INT. J. CONTROL, 1996, VOL. 65, NO. 2, 205–222

Dynamical adaptive pulse-width-modulation control of DC-to-DC power converters: a backstepping approach

HEBERTT SIRA-RAMÍREZ†, MAURICIO GARCÍA-ESTEBAN‡ and ALAN S. I. ZINOBER§

Adaptive regulation of pulse-width-modulation (PWM) controlled DC-to-DC power supplies is proposed using a suitable combination of dynamical input–output linearization and the 'backstepping' controller design method. A nominal parameter, input-dependent, state coordinate transformation of the average PWM converter models leads to a type of pure parameter feedback canonical form associated with the Fliess generalized observability canonical form of such average models. A backstepping design procedure can then be immediately devised which leads to a dynamical adaptive regulation scheme for the generation of the stabilizing duty ratio function. The validity of the proposed approach, regarding control objectives and robustness with respect to unmodelled, yet unmatched, and bounded stochastic perturbation inputs, is tested through digital computer simulations.

1. Introduction

Feedback regulation of switchmode DC-to-DC power converters is usually accomplished by means of pulse-width-modulation (PWM) feedback strategies. For the fundamental background of this important subject the reader is referred to conference proceedings (such as the yearly Power Specialist Conference Records, the multi-volume series edited by Middlebrook and Cúk (1981), or the remarkable collection of articles recently edited by Bose (1992). Also, useful material may be found in specialized books such as Kassakian *et al.* (1991), Severns and Bloom (1985) and Csaki *et al.* (1983).

PWM feedback regulation strategies for DC-to-DC power converters are usually based on perfect knowledge of the converter parameters (see, among many other authors, the articles by Sira-Ramírez and co-workers (1989, 1991, 1992). This fundamental assumption is sometimes invalid due to imprecise knowledge of the values of the converter circuit components as well as of the external voltage source. The situation is often due to either measurement errors, or unavoidable ageing effects on the circuit components. Automatic control problems which efficiently handle uncertainty in the system parameter values usually require adaptive solutions employing different forms of the so-called 'uncertainty equivalence principle' (Sastry and Bodson 1989). In other words, the controller is designed as if the system parameters were perfectly known, and then the values of the parameters appearing in the controller expression are regarded as tunable, in an online fashion. Parameter

Received 15 August 1994. Revised 13 July 1995.

† Departamento Sistemas de Control, Universidad de Los Andes, Mérida 5101, Venezuela. e-mail: isira@ing.ula.ve.

‡ Centro de Instrumentos, Universidad Nacional Autónoma de México, México, D.F., México. e-mail: imauricio@ing.ula.ve.

§ Applied Mathematics Section, School of Mathematics and Statistics, University of Sheffield, Sheffield S10 2TN, U.K. e-mail: A.Zinober@sheffield.ac.uk.

tuning is accomplished by the specification of an updating, or parameter adaptation, law designed to simultaneously guarantee the demands of the regulation objectives and the stability of the adaptation process.

Adaptive feedback control techniques for PWM controlled DC-to-DC power supplies have been explored by Sira-Ramírez *et al.* (1993 a, b). The approach in these contributions incorporated an extension of the results found in Sastry and Isidori (1989), for the adaptive stabilization of partially linearizable, minimum-phase, discontinuously controlled nonlinear systems.

In this paper a rather different adaptive feedback strategy is adopted by resorting to an approach inspired by the recently introduced adaptive backstepping controller design methodology. Backstepping adaptation was developed for the regulation of a large class of state linearizable nonlinear systems exhibiting constant, but otherwise unknown, parameter values. The basic ideas and rather useful variations, of the backstepping adaptive design procedure can be found in the excellent research articles of Kanellakopoulos *et al.* (1991 a, b, c) and by Krstić *et al.* (1992), to which the reader is referred for enlightening details.

We specifically assume that the circuit converter components are only nominally known and that their constant discrepancies from the given nominal values are totally unknown. The use of an input-dependent, nominal parameter based, input–output linearizing state coordinate diffeomorphism for the unknown system produces an imperfectly transformed system in generalized phase variables (Fliess 1989). The structure of the resulting systems, in generalized phase variables, strongly resembles the pure parameter feedback canonical form, presented by Kanellakopoulos (1991 a), except for the presence of the control inputs in some of the 'regressor vectors', as well as the control input time derivative in the transformed system equations.

Computation of the feedback controllers, and of the associated incremental parameter update laws, is then carried out by resorting to a backstepping calculation procedure applied to the obtained generalized pure incremental parameter feedback canonical form. The net result is that one yields adaptive dynamical duty ratio synthetizers, rather than traditional static feedback compensators. The advantage of dynamically generated duty ratio control signals lies in the enhanced smoothed character of this important feedback regulation signal during the actual (i.e. discontinuous) operation of the converters. Smoothing of the duty ratio function increases the precision, and qualitative performance features, of the closed-loop behaviour of the DC-to-DC power converter circuits.

Section 2 is devoted to revisiting, via a 'boost' converter example, the fundamentals of the input–output linearization of PWM controlled DC-to-DC power converters by means of a dynamical feedback duty ratio synthesizer. We assume that all parameters in the system are perfectly known. The input–output linearization scheme achieves indirect regulation of the average output capacitor voltage by means of average input inductor current regulation. This strategy, which essentially involves a 'change of output' effectively avoids the non-minimum phase problem in the direct regulation of the output capacitor voltage variable. This method has already been used by Sira-Ramírez *et al.* (1991, 1993) and it was later justified, from a general viewpoint, by Benvenuti *et al.* (1992) for nonlinear systems and by Fliess and Sira-Ramírez (1993) for linear systems.

Section 3 presents the developments leading to a dynamical adaptive PWM control strategy for DC-to-DC power supplies of the 'boost' and 'buck-boost' types with unknown incremental parameter values. Computer simulations are presented which clearly indicate the effectiveness, and robustness, of the proposed adaptive feedback regulation scheme with respect to unmodelled, and unmatched, external stochastic perturbation inputs of bounded nature. Section 4 contains the conclusions and suggestions for further work in this area.

2. A nominal input–output linearization strategy for DC-to-DC power converters

This section contains the developments leading to dynamical feedback duty ratio synthesizers for the PWM stabilization of nominal average models of DC-to-DC power converters. The scheme, already exploited by Sira-Ramírez (1991), is presented here only for the purpose of making the article self-contained. The fundamental idea is to obtain a generalized nonlinear phase variable representation of the input–output behaviour of the average circuit for which the control synthesis problem is straightforward. Due to non-minimum phase problems associated with the output capacitor voltage variable, the regulated output is chosen as the input inductor current. Thus, indirect output voltage regulation is achieved.

2.1. *Boost converter*

Consider the boost converter circuit shown in Fig. 1. This circuit is described by the state equation model

$$\left.\begin{aligned} \dot{I}(t) &= -\frac{1}{L}(1-u)\,V(t)+\frac{E}{L} \\ \dot{V}(t) &= \frac{1}{C}(1-u)\,I(t)-\frac{1}{RC}\,V(t) \\ y(t) &= I(t) \end{aligned}\right\} \tag{1}$$

where I and V represent, respectively, the input inductor current and the output capacitor voltage variables. The positive quantity E is the external input voltage. The variable u denotes the switch position function, acting as a control input which takes values in the discrete set $\{0, 1\}$. The output y of the system is represented by the input inductor current I.

We define

$$\Theta_1 = \frac{1}{L}, \quad \Theta_2 = \frac{1}{C}, \quad \Theta_3 = \frac{1}{RC}, \quad \Theta_4 = \frac{E}{L} \tag{2}$$

as the system parameters assumed to be nominally known.

A PWM feedback control strategy for the regulation of the boost converter circuit is typically given by the following prescription of the switch position function (Sira-Ramírez 1989a, Sira-Ramírez *et al.* 1991):

$$\left.\begin{aligned} u &= \begin{cases} 1, & \text{for } t_k \leqslant t < t_k+\mu(t_k)\,T \\ 0, & \text{for } t_k+\mu(t_k)\,T \leqslant t < t_k+T \end{cases} \\ t_{k+1} &= t_k+T, \quad k = 0, 1, \ldots \end{aligned}\right\} \tag{3}$$

where t_k represents a sampling instant; the parameter T is the fixed sampling period, also called the duty cycle; and the sampled values of the state vector $x(t)$ of the

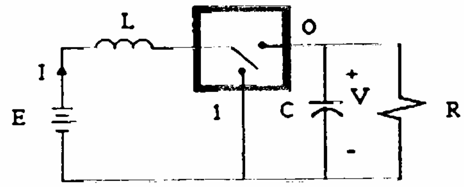

Figure 1. Boost converter circuit.

converter are denoted by $x(t_k)$. The function $\mu(\cdot)$ is the duty ratio function acting as a truly feedback policy. The value of the duty ratio function $\mu[t_k]$ determines at every sampling instant t_k the width of the upcoming 'pulse' (switch at the position $u = 1$) as $\mu[t_k]\,T$. The actual duty ratio function $\mu(\cdot)$ is evidently a function limited to the closed interval [0, 1] on the real line.

The control problem associated with the stabilization of the discontinuously controlled system (1) and (3) towards some (feasible) prespecified constant desired equilibrium point, consists of specifying the duty ratio function μ as a static, or dynamical, feedback control policy, i.e. as a function of the state vector x, or as the solution of a time-varying differential equation based on the measured values of the state x. As formulated, the problem of synthesizing a suitable duty ratio function μ is quite involved, owing to the difficulty in performing an exact discretization of the PWM system model (1) and (3). A conceptually useful, and practical, alternative consists of resorting to the infinite frequency average PWM model, also known as the state space average model of the PWM controlled converter (1) and (2) (Kassakian *et al.* 1991, Middlebrook and Ćuk 1981). The assumption of an infinite sampling frequency results in a smooth linear average system model of (1) in which the duty ratio function μ is readily interpreted as a control input to the average system in formal replacement of the switch position function u. In fact, the duty ratio function becomes the equivalent control input in the corresponding sliding mode (Utkin 1978) interpretation of the obtained idealization (Sira-Ramírez 1989 b).

The above idealization has the fundamental advantage of reducing the duty ratio synthesis problem to a standard nonlinear feedback control design problem in which the duty ratio function acts as the required feedback control input. Any of the well known static, or dynamical, feedback controller design procedures established in the recent literature (Isidori 1989, Rugh 1986, Fliess 1989) can be applied to the nonlinear average model of the circuit to obtain the required duty ratio function as a nonlinear feedback control law.

Consider, then, the following nominal average PWM model of the boost converter circuit:

$$\left.\begin{aligned} \dot{\zeta}_1 &= -\Theta_1(1-\mu)\zeta_2+\Theta_4 \\ \dot{\zeta}_2 &= \Theta_2(1-\mu)\zeta_1-\Theta_3\zeta_2 \\ \eta &= \zeta_1 \end{aligned}\right\} \tag{4}$$

where ζ_1 and ζ_2 represent the averaged values of the original state variables I and V. The average output variable ζ_1 is here denoted by η.

For a given constant value $\mu = U$ of the duty ratio function, the correponding equilibrium values of the average state variables of the circuit are obtained as

$$\zeta_1(U) = \frac{\Theta_3\Theta_4}{\Theta_1\Theta_2(1-U)^2}, \quad \zeta_2(U) = \frac{\Theta_4}{\Theta_1(1-U)} \tag{5}$$

Note that by straightforward elimination of the constant parameter U, in the set of equations (5), the equilibrium values for ζ_1 and ζ_2 are related by

$$\zeta_1(U) = \frac{\theta_3\theta_4}{\theta_2\theta_1}\zeta_2^2(U)$$

Hence, the prescription of a desired steady-state value for the average output capacitor voltage $\zeta_2(U)$ uniquely determines both the required constant value of the duty ratio function U and the corresponding average value for the input inductor current $\zeta_1(U)$. This simple fact allows the indirect regulation of the average output capacitor voltage of the converter through regulation of the average input inductor current. For this reason our control problem will be formulated in terms of achieving a desired steady-state equilibrium value, denoted by Y, for the average input inductor current ζ_1.

Consider, then, the following, locally invertible, nominal input-dependent state coordinate transformation of the nonlinear average model (4):

$$x_1 = \zeta_1, \quad x_2 = -\Theta_1(1-\mu)\zeta_2 + \Theta_4 \tag{6}$$

and its associated inverse transformation is

$$\zeta_1 = x_1, \quad \zeta_2 = \frac{\Theta_4 - x_2}{\Theta_1(1-\mu)} \tag{7}$$

Using the above state coordinate transformation (6) and (7), on the average circuit equations (4), one obtains the following Fliess generalized obserability canonical form of the average boost converter model:

$$\left.\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\theta_1\theta_2(1-\mu)^2 x_1 - \theta_3(x_2-\theta_4) - \frac{\dot{\mu}}{1-\mu}(x_2-\theta_4) \\ \eta &= x_1 \end{aligned}\right\} \tag{8}$$

The zero dynamics associated with the equilibrium point $x_1 = \zeta_1(U)$ and $x_2 = 0$ are given by the first-order dynamics,

$$\dot{\mu} = -\frac{\theta_3}{(1-U)^2}(1-\mu)(\mu-U)(2-\mu-U) \tag{9}$$

The zero dynamics exhibits two unstable equilibrium points, $\mu = U-2 < 0$ and $\mu = 1$. The only stable equilibrium point $\mu = U$ makes the average system minimum phase in the region of interest.

Let the desired behaviour of the transformed system (8) be prescribed by the following asymptotically stable second-order linear time-invariant dynamics:

$$\left.\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -2\xi\omega_n x_2 - \omega_n^2(x_1 - Y) \\ \eta &= x_1 \end{aligned}\right\} \tag{10}$$

where ξ and ω_n are design parameters that reflect our need for particular transient features of the average regulated output $\eta = x_1$. The constant Y represents a desired steady-state closed-loop equilibrium value of the average output variable η. In our case of perfectly known parameter values, the desired output is obtained as $Y = \zeta_1(U) = \Theta_3 \Theta_4 / \Theta_1 \Theta_2 (1-U)^2$.

From (8) and (10) one readily obtains an expression for the dynamical feedback controller yielding the required linearizing and stabilizing duty ratio function. In terms of the transformed variables x_1 and x_2 the duty ratio function μ is obtained as the solution of the time-varying differential equation

$$\dot{\mu} = \frac{1-\mu}{x_2 - \theta_4} [2\xi\omega_n x_2 + \omega_n^2 (x_1 - Y) - \theta_1 \theta_2 (1-\mu)^2 x_1 - \theta_3 (x_2 - \theta_4)] \tag{11}$$

In terms of the average state variables ζ_1 and ζ_2, the dynamical duty ratio synthesizer is equivalently obtained as

$$\dot{\mu} = \frac{1}{\theta_1 \zeta_2} [(\theta_1 \theta_2 (1-\mu)^2 - \omega_n^2) \zeta_1 + (2\xi\omega_n \theta_1 - \theta_1 \theta_3)(1-\mu) \zeta_2 - 2\xi\omega_n \theta_4 + \omega_n^2 Y] \tag{12}$$

The values of μ, obtained from the on-line solution of equation (11), actually represent the computed duty ratio function, which we still denote by μ. However, in order to implement this dynamical feedback control strategy on the actual (i.e. discontinuously regulated) converter system the values of μ must be necessarily limited to the closed interval [0, 1]. We then define the actual duty ratio function denoted by μ_a as

$$\mu_a(t) = \begin{cases} 1, & \text{for } \mu(t) \geqslant 1 \\ \mu(t), & \text{for } 0 < \mu(t) < 1 \\ 0, & \text{for } \mu(t) \leqslant 0 \end{cases} \tag{13}$$

Finally, it should be noted that when the sampling period T is sufficiently small, the actual values of the state variables V and I, rather than their average values, ζ_1 and ζ_2, may be used for the on-line solution of the computed duty ratio funtion μ. This procedure is precisely at the heart of the state average method for PWM designs. A theoretical justification of this procedure has been given by Sira-Ramírez *et al.* (1993).

Summarizing, a dynamical PWM controller achieving the asymptotic stabilization of the average input inductor current of the boost converter circuit (1) to a desired constant equilibrium value $y = Y$, is given by

$$\left. \begin{array}{l} u = \begin{cases} 1, & \text{for } t_k \leqslant t < t_k + \mu_a(t_k) T \\ 0, & \text{for } t_k + \mu_a(t_k) T \leqslant t < t_k + T \end{cases} \\ t_{k+1} = t_k + T, \quad k = 0, 1, \ldots \end{array} \right\} \tag{14}$$

where $\mu_a(t_k)$ represents the sampled values of the actual duty ratio function at time t_k, given by (13). The computed duty ratio function $\mu(t)$ is obtained from the on-line solution of

$$\dot{\mu} = \frac{1}{\theta_1 V(t)} [(\theta_1 \theta_2 (1-\mu)^2 - \omega_n^2) I(t) + (2\xi\omega_n \theta_1 - \theta_1 \theta_3)(1-\mu) V(t) - 2\xi\omega_n \theta_4 + \omega_n^2 Y] \tag{15}$$

3. An adaptive feedback control strategy for indirect output voltage regulation in DC-to-DC power converters

3.1. *Boost converter*

With reference to the boost converter circuit, consider the following version of the average boost converter model:

$$\left.\begin{aligned} \dot{\zeta}_1 &= -\theta_1(1-\mu)\zeta_2+\theta_4 \\ \dot{\zeta}_2 &= \theta_2(1-\mu)\zeta_1-\theta_3\zeta_2 \\ y &= \zeta_1 \end{aligned}\right\} \tag{16}$$

where the θ_i, $i = 1,2,3,4$, represent the actual parameter values, modelled by

$$\theta_i = \Theta_i+\Delta\theta_i, \quad i = 1,2,3,4 \tag{17}$$

with Θ_i, $i = 1,2,3,4$, being the nominal parameters, assumed to be perfectly known. The quantities $\Delta\theta_i$, $i = 1,2,3,4$, denote the corresponding constant, but unknown, incremental variations of the parameters from their nominal values.

Consider, then, the nominal input-dependent state coordinate transformation, used for exact input–output linearization of the average boost converter model in the preceding section:

$$\left.\begin{aligned} x_1 &= \zeta_1 \\ x_2 &= -\Theta_1(1-\mu)\zeta_2+\Theta_4 \end{aligned}\right\} \tag{18}$$

Clearly this control-parametrized transformation is invertible everywhere, except when the duty ratio function μ is identically equal to one. The associated inverse transformation is readily found to be

$$\left.\begin{aligned} \zeta_1 &= x_1 \\ \zeta_2 &= \frac{\Theta_4-x_2}{\Theta_1(1-\mu)} \end{aligned}\right\} \tag{19}$$

When the state coordinate transformation (18) and (19) is applied to the actual boost converter model (16) and (17), the transformed system is not quite in the Fliess generalized observability canonical form (8), but rather in what we call the generalized pure incremental parameter feedback canonical form. The transformed system is easily shown to be given by

$$\begin{aligned} \dot{x}_1 &= x_2+\Delta\theta^{\mathrm{T}}\gamma_1(x_1,x_2) \\ \dot{x}_2 &= \dot{\mu}\left(\frac{-x_2+\Theta_4}{(1-\mu)}\right)-\Theta_1\Theta_2(1-\mu)^2x_1-\Theta_3(x_2-\Theta_4)+\Delta\theta^{\mathrm{T}}\gamma_2(x_1,x_2,\mu) \\ y &= x_1 \end{aligned} \tag{20}$$

where

$$\begin{aligned} \gamma_1^{\mathrm{T}}(x_1,x_2) &= \begin{bmatrix} \dfrac{x_2-\Theta_4}{\Theta_1} & 0 & 0 & 1 \end{bmatrix} \\ \gamma_2^{\mathrm{T}}(x_1,x_2,\mu) &= [0 \quad -\Theta_1(1-\mu)^2x_1 \quad -x_2+\Theta_4 \quad 0] \\ \Delta\theta^{\mathrm{T}} &= [\Delta\theta_1 \quad \Delta\theta_2 \quad \Delta\theta_3 \quad \Delta\theta_4] \end{aligned} \tag{21}$$

For ease of reference we let

$$f(x,\mu,\dot{\mu},\Theta) = \dot{\mu}\left(-\frac{-x_2+\Theta_4}{1-\mu}\right) - \Theta_1\,\Theta_2(1-\mu)^2 x_1 - \Theta_3(x_2-\Theta_4) \tag{22}$$

The transformed system (20) strongly resembles the more traditional pure parameter feedback canonical form developed by Kanellakopoulos *et al.* (1991), except for the presence of the control input (duty ratio) μ in the regressor vector γ_2 and the presence of the first-order time derivative $\dot{\mu}$ of the control input in the second differential equation. This control input derivative will, in fact, act as the actual control input, whereas the control input μ may be regarded as playing the role of an additional state variable.

We now proceed to apply the adaptive backstepping algorithm, as developed by Kanellakopoulos *et al.* (1991), to the transformed model (20) and (21).

Step 0: Let Y be the desired steady-state equilibrium value of the output variable x_1 and define the stabilization error z_1 as

$$z_1 = x_1 - Y \tag{23}$$

Step 1: Consider the stabilization error equation

$$\dot{z}_1 = x_2 + \Delta\theta^T \gamma_1(x_1, x_2) \tag{24}$$

Suppose that the transformed variable x_2 can be used as a 'pseudo-control' in (24) and proceed to compute the required value of x_2 which stabilizes the error variable z_1 to zero. Computation of x_2 requires the unknown vector $\Delta\theta$. Using the 'certainty equivalence principle' (Kanellakopoulos *et al.* 1991) we replace the vector $\Delta\theta$ by an estimate in the fictitious stabilizing 'control law'. We proceed to devise also a parameter update law for the hypothesized estimate of the incremental parameter vector $\Delta\theta$, denoted here by $\widehat{\Delta\theta}^1$. This specification must result in simultaneous stable adaptation and convergence to zero of the error variable z_1. The superscript 1 will denote a first estimate of $\Delta\theta$. Let c_1 be a strictly positive design parameter. We then have as a plausible 'pseudo-control' action the following expression for x_2:

$$x_2 = -c_1 z_1 - \frac{\widehat{\Delta\theta}_1^1}{\Theta_1}(x_2-\Theta_4) - \widehat{\Delta\theta}_4^1, \quad c_1 > 0 \tag{25}$$

where $\widehat{\Delta\theta}_i^1$, $i = 1, 4$, denotes a first estimate of $\Delta\theta_i$, $i = 1, 4$. A simple Lyapunov stability argument shows that the pseudo controller (25) and the update law

$$\dot{\widehat{\Delta\theta}}_\theta^1 = {}_1\gamma_1(x_1, x_2) \tag{26}$$

yield a closed-loop stable system for which z_1 is guaranteed to converge to zero.

Since x_2 is not really a control input, one defines the pseudo-control error variable z_2 as the difference between x_2 and its required value, computed in (25). Let

$$z_2 = x_2 - \left[-c_1 z_1 - \frac{\widehat{\Delta\theta}_1^1}{\Theta_1}(x_2-\Theta_4) - \widehat{\Delta\theta}_4^1\right] \tag{27}$$

By solving for x_2 from (27) and using (23), one obtains a new state coordinate transformation defining, respectively, the stabilization error and the pseudo-control error variables, z_1 and z_2

$$
\begin{aligned}
z_1 &= x_1 - Y \\
z_2 &= c_1 z_1 + x_2\left(1 + \frac{\Delta\widehat{\Theta}_1^1}{\Theta_1}\right) - \frac{\Delta\widehat{\theta}_1^1}{\Theta_1}\Theta_4 + \Delta\widehat{\theta}_4^1
\end{aligned} \tag{28}
$$

The corresponding inverse transformation is simply obtained as

$$
\begin{aligned}
x_1 &= z_1 + Y \\
x_2 &= \frac{\Theta_1}{\Theta_1 + \Delta\widehat{\theta}_1^1}\left[z_2 - c_1 z_1 + \frac{\Delta\widehat{\theta}_1^1}{\Theta_1}\Theta_4 - \Delta\widehat{\theta}_4^1\right]
\end{aligned} \tag{29}
$$

The first equation of the transformed system may then be written as

$$
\begin{aligned}
\dot{z}_1 &= z_2 - c_1 z_1 \\
&\quad + (\Delta\theta_1 - \Delta\widehat{\theta}_1^1)\left[\frac{1}{\Theta_1 + \Delta\widehat{\theta}_1^1}\left[z_2 - c_1 z_1 + \frac{\Delta\widehat{\theta}_1^1}{\Theta_1}\Theta_4 - \Delta\widehat{\theta}_4^1\right] - \frac{\Theta_4}{\Theta_1}\right] + (\Delta\theta_4 - \Delta\widehat{\theta}_4^1)
\end{aligned} \tag{30}
$$

which can be briefly expressed as

$$
\dot{z}_1 = z_2 - c_1 z_1 + (\Delta\theta - \Delta\widehat{\theta}^1)^{\mathrm{T}} w_1(z_1, z_2, \Delta\widehat{\theta}^1) \tag{31}
$$

where

$$
w_1^{\mathrm{T}}(z_1, z_2, \Delta\widehat{\theta}^1) = \left[\left(\frac{1}{\Theta_1 + \Delta\widehat{\theta}_1^1}\left[z_2 - c_1 z_1 + \frac{\Delta\widehat{\theta}_1^1}{\Theta_1}\Theta_4 - \Delta\widehat{\theta}_4^1\right] - \frac{\Theta_4}{\Theta_1}\right) \quad 0 \quad 0 \quad 1\right] \tag{32}
$$

Note that the update laws corresponding to $\Delta\widehat{\theta}_2^1$ and $\Delta\widehat{\theta}_3^1$ will yield constant values for such estimates. Note, moreover, that these two estimated parameters are not needed in this first step of the backstepping calculation.

We let W_1^1 denote the first component of the regressor vector $w_1(z_1, z_2, \Delta\widehat{\theta}_\theta^1)$.

The first adaptation law (26) may then be rewritten, in terms of the new error variables z_1 and z_2, as

$$
\dot{\Delta\widehat{\theta}}_\theta^1 = z_1 w_1(z_1, z_2, \Delta\widehat{\theta}^1) \tag{33}
$$

Step 2: We proceed to complete the state coordinate transformation (28) and (29) of the original phase variables by considering now the differential equation for the pseudo controller error z_2. Using the definition of z_2 and z_1 and the first incremental parameter update laws for the involved components of the vector $\Delta\theta$, one obtains, after long but straightforward manipulations, the following expression:

$$
\begin{aligned}
\dot{z}_2 &= \left(1 + \frac{\Delta\widehat{\theta}_1^1}{\Theta_1}\right)\Bigg[-\frac{\dot{\mu}}{-\mu}(x_2(z_1, z_2, \Delta\widehat{\theta}^1) - \Theta_4) - \Theta_1\Theta_2(1-\mu)^2(z_1 + Y) \\
&\quad - \Theta_3(x_2(z_1, z_2, \Delta\widehat{\theta}^1) - \Theta_4) - \Delta\theta_2\Theta_1(1-\mu)^2(z_1 + Y) - \Delta\theta_3(x_2(z_1, z_2, \Delta\widehat{\theta}^1) - \Theta_4)\Bigg] \\
&\quad + c_1[z_2 - c_1 z_1 - \Delta\widehat{\theta}_1^1 W_1^1 - \Delta\widehat{\theta}_4^1] + z_1(1 + (W_1^1)^2) + c_1(\Delta\theta_1 W_1^1 + \Delta\theta_4)
\end{aligned} \tag{34}
$$

where $x_2(z_1, z_2, \Delta\widehat{\theta}^1)$ is given by the second equation of (29), which we do not substitute just to avoid lengthy intermediate equations. In the rest of this section x_2 stands for $x_2(z_1, z_2, \widehat{\Delta\theta}^1)$.

If we now equate the dynamics obtained in (34) for z_2 to the dynamics of an asymptotically stable behaviour for z_2, given by

$$\dot{z}_2 = -c_2 z_2, \quad c_2 > 0 \tag{35}$$

one can immediately solve for the required control input derivative $\dot{\mu}$ upon invoking, once more, the certainty equivalence principle. In this instance the unknown value of the vector $\Delta\theta$ will be replaced by a new vector of parameter estimates, denoted by $\widehat{\Delta\theta}^2$. One then obtains

$$\begin{aligned}\dot{\mu} = \frac{(1-\mu)\,\Theta_1}{(\Theta_1+\widehat{\Delta\theta}_1^1)(x_2-\Theta_4)}\Big\{ & c_2 z_2 + c_1[z_2 - c_1 z_1 - (\widehat{\Delta\theta}^1)^{\mathrm{T}} w_1(z_1, z_2, \widehat{\Delta\theta}^1)] \\ & + z_1[1+(W_1^1)^2] + c_1(\widehat{\Delta\theta}^2)^{\mathrm{T}} w_1(z_1, z_2, \widehat{\Delta\theta}^1) - \left(1+\frac{\widehat{\Delta\theta}_1^1}{\Theta_1}\right)[\Theta_1\,\Theta_2(1-\mu)^2(z_1+Y) \\ & + \Theta_3(x_2-\Theta_4) + \widehat{\Delta\theta}_2^2\,\Theta_1(1-\mu)^2(z_1+Y) + \widehat{\Delta\theta}_3^2(x_2-\Theta_4)]\Big\}\end{aligned} \tag{36}$$

where $\widehat{\Delta\theta}_j^2$, $j = 2, 3$, represent the new estimates of the incremental parameter vector components $\Delta\theta_j$, $j = 2, 3$, and x_2 is given by (29). The expression for the dynamically controlled error variable z_2 (i.e. the closed-loop behaviour of z_2) is found to be

$$\begin{aligned}\dot{z}_2 = -c_2 z_2 + \left(1+\frac{\widehat{\Delta\theta}_1^1}{\Theta_1}\right)[& -(\Delta\theta_2 - \widehat{\Delta\theta}_2^2)\,\Theta_1(1-\mu)^2(z_1+Y) \\ & -(\Delta\theta_3-\widehat{\Delta\theta}_3^2)(x_2-\Theta_4)] + c_1[(\Delta\theta_1-\widehat{\Delta\theta}_1^2)\,W_1^1 + (\Delta\theta_4 - \widehat{\Delta\theta}_4^2)]\end{aligned} \tag{37}$$

which can also be briefly expressed as

$$\dot{z}_2 = -c_2 z_2 + (\Delta\theta - \widehat{\Delta\theta}^2)^{\mathrm{T}} w_2(z_1, z_2, \mu, \widehat{\Delta\theta}^1, \widehat{\Delta\theta}^2) \tag{38}$$

The regressor vector for the new estimation process is thus given by

$$w_2(z_1, z_2, \mu, \widehat{\Delta\theta}^1, \widehat{\Delta\theta}^2) = \left(1+\frac{\widehat{\Delta\theta}_1^1}{\Theta_1}\right)\gamma_2(z_1, z_2, \mu, \widehat{\Delta\theta}^1, \widehat{\Delta\theta}^2) + c_1\, w_1(z_1, z_2, \widehat{\Delta\theta}^1) \tag{39}$$

Note that the dependence of w_2 on $\widehat{\Delta\theta}^2$ is implicit through its dependence on μ, as given by the solution of (36).

As in the previous step, an incremental parameter adaptation law for the vector of new estimates $\widehat{\Delta\theta}^2$ can be devised to achieve simultaneously a stable adaptation process and an asymptotic convergence to zero of the pseudo-control error variable z_2. Such a new incremental parameter update law is given by

$$\dot{\widehat{\Delta\theta}}{}^2 = z_2\, w_2(z_1, z_2, \mu, \widehat{\Delta\theta}^1, \hat{\Delta\theta}^2) \tag{40}$$

3.1.1. *Summary of adaptive controller expressions for the boost converter.* The adaptive PWM controller is next summarized in terms of the original state variables of the system. The constant Y stands for the desired value of the input inductor current $I(t)$. The constants c_1 and c_2 are positive design constants, satisfying $c_1, c_2 > 2$.

The adaptive feedback regulated switch position function is synthesized as

$$\left.\begin{aligned} u &= \begin{cases} 1, & \text{for } t_k \leqslant t < t_k + \mu_a(t_k)\,T \\ 0, & \text{for } t_k + \mu_a(t_k)\,T \leqslant t < t_k + T \end{cases} \\ t_k + T &= t_{k+1}, \quad k = 0, 1, 2, \ldots \end{aligned}\right\} \tag{41}$$

where $\mu_a(t)$ is obtained from a bounding operation carried out on the computed duty ratio μ in the following manner:

$$\mu_a(t) = \begin{cases} 1, & \text{if } \mu(t) \leqslant 1 \\ \mu(t), & \text{if } 0 < \mu(t) < 1 \\ 0, & \text{if } \mu(t) \leqslant 0 \end{cases} \tag{42}$$

The duty ratio function μ is obtained as the solution of the following time-varying differential equation from an initial condition which does not cause permanent saturation of the actual duty ratio $\mu_a(t)$.

$$\begin{aligned} \dot{\mu} = \frac{1}{V(t)(\Theta_1 + \widehat{\Delta\theta}{}_1^1)} \Big\{ & -(I(t) - Y)(1-\mu)^2 V^2(t) + (\Theta_1 + \widehat{\Delta\theta}{}_1^1)(1-\mu) \\ & \times [(\Theta_2 + \widehat{\Delta\theta}{}_2^2)(1-\mu)\, I(t) - (\Theta_3 + \widehat{\Delta\theta}{}_3^2)\, V(t)] - (I(t) - Y) \\ & - c_1[-(\Theta_1 + \widehat{\Delta\theta}{}_1^2)(1-\mu)\, V(t) + (\Theta_4 + \widehat{\Delta\theta}{}_4^2)] \\ & + c_2[(\Theta_1 + \widehat{\Delta\theta}{}_1^1)(1-\mu)\, V(t) - (\Theta_4 + \widehat{\Delta\theta}{}_4^1) - c_1(I(t) - Y)] \Big\} \end{aligned} \tag{43}$$

The estimated values of the controller parameters are obtained as the online solution of the following system of differential equations:

$$\begin{aligned} \dot{\widehat{\Delta\theta}}{}_1^1 &= -(I(t) - Y)(1-\mu)\, V(t) \\ \dot{\widehat{\Delta\theta}}{}_2^1 &= 0 \\ \dot{\widehat{\Delta\theta}}{}_3^1 &= 0 \\ \dot{\widehat{\Delta\theta}}{}_4^1 &= -(I(t) - Y) \\ \dot{\widehat{\Delta\theta}}{}_1^2 &= [-(Y_1 + \widehat{\Delta\theta}{}_1^1)(1-\mu)\, V(t) + (\Theta_4 + \widehat{\Delta\theta}{}_4^1) + c_1(I(t) - Y)][-c_1(1-\mu)\, V(t)] \\ \dot{\widehat{\Delta\theta}}{}_2^2 &= [-(Y_1 + \widehat{\Delta\theta}{}_1^1)(1-\mu)\, V(t) + (\Theta_4 + \widehat{\Delta\theta}{}_4^1) + c_1(I(t) - Y)][-(\Theta_1 + \widehat{\Delta\theta}{}_1^1)(1-\mu)^2 I(t)] \\ \dot{\widehat{\Delta\theta}}{}_3^2 &= [-(Y_1 + \widehat{\Delta\theta}{}_1^1)(1-\mu)\, V(t) + (\Theta_4 + \widehat{\Delta\theta}{}_4^1) + c_1(I(t) - Y)][(\Theta_1 + \widehat{\Delta\theta}{}_1^1)(1-\mu)\, V(t)] \\ \dot{\widehat{\Delta\theta}}{}_4^2 &= [-(\Theta_1 + \widehat{\Delta\theta}{}_1^1)(1-\mu)\, V(t) + (\Theta_4 + \widehat{\Delta\theta}{}_4^1) + c_1(I(t) - Y)]\, c_1 \end{aligned} \tag{44}$$

3.1.2. *Simulation results*. Simulations were carried out for a perturbed version of the boost converter model in conjunction with the adaptive controller described by (41)–(44). An unmodelled stochastic but bounded, yet unmatched, uncertain signal (denoted by $v(t)$) was hypothesized to be acting on the circuit through the external source voltage E. The designed dynamical adaptive PWM controller (41)–(44) was then directly used for the regulation of the input inductor current variable $I(t)$ of the converter using the actual discontinuously regulated state variables $I(t)$ and $V(t)$, rather than the averaged values ζ_1 and ζ_2.

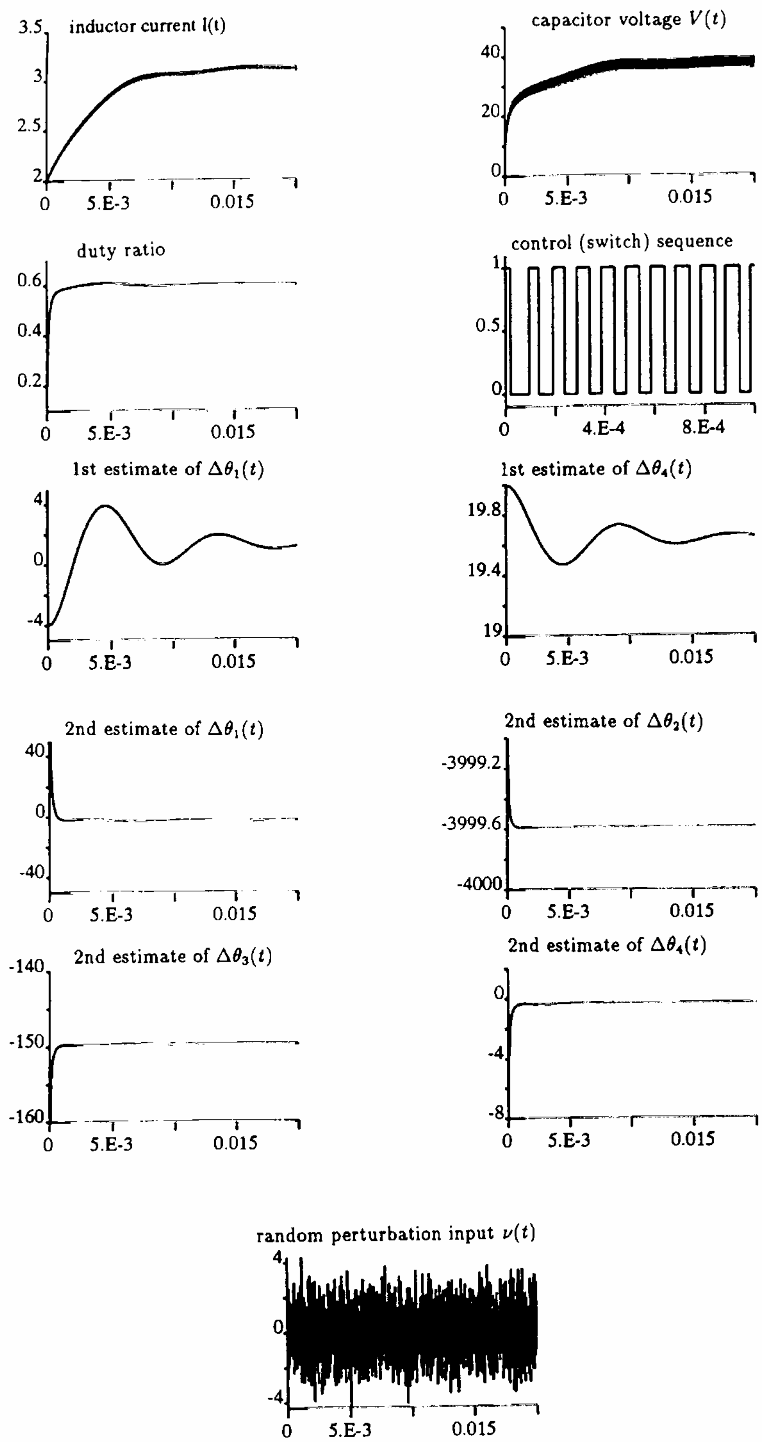

Figure 2. Adaptively controlled state trajectories of perturbed boost converter, evolution of controller incremental parameter estimates and perturbation noise signal.

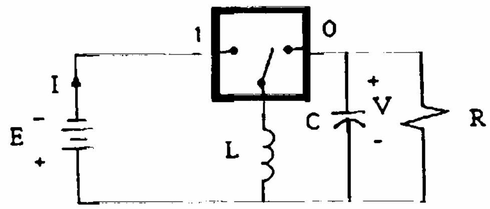

Figure 3. Buck–boost converter circuit.

The perturbed circuit model, used in the computer simulations, were taken to be

$$\left.\begin{aligned} \dot{I}(t) &= -\frac{1}{L}(1-u)\,V(t)+\left(\frac{E+\nu(t)}{L}\right) \\ \dot{V}(t) &= \frac{1}{C}(1-u)\,I(t)-\frac{1}{RC}V(t) \\ y &= I(t) \end{aligned}\right\} \tag{45}$$

The simulation results, depicting the behaviour of the controlled converter, are shown in Fig. 2. The nominal values of the converter parameters were chosen as $L = 20$ mH, $C = 20$ mF, $R = 30\,\Omega$ and $E = 15$ V. These values rendered: $\Theta_1 = 50$; $\Theta_2 = 5{\cdot}0 \times 10^4$; $\Theta_3 = 1{\cdot}667 \times 10^3$ and $\Theta_4 = 750$. The actual parameter values used in the simulations were set to $\theta_1 = 55$, $\theta_2 = 4{\cdot}5 \times 10^3$, $\theta_3 = 1{\cdot}5 \times 10^3$ and $\theta_4 = 825$. These values were, however, assumed to be completely unknown in the controller implementation. In Fig. 2 the response of the input inductor current $I(t)$ evolves towards the preassigned (nominal) equilibrium value $I(t) = Y = 3{\cdot}125$ A, which corresponds to a nominal output capacitor voltage, $V(t) = 37{\cdot}5$ V and a duty ratio $U = 0{\cdot}6$ V. The PWM sampling frequency was set to 10 kHz. The asymptotically stable evolution of the duty ratio function $\mu(t)$ towards its equilibrium value, $\mu = U = 0{\cdot}6$, along with a small portion (0·1 ms) of the PWM switching actions is also presented in this figure. The trajectories of the estimated incremental parameter values, $\widehat{\Delta\theta}{}_1^1(t)$, $\widehat{\Delta\theta}{}_4^2(t)$, $\widehat{\Delta\theta}{}_1^2$, $\widehat{\Delta\theta}{}_2^2\,\widehat{\Delta\theta}{}_3^2$ and $\widehat{\Delta\theta}{}_4^2$, are also depicted in this figure. Finally, a sample of the computer generated stochastic perturbation input $\nu(t)$ is shown at the end of Fig. 2.

3.2. *The buck–boost converter*

In this section we briefly summarize the controller expressions obtained from the backstepping calculation procedure applied to a nominally transformed parameter uncertain average PWM buck–boost converter model.

Consider the buck–boost converter circuit shown in Fig. 3.

State-space model of the buck–boost converter

$$\left.\begin{aligned} \dot{I}(t) &= \frac{1}{L}(1-u)\,V(t)+\frac{E}{L}u \\ \dot{V}(t) &= -\frac{1}{C}(1-u)\,I(t)-\frac{1}{RC}V(t) \\ y(t) &= I(t) \end{aligned}\right\} \tag{46}$$

where $I(t)$ and $V(t)$ represent, respectively, the input inductor current and the output capacitor voltage variables. The positive quantity E is the constant external input voltage. The variable u denotes the switch position function taking values in the discrete set $\{0, \}$. The output $y(t)$ of the system is represented by the input inductor current $I(t)$.

Nominal parameters definitions

$$\Theta_1 = \frac{1}{L}, \quad \Theta_2 = \frac{1}{C}, \quad \Theta_3 = \frac{1}{RC}, \quad \Theta_4 = \frac{E}{L} \tag{47}$$

Uncertainty model for the parameters

$$\theta_i = \Theta_i + \Delta\theta_i, \quad i = 1,2,3,4 \tag{48}$$

Average PWM model of the buck–boost converter

$$\left.\begin{aligned} \dot{\zeta}_1 &= \theta_1(1-\mu)\zeta_2 + \theta_4\mu \\ \dot{\zeta}_2 &= -\theta_2(1-\mu)\zeta_1 - \theta_3\zeta_2 \\ \eta &= \zeta_1 \end{aligned}\right\} \tag{49}$$

Nominal transformation of average PWM buck–boost converter model to Fliess' generalized observability canonical form

$$\left.\begin{aligned} x_1 = \zeta_1 \quad & \xi_2 = \Theta_1(1-\mu)\zeta_2 + \Theta_4\mu, \\ \zeta_1 = x_1, \quad & z_2 = \frac{x_2 - \Theta_4\mu}{\Theta_1(1-\mu)} \end{aligned}\right\} \tag{50}$$

Uncertain buck–boost converter model transformed to generalized phase variables

$$\left.\begin{aligned} \dot{x}_1 &= x_2 + \Delta\theta^{\mathrm{T}}\gamma_1(x_1, x_2) \\ \dot{x}_2 &= f(x_1, x_2, \mu, \dot{\mu}) + \Delta\theta^{\mathrm{T}}\gamma_2(x_1, x_2, \mu) \end{aligned}\right\} \tag{51}$$

where

$$\Delta\theta^{\mathrm{T}} = [\Delta\theta_1 \quad \Delta\theta_2 \quad \Delta\theta_3 \quad \Delta\theta_4]$$

$$\gamma_1^{\mathrm{T}}(x_1, x_2) = \left[\frac{x_2 - \Theta_4\mu}{\Theta_1} \quad 0 \quad 0 \quad \mu\right]$$

$$\gamma_2^{\mathrm{T}}(x_1, x_2, \mu) = [0 \quad -\Theta_1(1-\mu)^2 x_1 \quad -x_2 + \Theta_4\mu \quad 0]$$

$$f(x_1, x_2, \mu, \dot{\mu}) = \dot{\mu}\left(\Theta_4 - \frac{x_2 - \Theta_4\mu}{1-\mu}\right) - \Theta_1\Theta_2(1-\mu)^2 x_1 - \Theta_3(x_2 - \Theta_4\mu) \tag{52}$$

3.2.1. *Summary of adaptive controller expressions for the buck–boost converter*

$$\left.\begin{aligned} u &= \begin{cases} 1, & \text{for } t_k \leqslant t < t_k + \mu_a(t_k)T \\ 0, & \text{for } t_k + \mu_a(t_k)T \leqslant t < t_k + T \end{cases} \\ & t_k + T = t_{k+1}, \quad k = 0, 1, 2, \ldots \end{aligned}\right\} \tag{53}$$

where $\mu_a(t)$ is obtained from the following bounding operation:

$$\mu_a(t) = \begin{cases} 1, & \text{if } \mu(t) \leqslant 1 \\ \mu(t), & \text{if } 0 < \mu(t) < 1 \\ 0, & \text{if } \mu(t) \leqslant 0 \end{cases} \tag{54}$$

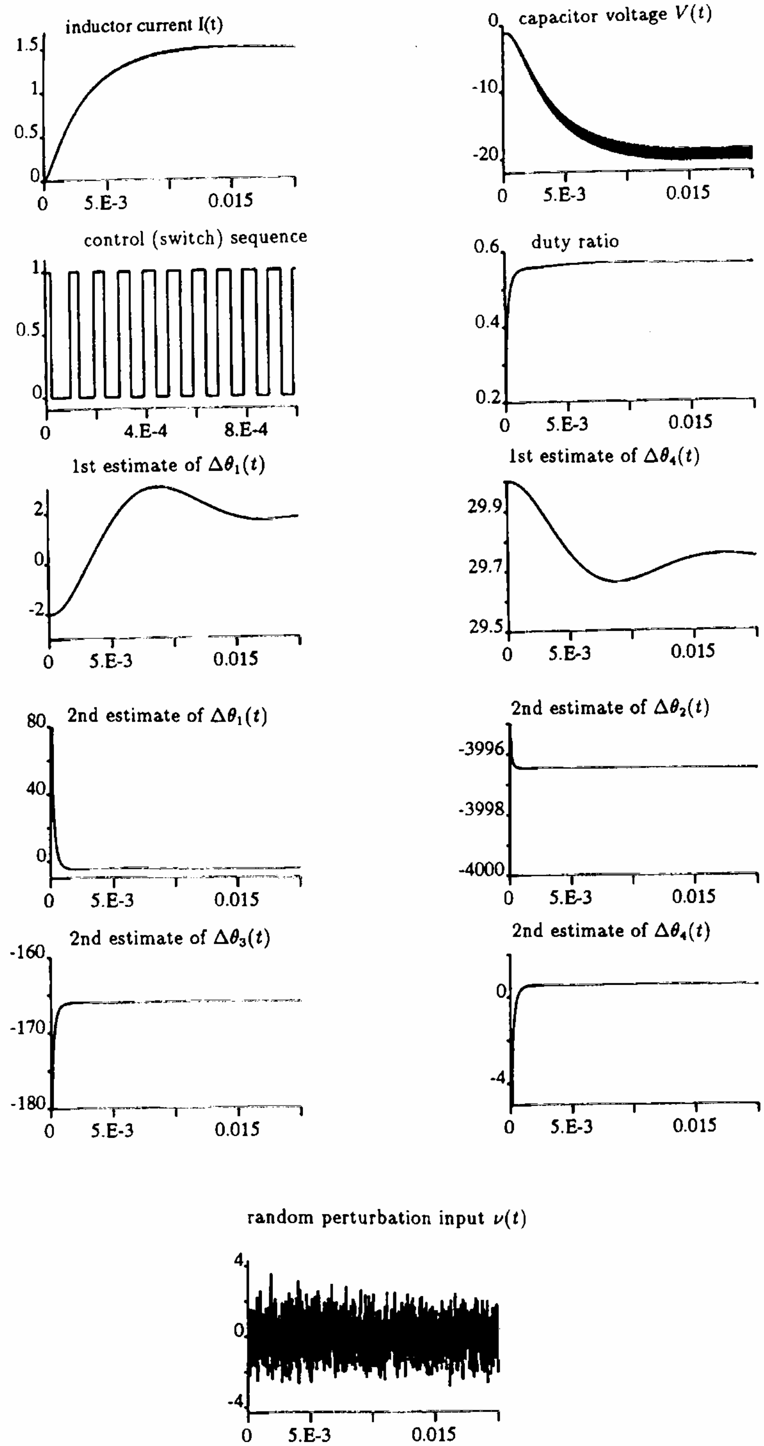

Figure 4. Adaptively controlled state trajectories of perturbed buck–boost converter, evolution of controller incremental parameter estimates and perturbation noise signal.

We let $\hat{\theta}_i^j$, $i = 1, \ldots, 4$, $j = 1, 2$, stand for $\Theta_i + \widehat{\Delta\theta}_i^j$ in the following expressions.

$$\dot{\mu} = \frac{1}{(\hat{\theta}_4^1 - \hat{\theta}_1^1 V(t))}\{-(I(t)-Y)(1-\mu)^2 V^2(t) + \hat{\theta}_1^1 \hat{\theta}_2^2 (1-\mu)^2 I(t) + \hat{\theta}_1^1 \hat{\theta}_3^2 (1-\mu) V(t) - (I(t)-Y)\mu - c_1(\hat{\theta}_1^2(1-\mu)V(t) + \theta_4^2 \mu) + c_2[\hat{\theta}_1^1(1-\mu)V(t) + \hat{\theta}_4^1 + c_1(I(t)-Y)]\} \tag{55}$$

$$\left.\begin{aligned}
\dot{\hat{\theta}}_1^1 &= (I(t)-Y)(1-\mu)V(t) \\
\dot{\hat{\theta}}_2^1 &= 0 \\
\dot{\hat{\theta}}_3^1 &= 0 \\
\dot{\hat{\theta}}_4^1 &= (I(t)-Y)\mu \\
\dot{\hat{\theta}}_1^2 &= [\hat{\theta}_1^1(1-\mu)V(t) + \hat{\theta}_4^1\mu + c_1(I(t)-Y)][c_1(1-\mu)V(t)] \\
\dot{\hat{\theta}}_2^2 &= [\hat{\theta}_1^1(1-\mu)V(t) + \hat{\theta}_4^1\mu + c_1(I(t)-Y)][-\hat{\theta}_1^1(1-\mu)^2 I(t)] \\
\dot{\hat{\theta}}_3^2 &= [\hat{\theta}_1^1(1-\mu)V(t) + \hat{\theta}_4^1\mu + c_1(I(t)-Y)][-\hat{\theta}_1^1(1-\mu)V(t)] \\
\dot{\hat{\theta}}_4^2 &= [\hat{\theta}_1^1(1-\mu)V(t) + \hat{\theta}_4^1\mu + c_1(I(t)-Y)]c_1\mu
\end{aligned}\right\} \tag{56}$$

3.2.2. *Simulation results.* Simulations were carried out for the following perturbed version of the buck–boost converter model:

$$\left.\begin{aligned}
\dot{I}(t) &= \frac{1}{L}(1-u)V(t) + \left(\frac{E+v(t)}{L}\right)u \\
\dot{V}(t) &= -\frac{1}{C}(1-u)I(t) - \frac{1}{RC}V(t) \\
y &= I(t)
\end{aligned}\right\} \tag{57}$$

in conjunction with the adaptive controller described by (53)–(56).

The simulation results, depicting the behaviour of the controller converter, are shown in Fig. 4. The nominal values of the converter parameters were chosen to be the same as for the boost converter case: $L = 20$ mH, $C = 20$ mF, $R = 30\ \Omega$, and $E = 15$ V. The actual parameter values used in the simulations were the same as before. The response of the input inductor current $I(t)$ is seen to evolve towards the preassigned (nominal) equilibrium value $I(t) = Y = 1{\cdot}5$ A, which corresponds to a nominal output capacitor voltage, $V(t) = -21{\cdot}38$ V. The PWM sampling frequency was also set at 10 kHz. The asymptotically stable evolution of the duty ratio function $\mu(t)$ towards its equilibrium value, $\mu = U = 0{\cdot}55$, along with a small portion of the switching action is also presented in this figure. The trajectories of the estimated incremental parameter values, $\widehat{\Delta\theta}_1^1(t)$, $\widehat{\Delta\theta}_4^2(t)$, $\widehat{\Delta\theta}_1^2$, $\widehat{\Delta\theta}_2^2$ $\widehat{\Delta\theta}_3^2$ and $\widehat{\Delta\theta}_4^2$, are also depicted in this figure. Finally, a sample of the computer-generated stochastic perturbation input $v(t)$ is shown at the bottom of Fig. 4.

4. Conclusions

An adaptive feedback control approach has been proposed which is based on nominal dynamical input–output linearization of the average model of PWM regulated DC-to-DC power converters and the backstepping algorithm. The approach achieves indirect averate output capacitor voltage regulation by considering the input inductor current as the regulated output. This procedure sidesteps the non-minimum phase problems associated with direct output capacitor voltage regulation. The simulated behaviour of the closed-loop system exhibits remarkable robustness with respect to unmatched and unmodelled external perturbation signals of bounded and stochastic nature.

Over-parametrization is implicit in the backstepping procedure when applied to systems in pure parameter feedback canonical form (Kanellakopoulos *et al.* 1991). This feature substantially contributes to increase the complexity of the controller. An alternative approach to the one presented here is constituted by the possibility of avoiding the over-parametrization associated with the incremental parameter update estimation process. The fundamental developments regarding this technique may be found in Krstić *et al.* (1992), and an alternative approach to that of this paper has been presented by Sira-Ramírez *et al.* (1995b).

Simulations show that the scheme is quite robust with respect to unmodelled stochastic, but bounded, external perturbation inputs of the unmatched type. This type of robust behaviour is inherited from: the underlying input–output viewpoint present in the generalized observability canonical form, used for the derivation of the dynamical feedback controller; and the robustness features traditionally associated with discontinuous feedback control policies of the pulse-width-modulation type.

A topic for further study is the direct output capacitor voltage regulation problem, which exhibits a non-minimum phase property, and hence an input–output linearization approach fails. In a recent work Sira-Ramírez *et al.* (1995a) proposed the possibility of handling the non-minimum phase case by means of a piecewise unstable dynamical compensator in which a controller output 'resetting' strategy is enforced. The adaptive version of this resetting controller did not use the backstepping algorithm.

Another very interesting development has been given by Karsenti and Lamnabhi-Lagarrigue (1995) in which the backstepping method is generalized to include sliding mode control strategies in systems with nonlinear parameter dependencies. Application of this latter technique to DC-to-DC power converters represents a welcome contribution, since a more realistic class of (nonlinear) incremental circuit parameter variations may be efficiently handled with such a method.

Acknowledgment

This research was supported by the Consejo de Desarrollo Científico, Humanístico and Tecnológico of the Universidad de Los Andes under Research Grant I-456-91.

References

Benvenuti, L., Di Benedetto, M. D., and Grizzle, J. W., 1992, Approximate output tracking for nonlinear non-minimum phase systems with applications to flight control. Report CGR-92-20, Michigan Control Group Reports, University of Michigan, Ann Arbor, U.S.A.

Bose, B., 1992, *Modern Power Electronics* (New York: IEEE Press).

Csaki, F., Ganski, K., Ipsitz, I., and Marti, S., 1983, *Power Electronics* (Budapest, Hungary: Akademia Kiado).

Fliess, M., 1989, Nonlinear control theory and differential algebra, in *Modeling and Adaptive Control*, edited by I. Byrnes and A. Khurzhansky, Lecture Notes in Control and Information Sciences, Vol. 105 (Berlin: Springer-Verlag).

Fliess, M., and Sira-Ramírez, H., 1993, Régimes glissants, structures variables linéaires et modules. *C. R. de la Acad. de Sci. Paris, Serie I, Automatique*, **317**, 703–706.

ISIDORI, A., 1989, *Nonlinear Control Systems*, second edition (New York: Springer-Verlag).

KANELLAKOPOULOS, I., KOKOTOVIĆ, P. V., and MORSE, A. S., 1991a, Systematic design of adaptive controllers for feedback linearizable systems. *IEEE Transactions on Automatic Control*, **36**, 1241–1253; 1991b, A toolkit for nonlinear feedback design, Report CCEC-91-0619, University of California, Santa Barbara, U.S.A.; 1991c, Adaptive output feedback control of a class of nonlinear systems. *Proceedings of the 30th IEEE Conference on Decision and Control*, Vol. 2, Brighton, U.K., pp. 1082–1087.

KARSENTI, L., and LAMNABHI-LAGARRIGUE, F., 1995, Adaptive feedback control for nonlinear systems using backstepping and sliding-mode designs, to be published.

KASSAKIAN, J. G., SCHLECHT, M. F., and VERGHESE, G. C., 1991, *Principles of Power Electronics* (Reading, Massachusetts, U.S.A.: Addison-Wesley).

KRSTIĆ, M., KANELLAKOPOULOS, I., and KOKOTOVIC, P. V., 1992, Adaptive nonlinear control without overparametrization. *Systems and Control Letters*, **19**, 177–185.

MIDDLEBROOK, R. D., and ĆUK, S., 1981, *Advances in Switchmode Power Conversion* (multivolume series) (Pasadena, California, U.S.A.: TESLA).

RUGH, W. J., 1989, The extended linearization approach for nonlinear systems problems, in *Algebraic and Geometric Methods in Nonlinear Control Theory*, edited by M. Fliess and M. Hazewinkel (Dordrecht, The Netherlands: Reidel), pp. 285–309.

SASTRY, S., and BODSON, M., 1989, *Adaptive Control: Stability, Convergence and Robustness* (Englewood Cliffs, New Jersey, U.S.A.: Prentice-Hall).

SASTRY, S., and ISIDORI, A., 1989, Adaptive control of linearizable systems. *IEEE Transactions on Automatic Control*, **34**, 1123–1131.

SEVERNS, R. P., and BLOOM, G., 1985, *Modern Dc-to-Dc Switch-Mode Power Converter Circuits* (New York: Van Nostrand-Reinhold).

SIRA-RAMÍREZ, H., 1989a, Switched control of bilinear converters via pseudolinearization. *IEEE Transactions on Circuits and Systems*, **36**, 858–865; 1989b, A geometric approach to pulse-width-modulated control in nonlinear dynamical systems. *IEEE Transactions on Automatic Control*, **34**, 184–187; 1991, Nonlinear P-I controller design for switch-mode DC-to-DC power converters. *IEEE Transactions on Circuits and Systems*, **38**, 410–417.

SIRA-RAMÍREZ, H., GARCIÁ-ESTEBAN, M., and LLANES-SANTIAGO, O., 1995a, Dynamical adaptive regulation of non-minimum phase PWM controlled DC-to-DC power converters, to be published.

SIRA-RAMÍREZ, H., and ILIC, M., 1989, Exact linearization in switch mode DC-to-DC power converters. *International Journal of Control*, **50**, No. 2, 511–524.

SIRA-RAMÍREZ, H., and LISCHINSKY-ARENAS, P., 1992, The differential algebraic approach in non-linear dynamical compensator design for Dc-to-Dc power converters. *International Journal of Control*, **54**, 111–133.

SIRA-RAMÍREZ, H., LISCHINSKY-ARENAS, P., and LLANES-SANTIAGO, O., 1993b, Dynamic compensator design in nonlinear aerospace systems. *IEEE Transactions on Aerospace and Electronic Systems*, **29** (2), 364–379.

SIRA-RAMÍREZ, H., and LLANES-SANTIAGO, O., 1993, On the adaptive control of PWM switch-regulated systems. *Proceedings of 10th IFAC World Congress*, Sidney, Australia.

SIRA-RAMÍREZ, H., and PRADA-RIZZO, M. T., 1992, Nonlinear feedback regulator design for the Ćuk converter. *IEEE Transactions on Automatic Control*, **37**, No. 8, 1173–1180.

SIRA-RAMÍREZ, H., RÍOS-BOLÍVAR, E. M., and ZINOBER, A. S. I., 1995b, A non-overparametrized backstepping controller approach for the PWM stabilization of DC-to-DC power converters. *International Journal of Robust and Nonlinear Control*, to be published.

SIRA-RAMÍREZ, H., TARANTINO-ALVARADO, R., and LLANES-SANTIAGO, O., 1993a, Adaptive feedback-stabilization in PWM controlled DC-to-DC power supplies. *International Journal of Control*, **57**, No. 3, 599–625.

UTKIN, V. I., 1978, *Sliding Regimes and Their Applications in Variable Structure Systems* (Moscow: MIR).