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Switching control strategy for the power system stabilization problem

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A switching control strategy is proposed for the stabilization of the simplest form of a power system. The underlying switching control strategy is represented by the pulsed addition of an inductive impedance to the transmission line, through a suitable and well-known arrangement of switching thyristors. It is proved that there exists a local stabilizing sliding mode. The energy function of the system is used as a Lyapunov function to derive a switching control which guarantees the reachability of the sliding surface from the stability region. The switching control scheme is also implemented via a reduced-order nonlinear observer based on exact error linearization achieved through nonlinear output injection. Numerical simulations are presented.

1. Introduction

Generators for an AC power system must rotate at exactly the same speed. If the generators are not synchronous then, simply, there is no average power flow. Any disturbance will cause the angles between generators to oscillate. For sufficiently small disturbances the oscillations will eventually decay to zero. For large disturbances, however, a loss of synchronism may arise that may require the breaking of the connection between the generators with the consequent loss of supply to customers. Analysis tools have been developed to predict the oscillations between generators, following disturbances such as lightning strikes to transmission lines (Anderson and Fouad 1977).

The traditional form of suppression of oscillations was based on control of the field current of the generators, which then influenced terminal voltage and hence power flow (DeMello and Concordia 1969). In contrast to the slow speed of response and the localization of excitation control to generator sites, the fast response, and flexible location, of Static Var Shunt Compensators (SVCs) makes them a desirable asset in the suppression of oscillations of groups of generators (O'Brien and Ledwich 1987). These SVCs can be based on phase control of inductors in parallel with capacitors (Torseng 1981). The system effectively becomes a variable reactance from the line to ground and its possible changes affect power flow by varying the effective series line impedance and the voltage at load buses. A more recent development has been the controlled series compensation of transmission lines. This alternative has been shown to be quite effective in the control of power flow and, hence, in achieving stability

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enhancement of the power system (Johnson *et al.* 1993). The control by shunt and series compensators has traditionally been accomplished by means of linear control laws with a limited range of control action, determined by the rating of the device. The nonlinear dependency of the power flow on line angles and the saturation effects have also been examined by means of extensive simulation and subsequent tuning of the parameters of the underlying linear control laws.

A nonlinear discontinuous feedback control design is examined in this paper, with the expectation of improved performance for a wide range of operating conditions. The model is that of a single machine depicted in Fig. 1 with a constant voltage behind machine reactance. We focus on the design of a nonlinear controller for suppression of oscillation in the post fault system.

Variable structure control has been extensively studied in the past and many of its applications have been published (Zinober 1990, Utkin 1977, DeCarlo *et al.* 1988). Variable structure control is characterized by a discontinuous feedback control law which causes the, so called, 'sliding mode' responses. Under variable structure control, the system trajectories are forced to lie within a certain prespecified manifold in the state space. When the system complies with sliding mode conditions, it is insensitive to a certain (matched) class of system parameter variations and independent external disturbances.

In this article we propose a variable structure control for the stabilization problem of the simplest power system. The design of a variable structure controller for a series compensator based on a second-order model of the line has been discussed by Wang *et al.* (1992). However, the design does not include any variable structure convergence checks but chooses a linear switching line with exponential convergence to the switching line.

We show that because of the discrete nature of the values of the possible control actions, the sliding mode behaviour can only exist in a local region of the phase space of the system. Whenever the closed system trajectories enter the region of existence of sliding motions, and criss-cross the switching line, the existence of a local stabilizing sliding mode behaviour will be guaranteed. The boundaries of the local region for existence of the sliding mode in the phase space are explicity determined. One of the benefits of using the variable structure control is that the existence of local sliding mode prevents the power system from undesirable oscillations.

We next show that outside the region of existence of a sliding mode, a switching control can still be employed to achieve reachability of the sliding region. The use of the energy function, as a Lyapunov function candidate, is well suited for the purposes of demonstrating attractivity of the region of existence of a sliding regime.

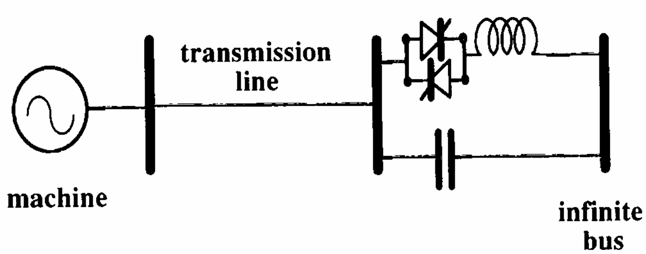

Figure 1. Single machine model.

The controller is represented by the pulsed addition of inductive impedance to the transmission line through a traditionally suitable switching arrangement of thyristors. The proposed control scheme can also be implemented via a reduced-order nonlinear observer, based on exact error linearization via non-linear output injection.

This article is organized as follows. Section 2 presents the main results of the article about the switching control strategy referred to the pulsed feedback stabilization of the power system. Section 3 contains the simulation results of the proposed switching control scheme based on a nonlinear state observation strategy. Conclusions and discussions are summarized in § 4.

2. A switching control solution to the power system stabilization problem

2.1. *Problem formulation*

The model of the single machine system shown in Fig. 1 is based on the difference between the electrical output power P_{elec} and the shaft power P_s generated by the turbine (Anderson and Fouad 1977). This, in turn, results in an acceleration of the combined rotating inertia. The electrical power flow betwen two a.c. voltages E_1 and E_2 at identical frequencies and separated by angle δ, over an effective reactance X_e, is given by

$$P_{\text{elec}} = \frac{E_1 E_2}{X_e} \sin \delta \tag{2.1}$$

The effective reactance of the line, plus compensator reactance, at mains frequency is the following

$$X_e = X + \frac{1}{\left(Y_L + \dfrac{1}{\omega C}\right)} \tag{2.2}$$

where X is the line reactance, and Y_L, is the effective variable admittance. Thanks to the appropriate firing of the compensating thyristors switching circuit arrangement, the variable admittance can take values between two possible values $Y_L = Y$ and $Y_L = 0$. The effective reactance X_e varies then between two corresponding possible values X and $X + \varepsilon$ where X is addressed as the fully compensated reactance and $X + \varepsilon$ is referred to as the uncompensated reactance. These quantities are immediately defined from (2.2).

The power flow can thus be modelled by the introduction of a discrete-valued control input variable u, representating the thyristor switching action, as

$$P_{\text{elec}} = \frac{E_1 E_2}{X}\left(1 - u\frac{\varepsilon}{X + \varepsilon}\right) \sin \delta \tag{2.3}$$

The control input u, thus, takes values in the set $\{0, 1\}$.

The angle δ is also the angle of the rotor of the machine so that the acceleration of the machine, from the normal synchronous frequency, is just $\ddot{\delta}$. For a rotational inertia, J, and provided we ignore the small damping component, obtained from the damper bars of the generator, the dynamics of the angle

δ are then given by

$$J\ddot{\delta} = P_s - P_{elec} = P_s - \frac{E_1 E_2}{X}\left(1 - u\frac{\varepsilon}{X + \varepsilon}\right)\sin\delta \tag{2.4}$$

Equation (2.4) is referred as the swing equation.

2.2. *A variable structure control strategy*

Consider the swing equation model described in the previous section

$$\begin{aligned} \dot{\delta}_1 &= \delta_2 \\ \dot{\delta}_2 &= \frac{P_s}{J} - \frac{E_1 E_2}{JX}\left(1 - u\frac{\varepsilon}{X + \varepsilon}\right)\sin\delta_1 \end{aligned} \tag{2.5}$$

The control input $u \in \{0, 1\}$ represents a switch position function related to the operation of the inductive impedance switching thyristors. For the sake of simplicity, we denote $\delta = (\delta_1, \delta_2)^T$ as the state of the swing equation (2.5).

Notice that the equilibrium points corresponding to the uncontrolled system (i.e. with $u = 0$), are given by $(\delta_s^n, 0)^T$, where

$$\delta_s^n = n\pi + (-1)^n \sin^{-1}\left(\frac{P_s X}{E_1 E_2}\right), \quad n = 0, \pm 1, \pm 2, \ldots \tag{2.6}$$

The equilibrium point for $n = 0$

$$\delta_s^0 = \sin^{-1}\left(\frac{P_s X}{E_1 E_2}\right) \tag{2.7}$$

is deemed as desirable for the steady-state operation of the system. Note that, necessarily, such an equilibrium actually exists whenever $P_s X < E_1 E_2$.

Note also that the equilibrium points with δ_s^{-1} ($\delta_s^{-1} = -\pi - \delta_s^0$) and δ_s^1 ($\delta_s^1 = \pi - \delta_s^1$) are unstable and actually represent saddle points. A phase portrait of the swing equation is illustrated in Fig. 2, which shows that there is a separatrix through δ_s^1 that delimits the closed orbits representing periodic solutions, from unbounded orbits (Drazin 1992). Since the saddle point δ_s^1 is connected with itself by the separatrix, the orbit is also called the *homoclinic*

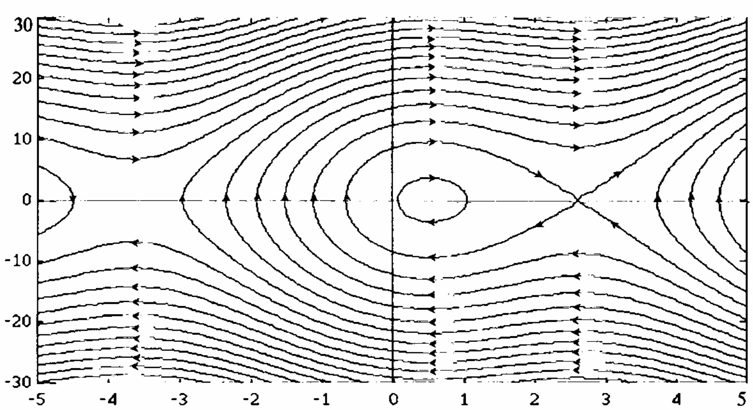

Figure 2. Phase portrait of the swing equation.

orbit. Therefore, the stability region we are concerned with, denoted as Ω_{sr}, is bounded within the range (δ_{sp}, δ_s^1), where δ_{sp} and δ_s^1 are the two intersection points of the homoclinic orbit with the axis $x_2 = 0$. Within this region, the orbits are bounded.

To develop a variable structure control, we first construct the sliding surface as

$$s = \delta_2 + \lambda(\delta_1 - \delta_s^0) \quad \lambda > 0 \tag{2.8}$$

and we have the following result.

Proposition 2.1: *If the variable structure control is given by*

$$u_{ss} = \begin{cases} 0 & \textit{for } (\sin\delta_1)s > 0 \\ 1 & \textit{for } (\sin\delta_1)s < 0 \end{cases} \tag{2.9}$$

then a sliding mode exists on the sliding surface $s = 0$, *defined in* (2.8), *only in the bounded region*

$$\Omega_{ss} = \{(\delta_1, \delta_2) | l_1(\delta) \leqslant 0, \, l_2(\delta) \geqslant 0\} \tag{2.10}$$

where the two delimiting lines are given by

$$l_1(\delta) = \frac{P_s}{J} - \frac{E_1E_2}{JX}\sin\delta_1 + \lambda\delta_2 = 0 \tag{2.11}$$

$$l_2(\delta) = \frac{P_s}{J} - \frac{E_1E_2}{J(X + \varepsilon)}\sin\delta_1 + \lambda\delta_2 = 0 \tag{2.12}$$

Proof: For the existence of a sliding mode on $s = 0$, it is sufficient that

$$s\dot{s} < 0 \tag{2.13}$$

Since

$$s\dot{s} = \frac{P_s}{J}s - \frac{E_1E_2}{JX}\left(1 - u\frac{\varepsilon}{X + \varepsilon}\right)(\sin\delta_1)s + \lambda\delta_2 s \tag{2.14}$$

for (2.13) to hold, if $(\sin\delta_1)\, s > 0$, we need the third term on the right-hand side of (2.14) to be as negative as possible, which means u must be chosen as 0. However, for $(\sin\delta_1)\, s < 0$, we need to eliminate the positiveness of the third term on the right-hand side of (2.14) such that (2.13) holds, the only choice is to set u to 1. Therefore, the control (2.9) is obtained. Because the control is one of the elements in the bounded set $\{0, 1\}$, the existence region of the sliding mode is therefore delimited by the two delimiting lines (2.11) and (2.12) that are obtained by setting $u = 0$ and $u = 1$ respectively in (2.14), such that $\dot{s} = 0$. The switching line $s = 0$ necessarily intersects the boundary lines and therefore the sliding regime is only local. □

The two delimiting lines $l_1(\delta) = 0$, $l_2(\delta) = 0$, both of which cross the axis $\delta_1 = 0$ at $P_s/(J\lambda)$, intersect the line $\delta_2 = 0$ at the points δ_s^0, and $\sin^{-1}(P_s(X + \varepsilon)/(E_1E_2))$ respectively. The width of the sliding region can be measured as the vertical length of the bounded region at the desirable point of equilibrium $(\delta_s^0, 0)$. This distance is, in absolute value, given by $(P_s\varepsilon)/(J\lambda(X + \varepsilon))$. Since all the parameters are fixed except the parameter for the sliding line λ, to enlarge the existence region one should choose the slope of the sliding line to be sufficiently small. This, in turn, reduces the convergence speed of the system towards the equilibrium point. So there is a need for a compromise between a large existence region of the sliding mode and a fast speed of stabilization. Figure 3 shows the existence region of a sliding mode. When the system behaves in a sliding model, it exhibits an exponentially decaying behaviour towards the prescribed equilibrium point.

2.3. *Asymptotic stability towards the region of existence*

It has been shown in § 2.2 that there only exists a local sliding mode. The locality of the sliding mode does not allow sudden unforeseen disturbances with large magnitude, albeit that the elimination of slow oscillations is desirable. We will show here that although the sliding mode is local, a switching control design enables reachability of the local sliding mode.

Energy functions such as the Lyapunov functions have long been used for determination of the critical clearing time for various faults and the assessment of stability of power systems (Pai 1981). The well-known Equal Area Criterion is actually an implicit application of the Lyapunov approach. In this section, an energy function is used as the Lyapunov function to obtain a switching control such that the system trajectories converge (or shrink) towards, and eventually enter, the existence region of sliding mode Ω_{ss}.

The Lyapunov function is chosen

$$V(\delta) = \frac{1}{2}J\delta_2^2 - \frac{E_1E_2}{X}(\cos\delta_1 - \cos\delta_s^0) - P_s(\delta_1 - \delta_s^0) \tag{2.15}$$

which is actually an energy function for (2.5), and can be obtained by integration of (2.5) (Drazin 1992).

The Lyapunov function (2.15) has the following property.

Property 2.2: *The Lyapunov function* (2.15) *is positive definite for* $\delta_1 \in (\delta_s^{-1}, \delta_s^1)$.

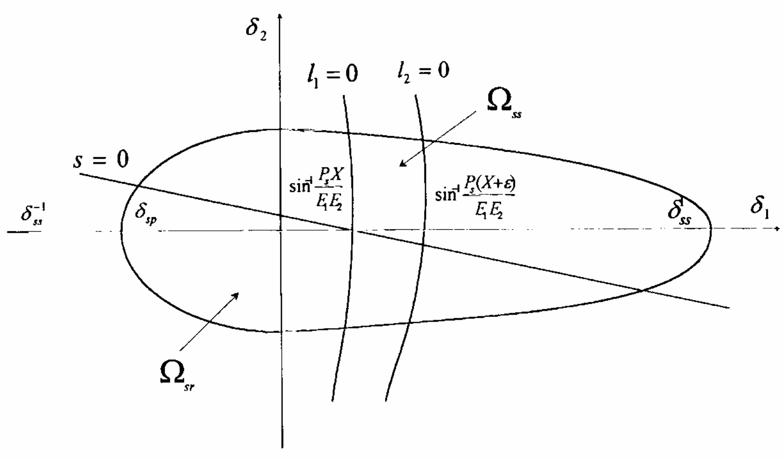

Figure 3. Construction of Ω_{sr} and Ω_{ss}.

Proof: It is obvious that the equilibrium point of $V(\delta)$ is $(\delta_s^0, 0)^T$ and

$$V(\delta) \geqslant \hat{V}(\delta_1) = -\frac{E_1 E_2}{X}(\cos \delta_1 - \cos \delta_s^0) - P_s(\delta_1 - \delta_s^0) \tag{2.16}$$

If $\hat{V}(\delta_1)$ is positive definite for $\delta_1 \in (\delta_s^{-1}, \delta_s^1)$, then $V(\delta)$ is positive definite. By a simple calculation, one can easily see that

$$\hat{V}(\delta_1) = \int_{\delta^0}^{\delta_1} \left(\frac{E_1 E_2}{X} \sin \tau - P_s \right) d\tau \tag{2.17}$$

For $\delta_s^0 \leqslant \delta_1 \leqslant \delta_s^1$, $(E_1 E_2 / X) \sin \tau - P_s > 0$, hence $\hat{V}(\delta_1) > 0$. For $\delta_s^{-1} \leqslant \delta_1 \leqslant \delta_s^0$, $(E_1 E_2/X) \sin \tau - P_s < 0$, hence $V(\delta_1) > 0$. Therefore, $\hat{V}(\delta_1)$ is positive definite over $(\delta_s^{-1}, \delta_s^1)$ and hence so is $V(\delta)$. □

Although the Lyapunov function (2.15) is positive definite on $\delta_1 \in (\delta_s^{-1}, \delta_s^1)$, the actual stability region we are concerned with is Ω_{sr}, in which the orbits of the Lyapunov function (2.15) are bounded. The value of the Lyapunov function (2.15) on the boundaries of Ω_{sr}, i.e. $V(\delta_{sr})$, where δ_{sr} is on the boundaries of Ω_{sr}, is called the *critical value*. For any δ such that $V(\delta) > V(\delta_{sr})$, when $t \to \infty$, the value of the Lyapunov function $V \to \infty$. This means that the power system will lose synchronism, and hence the stability.

The Lyapunov function (2.15) is then used to derive a switching control that stabilizes the power system (2.5).

Proposition 2.3: *If the switching control is given by*

$$u_{sr} = \begin{cases} 0 & \textit{for } (\sin \delta_1)\, \delta_2 > 0 \\ 1 & \textit{for } (\sin \delta_1)\, \delta_2 < 0 \end{cases} \tag{2.18}$$

then the system (2.5) *trajectories converge towards the equilibrium point* $(\delta_s^0, 0)$ *in the stability region* Ω_{sr}.

Proof: With the Lyapunov function (2.15), one can easily verify that its time derivative, along the solutions of (2.5), is given by

$$\dot{V}(\delta) = -\frac{E_1 E_2 \varepsilon}{X(X + \varepsilon)}(\sin \delta_1)\delta_2 u \tag{2.19}$$

With control (2.18), apparently $\dot{V} \leqslant 0$. Since $\dot{V} = 0$ does not contain any solutions of the swing equation, the system (2.5) is asympotically stable in Ω_{sr}. □

The switching control (2.18) may be unable to suppress oscillations fully during the transients. A combination of control (2.18) and (2.9) may result in asymptotic stability at large (in Ω_{sr}) and the local existence of a sliding mode behaviour such that the oscillations are eliminated.

Proposition 2.4: *If the switching control strategy is*

$$u = \begin{cases} u_{sr} & \textit{if } \delta \in \Omega_{sr} \backslash \Omega_{ss} \\ u_{ss} & \textit{if } \delta \in \Omega_{ss} \end{cases} \tag{2.20}$$

where $\Omega_{sr} \backslash \Omega_{ss}$ *represents the region of* Ω_{sr} *that excludes the overlapping region* *between Ω_{sr} and Ω_{ss}, then the system* (2.5) *is asymptotically stable and exhibits a sliding mode behaviour on* $s = 0$ *in the region* Ω_{ss}.

Proof: For $\delta \in \Omega_{sr} \backslash \Omega_{ss}$, that is, for δ in the stability region Ω_{sr} that excludes the existence region of sliding mode Ω_{ss}, the time derivative of $V(\delta)$ is actually $\dot{V}(\delta) \leq 0$. So from Proposition 2.3, the system state converges towards the equilibrium point $(\delta_s^0, 0)^T$. The asymptotic stability is guaranteed in the region $\Omega_{sr} \backslash \Omega_{ss}$.

The reachability of the existence region of sliding mode Ω_{ss} is guaranteed by the fact that outside the existence region the system trajectories are composed of portions of closed orbits for either $u = 0$ or $u = 1$, resulting in a spiralling trajectory towards the equilibrium point. Whenever the boundaries of the existence region of sliding mode Ω_{ss} are crossed, from Proposition 2.1, the trajectories will be ruled by u_{ss} and they will follow curves of types either $A'A''$ and $B'B''$ or $C'C''$ and $D'D''$ as depicted in Fig. 4, thus resulting in either reaching the equilibrium point or hitting the sliding line. When system trajectories cross the switching line, from Proposition 2.1, they will stay on $s = 0$, and a sliding mode occurs.

Note that in Ω_{ss}, for the trajectories following curves of types either $A'A''$ or $D'D''$, although the trajectories do not cross the switching line, they do make $\dot{V} = 0$ or $\dot{V} < 0$. For the curves of type $A'A''$ in the region of $(\sin \delta_1)\, s > 0$, the control is just $u = 0$, and hence $\dot{V}(\delta) = 0$. For the curves of type $D'D''$ in the region of $(\sin \delta_1)\, s < 0$, $u = 1$, and

$$\dot{V}(\delta) = -\frac{E_1 E_2 \varepsilon}{X(X + \varepsilon)} (\sin \delta_1) \delta_2 \tag{2.21}$$

from Fig. 4, $\delta_2 < 0$, therefore $\dot{V} < 0$. A combination of these arcs of trajectories, according to the switching control strategy, results in a spiralling orbit converging towards the equilibrium point. The asymptotical stability in Ω_{sr} towards the equilibrium point $(\delta_s^0, 0)$ is therefore guaranteed. □

The switching control strategy is illustrated in Fig. 5.

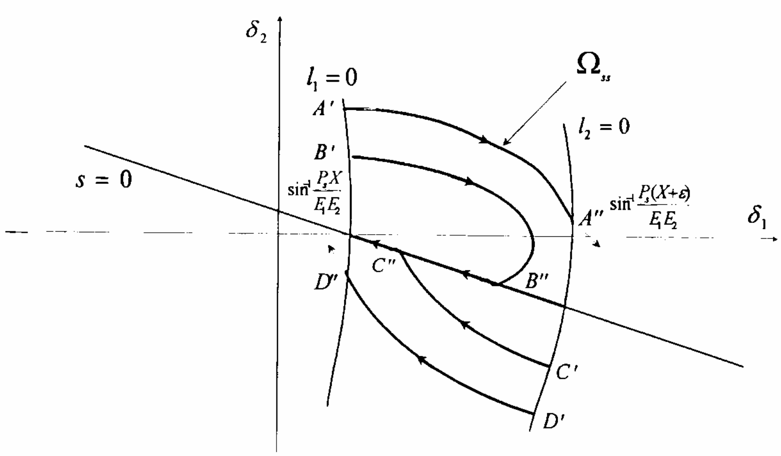

Figure 4. Phase portrait with the existence region of the sliding mode.

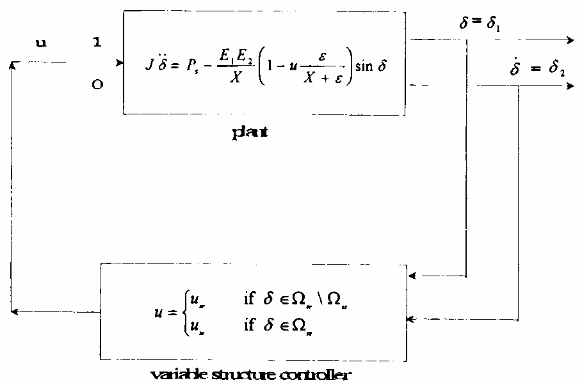

Figure 5. Switching control strategy.

2.4. *Reduced order nonlinear state observer design*

The previous developments assume full availability of the actual state vector components (δ_1, δ_2). In practice, however, it is not advisable to compute the angular velocity δ_2 from sampled (noisy) values of the angular position δ_1. For this reason we first propose a full-order nonlinear state observer based on estimation error exact linearization. Error linearization is achieved by means of nonlinear output injection. Since the full-order observer redundantly estimates the measured variable δ_1, use of its estimated value in the closed loop feedback scheme, rather than its actual measured value, generally results in performance degradation and expensive redundancy. We hence proceed to propose a reduced-order nonlinear state observer that only estimates the angular velocity δ_2. Note that recent developments in the use of Global Positioning System (GPS) signals make direct measurement of δ_1 possible (Pflieger and Clements 1992). The alternative path is to infer the value of δ_1 from measurement of power flow.

Consider the system (2.5) with output function y given by $y = \delta_1$. An asymptotic nonlinear observer with nonlinear output injection may be proposed as follows

$$\left.\begin{aligned} \dot{\hat{\delta}}_1 &= \hat{\delta}_2 + h_1(y - \hat{y}) \\ \dot{\hat{\delta}}_2 &= \frac{P_s}{J} - \frac{E_1E_2}{JX}\left(1 - u\frac{\varepsilon}{X + \varepsilon}\right)\sin y + h_2(y - \hat{y}) \\ \hat{y} &= \hat{\delta}_1 \end{aligned}\right\} \tag{2.22}$$

The estimation error is defined as $e_1 = \delta_1 - \hat{\delta}_1$ and $e_2 = \delta_2 - \hat{\delta}_2$. The estimation error dynamic equations turn out to be perfectly linear and they are given by

$$\left.\begin{aligned} \dot{e}_1 &= e_2 - h_1e_1 \\ \dot{e}_2 &= -h_2e_1 \end{aligned}\right\} \tag{2.23}$$

Evidently, the parameters h_1, h_2, which are constant observer gains, are to be chosen according to a desired (linear) asymptotically stable behaviour of the estimation error. To facilitate such a choice, note that the average angular reconstruction error $e = e_1 = \delta - \hat{\delta}_1$ satisfies the following second-order linear unforced equation

$$\ddot{e} + h_1\dot{e} + h_2 e = 0 \tag{2.24}$$

The observer gains h_2 and h_1 can therefore be suitably chosen to guarantee a non-oscillatory desirable exponential decay of the reconstruction error to zero.

In order to avoid the construction of a second-order nonlinear estimator, such as the one developed above, and in order to be able to use directly, for feedback purposes, the knowledge of the actually measured angular position, one frequently resorts to a reduced-order esimator. In the case at hand, all the state reconstruction effort is to be concentrated on obtaining an estimate of the angular velocity variable δ_2.

If δ_1 is precisely known, one may utilize the first differential equation of the second-order system (2.5) as a pseudo measurement equation; i.e. one considers the equation $y_a = \delta_2 = \dot{\delta}_1$ as an auxiliary linear output equation conveying some information on δ_2. A reduced-order observer could then be designed as

$$\dot{\hat{\delta}}_2 = \frac{P_s}{J} - \frac{E_1 E_2}{JX}\left(1 - u\frac{\varepsilon}{X + \varepsilon}\right)\sin y + h(y_a - \hat{\delta}_2) \tag{2.25}$$

Note that, the reconstruction error $e_2 = \delta_2 - \hat{\delta}_2$ satisfies

$$\dot{e}_2 = -h(y_a - \hat{\delta}_2) = -h(\dot{\delta}_1 - \hat{\delta}_2) = -he_2 \tag{2.26}$$

The gain h should be chosen as a positive constant in order to guarantee asymptotic exponential reconstruction of δ_2. The observer (2.24) is, however, not feasible due to the lack of availability of the $\dot{\delta}_1$ signal. Define, then, an auxiliary variable z as $z = \hat{\delta}_2 - hy = \hat{\delta}_2 - h\delta_1$ so that $\dot{z} = \dot{\hat{\delta}}_2 - hy_a$. Evidently $\hat{\delta}_2 = z + hy$. Using these expressions on the above observer equation one immediately obtains a reduced order nonlinear dynamic observer as

$$\left.\begin{aligned} \dot{z} &= \frac{P_s}{J} - \frac{E_1 E_2}{JX}\left(1 - u\frac{\varepsilon}{X + \varepsilon}\right)\sin y - hz - h^2 u \\ \hat{\delta}_2 &= z + hy \end{aligned}\right\} \tag{2.27}$$

Note that the estimation error equations for either the full order or the reduced order observer, are totally independent of input signal u. The output injection term is now obtained in terms of δ_1 rather than its average value δ_1. Figure 6 shows the complete variable structure control scheme using a reduced order observer.

3. Simulation results

For the simulation studies of the switching control strategy (2.20) for the simple power system (2.5), the following values of parameters were chosen in p.u.

$$P_s = 1, \quad J = 0{\cdot}06, \quad X = 0{\cdot}5, \quad E_1 = 1, \quad E_2 = 1, \quad \varepsilon = 0{\cdot}2 \tag{3.1}$$

The sampling rate was set to 10 ms to model the fact that the thyristor firing can only occur once every 10 ms for a 50 Hz power system. The values of $X = 0{\cdot}5$ and $\varepsilon = 0{\cdot}2$ correspond to effective line reactances varying in the range of 0·5 to 0·7 p.u.. The steady-state operating point $u = 0$ is chosen to give the minimum

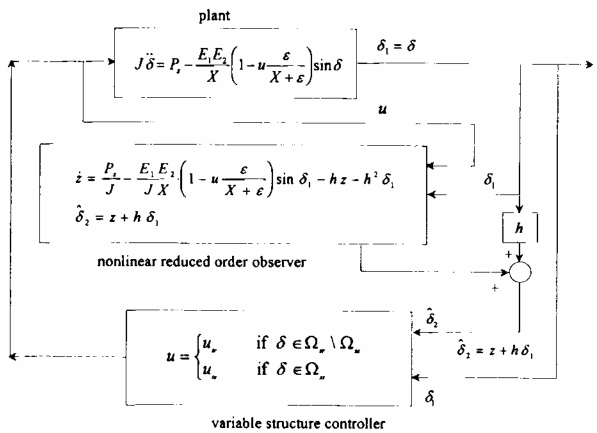

Figure 6. Switching control strategy for the power system stabilization problem including a reduced-order observer.

effective line reactance and hence the minimum prefault angle, thus minimizing the first swing angle for a given disturbance.

Figure 7 gives the simulation results showing the effectiveness of the switching control strategy (2.20) with the parameters for the ideal sliding mode

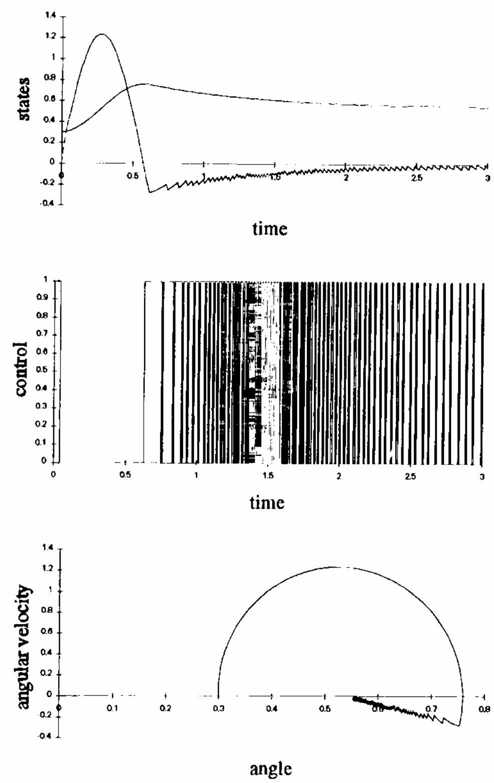

Figure 7. The power system with control u for initial state $\delta_1(0) = 0{\cdot}3$, $\delta_1(0) = 0$.

chosen as $\lambda = 1$. The initial state was chosen as $\delta_1(0) = 0{\cdot}3$, $\delta_2(0) = 0$. The equilibrium state is $(0{\cdot}5236, 0)$ and the sliding mode is achieved without saturation of the control signal. This is due to the fact that the initial state is within the existence region of the sliding mode. The system trajectory does not appear oscillatory except for some chattering, because of the relatively slow sampling frequency.

Figure 8 illustrates the simulation results showing the performance of the switching control strategy (2.20) with the same data as for Fig. 7 but with a different initial state $\delta_1(0) = 0{\cdot}1$, $\delta_2(0) = 0$. The system trajectory first travels in Ω_{sr} outside the existence region of the sliding mode, showing oscillations, but eventually enters the Ω_{ss} and converges to the equilibrium.

Figure 9 shows the results of the system with the switching control strategy (2.20) using the asymptotical nonlinear observer (2.22). The data were the same

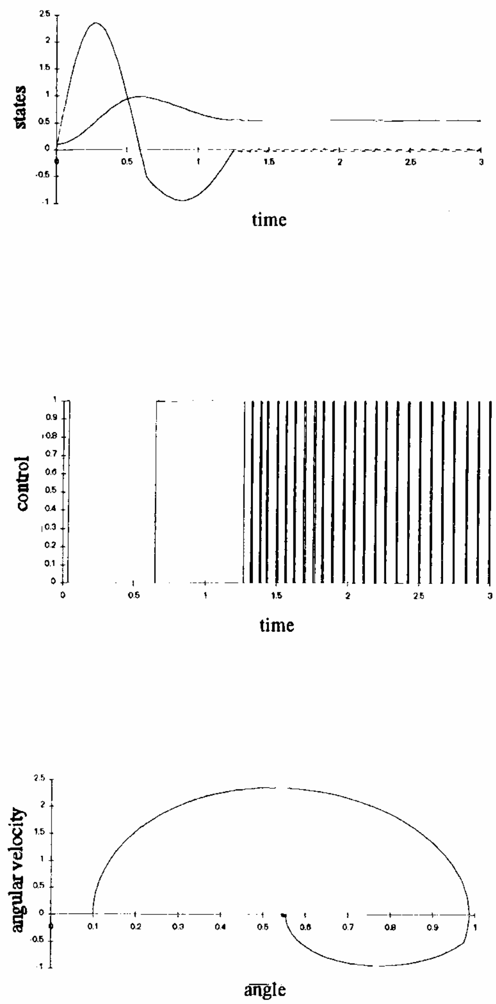

Figure 8. The power system with control u for initial state $\delta_1(0) = 0{\cdot}1$, $\delta_1(0) = 0$.

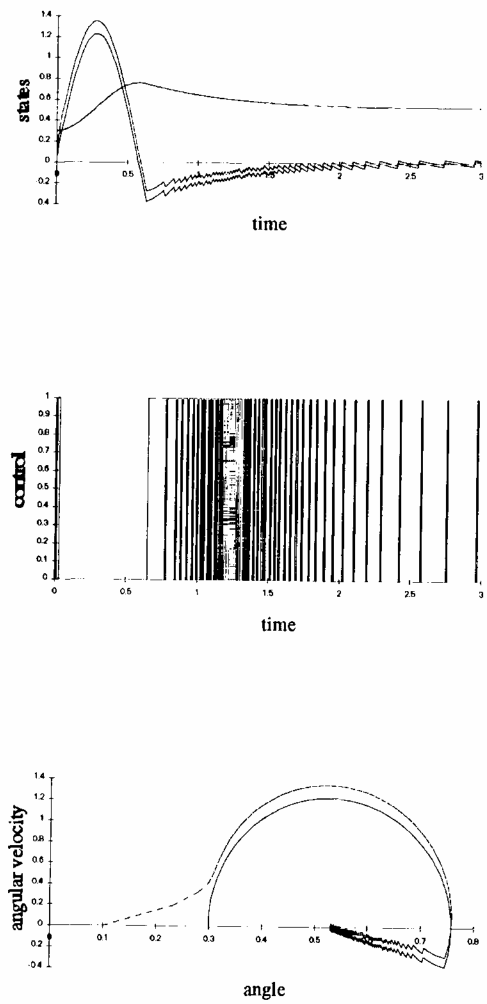

Figure 9. The power system with switching control u using the nonlinear observer for initial state $\delta_1(0) = 0{\cdot}3$, $\delta_2(0) = 0$, $\hat{\delta}_1(0) = 0{\cdot}2$, $\hat{\delta}_2(0) = 0$.

as for Fig. 7 with the parameters for the observer chosen as $h_1 = 9$, $h_2 = 6$. The inital state of the observer was chosen as $\hat{\delta}_1(0) = 0{\cdot}2$, $\hat{\delta}_2(0) = 0$. The trajectory approaches the equilibrium state $(0{\cdot}5236, 0)$ on the sliding line.

Figure 10 depicts the simulation results for the system with the switching control strategy (2.20) using the reduced observer (2.27). The data were the same as for Fig. 7. The parameter for the reduced observer was chosen as $h = 4$ with the initial state of the observer $\hat{\delta}_2(0) = 1{\cdot}2$. The system with the reduced-order observer performs well except for a little chattering.

However, from Figs 7–10 one can see that because of the relative slow sampling frequency, chattering occurs. To overcome the chattering problem, we can either increase sampling frequency or introduce a small disc around the equilibrium in which a linear control can be adopted to eliminate chattering. When the sampling frequency has to be low, introduction of a small disc around

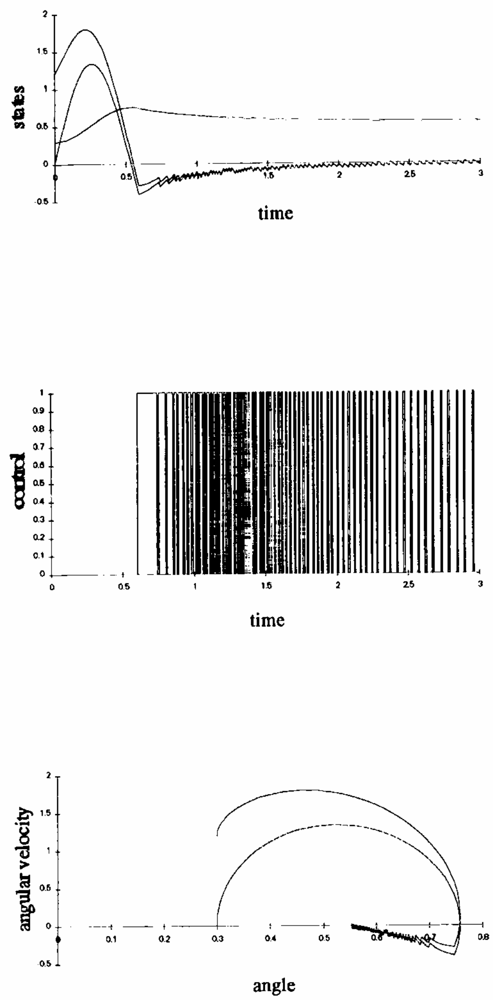

Figure 10. The power system with switching control u using the reduced nonlinear observer for initial state $\delta_1(0) = 0{\cdot}3$, $\delta_2(0) = 0$, $\hat{\delta}_2(0) = 1{\cdot}2$.

the equilibrium in which a linear control is used is quite acceptable in practice. In our case, for the power system (2.5), its linearized model around the equilibrium δ_s^0 is

$$\begin{aligned} \dot{\delta}_1 &= \delta_2 \\ \dot{\delta}_2 &= \frac{P_s \varepsilon}{J(X + \varepsilon)} u \end{aligned} \tag{3.2}$$

A linear control $u = (J(X + \varepsilon)/(P_s \varepsilon))(-k_1(\delta_1 - \delta_s^0) - k_2 \delta_2)$ with suitable k_1, k_2 will stabilize the system.

Figure 11 illustrates the effect of the introduction of a small disc $(\delta_1 - \delta_s^0)^2 + \delta_2^2 < 0{\cdot}025$. The initial state is $(0{\cdot}7, 0)$, and $k_1 = 1$, $k_2 = 2$ such that the eigenvalues of the linearized system are -1, -1. When the system trajectory

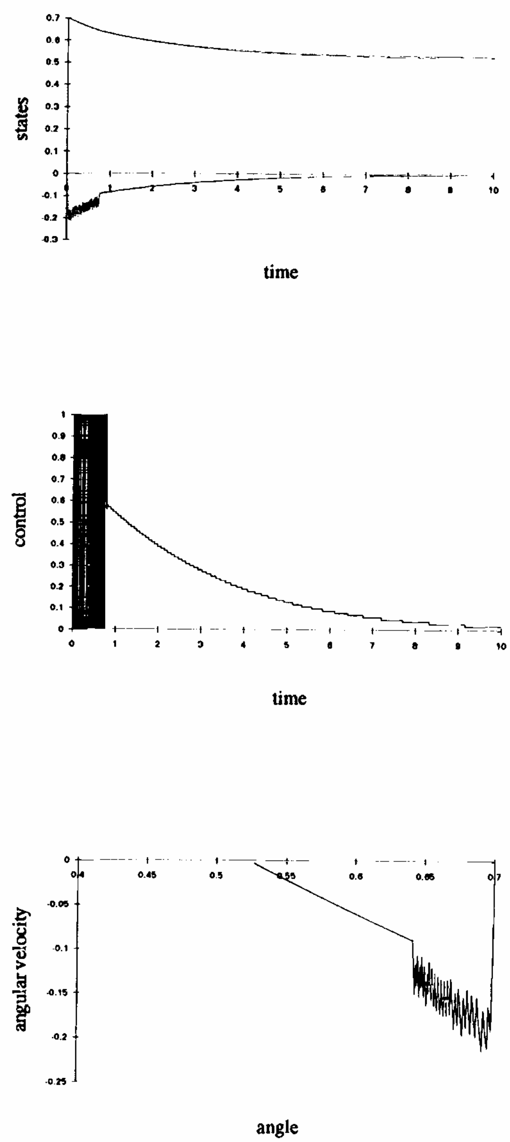

Figure 11. The power system with switching control u associated with a linear control in a small disc around the equilibrium for initial state $\delta_1(0) = 0{\cdot}7$, $\delta_2(0) = 0$.

enters the disc along the switching line, the linear control (which is designed so that $0 < u < 1$) is activated, and the trajectory converges towards the equilibrium smoothly.

4. Conclusions

In this article, a switching control strategy has been proposed for the stabilization of a simple power system consisting of a a.c. generator and a single transmission line. The feedback strategy is constituted by the pulse addition of inductive impedance to the transmission line through a suitable arrangement of thyristors. It has been proved that there only exists a local sliding mode. However, the proposed control strategy guarantees the asymptotic stability in the stability region as well as the local sliding mode. The design of full order, and reduced order, nonlinear observers has been proposed as a means of avoiding questionable swing angle velocity measurements, or computations based on noisy angle trajectory measurements.

Discontinuous feedback control techniques enjoy great robustness advantages and simplicity of implementation when compared with more traditional feedback controller design schemes. In forthcoming publications we shall extend the methodology developed to a multimachine system stabilization problem.

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