guaranteed. For example, LVQ1 cannot find a stable solution when we train a vector quantizer of size two in order to solve the problem of discriminating between two one-dimensional white Gaussian distributions having the same mean and different variances (see [2, p. 42] for the proof of this fact).

We point out also that, excepting some special recognition problems, it is not possible to reduce the goal of LVQ1 to the minimization of error probability, which is the ultimate goal of any smart classification strategy. Baras and La Vigna assess in [1] that the error probability associated with a slightly modified version of LVQ1 approaches the Bayes error probability as the number of codebook vectors becomes large. This claim is not really in contrast with ours. Actually it is quite expected, since the finer the quantization of the observation space is, the closer the classification performance is to the Bayes error probability.

Thus far, we have assumed the knowledge of the conditional distributions of the two classes or, equivalently, that an arbitrarily long random sequence is available. In practice, it often happens that the knowledge of the statistics is given by a training set consisting of a *finite* number of elements. Let $T = \{t_1, t_2, \cdots, t_S\}$. $S < \infty$, denote the training set and let us assume an indexing such that the label of t_i is A for $i = 1, 2, \cdots, S_A$, and is B for $i = S_A + 1, S_A + 2, \cdots, S$. We can make use of estimates based on the training set in place of the known distributions.

The probabilities of the two classes can be estimated by $P(A) = S_A/S$, $P(B) = 1 - P(A)$. The two conditional distributions can be estimated by Parzen's method

$$p(x|A) = \frac{1}{S_A} \sum_{i=1}^{S_A} h(x - t_i),$$

$$p(x|B) = \frac{1}{S - S_A} \sum_{i=S_A+1}^{S} h(x - t_i)$$

where $h(x)$ is some conveniently designed Parzen's window (see e.g., [4], sec. 6.1). According to this setting, the kth sample of the random sequence is $x^k = t^k + n^k$. Vector t^k is randomly picked with uniform probability from the training set,[2] while vector n^k is independently picked from a dense population whose pdf is $h(x)$. Obviously x^k inherits its label from t^k. In the practice, a delta function is often designed for $h(x)$, so $n^k = 0$ for any k. We note that, when S is comparable to the length of the random sequence used during the optimization, there are no substantial differences between known and unknown distributions.

III. Conclusion

The problem of recognizing the class (A or B) an observation vector (x) belongs to, can be solved by partitioning the x-space into regions such that, for any region and for any x belonging to the same region, either $P(A)p(x|A) \geq P(B)p(x|B)$ or $P(B)p(x|B) > P(A)p(x|A)$ holds true. Kohonen's idea of employing a labeled vector quantizer fits well this fact, in the sense that the structure of the vector quantizer is well suited to encode a partition as refined as one likes. Of course, one would like to obtain high performance with a small number of codebook vectors. In other words, the goal of vector quantization must be to *efficiently* encode the x-space. The work reported in this brief has made one thing clear: the efficiency of LVQ1 could be significantly improved by paying more attention to the optimality criterion the learning is based on. It is our feeling that the latter point is crucial to the successful use of Vector Quantizers in recognition tasks, and that much is to be done in this area. Results about the design of an adaptive VQ which minimizes error probability are available in [2], [3].

[2] A slightly different procedure adopted by some authors is to pick t^k cyclically from the training set.

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A Lagrangian Approach to Average Modeling of Pulsewidth-Modulation Controlled DC-to-DC Power Converters

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***Abstract*—A Lagrangian approach is used for obtaining the average model of a switch regulated dc-to-dc Power Converter of the "Boost" type undergoing a Pulsewidth-Modulation (PWM) feedback strategy. A set of average Euler–Lagrange (EL) parameters, modulated by the duty ratio function, is proposed which recovers, under duty ratio saturation conditions, the individual EL formulations of the intervening circuit topologies. The obtained average PWM model coincides with, both the *state average model* and the *infinite switching frequency* average model, previously proposed in the literature and derived from entirely different viewpoints.**

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I. Introduction

DC-to-DC Power Converters are frequently regulated by means of Pulsewidth-Modulation (PWM) feedback control policies whose complexity varies from traditional Proportional Derivative (P-D) schemes (see [1]) to sophisticated nonlinear adaptive feedback control options (see among many other authors, the work of Sira-Ramirez and colleagues [2]). In the feedback regulator synthesis problem for dc-to-dc converters governed by PWM feedback laws, *average models* play a most essential role (see [3]).

In this brief a Lagrangian dynamics approach is used for deriving a physically motivated model of the average behavior of PWM regulated dc-to-dc power converters. The approach consists in establishing the Euler–Lagrange (EL) parameters of the circuits associated with each one of the topologies corresponding to the two possible positions of the regulating switch. This consideration immediately leads one to realize that some EL parameters remain invariant under the switching action while some others are definitely modified by the addition of known quantities. An average PWM model of the noninvariant EL parameters can then be proposed by their suitable modulation through the duty ratio function. This modulation is done in a consistent fashion so that, under extreme duty ratio saturation conditions, the original EL parameters, corresponding to the two intervening circuit topologies, are exactly recovered. The average EL parameter considerations lead, through use of the classical Lagrangian dynamics equations, to systems of continuous differential equations, describing the average PWM converter behavior. These equations are interpretable in terms of ideal equivalent circuit realizations obtained by replacing the switching device by a suitable ideal transformer. This particular result is in accordance with well-known circuit equivalents of PWM switches already derived in [5] and [6]. The obtained average PWM models entirely coincide with the *state average models* of dc-to-dc Power Converters introduced in [3] and also with the *infinite switching frequency* model found in [4].

Section II contains some generalities about the average modeling of discontinuously controlled Euler–Lagrange systems. In Section III the general results are applied to the average modeling of dc-to-dc Power Converters of the "Boost" type, equipped with ideal switching devices. Section IV is devoted to present some conclusions and suggestions for further research.

II. Average Modeling of Switched Euler–Lagrange Systems

The Euler–Lagrange formulation of dynamical systems constitutes a thoroughly studied and developed chapter of Classical Mechanics. For the particular case of electrical and electromechanical systems, the reader is referred to the book by Meisel [7].

A. Euler–Lagrange Systems

An Euler–Lagrange system is classically characterized by the following set of nonlinear differential equations, known as *Lagrange equations*

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) - \frac{\partial \mathcal{L}}{\partial q} = -\frac{\partial \mathcal{D}}{\partial \dot{q}} + \mathcal{F}_q \quad (1)$$

where q is the vector of *generalized positions*, assumed to have n components, represented by $q_1, \cdots, q_n$, and $\dot{q}$ is the vector of *generalized velocities*. The scalar function $\mathcal{L}$ is the *Lagrangian* of the system, defined as the difference between the *kinetic energy* of the system, denoted by $\mathcal{T}(\dot{q}, q)$, and the *potential energy* of the system, denoted by $\mathcal{V}(q)$, i.e.,

$$\mathcal{L}(\dot{q}, q) = \mathcal{T}(\dot{q}, q) - \mathcal{V}(q). \quad (2)$$

The function $\mathcal{D}(\dot{q})$ is the *Rayleigh dissipation function* of the system. The vector $\mathcal{F}_q = (\mathcal{F}_{q_1}, \cdots, \mathcal{F}_{q_n})$ represents the ordered components of the set of *generalized forcing functions* associated with each generalized coordinate.

We refer to the set of functions $(\mathcal{T}, \mathcal{V}, \mathcal{D}, \mathcal{F})$ as the *Euler–Lagrange parameters* of the system and simply express a system Σ by the ordered cuadruple $\Sigma = (\mathcal{T}, \mathcal{V}, \mathcal{D}, \mathcal{F})$.

B. Switch Regulated Euler–Lagrange Systems

We are particularly interested in dynamical systems containing a single *switch*, regarded as the only *control function* of the system. The switch position, denoted by the scalar function u, is assumed to take values on a discrete set of the form $\{0, 1\}$. Our basic assumption is that for each one of the switch position values, the resulting system is still an Euler–Lagrange system (EL system for short) characterized by its corresponding EL parameters. In other words, we assume that when the switch position function takes the value, say, $u = 1$, the system, denoted by Σ_1, is characterized by the known set of EL parameters, $\Sigma_1 = (\mathcal{T}_1, \mathcal{V}_1, \mathcal{D}_1, \mathcal{F}_1)$. Similarly, when the switch position function takes the value $u = 0$, we assume the resulting system, denoted by Σ_0 is characterized by $\Sigma_0 = (\mathcal{T}_0, \mathcal{V}_0, \mathcal{D}_0, \mathcal{F}_0)$.

The previously described class of systems will be referred to as *switched EL systems* and they will be denoted by $\Sigma_u = \{\Sigma_1, \Sigma_0\} = u\Sigma_1 + (1-u)\Sigma_0$.

We assume that a Pulsewidth-Modulation feedback strategy is being imposed for the realization, in time, of the switch position function $u(t)$ acting as the only control variable of the switched EL system. A typical PWM regulation policy is specified as follows:

$$u(t) = \begin{cases} 1 & \text{for } t_k \le t < t_k + \mu(t_k)T \\ 0 & \text{for } t_k + \mu(t_k)T \le t < t_k + T \end{cases}$$
$$t_{k+1} = t_k + T; \quad k = 0, 1, \cdots \quad (3)$$

where t_k represents a sampling instant; the parameter T is the fixed sampling period, also called the *duty cycle*; the sampled values of the state vector $x(t)$ of the converter are denoted by $x(t_k)$. The function, $\mu(\cdot)$, is the *duty ratio function* acting as a truly feedback policy. The value of the duty ratio function, $\mu(t_k)$, determines, at every sampling instant, t_k, the width of the upcoming "pulse" (switch at the position $u = 1$) as $\mu(t_k)T$. The actual duty ratio function, $\mu(\cdot)$, is evidently a function limited to the closed interval [0, 1] of the real line.

C. An Average PWM Model of Switch-Regulated EL Systems

In order to provide some formalism to our concept of Average PWM system we introduce the following *criteria* for considering a given scalar function, such as the EL parameters, as a *reasonable* average PWM function of two given scalar functions.

Let $\mathcal{N}_1(q, \dot{q})$ and $\mathcal{N}_0(q, \dot{q})$ be a pair of scalar functions. Let t_k be a given arbitrary instant of time and let T be a fixed positive constant. Assume that a function $\mu(t)$ is known such that $\mu(t_k) \in [0, 1]$ $\forall k$. Suppose furthermore, as suggested by the PWM control policy (3), that during every time interval of the form, $[t_k, t_k + \mu(t_k)T]$, the function $\mathcal{N}_1[q(t), \dot{q}(t)]$ is valid, while the second function, $\mathcal{N}_0[q(t), \dot{q}(t)]$, is valid only during the remaining time intervals of the form, $[t_k + \mu(t_k)T, t_k + T]$. Let also $\mathcal{N}_M(q, \dot{q}) = \max\{\mathcal{N}_1(q, \dot{q}), \mathcal{N}_0(q, \dot{q})\}$; $\mathcal{N}_m(q, \dot{q}) = \min\{\mathcal{N}_1(q, \dot{q}), \mathcal{N}_0(q, \dot{q})\}$.

Definition 2.1: We consider a function $\mathcal{N}_\mu(q, \dot{q})$ to be an *average PWM function* of $\mathcal{N}_1(q, \dot{q})$ and $\mathcal{N}_0(q, \dot{q})$ whenever $\mathcal{N}_\mu(q, \dot{q})$ is a continuous function, parameterized by μ, which satisfies the following properties:

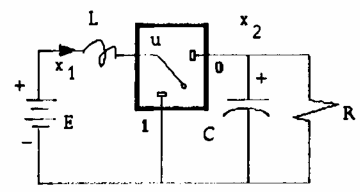

Fig. 1. "Boost" converter circuit.

1) $\mathcal{N}_\mu(q, \dot{q})$ fulfills the following *intermediacy* condition:

$$\mathcal{N}_m(q, \dot{q}) \leq \mathcal{N}_\mu(q, \dot{q}) \leq \mathcal{N}_M(q, \dot{q}) \quad \forall \mu \in (0, 1)$$

2) As a function of the generalized coordinates, and their corresponding velocities $(q, \dot{q})$ the function $\mathcal{N}_\mu(q, \dot{q})$ satisfies the following *consistency* condition:

$$\mathcal{N}_\mu(q, \dot{q})|_{\mu=1} = \mathcal{N}_1(q, \dot{q})$$

and

$$\mathcal{N}_\mu(q, \dot{q})|_{\mu=0} = \mathcal{N}_0(q, \dot{q}). \tag{4}$$

Definition 2.2: Given a switched EL system characterized by $\Sigma_u = \{\Sigma_1, \Sigma_0\}$ with $\Sigma_1 = (\mathcal{T}_1, \mathcal{V}_1, \mathcal{D}_1, \mathcal{F}_1)$ and $\Sigma_0 = (\mathcal{T}_0, \mathcal{V}_0, \mathcal{D}_0, \mathcal{F}_0)$, we consider Σ_μ to be an average PWM system of $\Sigma_u = \{\Sigma_1, \Sigma_0\}$ whenever the EL system characterized by the EL parameters $\Sigma_\mu = (\mathcal{T}_\mu, \mathcal{V}_\mu, \mathcal{D}_\mu, \mathcal{F}_\mu)$ is such that the EL parameters $(\mathcal{T}_\mu, \mathcal{V}_\mu, \mathcal{D}_\mu)$, and the scalar components of the vector $\mathcal{F}_\mu$ are constituted by *average PWM functions*, in the sense of the Definition 2.1, of the corresponding EL parameters of Σ_0 and Σ_1.

Definition 2.3: An EL parameter $\mathcal{P}(q, \dot{q})$ of a switched EL system $\{\Sigma_1, \Sigma_0\}$ is said to be *invariant* with respect to the switch position function u whenever, $\mathcal{P}(q, \dot{q}) = \mathcal{P}_0(q, \dot{q}) = \mathcal{P}_1(q, \dot{q}) \quad \forall (q, \dot{q})$.

III. A Lagrangian Approach to Average Modeling of the "Boost" Converter

Consider the switch-regulated "Boost" converter circuit of Fig. 1. The differential equations describing the circuit are given by

$$\begin{aligned} \dot{x}_1 &= -(1-u)\frac{1}{L}x_2 + \frac{E}{L}; \\ \dot{x}_2 &= (1-u)\frac{1}{C}x_1 - \frac{1}{RC}x_2 \end{aligned} \tag{5}$$

where x_1 and x_2 represent, respectively, the input inductor current and the output capacitor voltage variables. The positive quantity E represents the constant value of the external voltage source. The variable u denotes the switch position function, acting as a control input. Such a control input takes values in the discrete set $\{0, 1\}$.

It is assumed that a PWM based regulating policy for the switch position function u is specified as in (3). In order to use standard notation we refer to the input current x_1 in terms of the derivative of the circulating charge q_L, as $\dot{q}_L$. Also, the capacitor voltage x_2 will be written as q_C/C where q_C is the electrical charge stored in the output capacitor.

Consider $u = 1$. In this case two separate, or decoupled, circuits are clearly obtained and the corresponding Lagrange dynamics formulation can be carried out as follows.

Define $\mathcal{T}_1(\dot{q}_L)$ and $\mathcal{V}_1(q_C)$ as the kinetic and potential energies of the circuit, respectively. We denote by $\mathcal{D}_1(\dot{q}_C)$ the Rayleigh dissipation function of the circuit. These quantities are readily found to be

$$\begin{aligned} \mathcal{T}_1(\dot{q}_L) &= \tfrac{1}{2}L(\dot{q}_L)^2; \\ \mathcal{V}_1(q_C) &= \frac{1}{2C}q_C^2 \\ \mathcal{D}_1(\dot{q}_C) &= \tfrac{1}{2}R(-\dot{q}_C)^2 \\ \mathcal{F}^1_{q_L} &= E \\ \mathcal{F}^1_{q_C} &= 0 \end{aligned} \tag{6}$$

where $\mathcal{F}^1_{q_L}$ and $\mathcal{F}^1_{q_C}$ are the *generalized forcing* functions associated with the coordinates q_L and q_C, respectively.

Evidently, the Lagrange equations (1) used on these EL parameters immediately rederive (5), with $u = 1$, as it can be easily verified.

Consider now the case $u = 0$. Define $\mathcal{T}_0(\dot{q}_L)$ and $\mathcal{V}_0(q_C)$ as the kinetic and potential energies of the circuit, respectively. We denote by $\mathcal{D}_0(\dot{q}_L, \dot{q}_C)$ the Rayleigh dissipation function of the circuit. These quantities are readily found to be

$$\begin{aligned} \mathcal{T}_0(\dot{q}_L) &= \tfrac{1}{2}L(\dot{q}_L)^2; \\ \mathcal{V}_0(q_C) &= \frac{1}{2C}q_C^2 \\ \mathcal{D}_0(\dot{q}_L, \dot{q}_C) &= \tfrac{1}{2}R(\dot{q}_L - \dot{q}_C)^2; \\ \mathcal{F}^0_{q_L} &= E; \\ \mathcal{F}^0_{q_C} &= 0. \end{aligned} \tag{7}$$

Evidently, the Lagrange equations associated with these definitions immediately rederive (5), with $u = 0$.

The switching actions are seen to merely change the Rayleigh dissipation function between the values $\mathcal{D}_0(\dot{q}_C)$ and $\mathcal{D}_1(\dot{q}_L, \dot{q}_C)$. Therefore, the *dissipation structure* of the system is the only one affected by the switch position while the kinetic and potential energies remain invariant. One may then regard the switching action as a "damping injection," performed through the inductor current.

Note that, according to the PWM switching policy (3), on every sampling interval of period T, the Rayleigh dissipation function $\mathcal{T}_1(\dot{q}_C)$ is valid only a fraction of the time given by $\mu(t_k)T$ while the Rayleigh dissipation function $\mathcal{T}_0(\dot{q}_L, \dot{q}_C)$ is valid a fraction of the time equal to $[1 - \mu(t_k)]T$.

We propose the following set of average PWM EL parameters.

$$\begin{aligned} \mathcal{T}_\mu(\dot{q}_L) &= \tfrac{1}{2}L(\dot{q}_L)^2; \\ \mathcal{V}_\mu(q_C) &= \frac{1}{2C}q_C^2 \\ \mathcal{D}_\mu(\dot{q}_L, \dot{q}_C) &= \tfrac{1}{2}R[(1-\mu)\dot{q}_L - \dot{q}_C]^2; \\ \mathcal{F}^\mu_{q_L} &= E; \\ \mathcal{F}^\mu_{q_C} &= 0. \end{aligned} \tag{8}$$

Note that in the cases where μ takes the extreme values $\mu = 1$ and $\mu = 0$, one recovers, respectively, the dissipation functions $\mathcal{D}_1(\dot{q}_C)$ in (6) and $\mathcal{D}_0(\dot{q}_L, \dot{q}_C)$ in (7) from the proposed dissipation function, $\mathcal{D}_\mu(\dot{q}_L, \dot{q}_C)$, of (8). It is also easy to see that $\mathcal{D}_\mu(\dot{q}_L, \dot{q}_C)$ is "intermediate" between $\mathcal{D}_1(\dot{q}_C)$ and $\mathcal{D}_0(\dot{q}_L, \dot{q}_C)$.

The Lagrangian function, associated with the above defined EL parameters, is given by

$$\begin{aligned} \mathcal{L}_\mu &= \mathcal{T}_\mu(\dot{q}_L) - \mathcal{V}_\mu(q_C) \\ &= \frac{1}{2}L(\dot{q}_L)^2 - \frac{1}{2C}q_C^2. \end{aligned} \tag{9}$$

Using the Lagrange equations (1), one obtains the following set of differential equations defining the average system corresponding to the proposed average EL parameters (8)

$$\begin{aligned} L\ddot{q}_L &= -(1-\mu)R[(1-\mu)\dot{q}_L - \dot{q}_C] + E; \\ \frac{q_C}{C} &= R[(1-\mu)\dot{q}_L - \dot{q}_C]. \end{aligned} \tag{10}$$

These equations can be rewritten, after substitution of the second equation into the first, as

$$\ddot{q}_L = -(1-\mu)\frac{q_C}{LC} + \frac{E}{L};$$

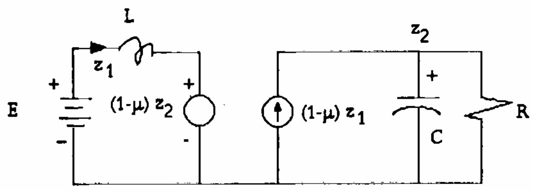

Fig. 2. Equivalent circuit of the average PWM model of the "Boost" converter circuit.

$$\dot{q}_C = -\frac{1}{RC} q_C + (1-\mu)\dot{q}_L. \tag{11}$$

Using $z_1 = \dot{q}_L$ and $z_2 = q_C/C$ one obtains

$$\dot{z}_1 = -(1-\mu)\frac{1}{L} z_2 + \frac{E}{L};$$
$$\dot{z}_2 = (1-\mu)\frac{1}{C} z_1 - \frac{1}{RC} z_2 \tag{12}$$

where we denote by z_1 and z_2 the *average input current* and the *average output capacitor voltage*, respectively, of the PWM regulated "Boost" converter. We establish this distinction with the nonaveraged variables x_1 and x_2 so that the average model and its associated variables is not mistakingly confused with the actual PWM regulated circuit variables.

The average PWM dynamics (12) is readily seen to have a circuit theoretic interpretation such as the one shown in Fig. 2.

Consider the isolated quadripole constituted by the ideal regulated sources in Fig. 2. Note that the (average) input power to the quadripole is given by the product of the average input current z_1 times the average (reflected) input voltage $(1-\mu)z_2$, i.e., $P_{\text{in}} = z_1(1-\mu)z_2$. On the other hand, the (average) output power, delivered by the quadripole, is given by the product of the average output current, $(1-\mu)z_1$ times the average output voltage z_2, i.e., $P_{\text{out}} = (1-\mu)z_1 z_2$. In other words, the quadripole is a lossless, ideal (average) power transferring device satisfying $P_{\text{in}} = P_{\text{out}}$. The switching element has thus been replaced by an *ideal transformer* with *turns ratio* parameter given by $(1-\mu)$. This result can also be found in [5] and [6].

IV. Conclusion

In this brief we have shown that well-known state average models of dc-to-dc Power Converters, such as that of the "Boost" converter, actually correspond to Euler–Lagrange system models. The average PWM model of the converter was derived using suitable averagings of the Euler Lagrange parameters associated with the intervening circuit topologies. The proposed approach can also be extended to a variety of other ideally switched power supplies and even to circuit models including more realistic switching devices characterized by combinations of ideal switches with conduction resistances and parasitic voltage sources (see [8]).

The nature of the modeling approach is consistent with recent trends in Automatic Control theory whereby a *passivity* based approach is emerging as an advantageous physically motivated controller design technique which exploits the energy structure of Euler–Lagrange systems (see [9] and the references therein). This brief thus constitutes a necessary initial step toward the development of a systematic nonlinear feedback controller design methodology, based on the passivity approach, for a variety of average models of dc-to-dc Power Converters.

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