

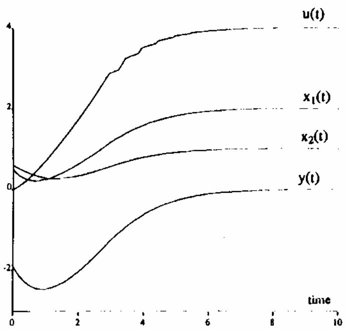


Fig. 5. Output, states and input variables trajectories ofdynamical PWM controlled CSTR.

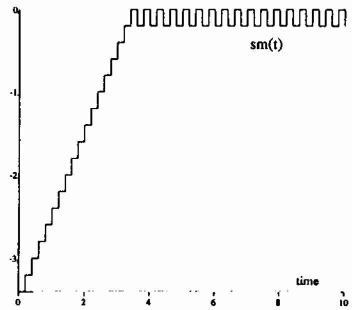


Fig. 8. Sampled slliding surface coordinate evolution fordynamical SSM controlled CSTR.

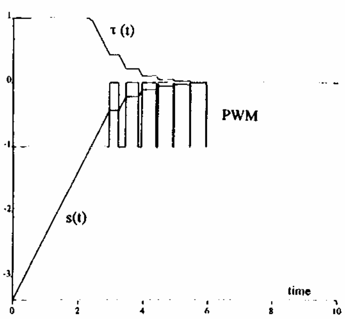


Fig. 6. Evolution of auxiliary output function and duty ratiofunction for dynamical PWM controlled CSTR.

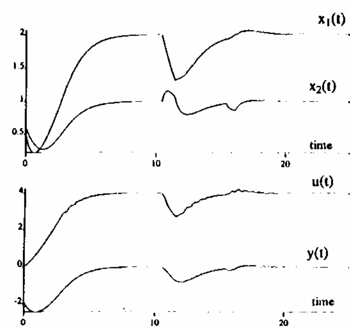


Fig. 9. Time responses of PWM controlled perturbed statestrajectories and the corresponding input and output signals.

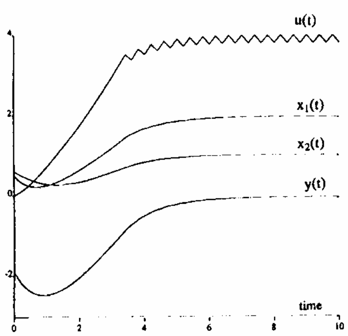


Fig. 7. Output, states and input variables trajectories ofdynamical SSM controlled CSTR.

turbation of about one second in duration for both D_{a1} and D_{a2} on different time intervals. The parameters D_{a1} and D_{a2} were allowed to take the value of 1.5 (which represents a 50 % variation from their nominal value of 1.0) on the time intervals $[10\text{sec}, 11\text{sec}]$ and $[15\text{sec}, 16\text{sec}]$, respectively. Fig. 9 shows the time responses of the control perturbed state trajectories, the corresponding output and the smoothed control input. The proposed controller is seen to quickly recover the desired stabilization features for the system. Fig. 10 depicts the corresponding duty ratio function, the auxiliary output function and the PWM signal. Simlar responses to parametric perturbations were obtained for the PFM and the SSM controllers

Dynamical PFM, PWM and SSM Control of a Double EffectEvaporator

The Double Effect Evaporator Model The following DEE model is taken fromMontano and Silva [17].

$$\begin{aligned} \dot{x}_1 &= \delta_1 F_0(c_0 - x_1) + \delta_2 x_1 u \\ \dot{x}_2 &= \delta_3 F_0(x_1 - x_2) + (\delta_4 x_1 + \delta_5 x_2) u \end{aligned} \tag{33}$$

where x_1 represents the product concentration in the first stage of the evaporator, while x_2 stands for the output concentration of the product at the second stage. The control input u is a positive quantity representing the steam flow generated by a boiler. The output variable of the system is the concentration error $y = x_2 - x_{2d}$, with x_{2d} being the required constant value of the output product concentration. The rest of the parameters in (31) are assumed to be known positive constants, except for δ_4 which is negative. We summarize below the necessary steps to obtain dynamical PFM, PWM and SSM regulators for the given system.

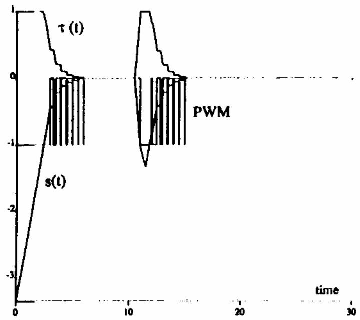


Fig. 10. Evolution of auxiliary output function, duty ratiofunction and PWM signal for parametrically perturbed PWMcontrolled CSTR.

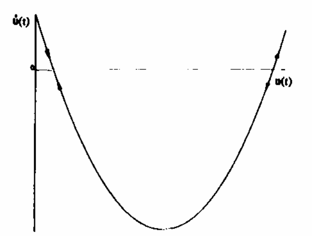


Fig. 11. Phase diagram for zero dynamics of DEE system.

Remark 3.1 A more realistic sixth order model of the DEEwas presented in Andre and Ritter [18]. The reduced second order model (31) has been extensively validated, from an experimental viewpoint, in Newell and Fisher [19], while further evaluations and comparisons were carried out in [17]. Our purposes, in using such a simplified model, are to illustrate the proposed dynamical controller designs and test the features, and possible advantages, of using such dynamical discontinuous feedback controllers.

Design of a Dynamical Discontinuous FeedbackRegulator for Concentration Control in a DEE In this section we let "DDC" (for Dynamical Discontinuous Control)stand for any of the three proposed discontinuous feedback control alternatives presented in this article. Namely,"DDC" stands for the PFM, PWM and SSM control options.

Rank condition on the output

$$\det \begin{bmatrix} \frac{\partial y}{\delta x} \\ \frac{\partial \dot{y}}{\delta x} \end{bmatrix} = \det \begin{bmatrix} 0 & 1 \\ \delta_3 F_0 + \delta_4 u & -\delta_3 F_0 + \delta_5 u \end{bmatrix} = -(\delta_3 F_0 + \delta_4 u) \quad (34)$$

The rank condition can only be violated by a (positive) constant value of thecontrol input u given by $u = -\delta_3 F_0/\delta_4$. It is easy to sec, from the second equation in (31) that this equilibrium value for the control input u has no physical significance since it implies that the product concentration in the first stage of the evaporator x_1 has no influence, whatsoever, on the product concentration x_2 at the second stage of the evaporator.

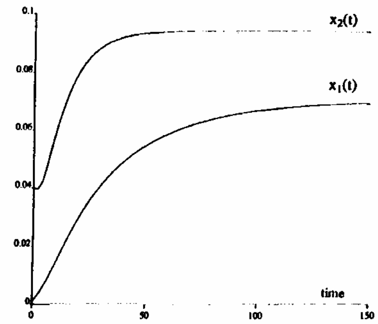


Fig. 12. Dynamical PFM controlled states responses fora DEE system.

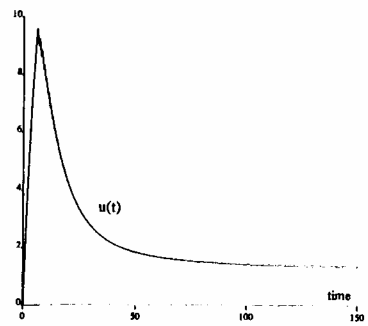


Fig. 13. Control input signal for dynamical PFM regulatedDEE system.

Input-dependent State Coordinate Transformation for the GOCF

$$\begin{aligned} z_1 &= y = x_2 - x_{2d}; \\ z_2 &= \dot{y} = \delta_3 F_0(x_1 - x_2) + (\delta_4 x_1 + \delta_5 x_2)\, u \\ x_1 &= \frac{z_2 - (\delta_5 u - \delta_3 F_0)(z_1 + x_{2d})}{\delta_3 F_0 + \delta_4 u}; \\ x_2 &= z_1 + x_{2d} \end{aligned} \quad (35)$$

Generalized Observability Canonical Form for the Plant

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= (\delta_5 u - \delta_3 F_0) z_2 + \frac{\delta_3 F_0(\delta_4 + \delta_5)(z_1 + x_{2d}) + \delta_4 z_2}{\delta_3 F_0 + \delta_4 u} \dot{u} \\ &\quad (\delta_2 u - \delta_1 F_0)[(\delta_3 F_0 - \delta_5 u)(z_1 + x_{2d}) + z_2)] \\ &\quad + c_0 \delta_1 F_0(\delta_3 F_0 + \delta_4 u) \\ y &= z_1 \end{aligned} \quad (36)$$

Zero Dynamics

$$\frac{\delta_3 F_0(\delta_4 + \delta_5) x_{2d}}{\delta_3 F_0 + \delta_4 u} \dot{u} + (\delta_2 u - \delta_1 F_0)[(\delta_3 F_0 - \delta_5 u) x_{2d}] + c_0 \delta_1 F_0(\delta_3 F_0 + \delta_4 u) = 0 \quad (37)$$

The equilibrium points of the zero dynamics are given by the realsolutions of the following quadratic algebraic equation:

$$(\delta_2 u - \delta_1 F_0)[(\delta_3 F_0 - \delta_5 u) x_{2d}] + c_0 \delta_1 F_0(\delta_3 F_0 + \delta_4 u) = 0$$

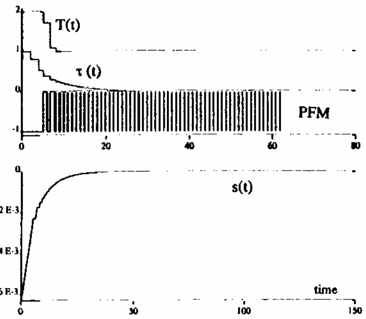


Fig. 14. Evolution of auxiliary output function, duty cycle andduty ratio functions for dynamical PFM controlled DEE system.

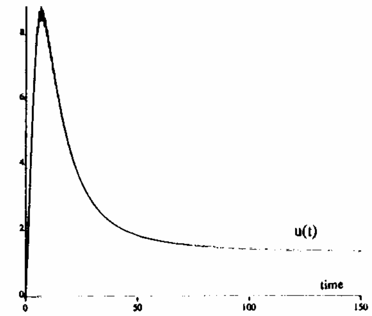


Fig. 16. Control input signal for dynamical PWM controlledDEE system.

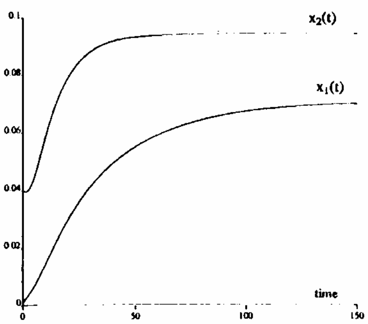


Fig. 15. Dynamical PWM controlled states trajectories forDEE system.

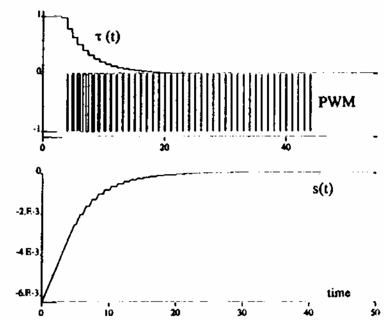


Fig. 17. Evolution of auxiliary output function and dutyratio function for dynamical PWM controlled DEE system.

Fig. 11 shows that one of the solutions of thequadratic algebraic equation (36) represents a stable (minimum phase) equilibrium point for the zero dynamics (35), while the second solution represents an unstable (non-minimum phase) equilibrium point for such a zero dynamics.

Auxiliary Output Function in Transformed and Original Coordinates

$$s(z) = z_2 + \gamma_1 z_1 \qquad (38)$$

$$s(x) = \delta_3 F_0(x_1 - x_2) + [\delta_4 x_1 + \delta_5 x_2]\, u + \gamma_1(x_2 - x_{2d})$$

Dynamical Discontinuous Controller (DDC) in Original Coordinates

$$\begin{aligned} \dot{u} = \frac{1}{\delta_4 x_1 + \delta_5 x_2} \cdot [&-(\delta_3 F_0 + \delta_4 u)(\delta_1 F_0 + \delta_4 u) \\ &\cdot [\delta_1 F_0(c_0 - x_1) + \delta_2 x_1 u] \\ &-(\delta_5 u - \delta_3 F_0)[\delta_3 F_0(x_1 - x_2) + (\delta_4 x_1 + \delta_5 x_2) u] \\ &-\gamma_1[\delta_3 F_0(x_1 - x_2) + (\delta_4 x_1 + \delta_5 x_2)\, u] \\ &- W\, \mathrm{DDC}\, (\delta_3 F_0(x_1 - x_2) + [\delta_4 x_1 + \delta_5 x_2]\, u \\ &+\gamma_1(x_2 - x_{2d}))] \end{aligned} \qquad (39)$$

In all three cases, *impasse points* for the dynamical controlleroccur on the line $\delta_4 x_1 + \delta_5 x_2 = 0$, which, from the second equation in (31), is seen to represent a region of *uncontrollability* of the product concentration on the second stage of the evaporator. This condition is entirely possible due to the negative value of the parameter δ_4 and the positive values of the concentration variables and the parameter δ_5. Results are, therefore, valid away from this *singularity* condition. We must emphasize that singularity avoidance has been extensively treated, in a related but somewhat different context, in [13].

Simulation Results

The following parameter values were used in the simulations:

$$\begin{aligned} &F_0 = 2.525[\mathrm{Kg/min}]\ ,\ c_0 = 0.04\ , \\ &\delta_1 = 0.00105\ , \delta_2 = 8.509 \times 10^{-3} \\ &\delta_3 = 9.523 \times 10^{-3}\ ,\ \delta_4 = -7.699 \times 10^{-3}\ , \\ &\delta_5 = 10.304 \times 10^{-3} \end{aligned}$$

With these parameter values, the physically meaningful equilibriumpoint is found to be $x_1 = 0.7$ and $x_2 = x_{2d} = 0.0939$.

According with the condition (12) of Proposition3, the PFM controller parameters were set to be:

$$\begin{aligned} &W = 8 \times 10^{-4}\ ;\ r_1 = 250\ ,\ r_2 = 300\ , \\ &r_3 = 400\ ;\ T_{max} = 2[\mathrm{min}] \end{aligned}$$

$$T_{min} = 1[\mathrm{min}]\ ;\ \gamma_1 = 0.1[\mathrm{min}^{-1}]$$

Figs. 12, 15 and 18 show thestate responses of the dynamical PFM, PWM and SSM controlled systems, asymptotically

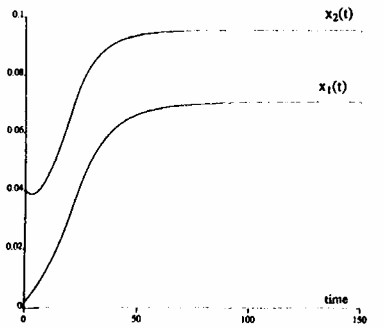


Fig. 18. Dynamical SSM controlled states responses for aDEE system.

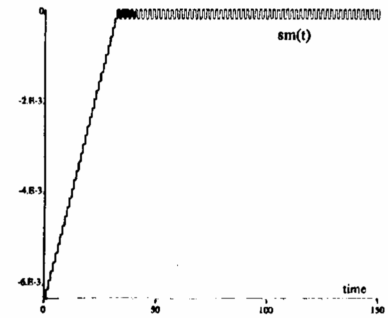


Fig. 20. Sampled sliding surface coordinate evolution fordynamical SSM controlled DEE system.

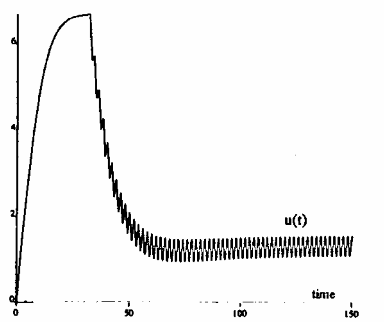


Fig. 19. Control input signal of dynamical SSM controlledDEE system.

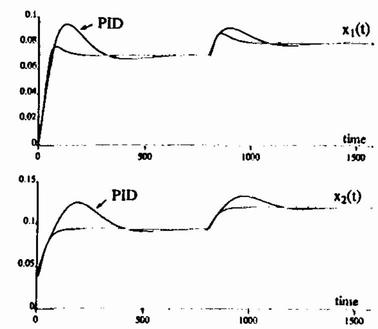


Fig. 21. Dynamical PFM controlled states responses andcorresponding PID control responses for aset-point change in a DEE system.

converging toward the desired equilibrium point given by $x_{2d} = 0.0939$ while the concentration x_1 converges to its equilibrium value 0.07. Figs. 13, 16 and 19 depict the smoothed control input trajectories for dynamical PFM, PWM and SSM controlled systems respectively. Fig. 14 depicts the evolution of the auxiliary output function s, the time response of the duty cycle function $T[s(t)]$, and the duty ratio function $\tau[s(t)]$ of the dynamical PFM controlled system. Fig. 17 portrays the evolution of the auxiliary output function s, the PWM signal and the duty ratio function $\tau[s(t)]$ for the PWM controlled system. Fig. 20 shows the auxiliary output function s for the SSM controlled system.

Set point changes, of about 30 % value, were provoked in thedesired output value of the dynamical PFM controlled system (from $x_{2d} = 0.0939$ to $x_{2d} = 0.12$). Fig. 21 depicts the state response of the closed-loop PFM system, under such a setpoint change. The dynamical PFM controlled state responses are jointly shown with the corresponding closed-loop state responses of a well tunned proportional-integra-derivative (PID) controller, designed on the basis of the linearized system trajectories. The PID controller parameters, taken from [17], were set so as to obtain a 5% overshoot and rise time of 110 min ($K_p = 28.067$, $T_i = 137.64$ and $T_d = 20.83$). The performance of the dynamical PFM controller, for such large setpoint changes is seen to be vastly superior to that of the designed PID controller, as far as overshoot and settling times are concerned.

Similar results were obtained for the performances of the PWM andSSM controllers when compared to those of the traditional PID controller.

IV. Conclusions

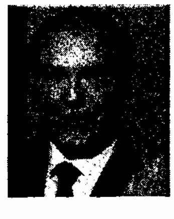
The feasibility of chattering-free discontinuous feedback controllershas been demonstrated via dynamical feedback strategies based on stabilization of suitably specified auxiliary output functions defined on the basis of Fliess's GOCF. In all cases the discontinuities are relegated to the controller's state space. Therefore, the hardware implementation is quite feasible using present day switching and electronics techniques. Stabilizing PFM, PWM and SSM controller design procedures for nonlinear plants are unified via this approach which is derived from basic facts of the *Differential Algebraic* viewpoint in systems dynamics [2]. It is necessary to remark that the benefits and robustness of these controllers has been studied, from a general viewpoint, in Sira-Ramírez ([20], and from the adaptive control viewpoint in Sira-Ramírez and Zribi [21]. In this study we have also tested the performance of some of the proposed dynamical discontinuous feedback controllers, in typical chemical process examples, with respect to plant parameter variations and with respect to large set point changes. The obtained results and comparisons with traditional PID regulator performances, are quite encouraging, from a simulations viewpoint. The results here presented are also extendable to multivariable nonlinear plants.

ACKNOWLEDGMENT

The authors are grateful to an annonymous reviewer for his manysuggestions to improve the article. The first author also benefitted from interesting discussions with Professors A. Montano and R. Kelly on chemical process control. Prof. A. Montano provided the authors with useful information about the double effect evaporator model.

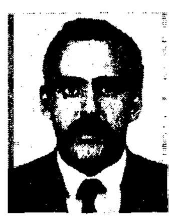
REFERENCES

[1] M. Fliess, "Nonlinear control theory and differential algebra," in *Modelling and Adaptive Contr.*. Ch. Byrnes and A. Khurzansky (Eds.), Lecture notes in control and information sciences, vol. 105, Springer-Verlag, 1990, pp. 134–145.
[2] M. Fliess, "Generalized canonical forms for linear and nonlinear systems," *IEEE Trans. on Automat. Contr.* vol. AC-35, no. 9, pp 994–1001, Sept., 1990.
[3] H. Sira-Ramírez, "A dynamical variable structure control strategy in asymptotic output tracking problems," *IEEE Trans. on Automat. Contr.* To be published..
[4] H. Sira-Ramírez, S. Ahmad and M. Zribi, "Dynamical feedback control of robotic manipulators with joint flexibility". *IEEE Trans. on Syst., Man and Cybernetics* vol. 22, no. 4, pp. 736–747, 1992.
[5] V.I. Utkin, *Sliding Regimes and Their Applications in Variable Structure Systems*. MIR Publishers, Moscow, 1978.
[6] H. Sira-Ramírez, "A geometric approach to pulse-width-modulation control of nonlinear systems." *IEEE Trans. on Automat. Contr.* vol. AC-34, pp. 184–187, 1989.
[7] C. Kravaris and S. Palanki, "Robust nonlinear state feedback under structured uncertainty". *Amer. Inst. of Chemical Engineers Journal* vol. 34, no. 7, pp. 1119–27, Jul. 1988.
[8] H. Sira-Ramírez and O. Llanes-Santiago, "Chattering-free pulse width modulation feedback control strategies in the regulation of robotic manipulators,"*Proc. of the IMACS/SICE Int'l Symp. on Robotics, Mechatronics and Mfg. Syst.* Kobe, Japan, Sept. 16–20, 1992, pp. 331–336.
[9] G. Conte *et al.*, "Un theoreme sur la representation entree-sortie d'un systeme non lineaire." *Comptes Rendus de la Academie des Sciences de Paris* vol. 307, serie I, pp. 363–366, 1988.
[10] A. Isidori *Nonlinear Control Syst.*. Second Edition, New York: Springer-Verlag, 1989.
[11] H. Sira-Ramírez and O. Llanes-Santiago, "Pulse-width-modulation control of some mechanical systems," *Dynamics and Contr.* vol. 3, no. 1, pp. 35–51, 1993.
[12] M. Fliess, "What the Kalman state variable representation is good for," *Proc. of the 29th IEEE Conf. on Decision and Contr.* vol. 3, pp. 1282–1287, Honolulu, Hawaii, Dec. 1990.
[13] S. Abu el Ata-Dos, A. Cofc, and M. Fliess, "Nonlinear predictive control by inversion: Discontinuities for critical behaviours," *Int'l J. of Control* vol. 55, no. 6, pp. 1521–1533, 1992.
[14] H. Sira-Ramírez, "Pulse-frequency-modulation control of nonlinear systems," *Control: Theory and Adv. Tech.* To be published.
[15] H. Sira-Ramírez and O. Llanes-Santiago, "Dynamical discontinuous feedback control of chemical processes," *3d. IFAC Symp. on Dynamics and Control of Chem. Reactors, Distillation Columns, and Batch Processes*, Apr. 26–29, College Park, Maryland USA, Jul. 1992.
[16] H. Sira-Ramírez, "Dynamical pulse-width- modulation control of nonlinear systems". *Syst. and Control Lett.* vol. 18, no. 2, pp. 223–231, 1992.
[17] A. Montano and G. Silva, "Design of a nonlinear control for a double effect evaporator," *Proc. of the First European Control Conf.*, Grenoble, France, Jul. 2-5, 1991, pp. 2256-2261.
[18] H. Andre and R. A. Ritter, "Dynamical response of a double effect evaporator," *The Canadian J. of Chem. Eng.*, vol. 46, no. 4, pp. 259–264, Aug. 1968.
[19] R. B. Newell and D. G. Fisher, "Model development, reduction and experimental evaluation for an evaporator," *Indust. Engg. Chem. Process Design and Devel.*, vol. 11, no. 2, pp 213–221, 1972.
[20] H. Sira-Ramírez, "On the sliding mode control of nonlinear systems," *Syst. and Control Letters* vol. 19, no. 4, pp. 302–312, 1992.
[21] H. Sira-Ramírez and M. Zribi, "Adaptive dynamical variable structure feedback regulation for nonlinear uncertain systems," *Int'l J. of Robust and Nonlinear Contr.*, To be published..



Dr. Hebertt Sira-Ramírez, obtained the Degree of IngenieroElectricista from the Universidad de Los Andes in Mérida-Venezuela in 1970. He pursued graduate studies from 1972 to 1977 at the Massachusetts Institute of Technology (MIT), Cambridge, MA, where he obtained the degree of Master of Science in Electrical Engineering (MSEE) and the degree of Electrical Engineer (EE) in 1974. He later obtained the degree of Doctor of Philosophy (PhD) in Electrical Engineering in 1977. He is currently a Full Professor in the Control Systems Department of the Systems Engineering School of the Universidad de Los Andes, where he has also held the positions of Head of the Control Systems Department from 1978 to 1980 and was elected Vice-president of the University for the term 1980-1984. He was appointed Head of the Graduate School in Control Engineering in 1992. Dr. Sira-Ramírez is a Senior Member of the Institute of Electrical and Electronics Engineers (IEEE), where he serves as a member of the IEEE International Committee. He is also a member of the International Federation of Automatic Control (IFAC), The Society of Industrial and Applied Mathematics (SIAM), Sigma Xi, The Planetary Society and of the Venezuelan College of Engineers (CIV).

He has held visiting positions in the Coordinated Science Laboratory of the University of Illinois at Urbana-Champaign, in the Department of Aeronautical and Astronautical Engineering and the Department Electrical Engineering of the same University in 1986. He has also held brief visiting positions in the School of Electrical Engineering of Purdue University in 1989, 1990 and 1991. During his sabbatical year in in 1992-1993, he was appointed Associate Research Director of the Laboratoire de Signaux et Systèmes (LSS) at the Ecole Superieure de Electricité (ESE) of the Centre National de la Recherche Scientifique (CNRS) in Plateau du Moulon, (Paris) France. Dr. Sira-Ramírez received the IEEE Distinguished Lecturer Award in 1993 and the Venezuelan College of Engineers Award for Scientific Research in 1987 as well as the Senior Researcher Scholarship Award from the Venezuelan National Council for Scientific and Technological Research (CONICIT) in 1990. Dr. Sira-Ramírez is interested in the theory and applications of discontinuous feedback control strategies for nonlinear dynamic systems.



Orestes Llanes-Santiago: received the degree of Ingeniero Electricista from the Instituto Superior Politécnico "José Antonio Echeverría", in 1981. He is currently an Assistant Professor in the Automatic Control Department of the Electrical Engineering Faculty of the Instituto Superior Politécnico "José Antonio Echeverría". Mr. Llanes-Santiago obtained the M.S. degree in Automatic Control Engineering in 1991 from the Universidad de Los Andes, Mérida, Venezuela, where he is currently working toward the Ph.D. degree. He is a student member of the Institute of Electrical and Electronics Engineers (IEEE). His area of interest is in the theory and applications of discontinuous control strategies for nonlinear systems.