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On the robust stabilization and tracking for robotic manipulators with artificial muscles

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The robustness of a dynamical sliding mode control strategy with respect to unmatched perturbation inputs and modeling errors is examined for robotic manipulators equipped with an arrangement of antagonistic artificial muscles, acting as actuators. Differential flatness, which in the case of single-input manipulators coincides with exact linearizability of the system, is exploited to obtain a discontinuous feedback control strategy. The approach naturally allows for the introduction of a first order servovalve-artificial muscle actuator model for sliding mode controller design purposes.

1. Introduction

The purpose of this paper is to examine the robustness of dynamical sliding mode feedback control of robotic manipulators, equipped with artificial muscles, in the context of stabilization and tracking tasks. The dynamic extension and differential flatness of the original system allow for the natural introduction of the dynamical actuator in the feedback loop as a part of a desired trajectory for the linearizing output, as well as the direct specification of a control strategy based on sliding modes. The robustness of the resulting dynamical feedback control scheme, with respect to mismatched perturbations and modelling errors, is first established and then tested by means of computer simulations.

Dynamical sliding mode control was introduced by Fliess and Messager (1990, 1991) in the context of linear dynamical systems. These results were later extended to nonlinear systems by Sira-Ramírez (1992 a, b, 1993 a, b). Application of dynamical sliding mode controllers to the regulation and tracking of flexible joint manipulators was also undertaken by Sira-Ramírez *et al.* (1992). A closely related development is represented by the possibilities of specifying dynamical pulse width modulation (PWN) feedback control strategies. For an example that deals with the dynamical regulation of rigid and flexible joint manipulators, using PWM control, the reader is referred to the work of Sira-Ramírez and Llanes-Santiago (1993). The mathematical basis of all these extensions, and developments, lies within the realm of the differential algebraic approach to control systems, introduced by M. Fliess (1989, 1990 a, b). An immediate consequence of this theoretical advance, within the sliding mode control area, was the justification, through the use of generalized state space (Fliess 1990 a) representations of the involved system, of smoothed sliding mode controllers directly obtained from inversion of the dynamical system. Such a smoothing strategy, fundamentally allowing for input-dependent sliding manifolds, represents a significant departure from the traditional high-gain approach (Slotine and Li 1991) and it is applicable to other kinds of discontinuous feedback control strategies, such as PWM and pulse frequency modulation feedback controllers (Sira-Ramirez and Llanes-Santiago 1994).

In this paper we adopt a slightly different posture by bringing into the design process the differential flatness of the dynamically extended manipulator system. Differentially flat systems constitute the simplest extension of the notion of linear controllability to the nonlinear systems case and this is equivalent to exact linearizability in the single input–single output case. Flat systems were introduced and developed, as an outcome of the differential algebraic approach, by Fliess and his coworkers (Fliess *et al.* 1992 a, b, 1993) and Rouchon *et al.* (1993). Single input–single output flat systems are characterized by the existence of an independent linearizing output (i.e. one which does not satisfy any algebraic differential equation independent of the control input), and such that all variables in the system, including the control input, are expressible as differential functions

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of such a linearizing output (a differential function of a linearizing output is one that has as its arguments the output and a finite number of its time derivatives). The highest derivative of the linearizing output in the control input expression is equal to the order of the system.

In § 2 we show that a rigid robotic manipulator and its first-order dynamical extension (Nijmeijer and van der Schaft 1990) are differentially flat. The additions of the artificial muscle actuator model (Nouri and Lopez 1993), and of the servovalve (Inoue 1987) not only do not destroy differential flatness, but they are even seen to be naturally incorporated as part of the desired linearizing output trajectory. In § 2 we design a dynamical sliding mode tracking controller in terms of an imposed trajectory for the linearizing output described in terms of an unforced differential equation with discontinuous right-hand side. The sliding mode controller design is purposefully carried out without consideration of unmatched external perturbation inputs and large (unmatched) modelling errors. Such modelling errors arise from an erroneously assumed independence of the generated torque with respect to the obtained angular position. In this section we present some simulations of the sliding mode controlled system that includes unmatched noisy perturbations and equally unmatched modelling errors. The robustness of the proposed feedback controller is obtained in terms of the perturbed input-dependent sliding surface coordinate evolution, and is then corroborated through computer simulations.

2. A dynamical sliding mode controller for a manipulator system equipped with an artifical muscle actuator

2.1. *Flatness of an extended model of a single-link robotic manipulator*

Consider the following simple model of a single-link rigid robotic manipulator (Khalil 1992):

$$\left.\begin{aligned} \dot{\theta} &= \omega, \\ \dot{\omega} &= -\frac{g}{L}\cos\theta - \frac{k}{M}\omega + \frac{1}{ML^2}\tau, \end{aligned}\right\} \tag{1}$$

where θ is the link angular position, measured with respect to the horizontal, ω is the corresponding angular velocity and τ is the applied torque. The constants g, k, L and M represent, respectively, the acceleration due to gravity, the viscous damping coefficient, the length of the manipulator arm and the mass of the link (assumed, for simplicity, to be concentrated at the tip of the manipulator).

Consider also the first-order dynamical extension of the given manipulator system (1)

$$\left.\begin{aligned} \dot{\theta} &= \omega, \\ \dot{\omega} &= -\frac{g}{L}\cos\theta - \frac{k}{M}\omega + \frac{1}{ML^2}\tau, \\ \dot{\tau} &= v, \end{aligned}\right\} \tag{2}$$

where v is a new external input to the system.

It is easy to see that all variables in the extended system, including the auxiliary control input v, can be expressed as a differential function of θ, i.e. as a function of the angular position θ and a finite number of its time derivatives; (angular velocity ω, angular acceleration α, jerk, etc). Indeed from (2) it follows immediately that if we let $y = \theta$ denote the linearizing output, then

$$\left.\begin{aligned} \theta &= y, \\ \omega &= \dot{y}, \\ \tau &= ML^2\left(\ddot{y} + \frac{g}{L}\cos y + \frac{k}{M}\dot{y}\right), \\ v &= ML^2\left(y^{(3)} - \frac{g}{L}\dot{y}\sin y + \frac{k}{M}\ddot{y}\right), \end{aligned}\right\} \tag{3}$$

i.e. the extended system is differentially flat (Fliess *et al.* 1992 a).

Note that since $y^{(3)}$ is completely free (hence the name flat coordinate for y), one may specify a trajectory for $y(t)$ in terms of, say, an unforced third-order linear time-invariant differential equation. A common procedure would then be to impose such a linear evolution on the linearizing output y and to compute the required auxiliary input v by using (3). The problem with such an exact linearization approach is the resulting lack of robustness of the obtained feedback controller with respect to small parameter variations and with respect to unavoidable external perturbation inputs acting on the actual system. In the next section we propose a dynamical sliding mode controller which eventually linearizes the closed-loop behaviour of the output y, or alternatively that of its associated tracking error, to a desired second-order system dynamics. The obtained design is shown to be robust with respect to significant unmatched perturbation inputs and reasonably large modelling errors.

2.2. *A model for the servovalve–artificial muscles actuator system*

The actuator considered is composed of two antagonistic muscles that develop a contracting force, in analogy with the force produced by a skeletal muscle arrangement (Nouri and Lopez 1993). The artificial muscles considered follow the same principles as the Japanese 'rubbertuator' (editorial article, Rubber

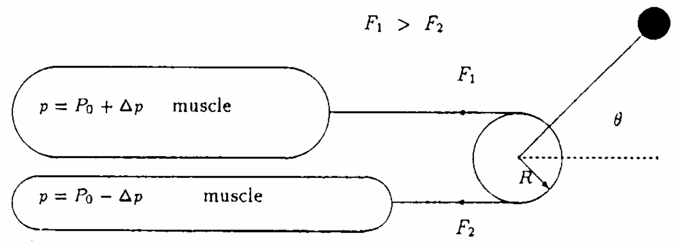

Figure 1. Antagonistic artificial muscles arrangement; muscles in current control position.

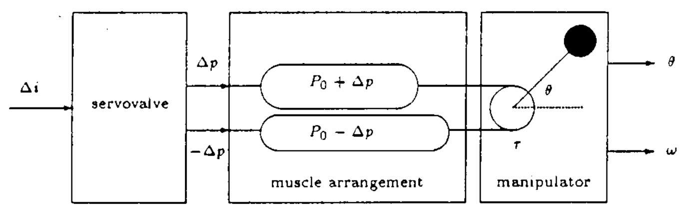

Figure 2. Block diagram of manipulator and artificial muscle–servovalve arrangement.

Development, Vol. 37, No. 4). The advantage of using such actuators lies in the natural compliance which can be obtained for robot motions. Simulations and actual experimental work have been extensively carried out on such a class of actuators, using different sliding mode control strategies (Hamerlain *et al.* 1991, Nouri *et al.* 1993, Nouri and Lopez 1993).

The antagonistic artificial muscle arrangement is shown in Fig. 1. It consists of two identical muscles joined by a chain through a free rotating wheel of radius R. Such a wheel provides the necessary link of the muscles for torque generation purposes. The rotating axis of the wheel is thus perpendicular to the 'muscles plane'. The actuator working principle is easily explained by referring to Fig. 1. At the 'rest state' the air pressure in the two muscles is equal to the atmospheric pressure; consequently, the muscles are not contracted. In the 'initial control state' the air pressures in the two muscles are equal and constant, of value P_0; thus, each muscle is equally contracted. The arrangement is such that the total applied torque to the joint pivot is zero and the link position rests at its inital state. In the 'current control state' an angular joint displacement is obtained by a simultaneous pressure increment of value Δp, in the 'antagonistic muscle' and a pressure decrement, by the same amount Δp, in the 'antagonistic muscle'.

The artificial muscles are modelled as cylinders which keep their cylindrical shape during contraction. In reality the muscles remain cylindrical only in their initial states, and during contraction they become nearly cone-shaped in the neighbourhood of the tips. This fact generates some nonlinear effects which, for the sake of simplicity, are not considered here.

Each muscle is supplied with the required air pressure from an electro-pneumatic converter, or current/pressure transducer. Figure 2 depicts a block diagram of the servovalve–muscle actuator and single-link manipulator system.

Consider the following dynamical first-order model of a servovalve (Inoue 1987, Miyazaki *et al.* 1989)

$$\frac{\mathrm{d}}{\mathrm{d}t}\Delta p = -\frac{1}{T_r}\Delta p + \frac{K_r}{T_r}\Delta i. \tag{4}$$

where Δp is the incremental variation of pressure in the two antagonistic muscles, with respect to a nominal pressure P_0 which keeps the manipulator arm in equilibrium at the position $\theta = 0$, and Δi is the corresponding variation of the input current in the servovalve, regarded as the effective control input.

The time constant of the servovalve–muscle combination is here denoted by T_r and the parameter K_r is the current-to-pressure gain. This first-order linear model for the servovalve has been fully documented in the literature and extensively used in connection with actual laboratory implementation of several feedback strategies (a sample of the available results using such a model is obtained from the works by Inoue (1987), Miyazaki *et al.* (1989), Hamerlain *et al.* (1991), Nouri and Lopez (1993), Nouri *et al.* (1993), as well as the references therein).

A model for the actuator can be deduced from the contracting force model considered for the artificial muscle. The following mathematical model for the contracting force F has been established by Nouri and Lopez (1993)

$$F = (\pi r_0^2)P[a(1-\epsilon)^2 - b], \tag{5}$$

where r_0 is the muscle radius at rest, P is the air pressure inside the muscle, ϵ is the current muscle contraction ratio, defined as

$$\epsilon = \frac{L - L_0}{L_0}$$

with L_0 the muscle length at rest and L the current muscle length. The parameters a and b are constant parameters related to the physical characteristics of the muscle. The torque τ, generated by the actuator, is given by

$$\tau = R(F_1 - F_2), \tag{6}$$

where R is the actuator wheel radius, F_1 is the agonistic force and F_2 is the antagonistic force (see Fig. 1).

Using the expression for the contracting force F in (5), one obtains, from a simple consideration of Fig. 1

$$\tau = R(\pi r_0^2)\left\{(P_0 + \Delta p)\left[a\left(1 - \epsilon_0 - \frac{R\theta}{L_0}\right)^2 - b\right] - (P_0 - \Delta p)\left[a\left(1 - \epsilon_0 + \frac{R\theta}{L_0}\right)^2 - b\right]\right\}, \tag{7}$$

where P_0 is the initial air pressure in the muscle, chosen to be half the maximum available pressure, ϵ_0 is the initial contraction ratio, chosen according to the muscle parameters and the desired joint motion range (10–15% being typical values), and Δp is the incremental commanded pressure. The following expression is deduced for the torque τ:

$$\tau = K_T\Delta p - K'_T\theta + K''_T\theta^2\Delta p. \tag{8}$$

The generated torque τ is thus assumed to be a function of the variation of pressure Δp, generated by the antagonistic muscle assembly, and of the angular position of the robot link θ. The constant parameters K_T, K'_T and K''_T are given by

$$\left.\begin{aligned} K_T &= 2\pi r_0^2 R[a(1-\epsilon_0)^2 - b],\\ K'_T &= \frac{4\pi r_0^2 R^2 a P_0}{L_0}(1-\epsilon_0),\\ K''_T &= \frac{2\pi r_0^2 R^3 a}{L_0^2}.\end{aligned}\right\} \tag{9}$$

It can be remarked that the term $-K'_T\theta$ in (8) indicates that the torque generated by the actuator provides a counteracting effect which explains the natural compliance of the artificial muscle actuator arrangement. We shall adopt, as a practical approximation for the generated torque, the simplified expression

$$\tau = K_T\Delta p - K'_T\theta. \tag{10}$$

This approximation is justified by the fact that for reasonably large values of the angular position θ, the contribution of the term $K''_T\theta^2\Delta p$ is quite small due to the fact that the constant K''_T is several orders of magnitude smaller than K_T. Note that if $y(t)$ is related to the physical control input signal Δi by means of

$$y^{(3)} = \frac{g}{L}\dot{y}\sin y - \frac{k}{M}\ddot{y} + \frac{1}{ML^2}\left[-\frac{1}{K_T T_r}\Delta p + \frac{K_r}{K_T T_r}\Delta i - \frac{K'_T}{K_T}\dot{y}\right], \tag{11}$$

then the auxiliary control input v is given by

$$v = -\frac{1}{K_T T_r}\Delta p + \frac{K_r}{K_T T_r}\Delta i - \frac{K'_T}{K_T}\dot{y}, \tag{12}$$

and the extended model (2) of the system (1) is naturally transformed into the physical model of the system, including now the approximate description of the artificial muscle–servovalve combination,

$$\left.\begin{aligned} \dot{\theta} &= \omega,\\ \dot{\omega} &= -\frac{g}{L}\cos\theta - \frac{k}{M}\omega + \frac{K_T}{ML^2}\Delta p - \frac{K'_T}{ML^2}\theta,\\ \dot{\Delta p} &= -\left(\frac{1}{T_r}\right)\Delta p + \left(\frac{K_r}{T_r}\right)\Delta i.\end{aligned}\right\} \tag{13}$$

These developments may be simply interpreted by the fact that the overall system is still differentially flat, i.e. linearizable. Indeed, the new state variable Δp and the actual input Δi are also expressible as differential functions of θ. From (3), (4) and (10) it follows that

$$\Delta p = \frac{ML^2}{K_T}\left(\ddot{y} + \frac{g}{L}\cos y + \frac{k}{M}\dot{y} + \frac{K'_T}{ML^2}y\right), \tag{14}$$

and

$$\Delta i = \frac{ML^2 T_r}{K_T K_r}\left[\frac{1}{T_r}\left(\ddot{y} + \frac{g}{L}\cos y + \frac{k}{M}\dot{y} + \frac{K'_T}{ML^2}y\right) + \left(y^{(3)} - \frac{g}{L}\dot{y}\sin y + \frac{k}{M}\ddot{y} + \frac{K'_T}{ML^2}\dot{y}\right)\right]. \tag{15}$$

One may also, alternatively, consider the incorporated model as part of a dynamical feedback controller for the original system (1), which synthesizes the applied torque τ as the solution of a controlled differential equation regulated by the control input current Δi. Clearly, from (10) and (13) one obtains

$$\dot{\tau} = -\frac{1}{T_r}\tau + \frac{K_T K_r}{T_r}\Delta i - \frac{K'_T}{T_r}\theta - K'_T\omega, \tag{16}$$

where the new control input Δi is to be synthesized as a static sliding mode based feedback control input.

2.3. *Sliding mode controller design for tracking and stabilization tasks*

Let $\Theta(t)$ denote a piecewise smooth function of time that represents a desirable angular position trajectory for the robotic manipulator. Consider the following sliding surface definition, written in terms of the linearizing output y:

$$s = \ddot{y} - \ddot{\Theta}(t) + 2\zeta\omega_n(\dot{y} - \dot{\Theta}(t)) + \omega_n^2(y - \Theta(t)). \qquad (17)$$

Evidently if the sliding surface s is forced to become zero, by means of a suitable control action, then the tracking error $e = y - \Theta(t)$ obeys the following second-order dynamics:

$$\ddot{e} + 2\zeta\omega_n\dot{e} + \omega_n^2 e = 0, \qquad (18)$$

where the coefficients ζ and ω_n may be suitably chosen to guarantee an asymptotically stable trajectory of the tracking error e to zero, with desired quality features.

The trajectories associated to the unforced discontinuous scalar dynamics

$$\dot{s} + W \operatorname{sign} s = 0, \qquad (19)$$

exhibit a finite time reachability of zero from any given initial condition $s(0) \in \mathbb{R}$, provided that the constant gain W is chosen to be strictly positive. It is therefore clear that it is highly desirable to impose on the sliding surface s, defined in (17), the autonomous dynamics (19).

The desired time-varying discontinuous dynamics for the linearizing output y are then given by

$$\begin{aligned} y^{(3)} = & -2\zeta\omega_n\ddot{y} - \omega_n^2\dot{y} + \Theta^{(3)}(t) + 2\zeta\omega_n\ddot{\Theta}(t) + \omega_n^2\dot{\Theta}(t) \\ & - W \operatorname{sign}[\ddot{y} - \ddot{\Theta}(t) + 2\zeta\omega_n(\dot{y} - \dot{\Theta}(t)) \\ & + \omega_n^2(y - \Theta(t))]. \end{aligned} \qquad (20)$$

A static sliding mode feedback controller, for the combinated third-order system, is readily obtained by substituting (20) into (3) and solving for Δi. Alternatively, one may consider the obtained expression for Δi as the static nonlinear feedback input signal to the dynamical feedback controller (16).

Note, however, that a controller thus derived must clearly perform according to the prescribed design demands, because no perturbations or significant modelling errors are being considered in the design model. One of the important characteristics of (traditional) static sliding mode control, which is shared by dynamical sliding mode controllers, is represented by the robustness of the feedback strategy. In the next section we derive a dynamical sliding mode controller on the basis of a simplified design model that contains (unmatched) modelling errors. The performance of the obtained controller is tested, via computer simulations, by using it as a feedback controller on a perturbed version of the physical model (14), regarded as the actual system model. Such a model also happens to include, unmatched, unmodelled external perturbation inputs of a stochastic nature.

2.4. *Robustness of a dynamical feedback sliding mode controller*

It is quite well known that sliding mode controllers are traditionally robust with respect to parameter variations and external perturbations, provided that some matching conditions are satisfied. These matching conditions, first developed in the work of Drazenovic (1969), are rightfully meaningful when considered in the context of a fixed state-space representation of the system. If an input–output approach is used, the matching conditions lose their meaning and they are, so to speak, automatically satisfied, provided that the time derivatives of the unmatched perturbation inputs are bounded. This is especially clear if a generalized state-space representation, of the generalized observability canonical form type, is used for the system model (Fliess and Messager 1990, Fliess 1989). This fact has also become evident in some recent theoretical work related to sliding mode control of linear systems that utilizes a module theoretic viewpoint (Fliess and Sirå-Ramírez 1993 a, b). Similar remarks are also applicable to sliding mode output feedback schemes using also discontinuous state vector reconstruction strategies (Sira-Ramírez and Spurgeon 1994, Sira-Ramírez *et al.* 1994).

For sliding mode controller design purposes, we now regard the model in (13) as the actual system model, and include a perturbation term $\vartheta(t)$ in the acceleration equation which represents unmodelled noisy signals that affect the actual behaviour of the system, i.e.

$$\left.\begin{aligned} \dot{\theta} &= \omega, \\ \dot{\omega} &= -\frac{g}{L}\cos\theta - \frac{k}{M}\omega + \frac{K_T}{ML^2}\Delta p - \frac{K_T'}{ML^2}\theta + \vartheta(t), \\ \dot{\Delta p} &= -\left(\frac{1}{T_r}\right)\Delta p + \left(\frac{K_r}{T_r}\right)\Delta i. \end{aligned}\right\} \qquad (21)$$

The scalar signal $\vartheta(t)$ represents an external, bounded, perturbation input to the system which also exhibits bounded time derivatives up to a first order. Note that the perturbation signal $\vartheta(t)$ is 'unmatched' with respect to the control input channel used by the input signal Δi.

To test the robustness of the control policy, generated by the proposed sliding mode dynamics (20), we compute the sliding mode control law for Δi under two simplifying assumptions that test its robust performance with respect to mismatched unmodelled perturbation terms and additive unmatched modelling errors.

(*a*) An intentional modelling error is introduced by enforcing a simplifying assumption into (10), describing the torque generated by the artificial muscle arrangement. We specifically assume that the torque τ is a function only of the incremental pressure Δp and that it does not depend on the angular position θ, i.e.

$$\tau = K_T \Delta p. \tag{22}$$

For sliding mode controller design purposes we thus ignore the counteracting effect of the angular position on the delivered torque.

(*b*) We also ignore the presence of external perturbation signals $\vartheta(t)$ in our simplified model.

The above assumptions result in a simplified dynamical model for the actuator–manipulator system, given now by

$$\left.\begin{aligned} \dot{\theta} &= \omega, \\ \dot{\omega} &= -\frac{g}{L}\cos\theta - \frac{k}{M}\omega + \frac{K_T}{ML^2}\Delta p, \\ \dot{\Delta p} &= -\left(\frac{1}{T_r}\right)\Delta p + \left(\frac{K_r}{T_r}\right)\Delta i. \end{aligned}\right\} \tag{23}$$

The resulting simplified dynamical sliding mode controller, corresponding to the sliding dyanics (20) and the adopted model (23), may be given, after the use of (22), in terms of the state vector $(\theta, \omega, \Delta p)$ by

$$\left.\begin{aligned} \dot{\Delta p} &= -\frac{1}{T_r}\Delta p + \frac{K_r}{T_r}\Delta i, \\ \Delta i &= \frac{T_r}{K_T K_r}\left\{\frac{K_T}{T_r}\Delta p + ML^2\left[-\frac{g}{L}\omega\sin\theta \right.\right. \\ &\quad + \left(\frac{k}{M} - 2\zeta\omega_n\right)\left(-\frac{g}{L}\cos\theta - \frac{k}{M}\omega + \frac{K_T}{ML^2}\Delta p\right) \\ &\quad - \omega_n^2\omega + \Theta^{(3)}(t) + 2\zeta\omega_n\ddot{\Theta}(t) \\ &\quad \left.\left. + \omega_n^2\Theta(t) - W \operatorname{sign} s\right]\right\}, \\ s &= -\frac{g}{L}\cos\theta - \frac{k}{M}\omega + \frac{K_T}{ML^2}\Delta p - \ddot{\Theta}(t) \\ &\quad + 2\zeta\omega_n(\omega - \dot{\Theta}(t)) + \omega_n^2(\theta - \Theta(t)). \end{aligned}\right\} \tag{24}$$

Note that by regarding the torque τ as $\tau = K_T\Delta p$ the proposed sliding surface $s = 0$ is actually an input torque dependent manifold. The discontinuous feedback control actions, present in the generated signal Δi, are integrated three times before explicitly appearing in the commanded angular position θ. This fact is responsible for the substantially smoothed output responses.

Substitution of the proposed simplified controller expression (24) on the actual perturbed system (21) and calculation of the sliding surface coordinate dynamics, results, after some algebraic manipulations, in the following expression for the sliding surface evolution:

$$\begin{aligned} \dot{s} = &-W \operatorname{sign} s + \dot{\vartheta}(t) - \left(\frac{k}{M} - 2\zeta\omega_n\right)\vartheta(t) \\ &- \frac{1}{ML^2}\left(\frac{K_T'}{T_r}\theta + K_T\omega\right). \end{aligned} \tag{25}$$

Because $\vartheta(t)$ and $\dot{\vartheta}(t)$ are assumed to be bounded, it is easy to see that for bounded ranges of the angular position θ and the angular velocity ω of the manipulator manoeuvres, there exists a sufficiently large positive value of the constant gain W, such that the sliding surface coordinate s is guaranteed to converge to zero in finite time. Hence, the desired dynamics (18) for the linearizing output tracking error e are achieved by the actual perturbed system. The ideal sliding dynamics are therefore obtained in spite of the (bounded) values of the perturbation input signal and the modelling errors adopted for the dynamical sliding mode controller calculation.

A similar analysis can be carried out if unknown variations in the mass M of the manipulator are allowed, whereas the simplified dynamical feedback controller (24) is still computed in terms of a certain nominal value, say M_0, of the manipulator mass M. The results are not reported here.

2.5. *Simulation results*

Simulations were performed for a stabilization task and a tracking task for the perturbed system (17).

The parameter values for the system were taken to be

$$M = 1{\cdot}0 \text{ kg}, \quad L = 1{\cdot}0 \text{ m}, \quad k = 0{\cdot}05 \text{ kg/s}, \quad g = 9{\cdot}8 \text{ m/s}^2.$$

The parameters of the artificial muscle model were obtained, in an approximate manner, from an experimental set-up. These parameters were set to be

$$K_T = 1{\cdot}0 \times 10^{-4} \text{ Nm/Pa}, \quad K_T' = 2{\cdot}0 \times 10^{-1} \text{ Nm/rad},$$
$$T_r = 25{\cdot}0 \text{ s}, \quad K_r = 2{\cdot}5 \times 10^8\, N^2 m^2 APa.$$

For the stabilization task, the sliding mode controller was designed with the following parameters:

$$\zeta = 0{\cdot}8, \quad \omega_n = 25 \text{ rad/s}, \quad \Theta = 0 \text{ rad}, \quad W = 600.$$

The graphs in Fig. 3 depict the behaviour of the controlled system for the stabilization task. The several figures show

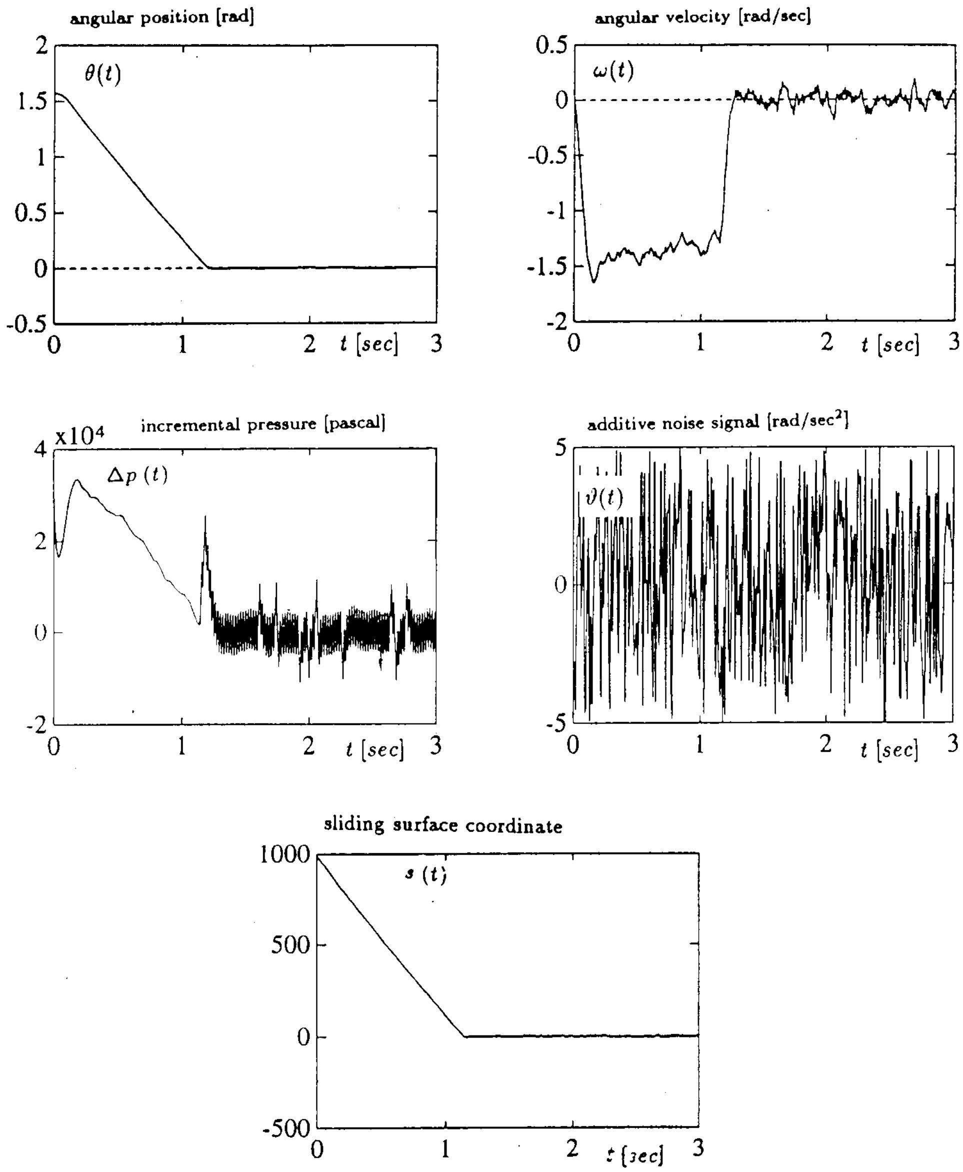

Figure 3. Sliding mode control stabilization of manipulator with artificial muscles.

the controlled evolutions of the state variables θ and ω and the incremental pressure Δp. All these variables evolve towards their desired equilibrium values. The sliding surface coordinate s reaches the value of zero in finite time and its evolution is sustained on such a condition in spite of the unmatched influence of the perturbation input $\vartheta(t)$. The mismatched noise signal $\vartheta(t)$, applied to the actual system model, (21) is also shown in this figure.

For the tracking task, the following piecewise smooth function is proposed, which produces a convenient transition, in time T, between an initial angular position Θ_0 and a final position Θ_1. The angular displacement is achieved with non-zero constant angular accelerations during the initial and final phases of the maneuver, of duration t_{acc}. The intermediate phase entitles a constant 'cruising' velocity of value V achieved at the end of the initial acceleration interval.

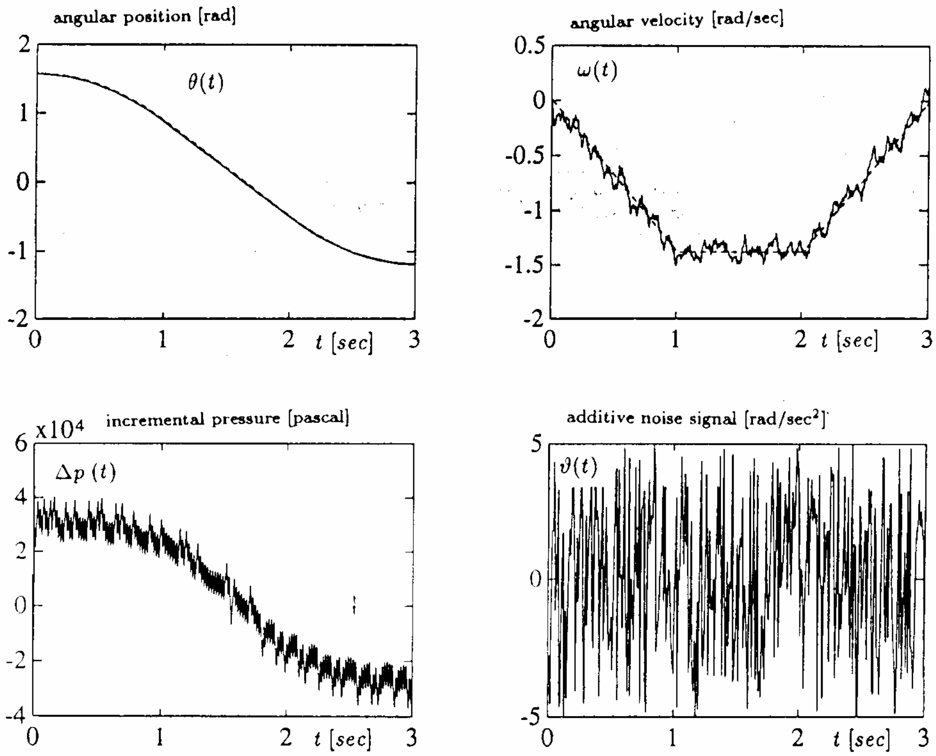

Figure 4. Tracking task for sliding mode controlled manipulator with artificial muscles.

$$\Theta(t) = \begin{cases} \dfrac{At^2}{2} + \Theta_0, & \text{for } 0 \le t \le t_{acc}, \\ V\left(t - \dfrac{t_{acc}}{2}\right) + \Theta_0, & \text{for } t_{acc} < t < T - t_{acc}, \\ -\dfrac{A(t-T)^2}{2} + \Theta_1, & \text{for } T - t_{acc} \le t \le T, \end{cases} \tag{26}$$

where $t_{acc} = |V/A|$ and $T = |\Theta_1 - \Theta_0/V| + |V/A|$, and the parameters Θ_0 and Θ_1 represent, respectively, the position of departure and arrival of the link angle θ. The quantity V is the crusing velocity and A is the allowed acceleration in the initial and final stages, respectively during the intervals $[0, t_{acc}]$ and $[T - t_{acc}, T]$. To obtain a physically meaningful solution for V and A, the initial and final values of θ must satisfy the condition

$$|\Theta_1 - \Theta_0| \ge \frac{V^2}{|A|}.$$

The preceding relations, and the careful choice of T and t_{acc}, allows one to uniquely compute the required values of V and A.

The sliding mode controller parameters W, ζ and ω_n, were taken to be the same as for the stabilization task. The initial and final angular positions were set to be

$$\Theta_0 = \frac{\pi}{2}, \quad \Theta_1 = -\frac{\pi}{6}.$$

These values, and the choice of

$$t_{acc} = 1{\cdot}0\ \text{s}, \quad T = 3{\cdot}0\ \text{s},$$

yield identical numerical values for the required acceleration and cruising velocity:

$$V = -1{\cdot}38\ \text{rad s}^{-1}, \quad A = -1{\cdot}38\ \text{rad s}^{-2}.$$

Figure 4 presents the performance of the perturbed controlled system to the tracking task. The evolutions of the controlled state variables are shown in this figure, as well as the sliding surface coordinate behaviour. The unmatched noise signal $\vartheta(t)$, applied to the system, is also depicted in Fig. 4.

3. Conclusions

In this paper, using the concept of differential flatness, which is equivalent to exact linearizability in the studied single-link robotic manipulator, a dynamical sliding mode controller for robust tracking and stabilization has been proposed. By using the differentially flat nature of the system, it was found that the dynamical discontinuous feedback controller easily assimilates a servovalve–artificial muscle model of first-order nature. Such an addition thus leads to a static sliding mode controller on the augmented order system. The addition of the artificial muscle and servovalve assembly model does not destroy flatness, and it is naturally incorporated as part of the prescribed linearizing output trajectory. The output trajectory is here taken to be autonomous third-order output error dynamics, robustly converging in finite time to a two-dimensional integral sliding surface in the error space. The robustness of the proposed scheme was mathematically proven and thoroughly tested in several computer simulations which included unmatched perturbation inputs and considerable (unmatched) modelling errors arising from simplifying assumptions on the actuator torque equations.

The performance of the controller was encouraging enough to attempt actual implementation on a laboratory facility located at INSA (Toulouse). Work in this direction is currently underway, and the results obtained will be reported elsewhere.

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