CONTROL-THEORY AND ADVANCED TECHNOLOGY
Vol. 10, No. 4, Part 3, pp. 1297–1316, September, 1995

C93120R
©MITA PRESS

Contributed Paper

A SLIDING MODE CONTROL APPROACH TO PREDICTIVE REGULATION*

H. SIRA-RAMÍREZ[1] AND M. FLIESS[2]

Abstract. In this article, a sliding mode feedback control scheme of dynamic nature is proposed as an efficient alternative to deal robustly with the intrinsic tracking problem associated with every Model Based Predictive Control strategy. The results, which are fundamentally based on system inversion, apply to nonlinear single-input single-output perturbed systems for which the tracking of a prespecified desirable output reference signal is required. The scheme is shown to handle efficiently large modeling errors and unmatched perturbation input.

Key Words—Predictive control, sliding mode control, dynamical feedback.

1. Introduction

The Model Based Predictive Control (MBPC) technique has received sustained attention, from both a theoretical as well as an applied viewpoint, ever since it was introduced by Richalet et al. (1978), fifteen years ago. The technique has been developed over the years by many authors, specially by Clark et al. (1987 a; b), Richalet (1990), Richalet et al. (1987) and Bitmead et al. (1991). The technique has received fundamental impetus towards its applicability in the chemical process industry by the research efforts of Morari (1993), Morari and Lee (1991), Morari et al. (1993) and García et al. (1992). On the theoretical side, extensions to the nonlinear case, in fruitful combination with the concept of system inversion, have been presented by Abu el Ata-Doss and Fliess (1989) and Abu el Ata-Doss et al. (1992). A recent book on the subject is that by Soeterboek (1992). More recently, interesting developments, related to nonlinear optimal control theory, have been presented in a series of works by Mayne and Michalska (1990 a; b; 1991).

Sliding mode control has been traditionally recognized as a high performance control technique with outstanding robustness features for both system stabilization and output tracking problems. The degree of development of the theoretical aspects of sliding mode control are well documented in the many books and research articles that have been written on the subject. The interested reader is referred to the books by Emelyanov (1969), Utkin (1978; 1992), Slotine and Li

* Received by the editors October 7, 1993 and in revised form May 11, 1994.
This research was supported by the Group d'Automatique et Robotique Industrielle (GARI) of the Institut National des Sciences Appliquées (I.N.S.A.) of Toulouse (France), by the Consejo de Desarrollo Científico, Humanístico and Tecnológico of the Universidad de Los Andes (Venezuela), under Research Grant I-358-91 and by the Laboratoire des Signaux et Systèmes of the Centre National de la Recherche Scientifique (CNRS), France.

[1] Departamento Sistemas de Control, Universidad de Los Andes, Mérida 5101, Venezuela.
[2] Laboratoire des Signaux et Systèmes, C.N.R.S. - E.S.E., Plateau du Moulon, 91192 Gif sur Ivette Cedex, France.

(1991), Zinober (1990; 1994) and Bühler (1986). A collection of articles of Sliding Mode Control, which indicates recent research trends and contains detailed surveys of the area, can be found in a recent Special Issue of the *International J. of Control*, edited by Prof. V.I. Utkin (1993).

In this article, we develop an approach that uses an advantageous combination of dynamical sliding mode control (see Sira-Ramírez, 1992 a) and input-output system inversion (Fliess, 1989) in MBPC schemes. These techniques naturally blend together to yield a robust solution to the nonlinear output tracking problem associated with any predictive control scheme and defined within a prespecified prediction interval. Initial steps in this direction have also been taken by Sira-Ramírez and Fliess (1993).

In Sec. 2 of this article, we present a general description of the Predictive Functional Control problem, using a dynamical, i.e., chattering-free, sliding mode control approach. The results in this section are based on a fundamental robustness result for discontinuously controlled scalar perturbed systems. The developments follow quite closely those found in Sira-Ramírez and Fliess (1993). Section 3 contains some illustrative examples along with digital computer simulations. One of the examples deals with the application of the synthesized predictive control scheme to an actual system that differs substantially from the model used. The sliding controller performs well even if control discontinuities have to be imposed in order to avoid inversion singularities. A second example includes the design of a predictive dynamical sliding mode controller for a nonlinear model of a field controlled d.c. motor. The robustness of the design is tested by the addition of unmodeled and unmatched, computer-generated, white noise perturbations. Section 4 presents conclusions and suggestions for further work in this area.

2. Robust Predictive Control via a Dynamical Sliding-mode Strategy

In this article, using techniques developed for dynamical sliding modes (see Sira-Ramírez, 1992 b; 1993), we propose an MBPC scheme based on dynamical sliding mode control and system inversion. In our setting, a prespecified smooth output reference trajectory $y_R(t)$ is prescribed as a desirable future output trajectory for a given nonlinear perturbed system. Our developments consider that the available system model has been placed in Generalized Observability Canonical Form (GOCF) (see Fliess, 1990). A desirable, asymptotically stable, reduced order linear tracking error dynamics is then proposed in a manner canonically determined by the order of the system model and standard asymptotic stability requirements. The specified linear tracking error dynamics ideally results in an asymptotic tracking of the proposed output trajectory within the current prediction interval. The prescribed reduced tracking error dynamics, in turn, uniquely specifies a linear algebraic relation between the phase variable coordinates in the error space. Such a relation yields the unambiguous definition of a suitable tracking error stabilizing sliding surface. A dynamical sliding mode controller, computed from the unperturbed system by means of standard system inversion techniques, is then proposed. The resulting variable structure control strategy is guaranteed to create a sliding regime on the computed sliding surface, in spite of all assumed realizations of the modeled perturbation signal. A model-based, robust asymptotically stable output tracking error may hence be obtained for the current prediction horizon.

Generally speaking, after application of the computed sliding mode policy to the actual system, noticeable tracking error discrepancies may be found between the available model and the actual plant performances evaluated at the end of the prediction interval. Under such circumstances, for the next prediction interval, the initial conditions of the reduced order prediction tracking error model must be reset according to the obtained actual system tracking error values. Also, the desired output reference trajectory, for the new horizon, may be suitably modified on the basis of the obtained tracking discrepancy. The new prediction horizon can be specified on the basis of the reaching time to the desired sliding surface and the slowest eigenvalue of the imposed linear output tracking error dynamics. The model-based control computation, its actual implementation and the performance reassessment process are then systematically repeated at the end of each proposed prediction interval. Reference trajectory modification, according to the obtained tracking error discrepancy between the prediction model and the plant may also be based on systematic procedures arising from the *self-compensation principle* (Richalet et al., 1987).

2.1 A sliding mode control result for scalar perturbed systems

The following result will be useful in the developments of this section (see also the Appendix).

Proposition 2.1. Let W and N represent strictly positive quantities, and let "sign" stand for the signum function. Suppose v is a scalar bounded perturbation signal such that $|v| \leq N$. Then, the perturbed scalar discontinuous system,

$$\dot{w} = v - W \operatorname{sign} w, \tag{2.1}$$

globally exhibits a sliding regime (Utkin, 1978) on $w = 0$, provided $W > N$. Furthermore, any trajectory starting on the initial value $w = w(0)$, at time $t = 0$, reaches the condition $w = 0$ in finite time T_r. An estimate of the reaching time T_r is given by

$$T_r \leq \frac{|w(0)|}{W - N}. \tag{2.2}$$

Proof. Immediate upon checking that globally, $w(dw/dt) < 0$, whenever $w \neq 0$ and $W > N$. This is a well known condition for the existence of a sliding regime (Utkin, 1978). The estimation of the reaching time in (2.2) is immediate upon integration of (2.1) and consideration of the most disfavorable perturbation case.

2.2 Predictive functional control via dynamical sliding modes Consider a nonlinear n-dimensional single-input single-output dynamical system, expressed in GOCF (Fliess, 1989),

$$\left.\begin{aligned} \dot{\eta}_1 &= \eta_2 \\ \dot{\eta}_2 &= \eta_3 \\ &\cdots \\ \dot{\eta}_n &= c(\eta, u, \dot{u}, \cdots, u^{(\alpha)}) + v \\ y &= \eta_1 \end{aligned}\right\}, \tag{2.3}$$

where the scalar signal function v comprises all known information about external bounded perturbation signals and an assessment of possible modeling errors. Note that a particular advantage of the GOCF is that perturbation signals are always *matched* with respect to the highest derivative of the control input, $u^{(\alpha)}$, which is taken as the effective control input signal in any dynamical feedback regulation scheme. This fact avoids the need for complying with the well known *matching conditions*, set almost twenty-five years ago in the work of Drazenovic (1969).

The integer α in (2.3) is considered to be a strictly positive integer. For systems which are *exactly input-output linearizable*, i.e., where $\alpha = 0$ (see Isidori, 1990), the same developments presented here are still applicable, except that, in order to obtain our proposed chattering-free responses, a first, or higher, order *dynamical extension* of the system becomes necessary (the concept of dynamical extension can be found in the book by Nijmeijer and van der Schaft (1990)).

The signal v is assumed to satisfy

$$\sup|v| \leq N. \tag{2.4}$$

Let $y_R(t)$ be a prescribed reference output function, assumed to be sufficiently smooth and defined over a given prediction interval $[0, T_p]$. Such an interval is determined below in Sec. 2.3.

Define a tracking error function, $e(t)$ as the difference between the actual system output, $y(t)$ and the output reference signal, $y_R(t)$,

$$e(t) = y(t) - y_R(t). \tag{2.5}$$

We then have

$$\left.\begin{aligned} e^{(i)}(t) &= \eta_{i+1} - y_R^{(i)}(t), \quad 0 \leq i \leq n-1 \\ e^{(n)}(t) &= \dot{\eta}_n - y_R^{(n)}(t) = c(\eta, u, \cdots, u^{(\alpha)}) - y_R^{(n)} + v \end{aligned}\right\}. \tag{2.6}$$

Defining $e_i = e^{(i-1)}$ $(i = 1, 2, \cdots, n)$ as components of an error vector $\boldsymbol{e}$, we may also express the tracking error system (2.5), (2.6) in GOCF as

$$\left.\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= e_3 \\ &\cdots \\ \dot{e}_n &= c(\boldsymbol{e} + \xi_R(t), u, \dot{u}, \cdots, u^{(\alpha)}) - y_R^{(n)}(t) + v \\ e &= e_1 \end{aligned}\right\} \tag{2.7}$$

with

$$\left.\begin{aligned} \xi_R(t) &= \operatorname{col}(y_R(t), y_R^{(1)}(t), \cdots, y_R^{(n-1)}(t)) \\ \boldsymbol{e} &= \operatorname{col}(e_1, e_2, \cdots, e_n) \end{aligned}\right\}. \tag{2.8}$$

The model-based predictive controller synthesis entitles the unambiguous specification of the (desired) system output tracking error, $e(t)$ within the specified prediction horizon $[0, T_p]$. This task is easily accomplished by prescribing a reduced order linear dynamical tracking error behavior, which is known to asymptotically converge to zero; i.e., we specify the desirable tracking error dynamics as

$$\left.\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= e_3 \\ &\cdots \\ \dot{e}_{n-1} &= -m_{n-1}e_{n-1} - \cdots - m_1 e_1 \\ e &= e_1 \end{aligned}\right\}, \tag{2.9}$$

where the set of real coefficients $\{m_1, \cdots, m_{n-1}\}$ is such that the following (characteristic) polynomial, in the complex variable "s", is Hurwitz:

$$p(s) = s^{n-1} + m_{n-1}s^{n-2} + \cdots + m_2 s + m_1 . \tag{2.10}$$

We denote by μ the smallest real part, in absolute value, of all the complex stable roots of the polynomial equation $p(s) = 0$, associated with (2.10). The parameter μ actually represents the smallest time constant associated with the asymptotical exponentially stable decay of the controlled tracking error response, under *ideal* sliding mode conditions (see Utkin, 1978). Such a design parameter is used in the computation of the current prediction interval $[0, T_p]$ and, evidently, it may be specified *a priori* during the design stage (see also Sec. 2.3).

The prescription of the desired linear tracking error dynamics (2.9), in turn, uniquely specifies a corresponding *sliding surface coordinate function* on the output tracking error phase space of the adopted model. In order to achieve such a desirable tracking error dynamics, the coordinate e_n must satisfy, according to (2.7) and (2.9),

$$e_n = -m_{n-1}e_{n-1} - \cdots - m_1 e_1 . \tag{2.11}$$

Motivated by this requirement, we next define an auxiliary scalar output variable w, in terms of the output tracking error coordinates e_i $(i = 1, \cdots, n)$ as

$$w = e_n + m_{n-1}e_{n-1} + \cdots + m_1 e_1 . \tag{2.12}$$

Note that if the auxiliary output function w is driven to zero by means of a suitable control action, say, in finite time, then the desired error dynamics, specified in (2.9), is accomplished, and asymptotic exponential stability of the tracking error towards zero is obtained.

A dynamical discontinuous controller inducing a robust sliding motion on the zero level set of the proposed sliding surface, $w = 0$, may be found by standard *system inversion* performed on the unperturbed version of system (2.7) (i.e., by setting $\nu = 0$). Consider, then, the following dynamical feedback controller in terms of an implicit ordinary differential equation with discontinuous right-hand side,

$$c(\xi_R + \boldsymbol{e}, u, \dot{u}, \cdots, u^{(\alpha)}) - y_R^{(n)} + \sum_{i=1}^{n-1} m_i e_{i+1} = -W \operatorname{sign}\left(\sum_{i=1}^{n} m_i e_i\right). \tag{2.13}$$

It easily follows, by taking the time derivative in (2.12) and using (2.7), that the controller (2.13) determines the following evolution of the auxiliary output function w:

$$\dot{w} = \nu - W \operatorname{sign}\left(\sum_{i=1}^{n} m_i e_i \right) = \nu - W \operatorname{sign} w \,. \tag{2.14}$$

According to the result of Proposition 2.1, the controlled values of w go to zero in finite time, and a sliding regime can be indefinitely sustained on the condition $w = 0$ provided $W > N$.

A truly *variable structure controller* is obtained from (2.13) since on each one of the regions, $w > 0$ and $w < 0$, a different *dynamic* feedback controller "structure" acts on the regulated system. The corresponding implicit differential equation (2.13) is to be independently solved for the controller u, on the basis of knowledge of the predicted error vector $\boldsymbol{e}$ and the vector of future desired output time derivative functions $\xi_R(t)$, computed, in turn, from knowledge of the future output reference trajectory $y_R(t)$. In light of the additional assumption that, locally, $\partial c / \partial u^{(\alpha)}$ is non zero in (2.13), then no singularities, of the *impasse* points type need be locally considered (Abu el Ata-Doss et al., 1992). If singularities do arise, they may be handled by the introduction of appropriate discontinuities on the dynamical controller output u (see Example 3.1).

Note that for $\alpha \geq 1$, the obtained sliding mode controller output u is actually *continuous*, rather than bang-bang. This result is nontypical in sliding mode control, where traditionally, bang-bang inputs and its associated *chattering* output responses are usually obtained (See Utkin, 1978).

After convergence to zero of the output tracking error, the dynamical controller exhibits the following *remaining dynamics*:

$$c(\xi_R, u, \dot{u}, \cdots, u^{(\alpha)}) = y_R^{(n)} \,. \tag{2.15}$$

It is assumed that the nonlinear time-varying dynamics (2.15) is globally stable for the given desired output reference function $y_R(t)$. The dynamics (2.15) is, evidently, coincident with the *zero dynamics* (see Fliess, 1990; Isidori, 1990) for those cases in which the desired value of the output function $y_R(t)$ is identically zero, or a given constant. In such cases, our previous assumption implies that the given system is locally, or globally, *minimum phase* (Isidori, 1990; Nijmeijer and van der Schaft, 1990). In this last class of systems, an asymptotically stable response is obtained, as a solution of (2.15) for the control input u, towards a stable equilibrium value.

2.3 The prediction interval The above procedure is evidently based on the validity of the available mathematical model for the system. Such a mathematical model, as usual, may be at variance with respect to the actual system behavior. In using the predictive dynamical discontinuous controller on the actual system, one may generally obtain, at the end of the prediction horizon, a nonzero tracking error, or a nonzero sliding surface coordinate function value. These nonzero values are unknown functions of the model mismatch. The predictive control technique then proposes a number of procedures for obtaining an improvement, in the actual closed loop system behavior, for the next prediction interval, $[T_p, T_p']$ (see Richalet, 1990; Richalet et al., 1978; 1987).

A reasonable choice for the setting of the new prediction horizon $[T_p, T_p']$ may be devised as

$$T_p' = \frac{|w(T_p)|}{W-N} + \frac{2}{\mu}; \tag{2.16}$$

i.e., the new prediction interval is comprised of the reaching time to the sliding surface, $w = 0$, computed from the sliding surface value based on the new reset tracking error initial conditions (see Eq. (2.12)) plus *twice* the slower time constant of the imposed linear error dynamics (this choice, roughly speaking, guarantees, at the end of the new prediction horizon, a theoretical decrease in absolute value of the slowest tracking error mode to about 13[%] of its initial value at the hitting of the proposed sliding surface).

The process described next is systematically repeated at the end of each prediction interval. Such process entitles:

1. Assessment of the actual values of the tracking error or of the proposed stabilizing sliding surface.
2. Re-initialization of the desirable error dynamics in accordance with the obtained actual tracking error performance (this step may include a redesign of the parameters defining the desired tracking errors).
3. Calculation of the new prediction interval and, by direct system inversion techniques, calculation of the required sliding mode control policy.
4. Implementation of the reassessed control policy and monitoring of the obtained response during the adopted planning horizon.

3. Some Illustrative Examples

In this section, we consider some illustrative design examples for the control scheme proposed in the preceeding section.

Example 3.1. (An academic example) We take, as our first example, the following nonlinear system, also found in Abu el Ata-Doss et al. (1992):

$$\ddot{y} + \dot{y} + y = u + 20u^2\dot{u}\,. \tag{3.1}$$

The system (3.1) has the following classical state space realization:

$$\left.\begin{aligned} \dot{\xi}_1 &= -\xi_1 + \xi_2 + \frac{20}{3}u^3 \\ \dot{\xi}_2 &= -\xi_1 + u \\ y &= \xi_1 \end{aligned}\right\}. \tag{3.2}$$

We utilize, however, the following GOCF realization of the system:

$$\left.\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1 - x_2 + u + 20u^2\dot{u} \\ y &= x_1 \end{aligned}\right\}. \tag{3.3}$$

It is desired to stabilize the output y of system (3.3) to the constant value $y = 1$.

We define the tracking, or more properly, the stabilization error as $e = y - 1$, i.e., $y_R(t) = 1$, $\forall t$. One then obtains the following error dynamics:

$$\left.\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= -1 - e_1 - e_2 + u + 20u^2\dot{u} \\ e &= e_1 \end{aligned}\right\}. \tag{3.4}$$

The system is evidently *minimum phase* around the equilibrium point $u = 1$ as found from the *zero dynamics* equation

$$20u^2\dot{u} + u - 1 = 0. \tag{3.5}$$

The desirable stabilization error dynamics is given by the reduced order system

$$\dot{e}_1 = e_2 = -m_1 e_1, \quad m_1 > 0. \tag{3.6}$$

The required sliding surface is thus given by

$$w = e_2 + m_1 e_1. \tag{3.7}$$

The corresponding dynamical sliding mode controller is obtained, by straightforward system inversion, from the error equation (3.4) and the imposed sliding dynamics $\dot{w} = -W \operatorname{sign} w$ on the sliding surface coordinate function w. Such a controller is given by

$$\dot{u} = \frac{e_1 + 1 + (1 - m_1)e_2 - u - W \operatorname{sign} w}{20u^2}. \tag{3.8}$$

The dynamic nature of the proposed predictive controller generates a continuous (or bang-bang free) control input signal u. The controller dynamics (3.8) is also seen to exhibit a singularity, or critical point, at $u = 0$. Using the techniques developed in Abu el Ata-Doss et al. (1992), for singularity avoidance, a discontinuity may be induced on the dynamical controller output u when the trajectory of such a generated control variable arrives at a prescribed vicinity of the singular value, $u = 0$.

The avoidance of the singularity point entitles a sudden change of sign of the controller output u and an instantaneous resetting of the initial conditions of the dynamical controller. The discontinuity imposed on the control input results, of course, in a continuous trajectory of the controlled system output y.

Figure 1 shows the controlled phase variable trajectories asymptotically converging towards the required set point. Figure 2 depicts the dynamically generated control input signal u, exhibiting a "jump" around the singular value of zero. Such a discontinuity on the control input trajectory effectively avoids the controller singularity. Figure 3 depicts the evolution of the sliding surface coordinate function w converging, in finite time, to zero. The design parameters for the dynamical sliding mode predictive controller were chosen as $W = 1$ and $m_1 = 2$.

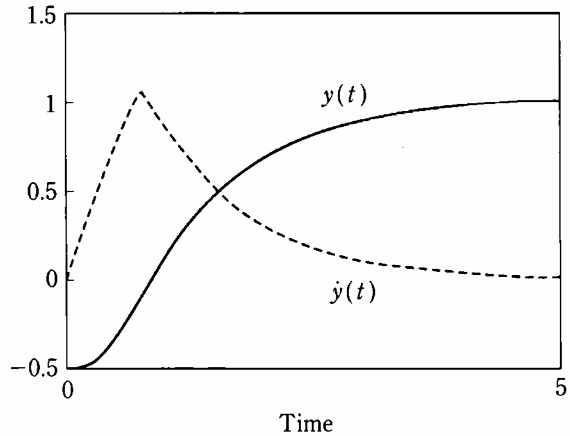


Fig. 1. Dynamical predictive sliding mode controlled trajectories for adapted model.

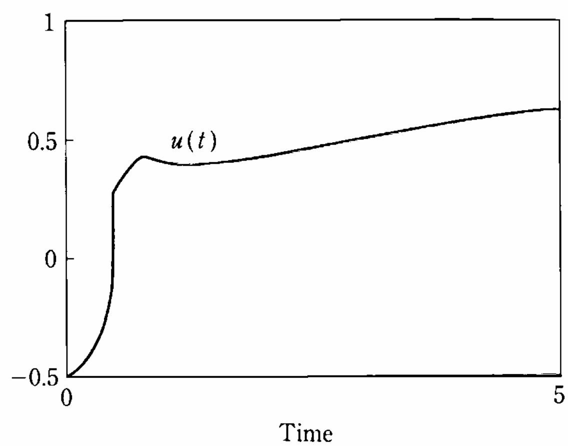


Fig. 2. Dynamically generated bang-bang free control input signal.

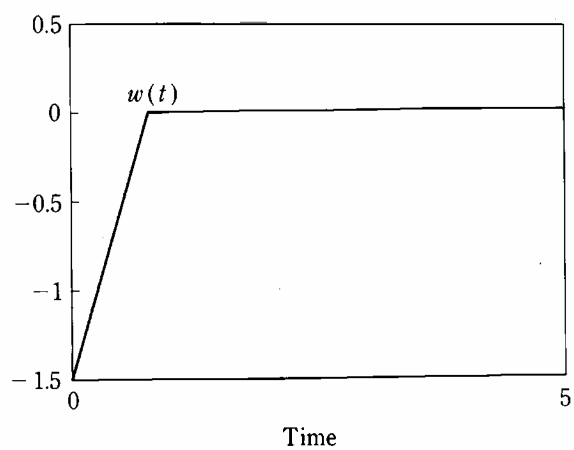


Fig. 3. Evolution of prescribed sliding surface coordinate function.

To test the robustness of the proposed discontinuous predictive feedback control scheme, the previously designed control law was also used on the following (actual) non-adapted model of the given system,

$$\ddot{y} + \dot{y} + y = u + (20u^2 - 2)\dot{u}, \tag{3.9}$$

which, evidently, includes a rather strong "structural" perturbation.

Figure 4 shows a comparison of the controlled output variable trajectories for the adapted and the non-adapted models (the non-adapted model response shown in dashed lines). Figure 5 shows the evolution of the corresponding controlled

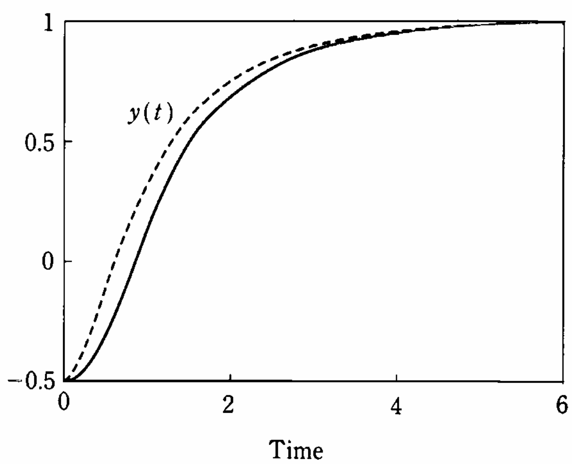


Fig. 4. Dynamical predictive sliding mode controlled output trajectories for adapted and non-adapted models.

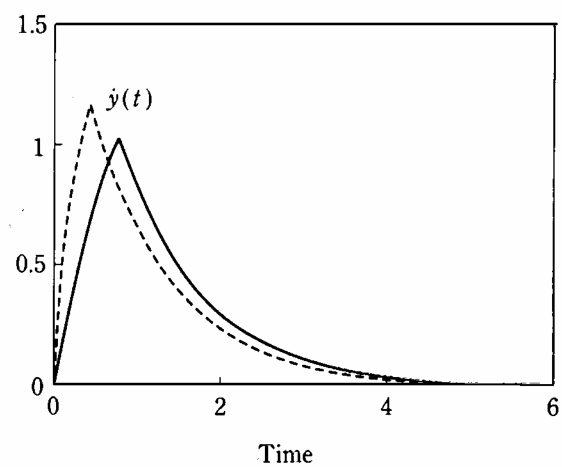


Fig. 5. Dynamical predictive sliding mode controlled output velocities for adapted and non-adapted models.

output velocities for both the adapted and the non-adapted models. The dynamical sliding mode predictive controller is seen to drive the actual system output trajectory y efficiently towards the desired set point value $y = 1$ with the expected zero final velocity. Figure 6 shows the corresponding dynamically generated input signals for both cases. The chosen initial conditions for this second case did not require a singularity avoidance, but the proposed scheme also works as expected when such singularity avoidance is necessary.

The performance of the control scheme is so efficient that no re-design is really necessary after each reassessment of the obtained tracking error trajectory at equally spaced prediction intervals of two units of time.

Example 3.2. (A d.c. motor example) Consider the following nonlinear model of a stator voltage controlled d.c. motor (see Isidori, 1990):

$$\left.\begin{aligned} \dot{x}_1 &= -\frac{R_r}{L_r}x_1 + \frac{V_r}{L_r} - \frac{K}{L_r}x_2 u \\ \dot{x}_2 &= -\frac{F}{J}x_2 + \frac{K}{J}x_1 u \\ y &= x_1 \end{aligned}\right\}, \tag{3.10}$$

where x_1 represents the armature circuit current, and x_2 is the angular velocity of the rotating axis. V_r is the fixed voltage applied to the armature circuit, while u is the field winding input voltage, acting as a control variable. The constants R_r, L_r and K represent, respectively, the resistance, the inductance in the armature circuit and the torque constant. The parameters F and J are the viscous damping coefficient and the moment of inertia associated with the rotor.

An input-output representation of the system is obtained by elimination of

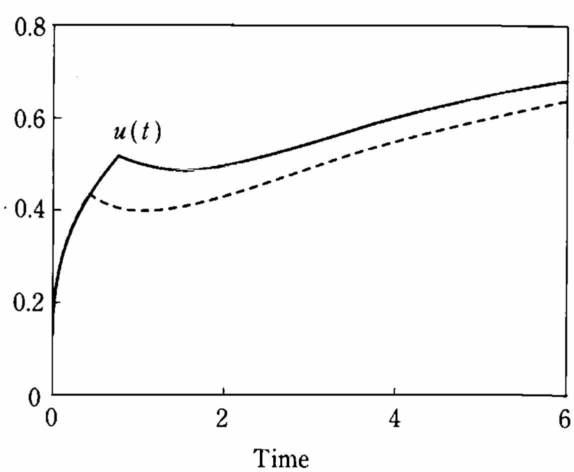


Fig. 6. Dynamically generated bang-bang free control input signal for adapted and non-adapted models.

the state vector (see Diop (1989) for general results):

$$\ddot{y} = -\frac{R_r F}{L_r J} y - \left(\frac{F}{J} + \frac{R_r}{L_r}\right)\dot{y} + \frac{KV_r}{L_r J} u - \frac{K^2}{L_r J} y u^2 + \frac{\dot{u}}{u}\left[\dot{y} + \frac{F}{J} y\right]. \quad (3.11)$$

Suppose it is desired to track a known angular velocity profile or reference trajectory, $y_R(t)$.

The zero dynamics for $y = 0$ degenerates into the algebraic condition $u = 0$. Since a common objective in velocity control is to track reference trajectories that eventually include constant angular velocities, we consider the zero dynamics of the system associated with such constant values of y. Such a zero dynamics is readily obtained after setting the output y to a constant equilibrium value, say Ω, and setting to zero the output derivatives $\dot{y}$, $\ddot{y}$. Hence, one obtains

$$\dot{u} - \frac{R_a}{L_a} u + \frac{KV_r}{\Omega L_r F} u^2 - \frac{K^2}{L_r J} u^3 = 0. \quad (3.12)$$

Aside from the trivial equilibrium point $u = 0$, there exists, for every constant angular velocity Ω two other physically meaningful equilibrium points for the zero dynamics, provided $V_a^2 > 4R_r F\Omega^2$. We denote here such equilibria by $u = U$. The minimum or non-minimum phase nature of a particular equilibrium point $u = U$ depends, respectively, on whether the quantity: $R_r F - K^2U^2$ exhibits a positive or negative value.

A GOCF representation for the tracking error dynamics, with state components defined by $e_1 = e = x_2 - y_R(t)$ and $e_2 = \dot{x}_2 - \dot{y}_R(t)$, is readily obtained as

$$\left.\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= -\frac{R_r F}{L_r J}(e_1 + y_R(t)) - \left(\frac{F}{J} + \frac{R_r}{L_r}\right)(e_2 + \dot{y}_R(t)) \\ &\quad + \frac{KV_r}{L_r J} u - \frac{K^2}{L_r J}(e_1 + y_R(t))u^2 \\ &\quad + \frac{\dot{u}}{u}\left[e_2 + \dot{y}_R(t) + \frac{F}{J}(e_1 + y_R(t))\right] - \ddot{y}_R(t) \end{aligned}\right\}. \quad (3.13)$$

A predictive dynamical sliding mode controller is next designed by considering the sliding surface w as

$$w = e_2 + m_1 e_1. \quad (3.14)$$

Note that such a sliding surface is a nonlinear, time-varying, input-dependent sliding manifold of the form

$$w = -\frac{F}{J} x_2 + \frac{K}{J} x_1 u - \dot{y}_R(t) + m_1(x_2 - y_R(t)). \quad (3.15)$$

Imposing the dynamics $\dot{w} = -W \operatorname{sign} w$ on such a sliding surface coordinate w, one obtains the following dynamical predictive controller by inversion of the tracking error system equations:

$$\dot{u} = \frac{u}{\left[e_2 + \dot{y}_R(t) + \frac{F}{J}(e_1 + y_R(t))\right]}\left\{\frac{R_r F}{L_r J}(e_1 + y_R(t)) + \left(\frac{F}{J} + \frac{R_r}{L_r}\right)(e_2 + \dot{y}_R(t)) - \frac{KV_r}{L_r J}u + \frac{K^2}{L_r J}(e_1 + y_R(t))u^2 + \ddot{y}_R(t) - m_1 e_2 - W \operatorname{sign} w\right\}. \quad (3.16)$$

Simulations were performed for a d.c. motor with the following parameter values:

$$\left.\begin{aligned} &R_r = 7.0\,[\text{Ohm}]; \quad L_r = 120.0\,[\text{mH}]; \quad V_r = 5.0\,[\text{V}] \\ &F = 6.04 \times 10^{-6}\,[N - m - s/\text{rad}] \\ &J = 1.06 \times 10^{-6}\,[N - m - s^2/\text{rad}]; \quad K = 1.41 \times 10^{-2}\,[N - m/\text{A}] \end{aligned}\right\}. \quad (3.17)$$

The following prescribed output trajectory, constituted by a piecewise linear function, was proposed as the velocity profile to be followed by the motor's shaft angular velocity x_2:

$$y_R(t) = \begin{cases} 300\,[\text{rad/sec}] & \text{for} \quad t \le 0.5\,[\text{s}], \\ 300 - 100(t - 0.5)\,[\text{rad/sec}] & \text{for} \quad 0.5\,[\text{s}] < t < 1.5\,[\text{s}], \\ 200\,[\text{rad/sec}] & \text{for} \quad t \ge 1.5\,[\text{s}]. \end{cases} \quad (3.18)$$

In order to test the robustness of the proposed discontinuous predictive control scheme, we devised simulation trials on two unmodeled perturbation input cases, both of them corresponding to the *unmatched* perturbation type. Thus, the above controller was used in combination with the following (actual) non-adapted, perturbed systems:

$$\left.\begin{aligned} \dot{x}_1 &= -\frac{R_r}{L_r}x_1 + \frac{V_r}{L_r} - \frac{K}{L_r}x_2 u + \vartheta(t) \\ \dot{x}_2 &= -\frac{F}{J}x_2 + \frac{K}{J}x_1 u \\ y &= x_1 \end{aligned}\right\}, \quad (3.19)$$

$$\left.\begin{aligned} \dot{x}_1 &= -\frac{R_r}{L_r}x_1 + \frac{V_r}{L_r} - \frac{K}{L_r}x_2 u \\ \dot{x}_2 &= -\frac{F}{J}x_2 + \frac{K}{J}x_1 u + \vartheta(t) \\ y &= x_1 \end{aligned}\right\}, \quad (3.20)$$

where the signal $\vartheta(t)$ was set to be a computer generated, normally distributed, white noise signal. The simulation results in both cases were highly encouraging, and the performance obtained was remarkably robust. We show only the simulation results corresponding to the performance of the system (3.19), controlled by a predictive scheme that includes the dynamic sliding mode regulator (3.16).

Figure 7 shows the angular velocity response of the system in comparison with the desired trajectory (3.18) (shown in dashed lines). In spite of the unmatched nature of the perturbation signal ϑ, the controlled trajectory $y(t)$ follows, quite closely, the required angular velocity profile. Figure 8 depicts the corresponding armature circuit current x_1, while Fig. 9 represents the bang-bang free control input signal u generated by the dynamical sliding mode predictive controller scheme. Figure 10 shows the applied perturbation input signal ϑ and, finally, Fig. 11 depicts the corresponding sliding surface evolution.

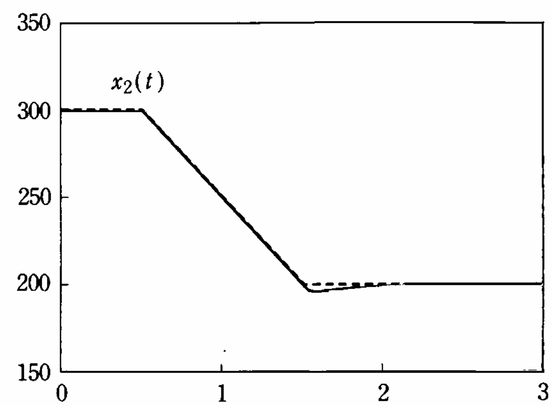


Fig. 7. Angular velocity response of predictive dynamical sliding mode controlled d.c. motor.

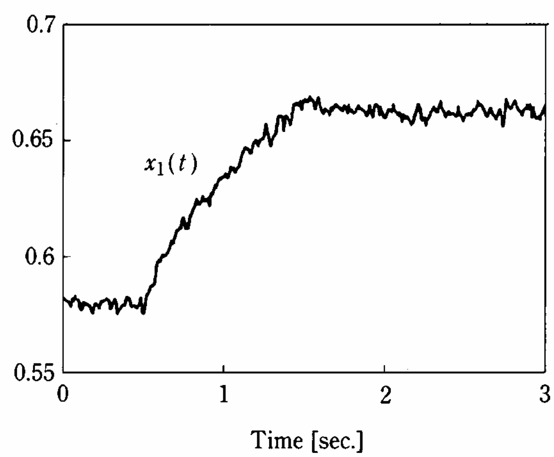


Fig. 8. Armature circuit current response of predictive dynamical sliding mode controlled d.c. motor.

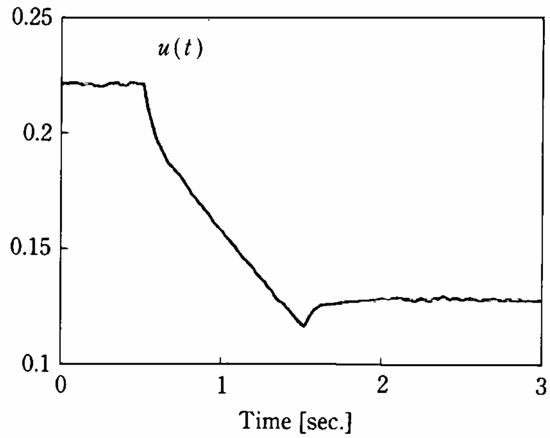


Fig. 9. Dynamically generated bang-bang free control input signal for the d.c. motor example.

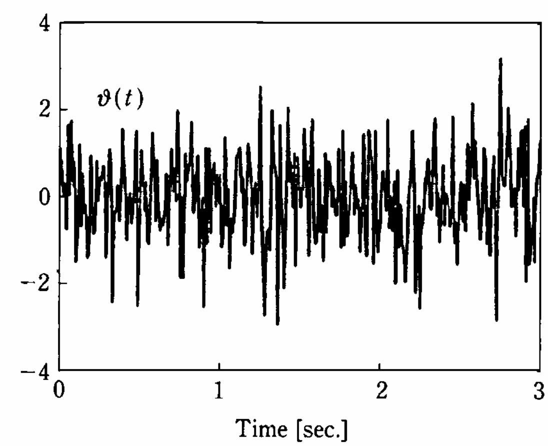


Fig. 10. Unmatched perturbation input signal to the d.c. motor.

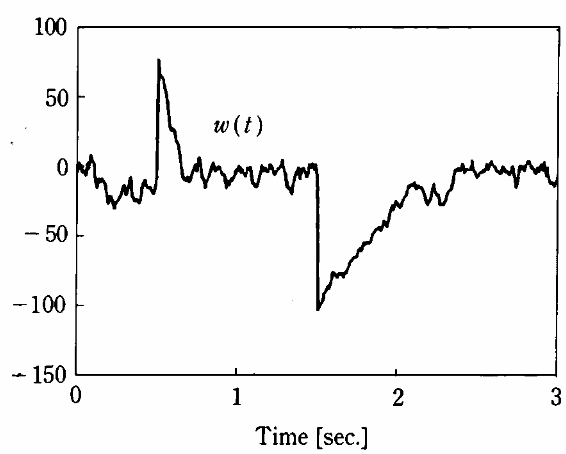


Fig. 11. Evolution of the sliding surface coordinate for predictive dynamical sliding mode controlled d.c. motor.

4. Conclusions

In this article, we have proposed a model-based predictive control scheme which combines the advantages of sliding mode control robustness and its traditional high performance features, with the conceptual simplicity of nonlinear system inversion techniques. The association of both techniques was proven to be particularly suitable for conceptually dealing with the associated output tracking problem present in every predictive feedback control scheme.

The adopted framework of designing sliding controllers via a GOCF of the nonlinear system model, which is, fundamentally, an input-output design approach, results in the possibility of effectively compensating for bounded unmatched uncertainties.

The obtained results may be extended to the case of decouplable, multivariable input-output systems. In this context, research efforts are being directed to relate the approach proposed here with the theory of *differentially flat systems* (see the work of Fliess and his colleagues in Fliess et al. (1992 a; b; 1993)). In forthcoming publications, we will show that the combination of sliding mode control, model-based predictive control and differentially flat systems results in a most natural, and rather general, way to formulate, and design, robust predictive regulators for controllable, nonlinear, multivariable systems.

Two illustrative single-input, single-output examples were presented, in which a dynamical sliding mode control strategy is devised for robust error stabilization on non-adapted systems. The results show the insensitivity of the sliding mode controller to rather large modeling errors and to (unmatched) random perturbation signals.

The results also show that even if control input discontinuities, additional to those already natural to the sliding mode control scheme, are necessary to avoid singularities during the transient performance of the system, the advantageous combination of sliding and predictive control results in an efficient feedback corrective scheme which accomplishes the desired control objectives.

Acknowledgments

The first author gratefully acknowledges the generosity of the *Minstere de la Recherche* of, France for its invitation to visit, during the last half of his sabbatical year 1993, the Groupe d'Automatique et Robotique Industrielle of the *Institut National des Sciences Appliquées de Tolouse*, (*I.N.S.A.*), where this research was carried out. The kindness of Professor Pierre Lopez of the I.N.S.A. is also gratefully acknowledged. The second author would like to acknowledge his friend, the late Salva Abu el Atta for having drawn his attention to the area of model-based predictive control.

References

Abu el Ata-Doss, S. and M. Fliess (1989). Nonlinear predictive control by inversion. *Preprints of IFAC Symposium on Nonlinear Control Systems Design*, Capri, June, 70–75.

Abu el Ata-Doss, S., A. Coc and M. Fliess (1992). Nonlinear predictive control by inversion: Discontinuities for critical behavior. *Int. J. Control*, **55**, 6, 1521–1533.

Bitmead, R., M. Gevers and V. Wertz (1991). *Adaptive Optimal Control.* Prentice-Hall, Englewood Cliffs, N.J.

Bühler, H. (1986). *Réglage par Mode de Glissement.* Presse Polytechnique Romande, Lausanne.

Clark, D.W., C. Mohtadi and P.S. Tuffs (1987 a). Generalized predictive control—Part I. The basic algorithm. *Automatica,* **23**, 2, 137–147.

Clark, D.W., C. Mohtadi and P.S. Tuffs (1987 b). Generalized predictive control—Part II. Extensions and interpretations. *Automatica,* **23**, 2, 148–160.

Diop, S. (1989). Elimination in control theory. *Mathematical Control Signals Systems,* **4**, 1, 17–32.

Drazenovic, B. (1969). The invariance conditions in variable structure systems. *Automatica,* **5**, 3, 287–295.

Emelyanov, S.V. (1969). *Automatische Regelsysteme mit veräderlicher Struktur.* Oldenbourg, München.

Filippov, A.F. (1988). *Differential Equations with Discontinuous Righthand Sides.* Kluwer Academic Publishers, Dordrecht, The Netherlands.

Fliess, M. (1989). Nonlinear control theory and differential algebra. Ch.I. Byrnes and A. Khurzhansky (eds.), *Modeling and Adaptive Control,* Lecture Notes in Control and Information Sciences, **105**, Springer-Verlag, N.Y.

Fliess, M. (1990). What the Kalman state representation is good for. *Proc. of the 29th IEEE Conference on Decision and Control,* **3**, Honolulu, Hawaii, December, 1282–1287.

Fliess, M., J. Lévine, P. Martin and P. Rouchon (1992 a). Sur les systèmes linéaires différentiellement plats. *C.R. Acad. Sci. Paris,* **315**, Serie I, 619–624.

Fliess, M., J. Lévine, P. Martin and P. Rouchon (1992 b). On differentially flat nonlinear systems. *Proc. of the IFAC-Symposium NOLCOS'92,* Bordeaux, 408–412.

Fliess, M., J. Lévine, P. Martin and P. Rouchon (1993). Défaut d'un système non linéaire et commande haute fréquence. *C. R. Acad. Sci. Paris,* **316**, Serie I, 513–518.

García, C.E., D.M. Prett and M. Morari (1992). Model predictive control: Theory and practice — A survey. *Automatica,* **25**, 3, 335–348.

Isidori, A. (1990). *Nonlinear Control Systems.* Second edition, Springer-Verlag, N.Y.

Mayne, D. and H. Michalska (1990 a). An implementable receding horizon controller for stabilization of nonlinear systems. *Proc. 29th IEEE Conference on Decision and Control,* Honolulu, Hawaii, 3396–3397.

Mayne, D. and H. Michalska (1990 b). Receding horizon control of nonlinear systems. *IEEE Trans. Automatic Control,* **AC-35**, 7, 814–824.

Mayne, D. and H. Michalska (1991). Model predictive control of nonlinear systems. *Proc of the American Control Conference,* Boston, 2343–2348.

Morari, M. (1993). Some control problems in the process industry. H.L. Trentelman and J.C. Willems (eds.), *Essays on Control: Perspectives in the Theory and its Applications,* Birkhäuser, Boston.

Morari, M. and J.H. Lee (1991). Model predictive control: The good, the bad and the ugly. Y. Arkun and W.H. Ray (eds.), *Proceedings of the Fourth International Conference on Chemical Process Control,* CPCIV, South Padre Island, TX,

Morari, M., C.E. García, J.H. Lee and D.M. Prett (1993). *Model Predictive Control.* Prentice-Hall, Englewood Cliffs, N.J.

Nijmeijer, H. and A. van der Schaft (1990). *Nonlinear Dynamical Systems.* Springer-Verlag, N.Y.

Richalet, J.A. (1990). Model based predictive control in the context of integrated design. J. Richalet and S. Tzafestas (eds.), *ESPRIT-CIM, Workshop on Computer Integrated Design of Controlled Industrial Systems,* Paris, March.

Richalet, J.A., J.L. Testud and J. Papon (1978). Model predictive heuristic control: Applications to industrial processes. *Automatica,* **14**, 5, 413–428.

Richalet, J.A., S. Abu el Ata-Doss, C. Arber, H.B. Kuntze, A. Jacubash and W. Schill (1987). Predictive functional control. Application to fast and accurate robots. *Proceedings of the 10th IFAC World Congress,* Munich.

Sira-Ramírez, H. (1992 a). Asymptotic output stabilization for nonlinear systems via dynamical variable structure control. *Dynamics and Control*, **2**, 1, 45–58.
Sira-Ramírez, H. (1992 b). Dynamical sliding mode control strategies in the regulation of nonlinear chemical processes. *Int. J. Control*, **56**, 1, 1–21.
Sira-Ramírez, H. (1993). A dynamical variable structure control strategy in asymptotic output tracking problems. *IEEE Trans. Automatic Control*, **AC-38**, 4, 615–620.
Sira-Ramírez, H. and M. Fliess (1993). A dynamical sliding mode control approach to predictive control by inversion. *Proc. of the 32nd IEEE Conference on Decision and Control*, San Antonio, TX, December, 1322–1323.
Slotine, J.J.E. and W. Li (1991). *Applied Nonlinear Control*. Prentice-Hall, Englewood Cliffs, N.J.
Soeterboek, R. (1992). *Predictive Control: A Unified Approach*. Prentice-Hall, N.Y.
Utkin, V.I. (1978). *Sliding Modes and Their Applications in Variable Structure Systems*. MIR, Moscow.
Utkin, V.I. (1992). *Sliding Modes in Control Optimization*. Springer-Verlag, Berlin.
Utkin, V.I. (ed.) (1993). Special Issue on Variable Structure Systems. *Int. J. Control*, **57**, 5.
Zinober, A.S.I. (1990). *Deterministic Control of Uncertain Systems*. Peter Peregrinus, London.
Zinober, A.S.I. (1994). Variable structure and Lyapunov control. Lecture Notes in Control and Information Sciences, **193**, Springer-Verlag, N.Y.

Appendix A

Here, and just for the purpose of self-containment, we shall briefly present some generalities about sliding motions of scalar controlled systems. The reader is referred to the references (Bühler, 1986; Emelyanov, 1969; Slotine and Li, 1991; Utkin, 1978; 1992; 1993; Zinober, 1990; 1994) for further details and results.

Consider the simple scalar controlled system,

$$\dot{w} = u, \tag{A.1}$$

where u is a scalar control input. Suppose it is desired to 1) drive the scalar state variable w to zero in *finite time* and 2) to maintain the motions of the controlled system state, in a robust fashion, at the value $w = 0$. By *robustness*, we mean that the presence of possible perturbation signals, appearing additively on the righthand side of Eq. (A.1), do not result in a significant, and definite, excursion of w from the desired condition $w = 0$.

The above tasks can be accomplished only through *discontinuous feedback control*. Indeed, the discontinuous feedback regulation policy,

$$u = -W \operatorname{sign} w, \tag{A.2}$$

results, for any arbitrary initial condition $w(0)$, in a monotone convergence of the closed loop trajectory towards the condition $w = 0$. Such a convergence is specifically characterized by a trajectory exhibiting constant slope of value $+W$, whenever $w(0) < 0$, or of value $-W$, whenever $w(0) > 0$. As a consequence, the finite time reachability requirement is always fulfilled in this case. Let T_r denote the first instant of time at which the state trajectory $w(t)$ reaches the value zero. Integration of the differential equation describing the closed loop system, (A.1), (A.2), results, for any time t previous to the reaching instant T_r, in

$$w(t) = w(0) - [W \operatorname{sign} w(0)]t, \quad t < T_r. \tag{A.3}$$

The condition $w(T_r) = 0$ is, therefore, satisfied at time T_r, given explicitly by:

$$T_r = \frac{w(0)}{W \operatorname{sign} w(0)} = \frac{|w(0)|}{W}. \tag{A.4}$$

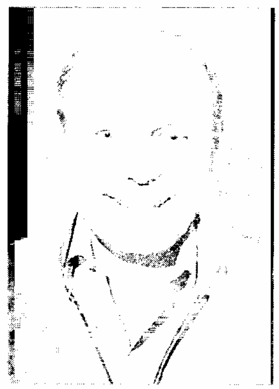
After the condition $w = 0$ is achieved, the solutions of the differential equation representing the feedback regulated system, (A.1), (A.2), can no longer be described, in traditional terms, using the standard concept of solutions of ordinary differential equations. The regulated motions on $w = 0$, of the above system are commonly addressed as *sliding motions*. In general, two methods have been proposed for describing mathematically the solution of differential equations with discontinuous righthand sides, such as (A.1), (A.2). The first method is known as *Filippov's concept of solution* (see Filippov, 1988), while the second is the *Method of the Equivalent Control* (see Utkin, 1978). For the simple case at hand, both methods are equivalent.

The method of the Equivalent Control assumes that after the "sliding surface" represented by $w = 0$, has been reached, an idealized description of the subsequent sliding motions can be made. Such an idealized description assumes that a *virtual continuous control input*, addressed as the *equivalent control* and denoted by u_{eq}, effectively drives the system response, maintaining valid the condition $w = 0$. In other words, the sliding motions of the controlled system, after T_r, are, ideally speaking (i.e., without the presence of perturbation inputs nor regard for switch imperfections, such as hysteresis and delays), described by

$$\dot{w} = u_{eq}. \tag{A.5}$$

The equivalent control is, therefore, assumed to be responsible for invariantly sustaining the motions on $w = 0$ by guaranteeing that no excursions take place from this condition; i.e., the condition $\dot{w} = 0$ is also being enforced. The two conditions, $w = 0$ and $\dot{w} = 0$, constitute the *invariance conditions* (Utkin, 1978) associated with the *sliding motion* taking place on $w = 0$. The equivalent control evidently satisfies then $u_{eq} = 0$ and, at least ideally, the system trajectories remain indefinitely on the sliding surface.

The third requirement, that of closed loop robustness, is presented in detail in Sec. 2.1.



H. Sira-Ramírez was born in San Cristóbal (Venezuela) in 1948. He obtained the Degree of Ingeniero Electricista from the Universidad de Los Andes in Mérida, Venezuela in 1970. He pursued graduate studies at the Massachusetts Institute of Technology (MIT), Cambridge, MA, where he obtained the degrees of Master of Science in E.E. and the Electrical Engineer degree in 1974. He later obtained the Ph.D. in E.E. in 1977. Currently, he is a Full Professor in the Control Systems Department of the Systems Engineering School of the Universidad de Los Andes, where he has also held the position of Head of the Control Systems Department and was elected as Vice-president of the University for the period 1980–1984. He was appointed Head of the Graduate School in Automatic Control Engineering in 1990.

Dr. Sira-Ramírez is a Senior Member of the Institute of Electrical and Electronics Engineers (IEEE), where he serves as a member of the IEEE International Committee. He is also a member of the International Federation of Automatic Control (IFAC), The Society of Industrial and Applied Mathematics (SIAM), The American Mathematical Society (AMS) and the Venezuelan College of Engineers (CIV).

Dr. Sira-Ramírez is an IEEE Distinguished Lecturer for the 1993–1996 period. Over the years, he has received several awards for his scientific work from Venezuelan institutions such as, FUNDACITE, CONICIT and the Venezuelan College of Engineers. Dr. Sira-Ramírez is interested in the theory and applications of discontinuous feedback control strategies for nonlinear dynamic systems with emphasis in Sliding Mode Control and Pulse Width Modulation Techniques.

Michel Fliess: Recent photograph and biography are not available.