Contributed Paper

ON THE SLIDING MODE CONTROL OF DIFFERENTIALLY FLAT SYSTEMS*

H. SIRA-RAMÍREZ[1]

Abstract. The design of static or dynamical sliding mode controllers for nonlinear systems is shown to be especially simple for the particular, but widespread, class of differentially flat systems. Several examples of differentially flat systems from widely different areas are presented. A complete static and dynamical, sliding mode controller design example is considered, along with simulations, for a chemical reactor system. An assessment is also furnished of the robustness of the proposed linearizing endogenous discontinuous feedback scheme with respect to external bounded perturbations.

Key Words—Sliding modes, differentially flat systems.

1. Introduction

Differentially flat systems constitute a widespread class of dynamical systems which represent the simplest possible extension of controllable linear systems to the nonlinear systems domain. Flat systems (in short) enjoy the property of possessing a finite set of *differentially independent outputs* (i.e., outputs which do not satisfy, by themselves, nonlinear differential equations), called *linearizing outputs*, such that all variables in the system, including the control input variables, can be written, exclusively, in terms of *differential functions* of such linearizing outputs (i.e., functions of the linearizing outputs and of a finite number of their time derivatives). Moreover, the linearizing outputs can, in turn, be expressed as differential functions of the system state variables and, possibly, a finite number of the control input derivatives. Flat systems are thus dynamical systems which are linearizable to a controllable linear system by means of *endogenous* feedback, i.e., one that does not require external variables to the system to be completely defined. This feature and the direct linearizability of the "flat" output coordinates make a linearizing feedback controller design task particularly simple for differentially flat nonlinear systems.

Flat systems were first introduced by Fliess et al. (1992 a) and further developed and characterized by Fliess et al. (1993). Practical examples of some mechanical systems, such as the truck and the trailer, the jumping robot and the crane were presented in Fliess et al. (1992 b; 1991). Uncontrollable systems, or

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[1] Departamento Sistemas de Control, Escuela de Ingeniería de Sistemas, Universidad de Los Andes, Mérida 5101, Venezuela.

systems *without* the so-called *strong accessibility property*, constitute typical examples of non-flat systems. The Kapitsa pendulum and the double inverted pendulum were shown to be non-differentially flat systems which can be "flatenned" by means of high-frequency vibratory control and averaging techniques (Fliess et al., 1993).

Endogenous feedback control strategies specified on the basis of desirable linearly decoupled behavior of the linearizing outputs are, generally speaking, non-robust with respect to unmodeled external perturbations and bounded parametric variations. Sliding mode control, whether static or dynamic, provides with a suitable and implementable alternative for the robust decoupled feedback linearization of differentially flat systems. Thus, in order to bestow enhanced robustness features to an endgenous feedback design, based on the differential flatness of the given system, it is here proposed to use alternatively *endogenous discontinuous feedback control strategies* based on static or dynamical sliding regimes imposed on suitable differential expressions of the linearizing outputs. While this scheme results in linearizations of reduced order for the evolution of the relevant regulated output system variables, the scheme is shown to enjoy advantageous insensitivity with respect to external (bounded) perturbations. In the dynamical sliding mode controlled case, the obtained behavior of the regulated output is substantially relieved from chattering while the traditional bang-bang control input is substituted for by a continuous signal (see Sira-Ramírez, 1992; 1993).

Section 2 is devoted to showing that several unrelated dynamical system examples, drawn from quite different areas, are differentially flat. Hamiltonian systems with control actions over every component of the momentum vector are differentially flat. Nonholonomically constrained systems which are transformable to "chained systems" are also flat. Many chemical process systems and even some well known controlled spacecraft with two actuators are indeed differentially flat. A landing spacecraft system is shown, however, not to be flat. From the presented examples in Sec. 2, we chose the perturbed version of a Continuously Stirred Tank Reactor system to develop completely in Sec. 3 a dynamical sliding mode feedback controller. Section 4 contains the conclusions.

2. Some Examples of Differentially Flat Systems

In this section, we provide a collection of nonlinear dynamical systems which are easily shown to be flat. An example of a non-flat system is also provided. The examples are taken from widely different areas to exhibit the prevailing nature of flat systems.

2.1 Hamiltonian control systems Hamiltonian control systems constitute a fascinating area of automatic control theory since they touch on well established aspects of classical mechanics and enjoy a rich differential geometrical treatment (see the book by Crouch and van der Schaft (1987)). We consider the simplest possible class of $2n$-dimensional Hamiltonian control systems including independent external control actions over every component of the vector of generalized momenta. We show that these systems are actually flat.

Consider, then, the following Hamiltonian system,

$$\left.\begin{aligned} \dot{q} &= \frac{\partial H}{\partial p}(q, p) \\ \dot{p} &= -\frac{\partial H}{\partial q}(q, p) + u \end{aligned}\right\}, \tag{2.1}$$

where u is an n-dimensional vector of control input functions and q represents the n-dimensional vector of *generalized positions*, while p represents the n-dimensional vector of *generalized momenta*. The scalar function $H(q, p)$ is the *Hamiltonian* of the system. Such a scalar function is here given by

$$H(p, q) = \frac{1}{2} p^T G(q) p + V(q), \tag{2.2}$$

where $G(q)$ is a *positive definite* matrix and $V(q)$ is a *potential function*. The Hamiltonian system (2.1) may then be written in the following form:

$$\left.\begin{aligned} \dot{q} &= G(q) p \\ \dot{p} &= -\frac{\partial V}{\partial q}(q) - \frac{1}{2}\frac{\partial}{\partial q}\operatorname{tr}[G(q) pp^T] + u \end{aligned}\right\}. \tag{2.3}$$

It is clear that the system is linearizable to a controllable linear system in *Brunovsky's canonical form* by means of static state feedback and state coordinate transformations. Indeed, the invertible transformation $z_1 = q$ and $z_2 = G(q) p$ and the static state feedback, defined by

$$u = -G^{-1}(q)[\dot{G}(q) p - v] + \frac{\partial V}{\partial q}(q) + \frac{1}{2}\frac{\partial}{\partial q}\operatorname{tr}(G(q) pp^T), \tag{2.4}$$

yield the linear controllable system

$$\left.\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= v \end{aligned}\right\}. \tag{2.5}$$

The linearizing set of (flat) outputs is then represented by the generalized position coordinates q of the system. All variables in the system can be expressed as differential functions of q. Indeed, let $y = q$; then, from the system equation (2.3), one readily obtains

$$\left.\begin{aligned} q &= y \\ p &= G^{-1}(y)\dot{y} \\ u &= \dot{G}^{-1}(y)\dot{y} + G^{-1}(y)\ddot{y} + \frac{\partial V}{\partial y}(y) \\ &\quad + \frac{1}{2}\frac{\partial}{\partial y}\operatorname{tr}[\dot{y}\dot{y}^T (G^{-1})^T (y)] \\ v &= \ddot{y} \end{aligned}\right\}. \tag{2.6}$$

Many controlled mechanical, electrical and electromechanical systems conform to the above special formulation of Hamiltonian control systems. Well known examples include robotic manipulators with several degrees of freedom, particles moving in potential fields, electrical circuits, etc. All such systems are flat.

2.2 Mechanical systems with nonholonomic velocity constraints Mechanical systems with non-integrable velocity constraints constitute a remarkably interesting class of flat systems. Such systems have been extensively studied in recent times by several authors (see the works of Murray and Sastry (1993)). The results below are taken from Bushnell et al. (1993)).

Consider the class of driftless n-dimensional systems described by

$$\dot{x} = \sum_{i=1}^{m} g_i(x) u_i, \tag{2.7}$$

where $g_i(x)$ $i = 1, \cdots, m$ are linearly independent, smooth, vector fields defined on an open set of $\mathcal{R}^n$. It is assumed that the m-dimensional distribution $\Delta(x)$, spanned vector fields $g_i(x)$, $i = 1, \cdots, m$, annihilates a given smooth m-dimensional co-distribution $\Omega(x) = \text{span}\{\omega^1(x), \cdots, \omega^m(x)\}$; i.e., the non-holonomic constraints are assumed to be given by $\omega^i(x) g_j(x) = 0$, $\forall \omega^i \in \Omega$, $\forall g_j \in \Delta$. Under involutivity conditions on certain distributions generated by the vector fields $g_j(x)$, $j = 1, \cdots, m$, the above class of systems can be transformed into the so-called m-input, $(m-1)$-chain, 1-generator chained form (see Bushnell et al., 1993) by means of state coordinate transformation and (static) control input redefinition,

$$\left.\begin{array}{llll} \dot{z}_1^0 = v_1, \quad \dot{z}_2^0 = v_2, & \dot{z}_3^0 = v_3, & \cdots, & \dot{z}_m^0 = v_m \\ \dot{z}_{21}^1 = z_2^0 v_1, & \dot{z}_{31}^1 = z_3^0 v_1, & \cdots, & \dot{z}_{m1}^1 = z_m^0 v_1 \\ \vdots & \vdots & \ddots & \vdots \\ \dot{z}_{21}^{n_2} = z_{21}^{n_2-1} v_1, & \dot{z}_{31}^{n_3} = z_{31}^{n_3-1} v_1, & \cdots, & \dot{z}_{m1}^{n_m} = z_{m1}^{n_m-1} v_1 \end{array}\right\}. \tag{2.8}$$

Proposition 2.1. Multivariable non-holonomically constrained systems which are transformable to m-input, $(m-1)$-chain, 1-generator chained form, are differentially flat. The linearizing coordinates are given by the m outputs constituted by the first transformed variable z_1^0 and the last state variable on every chain of the transformed system (2.8), $z_{21}^{n_2}, z_{31}^{n_3}, \cdots, z_{m1}^{n_m}$.

Proof. The proposed linearizing outputs do not satisfy, by themselves, ordinary differential equations; i.e., they are independent of each other. It is also quite straightforward to realize that each state variable on the jth chain, say $z_{j1}^{n_j - i}$, and every one of the inputs to the different chains v_j, $j = 1, \cdots, m$, can be computed in terms of expressions involving only time derivatives of the last state variable on the corresponding chain $y_j = z_{j1}^{n_j}$ and the first output $y_1 = z_1^0$. Indeed, letting z_{j1}^0 stand for z_j^0, $j = 1, \cdots, m$, one has

$$\begin{aligned} z_{j1}^{n_j - i} &= \left(\frac{1}{\dot{y}_1} \frac{d}{dt} \right)^{i-1} \left[\frac{\dot{y}_j}{\dot{y}_1} \right] \\ &= \frac{1}{\dot{y}_1} \frac{d}{dt} \left(\frac{1}{\dot{y}_1} \frac{d}{dt} \left(\cdots (i-1) \text{ times} \cdots \frac{1}{\dot{y}_1} \frac{d}{dt} \left(\frac{\dot{y}_j}{\dot{y}_1} \right) \cdots \right) \right), \quad i = 0, \cdots, n_j, \end{aligned} \tag{2.9}$$

The control input v_j to the jth chain is just the time derivative of the first state variable of that particular chain; i.e.,

$$v_j = \frac{d}{dt}\left(\frac{1}{\dot{y}_1}\cdot\frac{d}{dt}\right)^{n_j-1}\left[\frac{\dot{y}_j}{\dot{y}_1}\right], \quad j = 2, \cdots, m. \tag{2.10}$$

Evidently, the first control input v_1 is simply written as

$$v_1 = \dot{y}_1. \tag{2.11}$$

Example 2.1. (Bicycle kynematics) Consider the kinematics model of a bicycle moving on a horizontal plane. Let the state variables be defined as in Fig. 1. The describing equations are given by

$$\left.\begin{aligned} \dot{x} &= u_1 \cos\theta \\ \dot{y} &= u_1 \sin\theta \\ \dot{\theta} &= u_1 \frac{\tan\phi}{l} \\ \dot{\phi} &= u_2 \end{aligned}\right\}, \tag{2.12}$$

where u_1 is the forward velocity of the rear wheel and u_2 is the turn rate of the front (steering) wheel, taken as control input variables (see Murray and Sastry, 1993). The following state coordinate transformation,

$$z_1^0 = x, \quad z_2^0 = \frac{\tan\phi}{l\cos^3\theta}, \quad z_{21}^1 = \tan\theta, \quad z_{21}^2 = y, \tag{2.13}$$

and the redefinition of the input variables

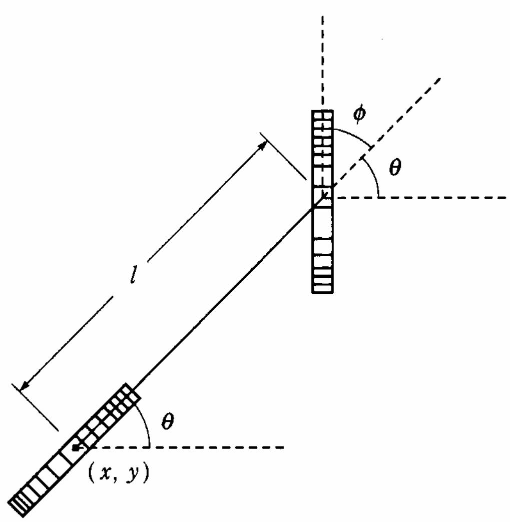

Fig. 1. Bicycle kinematic variables.

$$\left.\begin{aligned} v_1 &= u_1 \cos\theta \\ v_2 &= u_1 \frac{3\tan^2\phi \sin\theta}{l^2\cos^4\theta} + u_2 \frac{1}{l\cos^2\phi\cos^3\theta} \end{aligned}\right\} \qquad (2.14)$$

takes the system (2.12) into a 2-input, 1-chain, single generator chained system form (Murray and Sastry, 1993),

$$\left.\begin{aligned} \dot{z}_1^0 = v_1 \quad \dot{z}_2^0 &= v_2 \\ \dot{z}_{21}^1 &= z_2^0 v_1 \\ \dot{z}_{21}^2 &= z_{21}^0 v_1 \end{aligned}\right\}. \qquad (2.15)$$

All variables in the transformed system can be expressed as a differential function of $y_1 = z_1^0$ and $y_2 = z_{21}^2$ and its time derivatives (i.e., as a function of the position coordinates (x, y) of the contact point of the bicycle's rear wheel with the plane, and some of its time derivatives). Indeed,

$$\left.\begin{aligned} z_{21}^1 &= \frac{\dot{z}_{21}^2}{\dot{z}_1^0} = \frac{\dot{y}_2}{\dot{y}_1} \\ z_2^0 &= \frac{1}{\dot{y}_1}\frac{d}{dt}\left(\frac{\dot{y}_2}{\dot{y}_1}\right) = \frac{\ddot{y}_2\dot{y}_1 - \dot{y}_2\ddot{y}_1}{(\dot{y}_1)^3} \\ v_1 &= \dot{y}_1 \\ v_2 &= \frac{d}{dt} z_2^0 = \frac{d}{dt}\left[\frac{1}{\dot{y}_1}\frac{d}{dt}\left(\frac{\dot{y}_2}{\dot{y}_1}\right)\right] = \frac{d}{dt}\left(\frac{\ddot{y}_2\dot{y}_1 - \dot{y}_2\ddot{y}_1}{(\dot{y}_1)^3}\right) \\ &= \frac{y_2^{(3)}\dot{y}_1 - \dot{y}_2 y_1^{(3)}}{(\dot{y}_1)^3} + \frac{3(\dot{y}_2\ddot{y}_1\dot{y}_1 - \dot{y}_2(\ddot{y}_1)^2)}{(\dot{y}_1)^4} \end{aligned}\right\}. \qquad (2.16)$$

2.3 Systems linearizable by state coordinate transformations and static state feedback Systems which are linearizable by means of state coordinate transformations and static state feedback constitute the best studied class of flat systems. Consider the smooth n-dimensional multi-input system

$$\dot{x} = f(x, u), \quad x \in \mathcal{R}^n, \quad u \in \mathcal{R}^m, \qquad (2.17)$$

and assume an invertible state coordinate transformation and (regular) static state feedback of the form

$$z = \phi(x), \quad u = \vartheta(x, v) \qquad (2.18)$$

yields the following controllable linear system in *Brunovsky's canonical form*:

$$\left.\begin{aligned} \dot{z}_{i1} &= z_{i2} \\ &\vdots \\ \dot{z}_{i\gamma_i} &= v_i \\ y_i &= z_{i1}, \quad i = 1, \cdots, m \end{aligned}\right\}, \qquad (2.19)$$

where γ_i, $i = 1, \cdots, m$ are the *Brunovsky controllability indices* of the system. These indices satisfy: $\Sigma_{i=1}^{m} \gamma_i = n$. It should now be evident that the linearizing (i.e., flat) outputs are constituted by $y_i = z_{i1}$, $i = 1, \cdots, m$. Indeed, from the invertibility of the state coordinate transformation (2.18), one obtains

$$\begin{aligned} x &= \phi^{-1}(z) = \phi^{-1}(z_{11}, \cdots, z_{1\gamma_1}, \cdots, z_{m1}, \cdots, z_{m\gamma_m}) \\ &= \phi^{-1}(y_1, \cdots, y_1^{(\gamma_1 - 1)}, \cdots, y_m, \cdots, y_m^{(\gamma_m - 1)}); \end{aligned} \tag{2.20}$$

i.e., all state variables are expressible as differential functions of the linearizing outputs $y_1, \cdots, y_m$.

Each one of the new control inputs v_i, $i = 1, \cdots, m$, is expressible as the highest time derivative of the corresponding linearizing output y_i, i.e., $v_i = \dot{z}_{i\gamma_i} = y_i^{(\gamma_i)}$, $i = 1, \cdots, m$. It then easily follows, by virtue of (2.18), that the control input u can be expressed as

$$u = \vartheta[\phi^{-1}(z_{11}, \cdots, z_{1\gamma_1}, \cdots, z_{m1}, \cdots, z_{m\gamma_m}), \dot{z}_{1\gamma_1}, \cdots, \dot{z}_{m\gamma_m}]; \tag{2.21}$$

i.e.,

$$u = \psi(y_1, \cdots, y_1^{(\gamma_1)}, y_2, \cdots, y_2^{(\gamma_2)}, \cdots, y_m, \cdots, y_m^{(\gamma_m)}). \tag{2.22}$$

Example 2.2. (A Rigid spacecraft with two actuators) A popular nonlinear multivariable system is constituted by a third order kinematic model of a rigid spacecraft with two actuators along two principal axis (see Wen and Bernstein, 1992). The equations of motion, when the uncontrolled principal axis is not an axis of symmetry, are given by

$$\left.\begin{aligned} \dot{x}_1 &= u_1 \\ \dot{x}_2 &= u_2 \\ \dot{x}_3 &= x_1 x_2 \end{aligned}\right\}. \tag{2.23}$$

It is easy to show that system (2.23) is linearizable by means of a static state coordinate transformation and a redefinition of the control input vector. One immediately verifies that the required state and input space transformations are simply given by

$$\left.\begin{aligned} z_{11} &= x_1, \quad z_{21} = x_3, \quad z_{22} = x_1 x_2 \\ u_1 &= v_1, \quad u_2 = -\frac{x_2}{x_1} \cdot v_1 - \frac{1}{x_1} v_2 \end{aligned}\right\}. \tag{2.24}$$

The transformed system results in the following Brunovsky's canonical form:

$$\left.\begin{aligned} \dot{z}_{11} &= v_1 \\ \dot{z}_{21} &= z_{22} \\ \dot{z}_{22} &= v_2 \end{aligned}\right\}. \tag{2.25}$$

In other words, the linearizing (flat) coordinates are given by

$$y_1 = z_{11} = x_1, \quad y_2 = z_{21} = x_3. \tag{2.26}$$

Indeed, all variables in the system, including the control inputs, are differential functions of these two output coordinates,

$$\left.\begin{aligned} x_1 &= y_1, \quad x_2 = \frac{\dot{y}_2}{y_1}, \quad x_3 = y_2 \\ u_1 &= \dot{y}_1, \quad u_2 = \frac{\ddot{y}_2 y_1 - \dot{y}_2 \dot{y}_1}{y_1^2} \end{aligned}\right\}. \tag{2.27}$$

Since not all single-input single-output systems are exactly linearizable to a controllable linear system by means of coordinate transformations and static state feedback, examples of non-flat systems are primarily constituted by the class of uncontrollable systems, or systems which do not exhibit the so-called *strong accessibility* property.

Example 2.3. (Landing spacecraft) Consider the following model of a landing spacecraft (see Sira-Ramírez, 1991):

$$\left.\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= g - \left(\frac{\sigma\alpha}{M + x_3}\right)u \\ \dot{x}_3 &= -\sigma u \end{aligned}\right\}, \tag{2.28}$$

where x_1 is the vertical position of the spacecraft, x_2 is the downwards landing velocity and x_3 is the fuel mass contained in the spacecraft. The constants α and σ represent, respectively, the velocity of the exhaust gasses and the rate of fuel consumption. The constants M and g represent, respectively, the *dead* mass of the vehicle and the acceleration due to gravity. The control input u is usually assumed to vary smoothly in the closed interval [0, 1]. However, since this restriction is not crucially related to the flatness, or lack of flatness, of the system, we shall consider u to be a free (unrestricted) variable.

Introducing a *virtual* downwards velocity variable, defined by $z = x_2 - \alpha \log(M + x_3)$, one obtains the following "*normal form*" model for the landing spacecraft (2.28):

$$\left.\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= g - \sigma\alpha \exp\left(\frac{z - x_2}{\alpha}\right)u \\ \dot{z} &= g \end{aligned}\right\}. \tag{2.29}$$

The transformed system (2.29) is evidently uncontrollable and linearizable only to a second-order system by means of static state feedback. The system is, therefore, non-flat. Note that if the output y to be regulated happens to be the vertical height; i.e., $y = x_1$, then the system is also non-minimum phase, as the corresponding zero dynamics is clearly unstable. Discontinuous control is usually employed at the final stages of the landing maneuver to avoid fuel depletion (see Sira-Ramírez, 1991).

Note, however, that the system (2.28) is linearizable by means of *exogenous* dynamical feedback. Indeed, an input-output description of the system can be readily obtained by eliminating the state vector components x_2 and x_3 as follows:

$$y^{(3)} = -\left[\frac{\dot{u}}{u} + \frac{1}{\alpha}\cdot(g - \ddot{y})\right](g - \ddot{y}). \tag{2.30}$$

It should be evident that a nonlinear first-order dynamical controller, with state variable u, can now be obtained by equating the left-hand side of (2.30) to a (linear) expression in y, $\dot{y}$ and $\ddot{y}$. It is clear that the variable u cannot be obtained as a differential function of the linearizing output y, but only as the solution of a nonlinear time-varying differential equation. The construction of a linearizing dynamical controller thus entitles introducing variables which are not expressible in terms of the original system output variable and its time derivatives; i.e., the proposed controller is *exogenous.*

2.4 Extended systems of linearizable systems Here we consider single-input, single-output nonlinear systems. However, the assertions are easily extended to the case of multivariable nonlinear systems.

A basic fact that allows one to propose *dynamical sliding mode controllers* which are free of bang-bang input behavior, and thus exhibit substantially reduced "chattering" in the controlled variables, is constituted by the possibility of using Generalized State Space Canonical Forms of the *Controller* and *Observability* type (see Fliess, 1990). These generalized state canonical forms are particularly advantageous whenever, respectively, the *differential primitive element* and the output of the controlled system exhibit a *relative degree* which is *smaller* than the order of the system (see Sira-Ramírez, 1992; 1993). However, when the system is exactly linearizable by diffeomorphic state coordinate transformations and static state feedback, the linearizing output has the relative degree which equals to the order of the system, say n. In such cases, the sliding mode controller is necessarily static, and bang-bang inputs are, hence, obtained. Nevertheless, one may still propose a *dynamical feedback controller*, with smoothed control input response, by resorting to a *dynamical extension* of the linearizable system. This procedure entitles adding one or several integrators in front of the input channel to the system. It has been be shown above that if a system is exactly transformable to a controllable linear system, then the system is flat. It is then easy to see that the dynamical extensions of an exactly linearizable system are also flat. This simple but helpful result is summarized in the next proposition.

Proposition 2.2. Let the n-dimensional single-input (analytic) system

$$\dot{x} = f(x, u) \tag{2.31}$$

be differentially flat, with linearizing output given by an input-independent scalar function of the state $y = h(x)$. Then, the extended system

$$\left.\begin{aligned} \dot{x} &= f(x, x_{n+1}) \\ \dot{x}_{n+1} &= w \end{aligned}\right\} \tag{2.32}$$

is also differentially flat with y as the linearizing output.

Example 2.4. (A continuously stirred tank reactor system) Consider the following nonlinear model of a Continuously Stirred Tank Reactor (CSTR) system, taken from Kravaris and Palanki (1988),

$$\left.\begin{aligned} \dot{x}_1 &= -(1+Da_1)x_1 + u \\ \dot{x}_2 &= Da_1 x_1 - x_2 - Da_2 x_2^2 \\ z &= x_1 + x_2 - C \end{aligned}\right\}, \tag{2.33}$$

where x_1 represents the normalized concentration of a certain species P in the reaction, x_2 is the concentration of a second species Q called the reactant. The control input variable u is the volumetric feed rate of the first species. The output z of the system is the total concentration error from a desired constant concentration value C, while we let $\tilde{z}$ denote just the total concentration, $x_1 + x_2$. A linearizing output for system (2.33) is constituted by the concentration variable x_2. Indeed, let $y = x_2$. We then have

$$\left.\begin{aligned} x_1 &= \frac{1}{Da_1}(\dot{y} + y + Da_2 y^2) \\ u &= \frac{1}{Da_1}(\ddot{y} + \dot{y} + 2Da_2 y\dot{y}) + \frac{1+Da_1}{Da_1}(\dot{y} + y + Da_2 y^2) \\ z &= y + \frac{1}{Da_1}(\dot{y} + y + Da_2 y^2) - C \end{aligned}\right\}; \tag{2.34}$$

i.e., all variables in the system (2.33) are expressible as differential functions of y. The system is flat.

Consider now the extended system of the chemical reaction model (2.33),

$$\left.\begin{aligned} \dot{x}_1 &= -(1+Da_1)x_1 + x_3 \\ \dot{x}_2 &= Da_1 x_1 - x_2 - Da_2 x_2^2 \\ \dot{x}_3 &= w \\ z &= x_1 + x_2 - C \end{aligned}\right\}. \tag{2.35}$$

It is equally easy to verify that system (2.35) is also flat with the same linearizing output as before, $y = x_2$.

$$\left.\begin{aligned} x_1 &= \frac{1}{Da_1}(\dot{y} + y + Da_2 y^2) \\ x_3 &= \frac{1}{Da_1}(\ddot{y} + \dot{y} + 2Da_2 y\dot{y}) + \frac{1+Da_1}{Da_1}(\dot{y} + y + Da_2 y^2) \\ w &= \frac{1}{Da_1}(y^{(3)} + \ddot{y} + 2Da_2(\dot{y})^2 + 2Da_2 y\ddot{y}) \\ &\quad + \frac{1+Da_1}{Da_1}(\ddot{y} + \dot{y} + 2Da_2 y\dot{y}) \\ z &= y + \frac{1}{Da_1}(\dot{y} + y + Da_2 y^2) - C \end{aligned}\right\}. \tag{2.36}$$

3. Sliding Mode Control of Flat Systems

Discontinuous feedback strategies of the sliding mode type enjoy justified popularity because of their enhanced robustness and conceptual simplicity. Sliding mode control is a well established research area with great potential for practical applications (see the books by Utkin, 1978; 1992; Slotine and Li, 1991; Zinober, 1990). A collection of articles, which indicate recent research trends in Sliding Mode Control, is contained in a special issue of the *International J. of Control* (Utkin, 1993), edited by Prof. V.I. Utkin.

We consider here the simplest case of sliding mode control of monovariable flat systems, although the fundamental technique extends to multivariable systems. For simplicity, and also because, custormarily, at the sliding mode controller design stage, one totally and purposefully overlooks the presence of the external perturbations, we assume that the given system is unperturbed. However, in order to illustrate the robustness features of the proposed endogenous discontinuous feedback strategies, an externally perturbed system example is presented in this section. The endogenous sliding mode controller is obtained from the unperturbed version of the system, but its robust regulation effects are analyzed on the basis of the actual perturbed description of the system.

Consider, then, the following nonlinear n-dimensional, unperturbed, single-input single-output system,

$$\left.\begin{aligned} \dot{x} &= f(x, u) \\ z &= h(x) \end{aligned}\right\}, \tag{3.1}$$

and suppose it is differentially flat with linearizing coordinate given by either

$$y = \lambda(x) \quad \text{or} \quad y = \lambda(x, u, \dot{u}, \cdots, u^{(\beta)}). \tag{3.2}$$

If the linearizing output function coordinate y is expressible only as a function of x, this is indicative that the system is directly linearizable by means of a change of coordinates and static state feedback. If, on the other hand, the linearizing coordinate includes expressions in u, and some of its time derivatives, then the system is linearizable by means of dynamical state feedback. It is well known that if a system is linearizable by static feedback, then it is also linearizable by means of dynamical feedback (Charlet et al., 1988). We assume then that $\beta > 0$. From the flatness assumption, it follows that x is expressible as

$$x = \mathscr{A}(y, \dot{y}, \cdots, y^{(\alpha)}) \tag{3.3}$$

and

$$u = \mathscr{B}_0(y, \dot{y}, \cdots, y^{(\alpha+1)}). \tag{3.4}$$

It is clear then that for any $\gamma > 0$,

$$u^{(\gamma)} = \mathscr{B}_\gamma(y, \dot{y}, \cdots, y^{(\alpha+\gamma+1)}). \tag{3.5}$$

Moreover, the controlled output z is generally expressible as

$$z = \mathscr{C}(y, \dot{y}, \cdots, y^{(\delta)}), \tag{3.6}$$

where $\delta \leq \alpha$.

Suppose it is desired to drive the controlled output variable z to some constant value Z. Let, moreover, $\mu = \alpha - \delta + \gamma + 1$, where γ will be regarded as the order of the dynamical compensator. μ is assumed to be a strictly positive integer. Then, we have the following result.

Proposition 3.1. Let W be a strictly positive constant. For systems where $\delta < \alpha$, the following discontinuous dynamics, imposed on the controlled output variable z, results in an asymptotic convergence of z to the required value Z,

$$\begin{aligned} z^{(\mu)} &+ \sum_{i=1}^{\mu-1} a_{\mu-i-1} z^{(\mu-i)} \\ &= -W \operatorname{sign}\left[\left(\sum_{i=1}^{\mu-2} a_{\mu-i-1} z^{(\mu-i-1)}\right) + a_0(z - Z)\right], \end{aligned} \tag{3.7}$$

provided the constant coefficients $\{a_{\mu-2}, \cdots, a_0\}$ correspond to the coefficients of a Hurwitz polynomial in the complex variable p, given by

$$Q(p) = p^{\mu-1} + a_{\mu-2} p^{\mu-2} + \cdots + a_1 p + a_0 . \tag{3.8}$$

Moreover, the regulated dynamics for z is driven to satisfy, in finite time, the linear time-invariant asymptotically stable dynamics

$$z^{(\mu-1)} + a_{\mu-2} z^{(\mu-2)} + \cdots + a_1 \dot{z} + a_0(z - Z) = 0 . \tag{3.9}$$

Proof. The proof is immediate upon defining a sliding surface coordinate s as

$$s = \left(\sum_{i=1}^{\mu-2} a_{\mu-i-1} z^{(\mu-i-1)}\right) + a_0(z - Z). \tag{3.10}$$

It is then clear that if s is forced to go to zero in finite time, then the desired asymptotically stable dynamics (3.9) is also achieved in finite time. The result follows from the fact that using (3.7), the sliding surface coordinate s is seen to satisfy,

$$\dot{s} = -W \operatorname{sign} s . \tag{3.11}$$

Equation (3.11) is a well known sliding dynamics converging to zero in finite time T given by $T = |z(0)| / W$.

Remark 3.1: Note that if, in the previous proposition, $\delta = \alpha$ and $\gamma = 0$, then the value Z is achieved, in finite time, by means of the discontinuous dynamics

$$\dot{z} = -W \operatorname{sign}(z - Z). \tag{3.12}$$

It should now be clear that the required feedback controller may be found by means of some straight-forward, but possibly involved, algebraic manipulations. Such a controller is primarily found in terms of the linearizing output y and a number of its time derivatives. Flatness also allows us to place the controller expression back in the original state coordinates. The resulting controller may be, in general, of static or dynamical nature. We sketch in the following paragraph how to obtain the corresponding controller expression. The example presented in the next section should also clarify the procedure.

To obtain the discontinuous feedback controller expression, one simply substitutes the differential function $z = \mathscr{C}(y, \dot{y}, \cdots, y^{(\delta)})$, from Eq. (3.6), into the desired discontinuous dynamics (3.7) imposed on z. From the obtained expression, one solves for the highest order derivative of y, which is just $y^{\alpha+\gamma+1}$. This is the corresponding discontinuous dynamics for the linearizing output y, induced by the desired discontinuous dynamics (3.7) on the controlled output z. The obtained expression for $y^{\alpha+\gamma+1}$ is next substituted into the expression for the highest derivative of the control input $u^{(\gamma)}$ (3.5). The obtained expression for the highest derivative of the control input is then a differential function of the linearizing outputs, which includes discontinuities inherited from the prescribed dynamics for the controlled output z.

To obtain an implementable feedback controller, one proceeds to obtain its expression back in terms of the original state variables and, possibly, the control input and some of its lower order time derivatives, i.e., by using the differential function $y = \lambda(x, u, \dot{u}, \cdots, u^{(\beta)})$, in the expression found for the highest derivative of the control input. One generally obtains an *implicit*, time-varying differential equation for the control input u. Such an implicit differential equation also contains discontinuities represented by the "sign" terms. The switching surface is, generally speaking, an *input-dependent sliding surface* (see also Sira-Ramírez, 1992). The obtained feedback controller is, then, a truly *dynamical variable structure feedback control law.*

3.1 Static sliding mode control of an externally perturbed continuously stirred tank reactor Consider the following perturbed version of the CSTR example given in Eq. (2.33) in the previous section:

$$\left.\begin{aligned} \dot{x}_1 &= -(1 + Da_1)x_1 + u \\ \dot{x}_2 &= Da_1 x_1 - x_2 - Da_2 x_2^2 + \eta \\ z &= x_1 + x_2 - C \end{aligned}\right\}, \tag{3.13}$$

where η is now an unknown perturbation signal which is assumed to be bounded with bounded first-order time derivatives. Notice that the perturbation is of the "unmatched" type, i.e., its input channel field $[0 \;\; 1]'$, is not located on the image of the control input vector field $[1 \;\; 0]'$.

Suppose one is interested in driving the total measured concentration error variable z to zero in finite time, and in spite of the presence of the bounded perturbations.

According to the results of the previous section, one would proceed, totally ignoring the presence of perturbations, to impose the following discontinuous sliding mode dynamics on the controlled output z:

$$\dot{z} = -W \operatorname{sign} z\,. \tag{3.14}$$

This choice would result in the following unperturbed discontinuous dynamics for the linearizing output y:

$$\dot{y} + \frac{1}{Da_1}(\ddot{y} + \dot{y} + 2Da_2 y\dot{y})$$
$$+ W \operatorname{sign}\left[y + \frac{1}{Da_1}\cdot(\dot{y} + y + Da_2 y^2) - C\right] = 0. \quad (3.15)$$

Solving for the highest order time derivative of the linearizing output y in (3.15), and substituting the result in the control input expression found in (2.36), one obtains the following static sliding mode controller:

$$u = \frac{1 + Da_1}{Da_1}\cdot(\dot{y} + y + Da_2 y^2) - \dot{y}$$
$$- W \operatorname{sign}\left[y + \frac{1}{Da_1}\cdot(\dot{y} + y + Da_2 y^2) - C\right]; \quad (3.16)$$

i.e.,

$$u = x_1 + x_2 + D_{a2} x_2^2 - W \operatorname{sign}(x_1 + x_2 - C). \quad (3.17)$$

In spite of the fact that the controller (3.17) cannot be implemented in practice, due to the bang-bang nature of the required volumetric feed rate, it is interesting to note that if such a controller were used on the perturbed version of the system (3.13), one would obtain the following regulated dynamics for the total measured concentration error z:

$$\dot{z} = \frac{\dot{\eta}}{D_{a2}} - W \operatorname{sign} z. \quad (3.18)$$

Due to the assumed bounded nature of η and $\dot{\eta}$, the measured concentration error z is still seen to converge to zero in finite time, provided a sufficiently large value of W is used. For instance, if $|\eta| \leq N$ and $|\dot{\eta}| \leq N_1$, then it would suffice to let $W > N_1 / Da_2$.

Remark 3.2: In order to obtain Eq. (3.18), one proceeds to find the state variables of the system x_1, x_2, and of the control input, u, as differential functions of the linearizing output y and of the perturbation signal η, i.e., as

$$\left.\begin{aligned} x_1 &= \frac{1}{Da_1}\cdot(\dot{y} + y + Da_2 y^2) - \frac{\eta}{Da_2} \\ x_2 &= y \\ u &= \frac{1}{Da_1}\cdot(\ddot{y} + \dot{y} + 2Da_2 y\dot{y}) + \frac{1 + Da_1}{Da_1}\cdot(\dot{y} + y + Da_2 y^2) \\ &\quad - \left(\frac{1}{D_{a1}}\cdot\dot{\eta} + \frac{1 + D_{a1}}{Da_1}\cdot\eta\right) \end{aligned}\right\}. \quad (3.19)$$

The fact that the "perturbed" value of u in (3.19) is not being used to regulate the system and thus, one does not exactly cancel the effects of the perturbation signal, η, nor is one able to drive z to zero with constant slope W (due, precisely, to a lack of knowledge of η and $\dot{\eta}$), then, evidently, the "unperturbed" feedback action (3.17) results in an imperfectly regulated evolution of the output z. It is easy to see from simple algebraic manipulations of the above equations, in combination with the unperturbed controller expression (3.16), that the perturbed evolution of the total concentration error z is given by Eq. (3.18).

As pointed out before, discontinuous (bang-bang) feedback policies are not feasible as actual volumetric input rates to CSTR systems. For this reason, one should, then, resort to a *dynamical* sliding mode controller design. For such a synthesis task, it suffices to consider the dynamic extension (2.35) of the CSTR model. In so doing, we will trade off finite time reachability of the desired total concentration error z by continuity of the controller output signal u, which is demanded by physical implementation constraints.

3.2 Dynamical sliding mode control of an externally perturbed continuously stirred tank reactor Consider first, the process of obtaining a sliding mode controller for the unperturbed version of the extended system model, described by Eq. (2.35).

Take, as a sliding surface coordinate s, an expression of the total concentration error z such that, when s is forced to become zero, it corresponds to an asymptotically stable first order dynamics for z; i.e., $s = \dot{z} + \lambda z$, where λ is a strictly positive constant. The sliding surface coordinate s, in terms of the linearizing output $y = x_2$, is given by

$$s = \dot{y} + \frac{1}{Da_1}(\ddot{y} + \dot{y} + 2Da_2 y\dot{y}) + \lambda\left[y + \frac{1}{Da_1}(\dot{y} + y + Da_2 y^2) - C\right]. \quad (3.20)$$

Imposing now on s the sliding mode dynamics given by $\dot{s} = -W \operatorname{sign} s$, one obtains

$$\begin{aligned}
&\ddot{y} + \frac{1}{Da_1}[y^{(3)} + \ddot{y} + 2Da_2(\dot{y}^2 + y\ddot{y})] \\
&\quad + \lambda\left[\dot{y} + \frac{1}{Da_1}\cdot(\ddot{y} + \dot{y} + 2Da_2 y\dot{y})\right] \\
&= -W\operatorname{sign}\left[\dot{y} + \frac{1}{Da_1}(\ddot{y} + \dot{y} + 2Da_2 y\dot{y})\right. \\
&\quad \left. + \lambda\left(y + \frac{1}{Da_1}\cdot(\dot{y} + y + Da_2 y^2) - C\right)\right]. \quad (3.21)
\end{aligned}$$

Solving for $y^{(3)}$ from (3.21) and substituting its value into the expression found for the auxiliary control input w in (2.36), one obtains, after some simplifications, a static discontinuous feedback controller expression for $w = \dot{u}$ of the form

$$w = \left(\frac{1-\lambda}{Da_1}\right)\ddot{y} - \lambda\dot{y} + \left(\frac{1-\lambda+Da_1}{Da_1}\right)[\dot{y} + 2Da_2 y\dot{y}] - W\operatorname{sign} s. \quad (3.22)$$

The static controller (3.22) is to be viewed as a nonlinear, time-varying differential equation, with discontinuous right hand side, whose solution is the required control input function u. Using the expression for x_3 $(= u)$ in terms of the linearizing output y, found in (2.36), one obtains the following dynamical feedback controller:

$$\dot{u} = (1-\lambda)\left[u - \frac{1+Da_1}{Da_1}(\dot{y} + y + Da_2 y^2)\right] + (1 - \lambda + 2Da_2 y)\dot{y} - W \operatorname{sign} s. \tag{3.23}$$

The resulting controller output u is now a continuous function of time, rather than a bang-bang signal. The obtained dynamical sliding mode controller (3.23) is implementable, provided the linearizing output y and its first-order time derivative $\dot{y}$ are measurable. A simple nonlinear observer design which estimates such a time derivative, in terms of the original state variables and the measured linearizing output y can be easily designed for such a purpose.

In steady state conditions, the linearizing output y converges to an equilibrium concentration value, denoted by X_2 and the sliding surface value ideally satisfies the condition: $s = 0$. The remaining dynamics, or *zero dynamics*, associated with the closed loop system, is given by the following linear dynamics for the control input u:

$$\dot{u} = (1-\lambda)\left[u - \frac{1+Da_1}{Da_1}(X_2 + Da_2 X_2^2)\right]. \tag{3.24}$$

A minimum phase behavior may then be guaranteed for the closed loop system whenever the design parameter λ is chosen to satisfy $\lambda > 1$, so that (3.24) becomes an asymptotically stable system.

In terms of the original state variables of the system, one immediately writes the dynamical sliding mode controller as

$$\dot{u} = (1-\lambda)[u - x_1 - x_2 - Da_2 x_2^2] + 2Da_2 x_2(Da_1 x_1 - x_2 - Da_2 x_2^2) - W \operatorname{sign}[-(x_1 + x_2) - Da_2 x_2^2 + u + \lambda(x_1 + x_2 - C)]. \tag{3.25}$$

Note that the sliding surface coordinate s is, indeed, an input-dependent coordinate function, given by

$$s = -(1-\lambda)(x_1 + x_2) - Da_2 x_2^2 + u - \lambda C. \tag{3.26}$$

Consider now a corresponding "unmatched" perturbed version of the extended system,

$$\left.\begin{aligned} \dot{x}_1 &= -(1+Da_1)x_1 + x_3 \\ \dot{x}_2 &= Da_1 x_1 - x_2 - Da_2 x_2^2 + \eta \\ \dot{x}_3 &= w \\ z &= x_1 + x_2 - C \end{aligned}\right\}, \tag{3.27}$$

where the bounded perturbation signal η is now also assumed to exhibit bounded values in the first and in the second time derivatives (i.e., $|\eta| \le N$, $\dot{\eta} \le N_1$ and $\ddot{\eta} \le N_2$).

The state variables x_1, x_2, x_3 and the (extended) control input w, written as differential functions of the linearizing output y and the perturbation signal η, are found to be

$$\left.\begin{aligned} x_1 &= \frac{1}{Da_1}(\dot{y} + y + Da_2 y^2) - \frac{\eta}{D_{a2}} \\ x_2 &= y \\ x_3 &= \frac{1}{Da_1}(\ddot{y} + \dot{y} + 2Da_2 y\dot{y}) + \frac{1 + Da_1}{Da_1}(\dot{y} + y + Da_2 y^2) \\ &\quad - \left(\frac{1}{D_{a1}}\dot{\eta} + \frac{1 + D_{a1}}{D_{a1}}\eta\right) \\ w &= \frac{1}{Da_1}\cdot(y^{(3)} + \ddot{y} + 2Da_2(\dot{y})^2 + 2Da_2 y\ddot{y}) \\ &\quad + \frac{1 + Da_1}{Da_1}(\ddot{y} + \dot{y} + 2Da_2 y\dot{y}) - \left(\frac{1}{D_{a1}}\ddot{\eta} + \frac{1 + D_{a1}}{D_{a1}}\cdot\dot{\eta}\right) \end{aligned}\right\}. \tag{3.28}$$

Using the previous expressions, and that of the unperturbed dynamical sliding mode controller (3.23), it is easy to see that the sliding surface s now satisfies the following perturbed discontinuous closed loop dynamics:

$$\dot{s} = \frac{\ddot{\eta}}{D_{a2}} + \lambda\frac{\dot{\eta}}{D_{a2}} - W \operatorname{sign} s. \tag{3.29}$$

Under the previous assumptions about the bounded nature of the perturbation signal η, the sliding surface coordinate function s is driven to zero in finite time, provided that the constant gain W is set to satisfy

$$W > \frac{N_2}{D_{a2}} + \frac{\lambda N_1}{D_{a2}}. \tag{3.30}$$

Since s is robustly driven to zero, the linear, asymptotically stable dynamics $\dot{z} = -\lambda z$ is also robustly imposed for z. The total measured concentration error z converges to zero in spite of the effects of the bounded external perturbation signal, η, affecting the system.

3.2.1 Simulation results Simulations were performed for the unperturbed and the perturbed versions of the CSTR given by Eqs. (2.33) and (3.13), respectively. Both models were regulated by the "unperturbed" version of the dynamical sliding mode controller, given by Eq. (3.25). The system parameters were taken to be (Kravaris and Palanki, 1988)

$$Da_1 = 1.0, \quad Da_2 = 1.0, \quad C = 3.0.$$

The desired value for the controlled output z is $z = 0$, i.e., one corresponding to a total measured concentration of $\tilde{z} = C = 3$ (or, $x_1 + x_2 = 3$). From Eq. (2.33) or (2.36), one obtains, under equilibrium conditions ($y = Y$; $\dot{y} = \ddot{y} = 0$) that the nominal steady state value for y is given by $y = Y = 1$. The corresponding unperturbed equilibrium values for the state x_1 and the control input u are found to be $x_1 = 2$ and $u = 4$. The design values $\lambda = 2$ and $W = 4$ were used for the simulations. Figure 2 shows the chattering-free evolution of the unperturbed controlled state variables x_1 and x_2. The dynamically generated control input variable is seen to be continuous while the total concentration variable $\tilde{z}$ converges to the desired value $C = 3$. All of the variables thus converge towards their nominal equilibrium points. For the perturbed model, the bound on the computer generated noisy signal η was set to be $N = 1.2$. Figure 3 shows the perturbed evolution of the controlled state variables x_1 and x_2. The dynamically generated control input variable u, is also seen to be rather continuous for the perturbed case. In this figure, a sample trajectory of the perturbation signal η is also shown, while the total perturbed concentration variable $\tilde{z}$ is seen to converge, quite closely, to the desired value $C = 3$. All of the perturbed variables thus exhibit a satisfactory behavior towards their nominal equilibrium points, in spite of the large value of the perturbation signal.

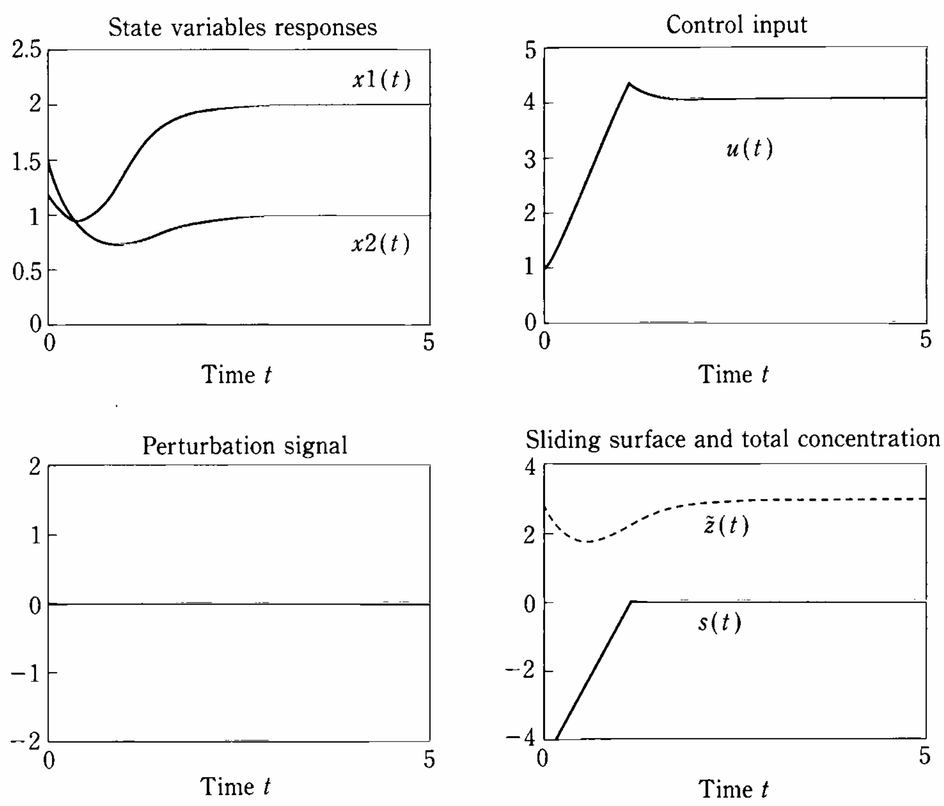

Fig. 2. Controlled responses for the (unperturbed) dynamical sliding mode feedback regulated CSTR system.

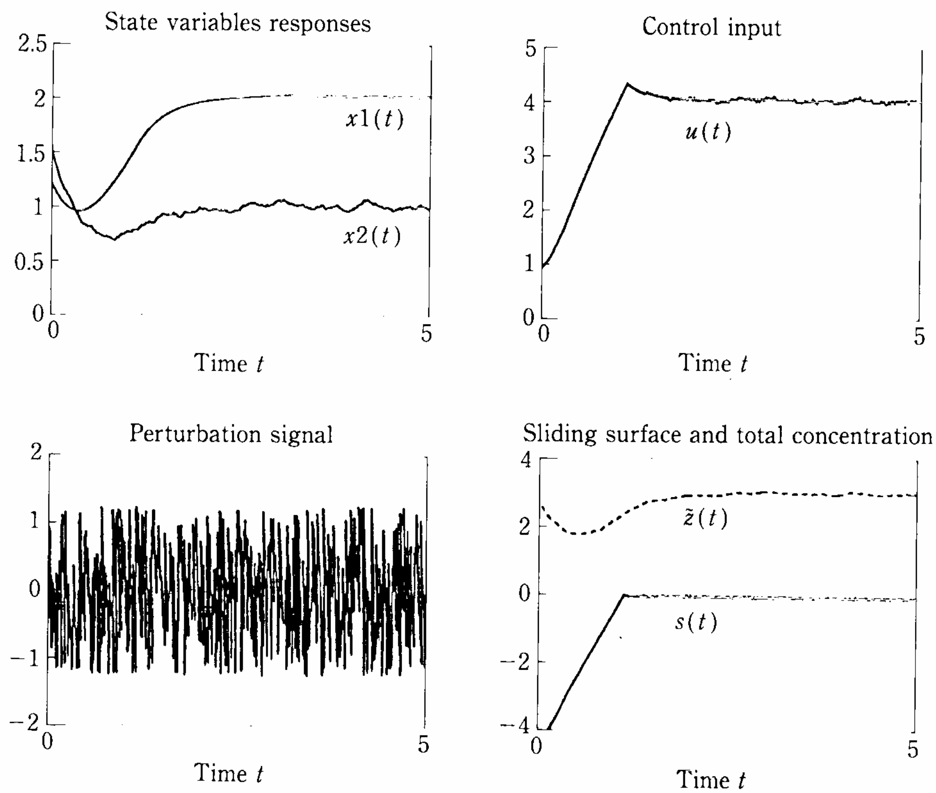

Fig. 3. Controlled responses for the (perturbed) dynamical sliding mode feedback regulated CSTR system.

4. Conclusions

The sliding mode controller design of differentially flat nonlinear systems becomes particularly simple, thanks to the possibilities of expressing sliding surfaces and imposed sliding mode dynamics in terms of the linearizing outputs. Dynamical sliding mode controller design for linearizable systems is also easily handled through dynamic extension of the original flat system.

In forthcoming publications, it will be shown that resorting to system flatness considerably simplifies the sliding mode controller design for multivariable nonlinear systems. This is explained by the fact that in multivariable nonlinear systems, flatness is explicitly related to decoupled linearizability.

The possibilities of prescribing *predictive control* strategies also seem to be particularly simple for flat systems, since, through differentiable functions of the linearizing coordinates, the control input can be immediately related to the desirable future output error trajectory in a most natural manner. A combination of predictive control techniques and dynamical sliding mode control for nonlinear systems has been initially explored in Sira-Ramírez and Fliess (1993). For the case of flat systems, such a combination would be particularly fruitful, given the practical importance and potentials, of both control research areas. This topic certainly requires and deserves further development.

Many theoretical and practical issues remain to be explored around the concept of flat systems. In this article, we have only attempted to present an introduction to the subject of flat systems and to explore its connections with sliding mode control through an illustrative example.

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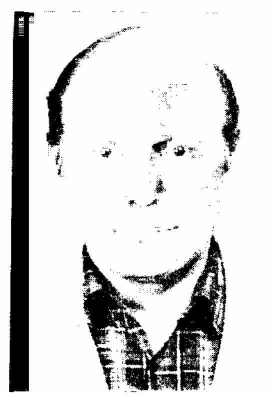

H. Sira-Ramírez was born in San Cristóbal (Venezuela) in 1948. He obtained the Degree of Ingeniero Electricista from the Universidad de Los Andes in Mérida-Venezuela in 1970. He pursued graduate studies at the Massachusetts Institute of Technology (MIT), Cambridge, MA, where he obtained the degrees of Master of Science, Electrical Engineer in 1974 and the Ph.D. in 1977, all in Electrical Engineering. He is currently a Full Professor in the Control Systems, Department of the Systems Engineering, School of the Universidad de Los Andes, where he has also held the positions of Head of the Control Systems Department and was elected as Vice-president of the University for the period 1980–1984. He was appointed Head of the Graduate School in Automatic Control Engineering in 1990.

Dr. Sira-Ramirez is a Senior Member of the Institute of Electrical and Electronics Engineers (IEEE), where he serves as a member of the IEEE International Committee. He is also a member of the International Federation of Automatic Control (IFAC), The Society of Industrial and Applied Mathematics (SIAM), The American Mathematical Society (AMS) and the Venezuelan College of Engineers (CIV).

Dr. Sira-Ramírez is an IEEE Distinguished Lecturer for the 1993–1996 period. Over the years, he has received several awards for his scientific work from Venezuelan institutions such as FUNDACION POLAR, FUNDACITE, CONICIT and the Venezuelan College of Engineers. Dr. Sira-Ramírez is interested in the theory and applications of discontinuous feedback control strategies for nonlinear dynamic systems, with emphasis in Sliding Mode Control and Pulse Width Modulation Techniques.