

## **A redundant dynamical sliding mode control scheme for an asymptotic space vehicle stabilization**

H. SIRA-RAMÍREZ<sup>†</sup> and H. B. SIGUERDIDJANE<sup>‡</sup>

A redundant dynamical sliding mode control scheme is proposed for the asymptotic stabilization of a rigid spacecraft. The input-dependent nonlinear sliding manifolds are derived from a smooth feedback control law, based on the use of nonlinear characteristic values and characteristic vectors. A scheme tolerating sudden perturbation failure is presented. Simulations studies are provided.

### **1. Introduction**

The use of dynamical sliding mode control policies for nonlinear systems, based on input-dependent manifolds, has resulted in a rather significant departure from traditional discontinuous feedback design schemes utilizing only state-dependent surfaces (Sira-Ramírez 1991, 1992). Continuous rather than bang-bang control input signals and substantially smoothed chattering-free trajectories have been shown to be some of the several advantageous properties exhibited by the use of such input-dependent manifolds as sliding surfaces.

Static sliding mode control schemes, using state-dependent sliding surfaces, have been proposed in the past for the stabilization of a rigid spacecraft. The work of Vadali proposed traditional linear manifolds for rest-to-rest reorientation manoeuvres. Dwyer and Sira-Ramírez (1988) used nonlinear manifolds which resulted in linearized kinematics regulation. The advantage of the sliding mode approach, for spacecraft stabilization, lies in the enhanced robustness features and the overall simplicity of the control scheme. The disadvantages are related to the bang-bang nature of the applied torques.

In this article, we propose to utilize a set of input-dependent sliding surfaces for the bang-bang free asymptotic stabilization of angular velocities in rigid spacecrafts. The multivariable input-dependent nonlinear sliding manifolds are directly suggested by a smooth feedback control law design, entirely based on the use of nonlinear eigenvalues and eigenvectors and the associated nonlinear characteristic equation (Siguerdidjane 1991, 1994). Such a collection of sliding surface coordinates has the interpretation of feedback control implementation error. Any deviation of the implemented input, from the required value generated by the smooth state feedback control law, yields a detectable error which triggers a dynamically generated feedback correction signal. Such redundant control signal forcefully imposes the designed feedback control law in a sliding mode manner. The implicit advantages of sliding mode control are then substantially enhanced in our proposed scheme, as it is also capable of tolerating sudden failures both in the main designed feedback loop and in the dynamical discontinuous portion of the controller.

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Received 1 August 1994. Revised 14 September 1995.

<sup>†</sup> Departamento Sistemas de Control, Universidad de Los Andes, Mérida, Venezuela.

<sup>‡</sup> Supélec, Plateau de Moulon, 91192 Gif-sur-Yvette, France.

The general feedback control scheme, based on utilizing a designed smooth feedback control law as an input-dependent sliding surface, is presented in §2 together with a derivation and analysis of some of the advantageous features of such an approach; §3 is devoted to an application of the proposed technique to rigid spacecraft stabilization; simulation studies are included; §4 contains the conclusions and suggestions for further research.

## 2. A redundant dynamical sliding mode control scheme

### 2.1. A dynamical sliding mode controller based on a prescribed smooth feedback control law

Consider a nonlinear  $n$ -dimensional multivariable smooth system of the form

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}) \quad (2.1)$$

where  $\mathbf{u}$  denotes the  $m$ -dimensional vector of control input components taking values in  $R$ .

Suppose that a smooth feedback controller has been designed, which locally stabilizes the trajectories of the control system (2.1) to a desired constant equilibrium point  $\mathbf{X}(U)$ , dependent upon a constant value of the input signal  $U$ , that is  $\mathbf{f}(\mathbf{X}(U), U) = 0$ . The stabilizing feedback control law is assumed to be explicitly given by

$$\mathbf{u} = -\mathbf{k}(\mathbf{x}) \quad (2.2)$$

In other words, the closed-loop system

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, -\mathbf{k}(\mathbf{x})) \quad (2.3)$$

is assumed locally to exhibit desirable asymptotic stability features towards the equilibrium point. In equilibrium, the value of the feedback signal  $-\mathbf{k}(\mathbf{X}(U))$  is compatible with the equilibrium value for the vector  $\mathbf{u}$ , that is  $U = -\mathbf{k}(\mathbf{X}(U)) \neq 0$ .

Suppose now that an auxiliary multivariable input-dependent function of the form

$$\mathbf{s}(\mathbf{x}, \mathbf{u}) = \mathbf{u} + \mathbf{k}(\mathbf{x}) \quad (2.4)$$

is synthesized, and its components are proposed as *sliding surface candidates* on which the following discontinuous dynamics are imposed:

$$\dot{\mathbf{s}}(\mathbf{x}, \mathbf{u}) = -\mathbf{W} \text{Sgn}[\mathbf{s}(\mathbf{x}, \mathbf{u})] \quad (2.5)$$

with  $\mathbf{W} = \text{diag}(W_i)$  being a strictly positive definite diagonal matrix with sufficiently large constant entries. The vector  $\text{Sgn}[\mathbf{s}(\mathbf{x}, \mathbf{u})]$  stands for the vector of scalar signum functions applied to each component  $s_i(\mathbf{x}, \mathbf{u})$ . Note that the trajectories of the components  $s_i(\mathbf{x}, \mathbf{u})$  of (2.5) independently reach the condition  $s_i(\mathbf{x}, \mathbf{u}) = 0$ , in *finite time*  $T_i$ , given by  $T_i = |s_i(\mathbf{x}(0), \mathbf{u}(0))|/W_i$  ( $i = 1, 2, 3$ ).

Upon reaching the condition  $\mathbf{s}(\mathbf{x}, \mathbf{u}) = 0$ , a sliding motion (Utkin 1978) is collectively created on the intersection of such a set of input-dependent manifolds. The sliding mode condition is then sustained in an indefinite manner. The *ideal sliding motions* (Sira-Ramírez 1993) associated with the sliding regime, thus created, imply that the control input vector  $\mathbf{u}$  precisely complies with the designed feedback control law. In other words, ideally speaking, under the sliding mode condition one has  $\mathbf{u} = -\mathbf{k}(\mathbf{x})$ .

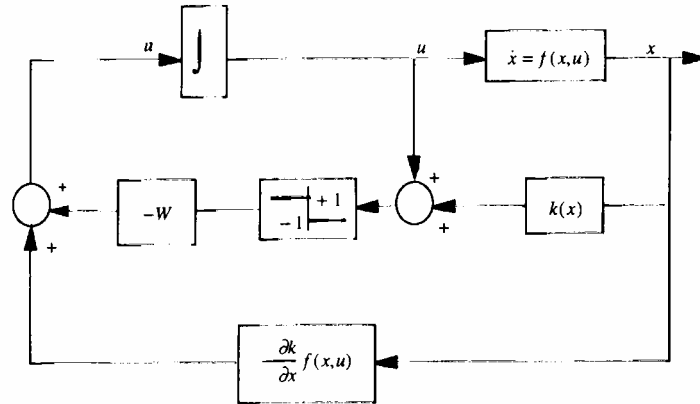


Figure 1. Dynamical sliding mode controller enforcing a designed smooth feedback control law.

Replacing (2.4) into (2.5) leads to the following dynamical discontinuous sliding mode controller for the nonlinear system:

$$\dot{u} = -\frac{\partial k(x)}{\partial x} f(x, u) - W \text{Sgn}[u + k(x)] \quad (2.6)$$

Equation (2.6) represents a time-varying nonlinear first-order differential equation for the control input vector  $u$  with a discontinuous right-hand side. The additional complication incurred in building such a dynamical discontinuous feedback controller is superseded by the many advantages that it bestows on the closed loop features of the controlled system.

A block diagram of the feedback controller (2.6) is shown in Fig. 1. A straightforward integration of the above expression (2.6) allows for the re-interpretation of the controller in terms of a redundant 'hybrid' controller comprising the original feedback law (2.3) implemented in parallel to an integrated (i.e. smoothed) discontinuous feedback vector signal, triggered by the signs of the feedback errors  $u + k(x)$ . Indeed, integration of (2.6) yields

$$\begin{aligned} u &= -\int_0^t \left( \frac{\partial k(x(\sigma))}{\partial x} f(x(\sigma), u(\sigma)) + W \text{Sgn}[u(\sigma) + k(x(\sigma))] \right) d\sigma + u(0) \\ &= -k(x(t)) - \int_0^t W \text{Sgn}[u(\sigma) + k(x(\sigma))] d\sigma + [u(0) + k(x(0))] \\ &= -k(x(t)) - W \int_0^t \text{Sgn}[u(\sigma) + k(x(\sigma))] d\sigma + s(x(0), u(0)) \end{aligned}$$

A block diagram depicting this reinterpretation of the controller (2.6) is shown in Fig. 2. From the previous expression, one also immediately obtains, upon rearrangement,

$$s(x(t), u(t)) = s(x(0), u(0)) - W \int_0^t \text{Sgn}[u(\sigma) + k(x(\sigma))] d\sigma \quad (2.7)$$

from where it easily follows that, regardless of the initial values of  $s(x(0), u(0))$  of the sliding surface vector  $s(x, u)$ , the condition  $s(x(T), u(T)) = 0$  is indeed reached in finite time  $T$ .

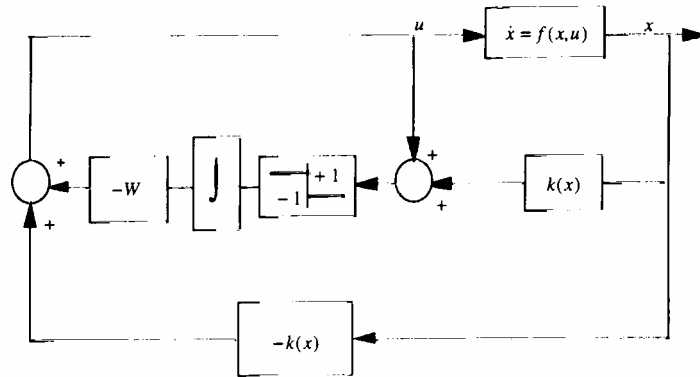


Figure 2. Reinterpretation of dynamical sliding mode controller enforcing a designed smooth feedback control law.

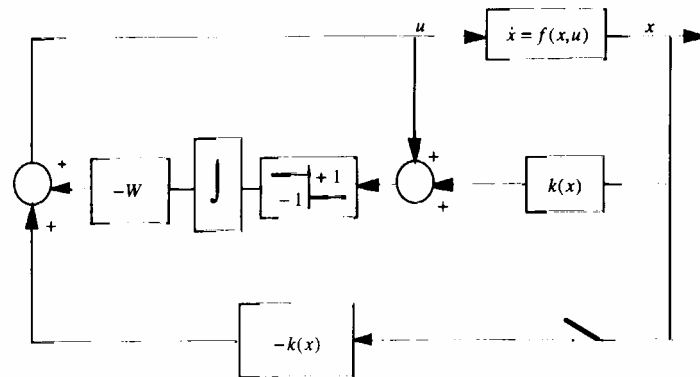


Figure 3. A feedback signal failure in the smooth portion of the redundant controller.

By virtue of the above developments, we finally rewrite the multivariable dynamical feedback controller (2.6) as

$$\left. \begin{aligned} u &= v - k(x) \\ \dot{v} &= -W \operatorname{Sgn}[u + k(x)] \end{aligned} \right\} \quad (2.8)$$

## 2.2. Some properties of the proposed dynamical discontinuous controller

The dynamical controller (2.6) exhibits several advantageous properties which are summarized below.

2.2.1. The discontinuities associated to the underlying sliding motion, taking place along the input-dependent manifold,  $s(x, u) = 0$  are relegated to the first-order time derivative of the control input signals components of  $u$ . Hence, the resulting controller is, indeed, *continuous*. Bang-bang input signals, otherwise characteristic of sliding mode control schemes (Utkin 1978), are thus effectively suppressed by the dynamic nature of the proposed controller (Sira-Ramírez 1991, 1992).

2.2.2. Suppose that at a certain time  $t = T_f$  the smooth portion of the feedback loop, feeding the signal components  $-k(x)$  to the control input  $u$ , fails for an indefinite period of time (Fig. 3). Assume, furthermore, that at the failure time  $T_f$  the

discontinuous portion of the controller was currently exhibiting a sliding mode behaviour (i.e. ideally  $s(x(T_f), u(T_f)) = 0$ ). Suppose also that the system's state was already stabilized at its equilibrium value  $x = X(U)$  and, hence,  $u(T_f) = -k(X(U)) = U$ . The feedback control law being enforced at any time  $t > T_f$ , after the feedback failure, satisfies

$$u = -W \int_{T_f}^t \text{Sgn}[u(\sigma) + k(x(\sigma))] d\sigma \quad (2.9)$$

that is

$$\dot{u} = -W \text{Sgn}[u(t) + k(x(t))] \quad (2.10)$$

It follows that the value of the components of  $u(t)$  are instantaneously set to zero at  $T_f +$ . It easily follows from (2.9) and (2.10) that  $u$  evolves, for  $t > T_f$ , in such a fashion that the norm of the feedback error signals,  $s(x, u) = u + k(x)$ , is being constantly diminished. The discontinuous part of the controller locally sustains the motions of the system in a sliding regime around the manifold,  $u + k(x) = 0$ , thus recovering, on the average, the original feedback control law  $u = -k(x)$ . Indeed, consider the value of the product  $s(x(t), u(t))^T \dot{s}(x(t), u(t))$  for  $t > T_f$ :

$$\begin{aligned} s(x(t), u(t))^T \dot{s}(x(t), u(t)) &= s(x(t), u(t))^T \left( \dot{u} + \frac{\partial k}{\partial x} f(x, u) \right) \\ &= s(x(t), u(t))^T \left( -W \text{Sgn}[s(x(t), u(t))] + \frac{\partial k}{\partial x} f(x, u) \right) \\ &= -W \sum_{i=1}^m |s_i(x(t), u(t))| + s(x(t), u(t))^T \left( \frac{\partial k}{\partial x} f(x, u) \right) \quad (2.11) \end{aligned}$$

Thus, for a sufficiently large value of the constant diagonal elements of  $W$ , the sign of the above expression can always be made negative on an open neighbourhood defined around  $s(x, u) = 0$ , and, hence, a sliding regime is seen to locally exist on  $s(x, u) = 0$  (Utkin 1978).

**Remark:** It should be pointed out that the region of attraction of the sliding manifold  $s(x, u) = 0$ , for control laws satisfying (2.10) must be precisely determined in each case. It may very well happen that after a feedback loop failure, such as that described above, the surviving discontinuous portion of the controller is incapable of achieving a sliding motion on the zero error manifold,  $s(x, u) = 0$ . Such a possibility is highly dependent upon the possibility of complying with the negativity of the final expression in (2.11).

2.2.3. Sliding mode controllers are known to be highly insensitive to external perturbation signals and to modelling errors. Thus the above scheme always imposes, in a robust fashion, 'the right feedback control law'. Changes in state due to external perturbation inputs to the system result in corresponding changes in the feedback control law  $-k(x)$ , at both the smooth and the discontinuous portions of the proposed controller. If the designed smooth control law is known to enjoy robustness properties with respect to a certain class of perturbation input signals, the proposed

controller simply inherits those properties and results in a forceful imposition, on the average, of the required smooth control law. The abundance of results in the area of sliding mode control schemes robustness makes this important feature of discontinuous feedback control schemes transparently obvious.

### 3. Dynamical sliding mode control stabilization of a rigid spacecraft

#### 3.1. A rigid spacecraft model

Consider a rigid body in an inertial reference frame. Let  $\omega_1, \omega_2$  and  $\omega_3$  denote the components of the angular velocity vector and denote by  $I_1, I_2$  and  $I_3$  the moments of inertia about the principal axes, here assumed to coincide with the body axes. Set  $x_i = \omega_i$  ( $i = 1, 2, 3$ ). The dynamics of the motion under the influence of external torques  $u_1, u_2$  and  $u_3$  are described by the following set of Euler equations:

$$\left. \begin{aligned} \dot{x}_1 &= \frac{I_2 - I_3}{I_1} x_2 x_3 + u_1 \\ \dot{x}_2 &= \frac{I_3 - I_1}{I_2} x_3 x_1 + u_2 \\ \dot{x}_3 &= \frac{I_1 - I_2}{I_3} x_1 x_2 + u_3 \end{aligned} \right\} \quad (3.1)$$

A stable constant equilibrium point for system (3.1) is given by

$$u_i = U_i = 0, \quad x_i(U_i) = 0 \quad (i = 1, 2, 3) \quad (3.2)$$

#### 3.2. A smooth controller design for the angular velocity stabilization

In this section we summarize the derivation of an explicit stabilizing nonlinear feedback controller, synthesized by means of the nonlinear characteristic equation method (Siguerdidjane 1991, 1992). For further details, the reader is referred to these references.

Suppose the following desired dynamics, which are directly obtained from an eigenvalue–eigenvector analysis of the closed-loop system equation, is to be imposed on the spacecraft system (3.1):

$$\left. \begin{aligned} x_1 &= imv_1 \omega_0 e^{\beta t} \operatorname{cn} \left( \frac{\omega_0 \lambda e^{\beta t}}{\beta} \right) \\ x_2 &= -mv_2 \omega_0 e^{\beta t} \operatorname{sn} \left( \frac{\omega_0 \lambda e^{\beta t}}{\beta} \right) \\ x_3 &= iv_3 \omega_0 e^{\beta t} \operatorname{dn} \left( \frac{\omega_0 \lambda e^{\beta t}}{\beta} \right) \end{aligned} \right\} \quad (3.3)$$

where  $\operatorname{cn}()$ ,  $\operatorname{sn}()$  and  $\operatorname{dn}()$  are the Jacobi elliptic functions of pole  $n$ . The constant  $\beta$  is a negative constant.  $m$  is the so-called modulus of the Jacobi functions. The

parameter  $\omega_0$  is an arbitrary constant, and  $i$  is the imaginary number. The  $v_i$ 's are the components of the nonlinear eigenvector  $\mathbf{v}$  of the closed-loop system which is easily seen to satisfy

$$\left. \begin{aligned} \lambda v_1 &= \frac{I_2 - I_3}{I_1} v_2 v_3 \\ \lambda v_2 &= \frac{I_3 - I_1}{I_2} v_3 v_1 \\ \lambda v_3 &= \frac{I_1 - I_2}{I_3} v_1 v_2 \end{aligned} \right\} \quad (3.4)$$

$\lambda$  is the nonlinear eigenvalue associated with  $\mathbf{v}$ .

Making the proposed solution (3.3) compatible with the system equation (3.1) and the required steady-state conditions, one finds, after tedious manipulations involving the formulae for the derivatives of the Jacobi elliptic functions, that the control input components must satisfy the following strikingly simple state feedback control laws:

$$\left. \begin{aligned} u_1(x) &= \beta x_1 \\ u_2(x) &= \beta x_2 \\ u_3(x) &= \beta x_3 \end{aligned} \right\} \quad (3.5)$$

Siguerdidjane (1991) has shown that the feedback control law (3.5) yields asymptotically stable closed-loop trajectories. The only limitation associated with such a control law rests in the fact that the argument of the Jacobi elliptic functions may not be allowed to take the value  $K$  representing the so-called 'real quarter-period'.

### 3.3. Redundant dynamical sliding mode controller design

According to the results of §2, we choose as sliding surface coordinate components the following input-dependent expressions:

$$\left. \begin{aligned} s_1(x) &= u_1(x) - \beta x_1 \\ s_2(x) &= u_2(x) - \beta x_2 \\ s_3(x) &= u_3(x) - \beta x_3 \end{aligned} \right\} \quad (3.6)$$

By imposing the discontinuous dynamics (2.5) on each one of the components of the vector  $\mathbf{s}(\mathbf{x}, \mathbf{u}) = (s_1(\mathbf{x}, \mathbf{u}), s_2(\mathbf{x}, \mathbf{u}), s_3(\mathbf{x}, \mathbf{u}))$  above, one obtains the following set of dynamical sliding mode controllers:

$$\left. \begin{aligned} \dot{u}_1 - \beta u_1 &= \beta \frac{I_2 - I_3}{I_1} x_2 x_3 - W_1 \text{Sgn}(u_1 - \beta x_1) \\ \dot{u}_2 - \beta u_2 &= \beta \frac{I_3 - I_1}{I_2} x_3 x_1 - W_2 \text{Sgn}(u_2 - \beta x_2) \\ \dot{u}_3 - \beta u_3 &= \beta \frac{I_1 - I_2}{I_3} x_1 x_2 - W_3 \text{Sgn}(u_3 - \beta x_3) \end{aligned} \right\} \quad (3.7)$$

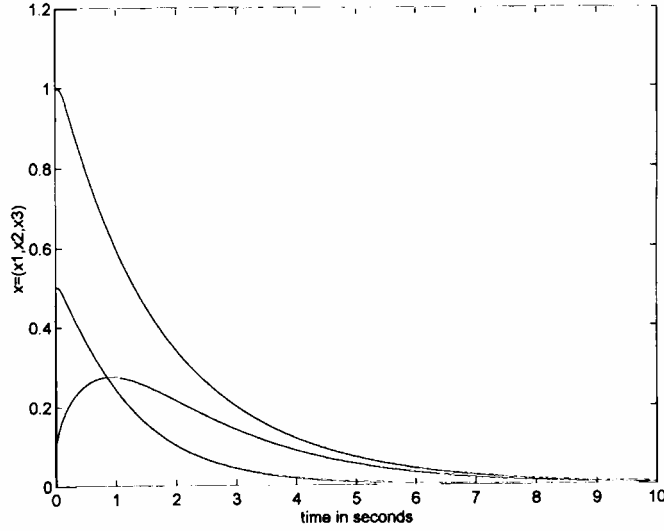


Figure 4. The state response: the angular velocities in radians per second.

Upon stabilization of the angular velocities, the zero dynamics, associated with the controlled system, are simply given by the following set of asymptotically stable motions:

$$\dot{u}_1 = \beta u_1, \quad \dot{u}_2 = \beta u_2, \quad \dot{u}_3 = \beta u_3 \quad (3.8)$$

which states that the proposed feedback control scheme leads to a minimum phase behaviour of the controlled angular velocities.

In terms of the reinterpretation (2.8) the above controller is simply written as

$$\left. \begin{aligned} u_1 &= v_1 + \beta x_1, & \dot{v}_1 &= -W_1 \text{Sgn}(u_1 - \beta x_1) \\ u_2 &= v_2 + \beta x_2, & \dot{v}_2 &= -W_2 \text{Sgn}(u_2 - \beta x_2) \\ u_3 &= v_3 + \beta x_3, & \dot{v}_3 &= -W_3 \text{Sgn}(u_3 - \beta x_3) \end{aligned} \right\} \quad (3.9)$$

### 3.4. Simulations

Simulations were performed for the system (3.1) with the dynamical controller (3.9). The numerical values adopted for the system parameters were chosen as the parameters of the satellite SPOT 4:  $I_1 = 2500 \text{ kg m}^2$ ,  $I_2 = 6500 \text{ kg m}^2$  and  $I_3 = 8000 \text{ kg m}^2$ .

The parameter  $\beta$  in the smooth controller (3.5) was set to be  $\beta = -0.5$ , while the dynamical sliding mode controller parameter matrix  $W$  was chosen as

$$W = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix} \quad (3.10)$$

Figure 4 shows the state response of the dynamically sliding mode controlled system asymptotically converging to their corresponding equilibrium points. Figure 5



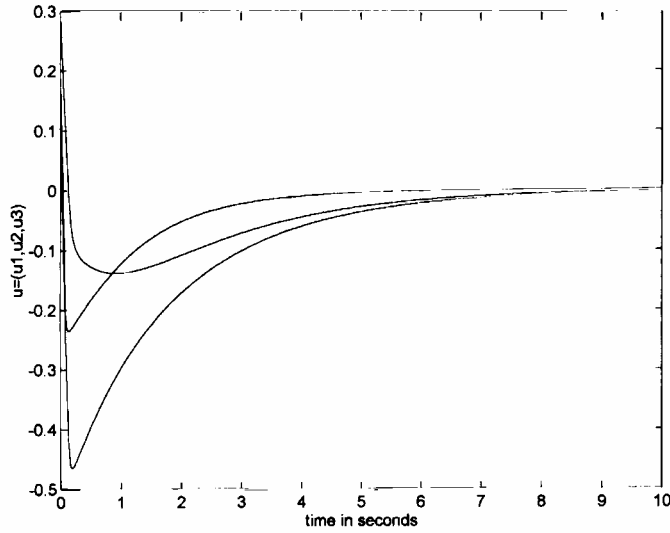


Figure 5. The continuous trajectory of the control input components.

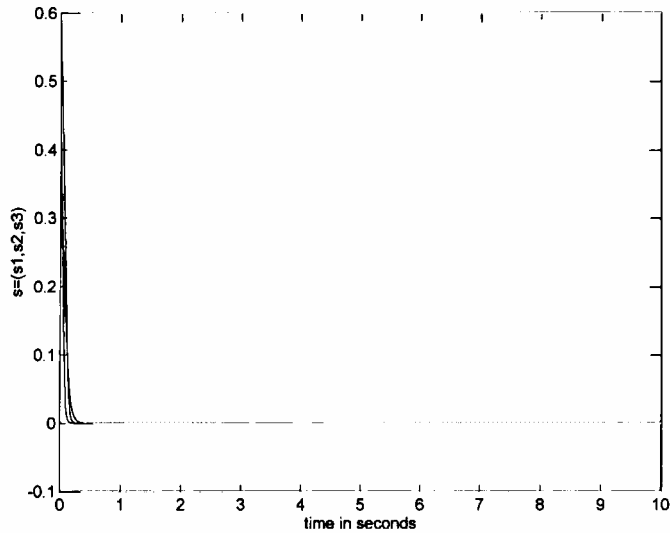


Figure 6. The input-dependent sliding surface coordinate components.

depicts the continuous trajectory of the control input components while Fig. 6 shows the evolution of the input-dependent sliding surface coordinate components  $s(x, u)$ , converging to zero in finite time.

In order to check the robustness of the proposed dynamical sliding mode control scheme with respect to sudden failures in the originally designed smooth feedback loop, we simulated the performance of the system with the following dynamical discontinuous feedback controller:

$$\left. \begin{aligned} u_1 &= v_1 + \kappa \beta x_1, & \dot{v}_1 &= -W_1 \operatorname{Sgn}(u_1 - \beta x_1) \\ u_2 &= v_2 + \kappa \beta x_2, & \dot{v}_2 &= -W_2 \operatorname{Sgn}(u_2 - \beta x_2) \\ u_3 &= v_3 + \kappa \beta x_3, & \dot{v}_3 &= -W_3 \operatorname{Sgn}(u_3 - \beta x_3) \end{aligned} \right\} \quad (3.11)$$

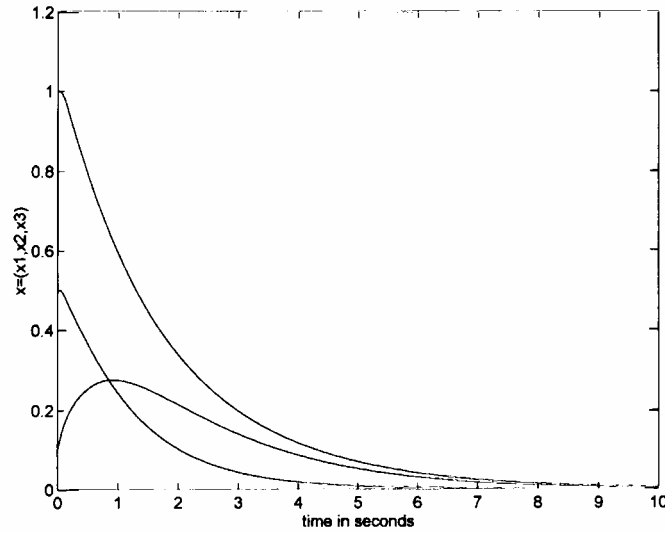


Figure 7. The state responses subject to a sudden failure at time  $T_f = 0.5$  s.

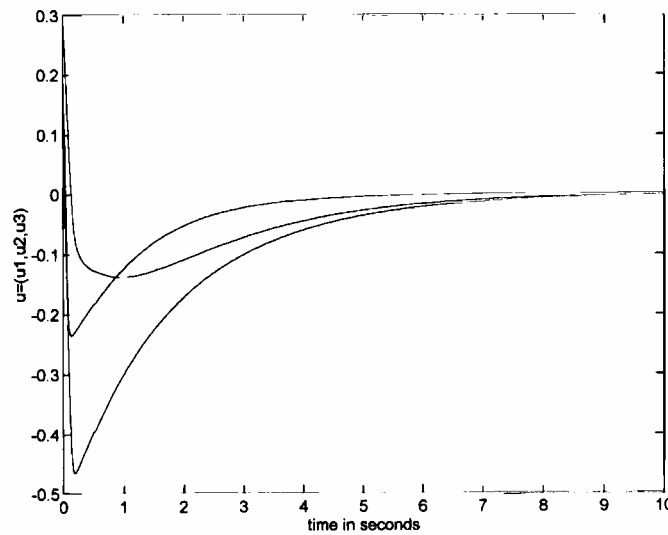


Figure 8. The trajectory of the failed control input.

where the variable  $\kappa$ , simulating the feedback loop failure, was allowed to be

$$\kappa = \begin{cases} 1, & \text{for } t \leq T_f \\ 0, & \text{for } t > T_f \end{cases} \quad (3.12)$$

with  $T_f = 0.5$  s.

Figure 7 shows the state responses of the dynamically sliding mode controlled system subject to the sudden failure of the form (3.11). The sliding mode controller is seen to restabilize the state trajectories to their corresponding equilibrium points. Figure 8 depicts the corresponding trajectory of the failed control input, showing, at failure time  $T_f$ , the instantaneous resetting to the value zero of the control input signal and its subsequent recovery to a sliding mode regulation of the system. Note that the

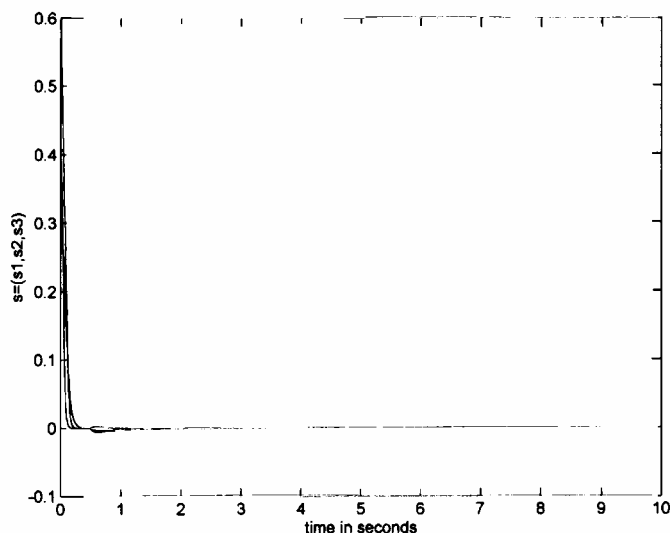


Figure 9. The behaviour of the input-dependent sliding surface coordinate components emphasizing the failure time  $T_f = 0.5$  s.

surviving portion of the controller still generates a continuous feedback signal. Figure 9 shows the behaviour of the input-dependent sliding surface coordinate function  $s(x, u)$  before and after the feedback signal failure.

#### 4. Conclusions

A robust redundant feedback control scheme, based on dynamical sliding mode control, has been proposed for the robust stabilization of angular velocities in a rigid spacecraft. The proposed scheme utilizes the expressions of a designed multivariable smooth feedback control policy as sliding surfaces and proceeds to impose forcefully those desirable relations by means of appropriately induced sliding regimes. The resulting dynamical controller is then reinterpreted in terms of two subsystems: one is the smooth portion of the controller, represented by the originally designed stabilizing static feedback control law, and the other is a parallel regulator based on dynamically generated (i.e. smoothed) discontinuous control actions of the sliding mode (i.e. relay) type. The scheme was shown to be advantageous in several respects, among which we found local robustness with respect to sudden failures in the static portion of the proposed feedback controller. The scheme is trivially robust with respect to failures in the discontinuous complementary controller. The basic features of the proposed control scheme were illustrated by means of simulations.

#### REFERENCES

- DWYER, T. A. W., and SIRA-RAMIREZ, H., 1988, Variable structure control of spacecraft attitude maneuvers. *Journal of Guidance, Dynamics and Control*, **11**, 262–270.
- SIGUERDIDJANE, H., 1991, Further results on the optimal regulation of satellite angular momentum. *Optimal Control: Applications and Methods*, **12**, 273–278; 1994, On the characteristic modes of a rigid body under forces. *Kybernetika*, **30**, 489–497; 1992, Stabilization of a rigid spacecraft on the nonlinear feedback construction. *12th IFAC Symposium on Aerospace Control*, pp. 247–249.

- SIRA-RAMÍREZ, H., 1991, Nonlinear dynamically feedback controlled descent on a non atmosphere-free planet: a differential algebraic approach. *Control Theory and Advanced Technology*, **7**, 301–320; 1992, Asymptotic output stabilization for nonlinear systems via dynamical variable control. *Dynamics and Control*, **2**, 45–58; 1993, A dynamical variable structure control strategy in asymptotic output tracking problems. *IEEE Transactions on Automatic Control*, **38**, 615–620.
- UTKIN, V., 1978, *Sliding Modes and Their Applications in Variable Structure Systems* (Moscow: Mir).