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Passivity-based control of nonlinear chemical processes

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Through a process of 'feedback passivization', a large class of nonlinear monovariable systems, such as those describing nonlinear chemical processes, may benefit from systematic controller design techniques available for the regulation of passive systems. Several illustrative design examples from the chemical process control area are presented including digital computer simulations.

1. Introduction

Passive systems are characterized by the fact that the net increase in the stored energy, in any given time interval, is always lower or, at most, equal to the energy delivered to the system during the same period. As a consequence, for systems with positive definite storage functions, a zero-input stability as well as a minimum phase property can be immediately inferred for passive systems. Passive systems have been shown to be stabilizable by a simple static output feedback control law provided a certain detectability property is satisfied. Generally speaking, passivity properties are often deemed as attractive. This is especially so, in connection with a possibly simplified feedback controller design strategy.

We show that by rendering the system passive, nonlinear systems such as those describing chemical, biological and level control processes, may benefit from a systematic feedback controller design procedure already available for nonlinear passive systems. As a first step, it is shown that nonlinear systems with non-zero constant equilibrium states, can be transformed into passive systems by means of a suitable state-dependent input coordinate transformation (i.e. affine state feedback). The crucial requirement for such a possibility is that the system must have a storage function which is locally strictly relative degree one in a region containing the equilibrium state. Although this result is independent of the minimum, or non-minimum, phase character of the system and, also, independent of the output relative degree, passivization of non-minimum phase systems will result in a unfeasible growth of the state coordinate transformation, when the system motions are sustained at the required equilibrium point. Hence, in connection with stabilizing controller designs, the technique developed here is only applicable to minimum phase systems.

Studies about passivity arose from the treatment of a closely related, and more general, property termed *dissipativity*. The main developments in this area, and in some other related topics, were first presented by Willems (1971), in the context of abstract system operators. The extension of those results to the case of nonlinear

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systems, which are affine in the control input, was given in the work by Hill and Moylan (1976). A geometric treatment of passive systems and feedback equivalence to passivity was given in the work of Byrnes *et al.* (1991). Fundamental developments can also be found in an article by Kokotovic and Sussman, (1989) and also in the work of Lin (1996). Non-trivial applications of passivity-based control, to the areas of robotics, synchronous motors and power electronics, have been given by Ortega and his co-workers over the years (Ortega *et al.* 1995, Ortega and Spong, 1989 and Sira-Ramírez *et al.* 1997). In the area of robotics, the work by Berghuis and Nijmeijer (1993), incorporated nonlinear observers to the passivity-based regulation approach. Extensions to include adaptive feedback control schemes can be found in an article by Kelly *et al.* (1989) and in that of Landau and Horowitz (1989). Interesting theoretical issues connecting passivity and thermodynamics appear in Alonso and Ydstie (1996). The reader may explore a complete and excellent exposition about passivity, and related topics, in the recent book by A. van der Schaft (1996). A very clear and enjoyable presentation is that contained in the book of Khalil (1996).

In this article, the geometric features of passivization are studied in connection with the system's defining drift and input vector fields in their relation to a family of smooth manifolds, representing constant values of the storage function. A decomposition of the system's drift vector field is proposed which identifies the dissipative, the non-dissipative, and the invariant components of such a vector field. We show that the proposed 'passivifying' input coordinate transformation renders lossless (i.e. invariant), with respect to the storage function level sets, the non-dissipative component of such a drift vector field. The proposed input coordinate transformation renders, in a loose sense, a general 'canonical' form of nonlinear passive systems. This canonical form is intimately related to the traditional passivity-based feedback regulation design procedure, performed through stored energy modifications and feedback-damping injections (Ortega *et al.* 1995).

Section 2 presents the background definitions of dissipativity, losslessness and passivity. We also present in this section, the geometric aspects of the proposed feedback passivization scheme for single-input single-output (SISO) systems. Section 3 proposes a general state space 'canonical form' for passive nonlinear systems. Section 3 also revisits the 'energy modification plus damping injection' controller design methodology in light of the derived passivity canonical form. Section 4 is devoted to presenting some illustrative design examples for SISO nonlinear continuous processes. The first example is concerned with a series of two gravity-tanks/pipe system. The second example, drawn from the biological process control area, is concerned with the regulation of a bioreactor system. The third example considers a regulation problem defined on an exothermic continuously stirred tank reactor (CSTR) system. The fourth example deals with the stabilization of an isothermic CSTR system. Section 5 contains the conclusions and suggestions for further research in this field.

2. Passiviization of SISO systems

2.1. *Passivity: background definitions*

Consider the system

$$\dot{x} = f(x) + g(x)u; \quad y = h(x) \tag{2.1}$$

where $x \in \mathcal{X} \subset R^n$ is the state vector, $u \in \mathcal{U} \subset R$ is the control input and the scalar function $y \in \mathcal{Y} \subset R$ is the output function of the system. The vector fields $f(x)$ and $g(x)$ are assumed to be smooth vector fields on $\mathcal{X}$. For simplicity we assume the existance of an isolated non-zero state of interest, $x = x_e \in \mathcal{X}$, where $f(x_e) + g(x_e)\bar{u} = 0$, for some non-zero constant $\bar{u}$. The region $\mathcal{X} \subset R^n$ is the operating region of the system which strictly contains x_e. All our results are, henceforth, local for as long as $\mathcal{X}$ cannot be assumed to be all of R^n. We refer to system (2.1) as the triple (f, g, h).

Associated with system (2.1) an energy storage function, $V : \mathcal{X} \mapsto R^+$ is assumed to exist, which may be zero outside of $\mathcal{X}$ (at the origin, for instance). The supply rate function is defined as a function $s : \mathcal{U} \times \mathcal{Y} \mapsto R$.

We introduce some well-known background definitions about dissipative, lossless and passive systems (see Byrnes *et al.* 1991 and van der Schaft 1996, for further details).

Definition 2.1 (van der Schaft 1996): System (2.1) is said to be *dissipative* with respect to the supply rate $s(u, y)$ if there exists a storage function $V : \mathcal{X} \mapsto R^+$, such that for all $x_0 \in \mathcal{X}$ and for all $t_1 \geq t_0$, and all input functions u, the following relation holds

$$V(x(t_1)) - V(x(t_0)) \leq \int_{t_0}^{t_1} s(u(t), y(t))\, \mathrm{d}t \tag{2.2}$$

with $x(t_0) = x_0$ and $x(t_1)$ is the state resulting, at time t_1, from the solution of system (2.1) taking as initial condition x_0 and as control input function $u(t)$. Inequality (2.2) is equivalent to (Byrnes *et al.* 1991)

$$\dot{V} \leq s(u(t), y(t)) \tag{2.3}$$

The system is *lossless* if the inequalities (2.2), or (2.3), are in fact, equalities. □

Definition 2.2 (van der Schaft 1996): System (2.1) is *passive* if it is dissapative with respect to the supply rate $s(u, y) = uy$. The system is strictly *input passive* if there exists $\delta > 0$ such that the system is dissipative with respect to $s(u, y) = uy - \delta u^2$. The system is strictly *output passive* if there exist a $\gamma > 0$ such that the system is dissipative with respect to $s(u, y) = uy - \gamma y^2$. □

The following definition and result constitute a generalization of the classical Kalman–Yakubovich–Popov property and associated lemma for positive real linear systems (Byrnes *et al.* 1991).

Definition 2.3: A system (f, g, h) has the *Kalman–Yakubovich–Popov (KYP) property* if there exists a continuously differentiable non-negative function $V: \mathcal{X} \mapsto R$, with $V(0) = 0$, such that

$$L_f V(x) \leq 0$$
$$L_g V(x) = h(x)$$

for all $x \in \mathcal{X}$. □

The following result follows directly from the definition of passivity and the fact that $V(x)$ is a relative degree one function of the system.

Proposition 2.4 (Byrnes *et al.* 1991): *A system which has the KYP property is passive with storage function V. Conversely a passive system having a continuously differentiable storage function has the KYP property.*

We shall consider means of rendering a system of the form (2.1) passive, or at least 'lossless', by means of state feedback. We therefore introduce a definition of a 'passifiable' system in the following terms.

Definition 2.5: System (2.2) is said to be *passifiable* with respect to the storage function V if there exists a regular affine feedback law of the form

$$u = \alpha(x) + \beta(x)v; \alpha(x) \in R; \beta(x) \in R \tag{2.4}$$

where $\beta(x)$ is a non-zero scaler function in $\mathcal{X}$, and such that the closed loop system (2.1)–(2.4) becomes passive with new scalar control input v.

Analogous definitions apply for the strict input and strict output passivization of the systems of the form (2.1).

Definition 2.6: Consider a smooth drift vector field $\phi(x)$. Let $L_\phi V$ stand for the *Lie derivative* of V in the direction of ϕ. In local coordinates $L_\phi V(x) = (\partial V/\partial x)\phi(x)$.

We say that the drift vector field $f(x)$ of (2.1) has a *natural decomposition* with respect to the storage function V, whenever $f(x)$ can be expressed as the sum of three components

$$f(x) = f_{\rm d}(x) + f_{\rm nd}(x) + f_{\rm I}(x)$$

such that

$$L_{f\rm d} V(x) \leq 0; \forall x \in \mathcal{X}$$

$$L_{f_{\rm nd}} V(x) \quad \begin{cases} \text{is either sign-undefined in } \mathcal{X} \\ \text{or else it is non-negative in } \mathcal{X} \end{cases}$$

$$L_{f_{\rm I}} V(x) = 0; \forall x \in \mathcal{X}$$

We address $f_{\rm d}(x)$ as the *dissipative* component of $f(x)$. Similarly $f_{\rm nd}(x)$ will be termed the *non-dissipative* component of $f(x)$ and finally $f_{\rm I}(x)$ is the *invariant* component of $f(x)$. We also address the triple of vector fields $(f_{\rm d}(x), f_{\rm nd}(x), f_{\rm I}(x))$ as the natural components of $f(x)$ with respect to V. □

2.2. *Feedback passivization*

Consider the system (2.1) with V being, locally in $\mathcal{X}$, a strict relative degree one function, i.e. $L_g V(x) \neq 0 \forall x \in \mathcal{X}$. Then, for any given control input u and any initial state x_0, the time derivative of the storage function V, along the solutions of (2.1), is given by

$$\dot{V} = \frac{\partial V}{\partial x} f(x) + \left(\frac{\partial V}{\partial x} g(x) \right) u = L_f V(x) + [L_g V(x)] u$$

Suppose that the vector field $f(x)$ has natural components, $f_{\rm d}(x), f_{\rm nd}(x), f_{\rm I}(x)$, with respect to the storage function V. The time derivative of the energy storage function, along the solutions of the system, is then given by

$$\dot{V} = L_{f_{\rm d}} V(x) + L_{f_{\rm nd}} V(x) + [L_g V(x)] u \tag{2.5}$$

Note that the previous expression may be rewritten as

$$\dot{V} = L_{f_{\rm d}} V(x) + [L_g V(x)] \left[\frac{L_{f_{\rm nd}} V(x)}{L_g V(x)} + u \right] \tag{2.6}$$

Define the following state dependent input coordinate transformation

$$u = \frac{h(x)}{L_g V(x)} v - \frac{L_{f_{nd}} V(x)}{L_g V(x)} - \gamma \frac{h^2(x)}{L_g V(x)} \tag{2.7}$$

where γ is an arbitrary strictly positive scalar.

It is seen, upon substitution of expression (2.7) into equation (2.6), that the time derivative of the energy storage function satisfies the following string of relations

$$\dot{V} = L_{f_d} V + h(x)v - \gamma h^2(x) \leq yv - \gamma y^2 \leq yv$$

In other words, if the system is such that $L_g V$ is locally non-zero, then the input coordinate of the system may be transformed in such a way that the partially closed loop system will exhibit a strictly output passive behaviour between the external control input v and the original scalar output $y = h(x)$.

We have, therefore, proven the following result.

Proposition 2.7: *System* (2.1) *is locally strictly output passivifiable with respect to the storage function V, by means of affine feedback of the form* (2.4) *if and only if*

$$L_g V(x) \neq 0 \quad \forall x \in \mathcal{X}$$

The affine feedback law, or state dependent input coordinate transformation, which achieves strict output passivization, is given by expression (2.7).

A different and simple way to prove the above result is to consider the closed-loop system (2.1)–(2.7) as

$$\begin{aligned}\dot{x} &= f_d(x) + f_I(x) + f_{nd}(x) - g(x)\frac{L_{f_{nd}} V(x)}{L_g V(x)} - g(x)\frac{\gamma h(x)^2}{L_g V(x)} + g(x)\frac{h(x)}{L_g V(x)} u \\ &= \tilde{f}(x) + \tilde{g}(x)v\end{aligned}$$

with

$$\begin{aligned}\tilde{f}(x) &= f_d(x) + f_I(x) + f_{nd}(x) - \frac{L_{f_{nd}} V(x)}{L_g V(x)} g(x) - \frac{\gamma h(x)^2}{L_g V(x)} g(x) \\ \tilde{g}(x) &= \frac{h(x)}{L_g V(x)} g(x)\end{aligned}$$

This system satisfies the KYP property. Indeed

$$\begin{aligned}L_{\tilde{f}} V(x) &= L_{f_d} V(x) + L_{f_{nd}} V(x) - \frac{L_{f_{nd}} V(x)}{L_g V(x)} L_g V(x) - \gamma \frac{h(x)^2}{L_g V(x)} L_g V(x) \\ &= L_{f_d} V(x) - \gamma h(x)^2 \leq 0 \\ L_{\tilde{g}} V(x) &= \frac{h(x)}{L_g V(x)} L_g V(x) = h(x)\end{aligned}$$

If the natural components $f_d(x), f_I(x)$, do not exist, or, alternatively, if it proves difficult to identify such components of $f(x)$ in a given operating region $\mathcal{X}$ of the state space, then we may regard the vector field $f(x)$ itself as the non-dissipative component, $f_{nd}(x)$, for the synthesis of the above affine feedback control law. It follows, then, that the given system may still be rendered passive (by taking $\gamma \neq 0$, in

(2.7)), or, at least, 'lossless' with respect to the storage function $V(x)$ (by taking $\gamma = 0$, in (2.7)).

The class of systems that can be (strictly output) passivified by static feedback corresponds to those systems where the control input exhibits enough 'authority' over the available energy storage function. Such an authority is to be understood in the sense that the first-order time derivative of the storage function V is directly influenced by the control input u and this explicit influence never vanishes in the operating region of interest.

Proposition 2.8: *System* (2.1) *is strictly input passivifiable with respect to the storage function V, by means of a nonlinear state feedback if and only if*

$$L_g V(x) \neq 0 \quad \forall x \in \mathcal{X}$$

The nonlinear state feedback law that achieves strict input passivization is given by

$$u = -\frac{L_{f_{nd}} V(x)}{L_g V(x)} + \left(\frac{h}{L_g V} - \delta \frac{v}{L_g V} \right) v$$

where δ is a strictly positive constant.

Remark 2.9: All of the above results are independent of the output relative degree (see Isidori 1995) and, moreover, they are, in principal, independent of the minimum, or non-minimum, phase character of the given system. These facts are in no contradiction with the results reported in Byrnes *et al.* (1991) where it was found that a necessary and sufficient condition for having a system passivifiable is that it is output relative degree one and weakly minimum phase. Our results do not apply to the class of systems considered in Byrnes *et al.* (1991), which have an equilibrium at the origin and are such that the storage function is also zero at this equilibrium point. By considering systems which have non-zero constant equilibria for constant controls and storage functions strictly relative degree one, the above restrictions are lifted. □

2.3. *A geometric interpretation of passivization by feedback*

Suppose a system of the form (2.1), with a natural decomposition of the vector field $f(x)$, is passivifiable, i.e. $L_g V \neq 0 \forall x \in \mathcal{X}$. Suppose that an input coordinate transformation of the form $u = (h/L_g V)v - L_{f_{nd}} V(x)/L_g V(x) - \gamma h^2/L_g V$ has been applied to the system.

In transformed input coordinates, the system (2.1) is given, upon some simple algebraic manipulations and use of the definition of the Lie derivative, by

$$\dot{x} = f_d(x) + f_I(x) + \left[I - g(x) \frac{\partial V(x)/\partial x}{L_g V(x)} \right] f_{nd}(x) + \frac{h}{L_g V} g(x) v - \gamma \frac{h^2(x)}{L_g V} g(x) \quad (2.8)$$

The geometric interpretation of the several components in the transformed equation (2.8) is given in Fig. 1.

We clearly identify four types of terms in the five summands of the right-hand side of (2.8). The first summand is, according to its definition, a naturally dissipative term. The second and third summands are the workless terms or invariant terms, the fourth summand is the power aquisition term responsible for the 'supply rate' in terms of the new control input and, finally, the fifth summand is an artificially induced dissipation term making use of nonlinear (quadratic) output feedback.

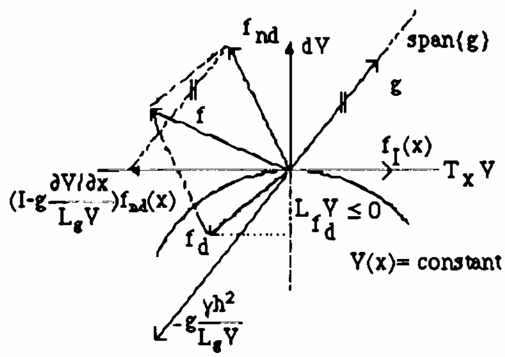

Figure 1. A geometric interpretation of passivization.

Note that the matrix

$$M(x) = \left[I - g(x) \frac{\partial V(x)/\partial x}{L_g V(x)} \right]$$

is a projection operator onto the tangent space to the level surface $V(x) = \text{constant}$, along the distribution span $\{g\}$. This projection operator 'hides' all destabilizing components of $f_{nd}(x)$ by making the vector $M(x)f_{nd}(x)$ a tangent to the level surfaces of constant stored energy, i.e. to the family of sets (or foliation) $\{x \text{ s.t. } V(x) = \text{constant}\}$. Thus, any unstable behaviour contained in $f_{nd}(x)$ does not increment, nor diminish, the value of the energy function $V(x)$ along the controlled trajectories of the transformed system. However, it should be made clear that passivization of non-minimum phase systems would be achieved at the expense of possibly unbounded (i.e. unfeasible) feedback control actions. The requirement of a minimum phase system for passivization is, hence, natural and convenient.

It is easy to verify that $M(x)$ satisfies the following properties, which are characteristic of projection operators onto tangent spaces, along the span of a given vector g

$$M(x)g(x) = 0 \qquad \forall x \in \mathcal{X}$$

$$\frac{\partial V}{\partial x} M(x) = 0 \qquad \forall x \in \mathcal{X}$$

$$M^2(x) = M(x) \quad \forall x \in \mathcal{X}$$

3. Passivity-based feedback controller design

We begin by revisiting a systematic procedure for the synthesis of passivity-based feedback controllers. This procedure, based on storage function modification and damping injection through feedback, has been extensively used in the area of mechanical, electromechanical and electric systems. The reader may find further details in Sira-Ramírez *et al.* 1997) and in the references therein.

3.1. *Towards a canonical form for passive systems*

Suppose that system (2.1) is passivifiable and assume $f_d(x), f_{nd}(x), f_I(x)$ are the natural components of $f(x)$ with respect to the storage function $V(x)$. Suppose furthermore, that $V(x)$ is given in its simplest form

$$V(x) = \frac{1}{2}x^{\mathrm{T}}x$$

Assuming that $L_g V = x^{\mathrm{T}}g(x) \neq 0$ in the operating region $\mathcal{X}$ of the state space, then the passivified system (2.8) can be further specialized to be written as

$$\dot{x} = f_{\mathrm{d}}(x) + f_{\mathrm{I}}(x) + \left[I - g(x)\frac{x^{\mathrm{T}}}{x^{\mathrm{T}}g(x)}\right]f_{\mathrm{nd}}(x) + \frac{h(x)}{x^{\mathrm{T}}g(x)}g(x)v - \gamma\frac{h^2(x)}{x^{\mathrm{T}}g(x)}g(x) \quad (3.1$$

As integrating parts of the time derivative of V one has the following terms.

$$x^{\mathrm{T}}\left[f_{\mathrm{d}}(x) - \gamma g(x)\frac{h(x)^2}{x^{\mathrm{T}}g(x)}\right] \leq 0$$

$$x^{\mathrm{T}}\{f_{\mathrm{I}}(x) + \left[I - g(x)\frac{x^{\mathrm{T}}}{x^{\mathrm{T}}g(x)}\right]f_{\mathrm{nd}}(x)\} = 0$$

it then follows, by straightforward factorization of the state vector components in the nonlinear entries of the different fields comprising expression (3.1), such that a system may always be rewritten in the following form

$$\dot{x} = -\mathcal{R}(x)x - \mathcal{J}(x)x + \mathcal{M}(x)v; \quad y = h(x) \quad (3.2)$$

with $\mathcal{R}(x)$ being a positive semidefinite matrix in $\mathcal{X}$, and $\mathcal{J}(x)$ being an anti-symmetric matrix. This implies the following identifications

$$f_{\mathrm{d}}(x) - \gamma\frac{h^2(x)}{x^{\mathrm{T}}g(x)}g(x) = -\mathcal{R}(x)x$$

$$f_{\mathrm{I}}(x) + \left[I - g(x)\frac{x^{\mathrm{T}}}{x^{\mathrm{T}}g(x)}\right]f_{\mathrm{nd}}(x) = -\mathcal{J}(x)x$$

$$\frac{h}{x^{\mathrm{T}}g(x)}g(x) = \mathcal{M}(x)$$

3.2. *Feedback controller design via energy modification and damping injection*

A passivity-based controller can now be proposed for systems of the form (3.1), or (3.2), by considering the following modified storage function

$$V_{\mathrm{d}}(x, x_{\mathrm{d}}) = \frac{1}{2}(x - x_{\mathrm{d}})^{\mathrm{T}}(x - x_{\mathrm{d}})$$

where x_{d} is an auxiliary state vector to be defined later.

Along the solutions of the system (3.2), the function $V_{\mathrm{d}}(x, x_{\mathrm{d}})$ exhibits the following time derivative

$$\dot{V}_{\mathrm{d}}(x, x_{\mathrm{d}}) = (x - x_{\mathrm{d}})^{\mathrm{T}}[-\mathcal{R}(x)x - \mathcal{J}(x)x + \mathcal{M}(x)v - \dot{x}_{\mathrm{d}}]$$

Completing squares in the right-hand side and adding a damping injection term of the form $-\mathcal{R}_{\mathrm{di}}(x)x$, so that $\mathcal{R}_{\mathrm{m}}(x) = \mathcal{R}(x) + \mathcal{R}_{\mathrm{di}}(x)$ is a negative definite matrix for all $x \in \mathcal{X}$, one obtains

$$\dot{V}_d(x,x_d) = (x - x_d)^T[-(\mathcal{R}(x) + \mathcal{R}_{di}(x))(x - x_d) - \mathcal{J}(x)(x - x_d) - \dot{x}_d - \mathcal{R}(x)x_d - \mathcal{J}(x)x_d + \mathcal{R}_{di}(x)(x - x_d) + \mathcal{M}(x)v]$$

Note that if we let the auxiliary vector $x_d(t)$, satisfy the following system of differential equations

$$\dot{x}_d = -\mathcal{R}(x)x_d - \mathcal{J}(x)x_d + \mathcal{R}_{di}(x)(x - x_d) + \mathcal{M}(x)v \tag{3.3}$$

then the time derivative of $V_d(x, x_d)$ satisfies

$$\begin{aligned}\dot{V}_d(x,x_d) &= -(x - x_d)^T\mathcal{R}_m(x)(x - x_d) \leq -\frac{a}{b}(x - x_d)^T(x - x_d)\\ &= -\frac{a}{b}V(x,x_d) \leq 0\end{aligned}$$

where, in terms of the minimum and maximum eigenvalues $(\lambda_{min}, \lambda_{max})$ of $R_m(x)$, a and b are given by

$$a = \inf_{x\in\mathcal{X}} \lambda_{min}\,(R_m(x)) > 0; \quad b = \sup_{x\in\mathcal{X}} \lambda_{max}(R_m(x)) > 0$$

It follows that the vector $x(t)$ exponentially asymptotically converges towards the auxiliary vector trajectory $x_d(t)$.

Notice that (3.3) is a time varying linear system for the auxiliary state x_d. This system of equations gives us enough freedom for the synthesis of a feedback controller. Typically, one sets for a particular component of the vector x_d a desired constant equilibrium value. This is made in correspondence with the component value in the equilibrium state x_e of the original state vector. The objective of such a particularization is to obtain a feedback expression for the external control input v in terms of the available state vector x, as well as the rest of the auxiliary variables in the vector x_d. The differential equations defining the remaining auxiliary variables in x_d, are to be regarded as state components of a dynamical feedback compensator (see Sira-Ramírez *et al.* 1997).

4. Illustrative examples

4.1. *A series of two gravity-flow tanks—pipeline system*

Consider the following series arrangement of two identical gravity-flow tanks equipped with outlet pipes (see Fig. 2). The single tank version of this model can be found in Karjala and Himmelblau (1996). As in Smith and Corripio (1985), our model includes an elementary static model for an 'equal percentage valve'

$$\left.\begin{aligned}\dot{x}_1 &= \frac{A_p g}{L}x_2 - \frac{K_f}{\rho A_p^2}x_1^2\\ \dot{x}_2 &= \frac{1}{A_t}(F_{Cmax}\alpha^{-(1-u)} - x_1)\\ \dot{x}_3 &= \frac{A_p g}{L}x_4 - \frac{K_f}{\rho A_p^2}x_3^2\\ \dot{x}_4 &= \frac{1}{A_t}(x_1 - x_3)\\ y &= x_4\end{aligned}\right\} \tag{4.1}$$

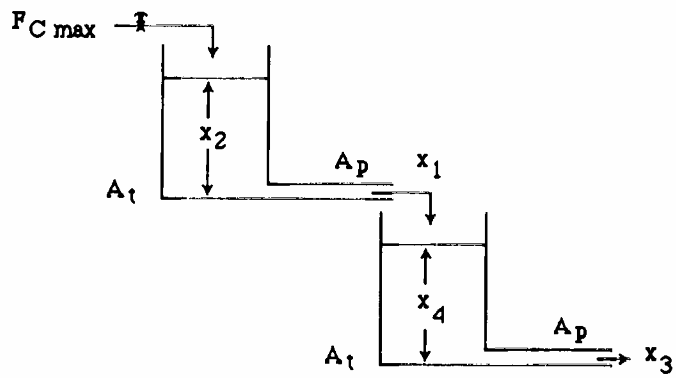

Figure 2. A series of two gravity flow tanks/ pipe system.

where x_1 and x_3 are the volumetric flow rates of liquid leaving the tanks via the pipes and x_2 and x_4 are the heights of the liquid in the tank, respectively. F_{Cmax} is the maximum value of the volumetric rate of fluid entering the first tank, g is the gravitational acceleration constant, L is the length of the pipes while K_f is the friction factor and ρ the density of the liquid, A_p is the cross-sectional area of the pipes, and A_t is the cross-sectional area of each of the tanks. The parameter α is the rangeability parameter of the valve and the control input, u, is the valve position, taking in the closed interval $[0, 1]$.

For a constant value $\bar{u} \in [0, 1]$ of the control input u, the system had an equilibrium point given by

$$\bar{x}_1 = \bar{x}_3 = F_{\mathrm{Cmax}} \alpha^{-(1-\bar{u})}; \quad \bar{x}_2 = \bar{x}_4 = \frac{LK_f}{A_p^3 g \rho} \bar{x}_1^2$$

i.e. the system has a constant non-zero equilibrium point for a constant control input. In order to avoid unnecessary complications we regard the control input term via the following auxiliary variable ω

$$\omega = F_{\mathrm{Cmax}} \alpha^{-(1-u)}$$

In reference to the vector fields description of the system as in (2.1), we have

$$f(x) = \begin{bmatrix} \dfrac{A_p g}{L} x_2 - \dfrac{K_f}{\rho A_p^2} x_1^2 \\ -\dfrac{1}{A_t} x_1 \\ \dfrac{A_p g}{L} x_4 - \dfrac{K_f}{\rho A_p^2} x_3^2 \\ \dfrac{1}{A_t}(x_2 - x_3) \end{bmatrix}; \quad g(x) = \begin{bmatrix} 0 \\ \dfrac{1}{A_t} \\ 0 \\ 0 \end{bmatrix}$$

The system output has relative degree equal to three.

The operating region for system (4.1) is given by points strictly located in the position orthant of R^4. In other words, $\mathcal{X}$ as the set.

$$\mathcal{X} = \{x = (x_1, x_2, x_3, x_4)^{\mathrm{T}} \in R^4, \ \text{s.t. } x_i > 0; i = 1, 2, 3, 4\}$$

Consider the following energy storage function

$$V = \frac{1}{2}(x_1^2 + x_2^2 + x_3^2 + x_4^2)$$

The condition, $L_g V \neq 0$, results, in this case, in

$$L_g V = \frac{x_2}{A_t} \neq 0$$

which is satisfied over the operating region $\mathcal{X}$.

The system is, then, clearly passivifiable with storage function $V(x)$. The time derivative of V along the regulated evolution of the system is given by

$$\begin{aligned}\dot{V} &= \frac{A_p g}{L} x_1 x_2 - \frac{K_f}{\rho A p^2} x_1^3 + \frac{1}{A_t} x_2(\omega - x_1)\\ &\quad + \frac{A_p g}{L} x_4 x_3 - \frac{K_f}{\rho A p^2} x_3^3 + \frac{1}{A_t} x_4(x_1 - x_3)\\ &\leq \frac{A_p g}{L} x_1 x_2 + \frac{1}{A_t} x_2 \omega + \frac{A_p g}{L} x_3 x_4 + \frac{1}{A_t} x_1 x_4 \end{aligned} \tag{4.2}$$

where the last inequality is obtained under the assumption that the system evolution takes place on the operating region $\mathcal{X}$ and, hence, x_1 through x_4 are strictly positive for all times. It follows that, in $\mathcal{X}$, the natural decomposition of the drift vector field $f(x)$ is given by

$$f_d(x) = \begin{bmatrix} -\frac{K_f}{\rho A_p^2} x_1^2 \\ -\frac{1}{A_t} x_1 \\ -\frac{K_f}{\rho A_p^2} x_3^2 \\ -\frac{1}{A_t} x_3 \end{bmatrix}; \quad f_{nd}(x) = \begin{bmatrix} \frac{A_p g}{L} x_2 \\ 0 \\ \frac{A_p g}{L} x_4 \\ \frac{1}{A_t} x_1 \end{bmatrix}$$

Define a state-dependent input coordinate transformation of the form

$$\omega = -\frac{A_p g A_t}{L} x_1 - \frac{A_p g A_t}{L} \frac{x_3 x_4}{x_2} - \frac{x_1 x_4}{x_2} + \frac{A_t x_4}{x_2} v - \gamma A_t \frac{x_4^2}{x_2} \tag{4.3}$$

Transformation (4.3) results in a strictly output passive system operator relating the new input v and the output variable x_4. Substituting the control law (4.3) into the system equations (4.2) one obtains, after evaluation and integration of the time derivative of V, the following passivity inequality

$$V(x(t)) - V(x(0)) \leq \int_0^t x_4(\sigma) v(\sigma)\, d\sigma$$

The input transformed system is rewritten as

$$
\begin{aligned}
\dot{x}_1 &= \frac{A_p g}{L} x_2 - \frac{K_f}{\rho A_p^2} x_1^2 \\
\dot{x}_2 &= \frac{x_4}{x_2} v - \gamma \frac{x_4^2}{x_2} - \frac{A_p g}{L} x_1 - \frac{A_p g}{L} \frac{x_4 x_3}{x_2} - \frac{1}{A_t} x_1 - \frac{1}{A_t} x_1 \frac{x_4}{x_2} \\
\dot{x}_3 &= \frac{A_p g}{L} x_4 - \frac{K_f}{\rho A_p^2} x_3^2 \\
\dot{x}_4 &= \frac{1}{A_t} x_1 - \frac{1}{A_t} x_3 \\
y &= x_4
\end{aligned}
$$

The transformed system may be placed in the form (3.2)

$$
\dot{x} = -\mathcal{J}(x)x - \mathcal{R}(x)x + \mathcal{M}v
$$

where $x^{\mathrm{T}} = [x_1 \ x_2 \ x_3 \ x_4]$, and

$$
-\mathcal{J}(x) = \begin{bmatrix} 0 & \frac{A_p g}{L} & 0 & 0 \\ -\frac{A_p g}{L} & 0 & -\frac{A_p g}{L}\frac{x_4}{x_2} & -\frac{1}{A_t}\frac{x^1}{x_2} \\ 0 & \frac{A_p g}{L}\frac{x_4}{x_2} & 0 & 0 \\ 0 & \frac{1}{A_t}\frac{x_1}{x_2} & 0 & 0 \end{bmatrix}; \quad \mathcal{M} = \begin{bmatrix} 0 \\ \frac{x_4}{x_2} \\ 0 \\ 0 \end{bmatrix}
$$

$$
-\mathcal{R}(x) = \begin{bmatrix} -\frac{K_f}{\rho A_p^2} x_1 & 0 & 0 & 0 \\ 0 & -\frac{1}{A_t}\frac{x_1}{x_2} - \gamma \frac{x_4^2}{x_2^2} & 0 & 0 \\ 0 & 0 & -\frac{K_f}{\rho A_p^2} x_3 & 0 \\ 0 & 0 & 0 & -\frac{1}{A_t}\frac{x_3}{x_4} \end{bmatrix}
$$

where $\mathcal{J}^{\mathrm{T}}(x) + \mathcal{J}(x) = 0$, and $\mathcal{R}(x) = \mathcal{R}^{\mathrm{T}}(x) > 0$.

4.1.1. *Controller design.* Consider a modified energy function V_d, in terms of an auxiliary state vector $x_d = [x_{1d} \ x_{2d} \ x_{3d} \ x_{4d}]^{\mathrm{T}}$, representing the desired state vector. Let V_d be given by

$$
\begin{aligned}
V_d(x, x_d) &= \tfrac{1}{2}(x - x_d)^{\mathrm{T}}(x - x_d) \\
&= \tfrac{1}{2}[(x_1 - x_{1d})^2 + (x_2 - x_{2d})^2 + (x_3 - x_{3d})^2 + (x_4 - x_{4d})^2]
\end{aligned}
$$

From the results of § 3.2, the following set of auxiliary controlled differential equations yield $\dot{V}_d(x, x_d)$ negative definite

$$\dot{x}_{1d} = \frac{A_p g}{L} x_{2d} - \frac{K_f}{\rho A_p^2} x_1 x_{1d} + R_1(x_1 - x_{1d})$$

$$\dot{x}_{2d} = \frac{x_4}{x_2} v - \gamma \frac{x_4^2}{x_2^2} x_{2d} - \frac{A_p g}{L} x_{1d} - \frac{A_p g}{L} \frac{x_4}{x_2} x_{3d} - \frac{1}{A_t} \frac{x_1}{x_2} x_{4d} - \frac{1}{A_t} \frac{x_1}{x_2} x_{2d} + R_2(x_2 - x_{2d})$$

$$\dot{x}_{3d} = \frac{A_p g}{L} \frac{x_4}{x_2} x_{2d} - \frac{K_f}{\rho A_p^2} x_3 x_{3d} + R_3(x_3 - x_{3d})$$

$$\dot{x}_{4d} = \frac{1}{A_t} \frac{x_1}{x_2} x_{2d} - \frac{1}{A_t} \frac{x_3}{x_4} x_{4d} + R_4(x_4 - x_{4d})$$

where R_1 through R_4 are the diagonal components of the positive definite matrix $\mathcal{R}_{di}(x)$ which, for simplicity, will be taken to be a constant matrix, $\mathcal{R}_{di}(x) = \mathcal{R}_{di} = \mathrm{diag}\,[R_1 \;\; R_2 \;\; R_3 \;\; R_4]$.

Suppose it is desired to regulate the height of the liquid, x_4, in the second tank to a predetermined constant equilibrium value $\bar{x}_4$. This value correponds, according to the equilibrium condition for the system, to $\bar{x}_2 = \bar{x}_4$. Letting, thus, $x_{2d} = \bar{x}_2 =$ constant, one obtains the following dynamical controller expression for the transformed input v

$$v = \frac{x_2}{x_4} \left[\frac{A_p g}{L} \xi_1 + \frac{1}{A_t} \frac{x_1}{x_2} \xi_4 + \frac{A_p g}{L} \frac{x_4}{x_2} \xi_3 + \left(\frac{1}{A_t} \frac{x_1}{x_2} + \gamma \frac{x_4^2}{x_2^2} \right) \bar{x}_2 - R_2(x_2 - \bar{x}_2) \right]$$

$$\dot{\xi}_1 = \frac{A_p g}{L} \bar{x}_2 - \frac{K_f}{\rho A_p^2} x_1 \xi_1 + R_1(x_1 - \xi_1)$$

$$\dot{\xi}_3 = \frac{A_p g}{L} \frac{x_4}{x_2} \bar{x}_2 - \frac{K_f}{\rho A_p^2} x_3 \xi_3 + R_3(x_3 - \xi_3)$$

$$\dot{\xi}_4 = \frac{1}{A_t} \frac{x_1}{x_2} \bar{x}_2 - \frac{1}{A_t} \frac{x_3}{x_4} \xi_4 + R_4(x_4 - \xi_4)$$

where the variables ξ_1, ξ_3, ξ_4, acting as the dynamical controller states, have replaced the auxiliary state variables x_{1d}, x_{3d} and x_{4d}, respectively.

The output of the dynamic feedback controller takes the form

$$\omega = A_t \Bigg[-\frac{A_p g}{L}(x_1 - \xi_1) - \frac{1}{A_t} \frac{x_1}{x_2}(x_4 - \xi_4) - \frac{A_p g}{L} \frac{x_4}{x_2}(x_3 - \xi_3) + \frac{1}{A_t} \frac{x_1}{x_2} \bar{x}_2 - R_2(x_2 - \bar{x}_2) - \gamma \frac{x_4^2}{x_2^2}(x_2 - \bar{x}_2) \Bigg]$$

Notice that the actual control input is the valve position u. This is given by

$$u = 1 + \frac{1}{\log \alpha} \log \left(\frac{\omega}{F_{\mathrm{Cmax}}} \right)$$

We take, after Karjala and Himmelblau (1996), the following system parameters for the simulation of the controlled gravity-tank/pipe system,

$$g = 9{\cdot}81\ \mathrm{m\,s^{-2}}; \quad L = 914\ \mathrm{mt}; \quad K_f = 4{\cdot}41\ \mathrm{N\,s^2\,m^{-3}}; \quad \rho = 998\ \mathrm{kg\,m^{-2}}$$
$$A_p = 0{\cdot}653\ \mathrm{m^2}; \quad A_t = 10{\cdot}5\ \mathrm{m^2}; \quad \alpha = 5; \quad F_{\mathrm{Cmax}} = 2\ \mathrm{m^3 s^{-1}}$$

The required equilibrium point for x_4 was set to be $\bar{x}_4 = \bar{x}_2 = 5{\cdot}0$ m, while that of x_1

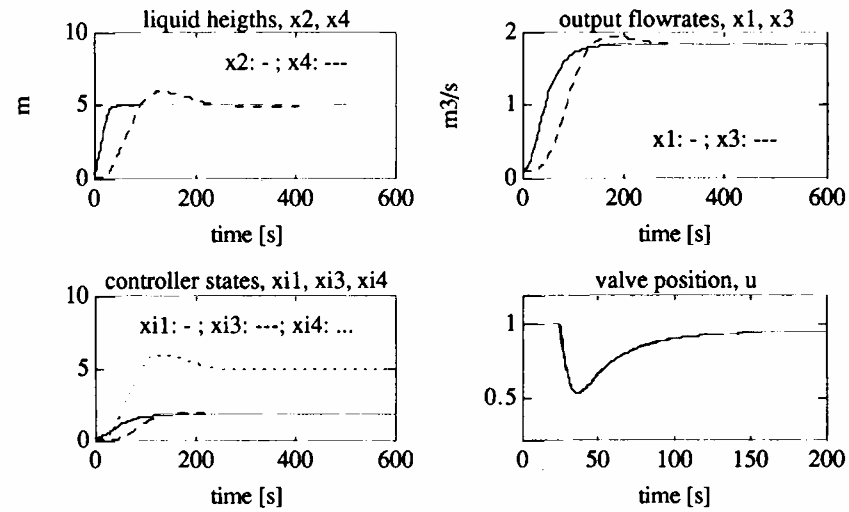

Figure 3. Simulation results of the passivity-based regulated gravity-tanks/pipe system.

was set to be $\bar{x}_1 = \bar{x}_3 = 1{\cdot}8389\ \mathrm{m^3\,s^{-1}}$. This corresponds to a steady state value of the control input $\bar{u} = 0{\cdot}9478$. The design parameters were chosen to be

$$R_1 = 0{\cdot}1; \quad R_2 = 0{\cdot}2; \quad R_3 = 0{\cdot}1; \quad R_4 = 0{\cdot}2; \quad \gamma = 0{\cdot}5$$

Figure 3 shows the closed loop response of the gravity-tank/pipe system with nice stabilization features and an overshoot, for x_4, of 21% with a settling time of about 200 s. This is to be compared with the settling time, of about 400 s, and the overshoot of 49·7% for the open loop response.

4.2. *A bioreactor system*

Consider the following bioreactor tank system, thoroughly discussed by Agrawal *et al.* (1982)

$$\begin{aligned} \dot{x}_1 &= -ux_1 + x_1(1-x_2)\exp\left(\frac{x_2}{\gamma}\right) \\ \dot{x}_2 &= -ux_2 + x_1(1-x_2)\frac{1+\beta}{1+\beta-x_2}\exp\left(\frac{x_2}{\gamma}\right) \\ y &= x_1 \end{aligned} \tag{4.4}$$

where x_1 and x_2 are the number of cells and the nutrient concentration at the time t respectively. The system parameters β and γ are assumed to be known constants. These scalar quantities represent the growth rate and the nutrient inhibition parameters, respectively. The input variable u represents the flow rate through the tank. The cell and nutrient concentration variables x_1 and x_2 are assumed to evolve on the open intervals (0, 1), while the control input flow rate is assumed to be restricted to the closed interval [0, 2]. The equilibrium point for the nutrient concentration x_2, corresponding to a given constant value $\bar{u}$ of the input flow rate variable u is given by the (positive) solution of

$$(1-\bar{x}_2) = \bar{u}\exp\left(-\frac{\bar{x}_2}{\gamma}\right)$$

Once the steady state value $\bar{x}_2$ is computed from the previous implicit equation, the corresponding equilibrium value for the cell concentration x_1 is given by

$$\bar{x}_1 = \frac{\bar{u}\bar{x}_2}{1-\bar{x}_2}\frac{1+\beta-\bar{x}_2}{1+\beta}\exp\left(\frac{\bar{x}_2}{\gamma}\right)$$

The system, thus, has a constant non-zero equilibrium point for a constant control input.

In terms of the vector fields description (2.1) for the above affine system we have

$$f(x) = \begin{bmatrix} x_1(1-x_2)\exp\left(\dfrac{x_2}{\gamma}\right) \\ x_1(1-x_2)\exp\left(\dfrac{x_2}{\gamma}\right)\dfrac{1+\beta}{1+\beta-x_2} \end{bmatrix}; \quad g(x) = \begin{bmatrix} -x_1 \\ -x_2 \end{bmatrix}$$

Clearly, the system has relative degree one and, hence, it has one-dimensional zero dynamics.

The physically meaningful operating region for system (4.4) is given by points strictly contained in the unit square $[0,1]\times[0,1]$ of the first quadrant in R^2 where the system evolution is defined. In other words we regard $\mathcal{X}$ as the set

$$\mathcal{X} = \{x = (x_1, x_2)^{\mathrm{T}} \in R^2, \text{ s.t. } x_1 \in (0,1) \text{ and } x_2 \in (0,1)\}$$

Consider the following energy storage function

$$V = \frac{1}{2}(x_1^2 + x_2^2)$$

The crucial condition, $L_g V \neq 0$, results, in this case, in

$$L_g V = -(x_1^2 + x_2^2) \neq 0$$

which is, in fact, satisfied over the operating region $\mathcal{X}$.

The system is, thus, passivifiable with respect to the storage function $V(x)$. The time derivative of V along the controlled motions of the system is given by

$$\begin{aligned}
\dot{V} &= -u(x_1^2+x_2^2) + x_1^2(1-x_2)\exp\left(\frac{x_2}{\gamma}\right) + x_1x_2(1-x_2)\exp\left(\frac{x_2}{\gamma}\right)\frac{1+\beta}{1+\beta-x_2} \\
&= -u(x_1^2+x_2^2) - x_1^2x_2\exp\left(\frac{x_2}{\gamma}\right) - x_1x_2^2\exp\left(\frac{x_2}{\gamma}\right)\frac{1+\beta}{1+\beta-x_2} \\
&\quad + x_1^2\exp\left(\frac{x_2}{\gamma}\right) + x_1x_2\exp\left(\frac{x_2}{\gamma}\right)\frac{1+\beta}{1+\beta-x_2} \\
&\leq -u(x_1^2+x_2^2) + x_1^2\exp\left(\frac{x_2}{\gamma}\right) + x_1x_2\exp\left(\frac{x_2}{\gamma}\right)\frac{1+\beta}{1+\beta-x_2}
\end{aligned} \tag{4.5}$$

where the last inequality is obtained under the assumption that the variables x_1 and x_2 are strictly positive for all times, which is, evidently, the above established physically meaningful restriction. In other words, a decomposition of the vector $f(x)$ is possible, including a locally dissipative component $f_d(x)$ in the region $\mathcal{X}$ of the state space. The decomposition of f in $\mathcal{X}$ into dissipative and non-dissipative components is clearly given by

$$f_{\rm d}(x) = \begin{bmatrix} -x_1x_2 \\ -x_1x_2\dfrac{1+\beta}{1+\beta-x_2} \end{bmatrix} \exp\left(\frac{x_2}{\gamma}\right); \quad f_{\rm nd}(x) = \begin{bmatrix} x_1 \\ x_1\dfrac{1+\beta}{1+\beta-x_2} \end{bmatrix} \exp\left(\frac{x_2}{\gamma}\right)$$

Define a state-dependent input coordinate transformation of the form

$$v = -u\left(x_1 + \frac{x_2^2}{x_1}\right) + x_1 \exp\left(\frac{x_2}{\gamma}\right) + x_2 \exp\left(\frac{x_2}{\gamma}\right)\frac{1+\beta}{1+\beta-x_2} \tag{4.6}$$

Transformation (4.6) yields a passive system between the external input v and the output variable x_1. Indeed, substituting the control law (4.6) in (4.5) and integrating both sides of the resulting relation leads one to obtain the following passivity inequality

$$V(x(t)) - V(x(0)) \leq \int_0^t x_1(\sigma)v(\sigma)\,{\rm d}\sigma$$

The partially closed loop system is rewritten as

$$\begin{aligned}
\dot{x}_1 &= -x_1x_2 \exp\left(\frac{x_2}{\gamma}\right) + x_1 \exp\left(\frac{x_2}{\gamma}\right)\frac{x_2 - x_1(1+\beta)/(1+\beta-x_2)}{x_1^2+x_2^2}x_2 + \frac{x_1^2}{x_1^2+x_2^2}v \\
\dot{x}_2 &= -x_1x_2 \exp\left(\frac{x_2}{\gamma}\right)\frac{1+\beta}{1+\beta-x_2} - x_1^2 \exp\left(\frac{x_2}{\gamma}\right)\frac{x_2 - x_1(1+\beta)/(1+\beta-x_2)}{x_1^2+x_2^2} \\
&\quad + \frac{x_2x_1}{x_1^2+x_2^2}v \\
y &= x_1
\end{aligned}$$

The transformed system may be placed in the form (3.2)

$$\dot{x} = -\mathcal{J}(x)x - \mathcal{R}(x)x + \mathcal{M}v$$

where $x^{\rm T} = [x_1 \ \ x_2]$, and

$$-\mathcal{J}(x) = \begin{bmatrix} 0 & x_1 \exp\left(\dfrac{x_2}{\gamma}\right)\dfrac{x_2-x_1(1+\beta)/(1+\beta-x_2)}{x_1^2+x_2^2} \\ -x_1 \exp\left(\dfrac{x_2}{\gamma}\right)\dfrac{x_2-x_1(1+\beta)/(1+\beta-x_2)}{x_1^2+x_2^2} & 0 \end{bmatrix}$$

$$-\mathcal{R}(x) = \begin{bmatrix} -x_2 \exp\left(\dfrac{x_2}{\gamma}\right) & 0 \\ 0 & -x_1 \exp\left(\dfrac{x_2}{\gamma}\right)\dfrac{1+\beta}{1+\beta-x_2} \end{bmatrix}; \quad \mathcal{M} = \frac{x_1}{x_1^2+x_2^2}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

where $\mathcal{J}^{\rm T}(x) + \mathcal{J}(x) = 0$, and $\mathcal{R}(x) = \mathcal{R}^{\rm T}(x) > 0$, as may easily be verified.

4.2.1. *Controller design.* Consider the modified energy function $V_{\rm d}$, defined with the aid of an auxiliary state vector $x_{\rm d} = [x_{1\rm d} \ \ x_{2\rm d}]^{\rm T}$, representing the desired state vector, to be determined later.

Let V_d be given by

$$V_d(x, x_d) = \frac{1}{2}(x - x_d)^T(x - x_d)$$
$$= \frac{1}{2}[(x_1 - x_{1d})^2 + (x_2 - x_{2d})^2]$$

From the results of the previous section, the following set of auxiliary controlled differential equations yields $\dot{V}_d(x, x_d)$ negative definite

$$\dot{x}_{1d} = -x_2 \exp\left(\frac{x_2}{\gamma}\right) x_{1d} + x_1 \exp\left(\frac{x_2}{\gamma}\right) \frac{x_2 - x_1(1 + \beta/(1 + \beta - x_2)}{x_1^2 + x_2^2} x_{2d}$$
$$+ \frac{x_1^2}{x_1^2 + x_2^2} v + R_1(x_1 - x_{1d})$$

$$\dot{x}_{2d} = -x_1 \exp\left(\frac{x_2}{\gamma}\right) \frac{1 + \beta}{1 + \beta - x_2} x_{2d} - x_1 \exp\left(\frac{x_2}{\gamma}\right) \frac{x_2 - x_1(1 + \beta)/(1 + \beta - x_2)}{x_1^2 + x_2^2} x_{1d}$$
$$+ \frac{x_1 x_2}{x_1^2 + x_2^2} v + R_2(x_2 - x_{2d})$$

where R_1 and R_2 are the components of a diagonal positive definite matrix $\mathcal{R}_{di}(x)$ which, for simplicity, will be taken to be a constant matrix, $\mathcal{R}_{di}(x) = \mathcal{R}_{di} = \text{diag}\,[R_1 \;\; R_2]$.

Letting $x_{1d} = \bar{x}_1 = \text{constant}$, one obtains the following dynamic controller expression, where x_{2d} has been substituted by the controller state variable ξ

$$v = \frac{x_1^2 + x_2^2}{x_1^2}\left[x_2 \exp\left(\frac{x_2}{\gamma}\right)\bar{x}_1 - x_1 \exp\left(\frac{x_2}{\gamma}\right)\frac{x_2 - x_1(1+\beta)/(1+\beta-x_2)}{x_1^2+x_2^2}\xi - R_1(x_1 - \bar{x}_1)\right]$$

$$\dot{\xi} = -x_1 \exp\left(\frac{x_2}{\gamma}\right) \frac{1 + \beta}{1 + \beta - x_2} \xi - x_1 \exp\left(\frac{x_2}{\gamma}\right) \frac{x_2 - x_1(1 + \beta)/(1 + \beta - x_2)}{x_1^2 + x_2^2} \bar{x}_1$$
$$+ \frac{x_1 x_2}{x_1^2 + x_2^2} v + R_2(x_2 - \xi)$$

Digital computer simulations were performed to test the behaviour of the closed loop system. We took, as in Agrawal *et al.* (1982), the following system parameters for the simulation of the regulated system

$$\gamma = 0{\cdot}48; \quad \beta = 0{\cdot}02$$

Corresponding to a constant value of the control input u given by $\bar{u} = 1{\cdot}3245$, the equilibrium value for x_2 is found to be $\bar{x}_2 = 0{\cdot}3200$, and the corresponding equilibrium for x_1 results in $\bar{x}_1 = 0{\cdot}2196$.

The design parameters were chosen to be

$$R_1 = R_2 = 1{\cdot}7$$

Figure 4 shows the closed loop behaviour of the system states, controller state and the synthesized control input. The system trajectories evolve exhibiting nice stabilization features (no overshoot) towards the steady state equilibrium and a settling interval of less that 5 time units.

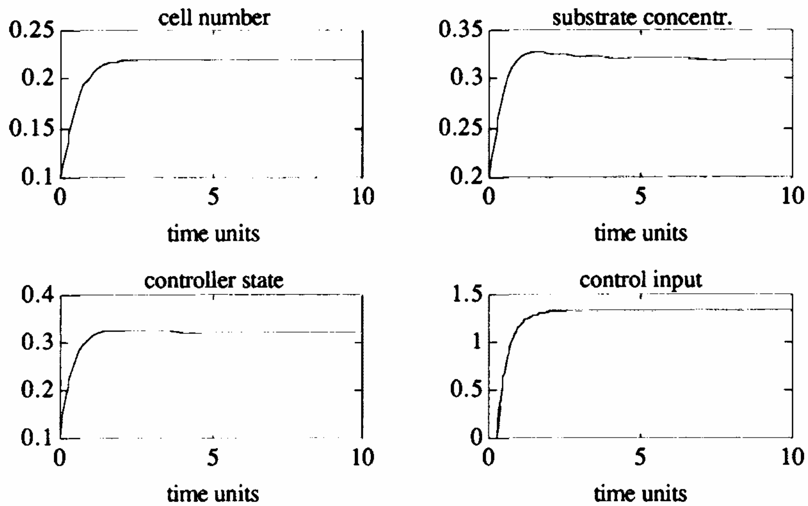

Figure 4. Simulation results of the passivity-based regulated bioreactor system.

4.3. *An exothermic continuously stirred tank reactor system*

We now consider a jacketed continuously stirred tank reactor system. The following model, taken from Smith and Corripio (1985), assumes that the reactor and the jacket are perfectly mixed, that all involved volumes as well as the relevant physical properties remain constant and that the heat losses may be neglected

$$\dot{x}_1 = \frac{F}{V}(C_{Ai} - x_1) - k_0 \exp\left\{ -\frac{E}{R(x_2 + 273{\cdot}16)} \right\} x_1^2$$

$$\dot{x}_2 = \frac{F}{V}(T_i - x_2) - \frac{\Delta H_R}{\rho C_p} k_0 \exp\left\{ -\frac{E}{R(x_2 + 273{\cdot}16)} \right\} x_1^2 - \frac{UA}{V_\rho C_p}(x_2 - x_3)$$

$$\dot{x}_3 = \frac{UA}{V_C \rho_C C_{pC}} \cdot (x_2 - x_3) - \frac{F_{C\max} \alpha^{-u}}{V_C}(x_2 - T_{C_i})$$

$$y = x_1$$

where x_1 is the concentration of the reactant in the reactor (kgmole m^{-3}), x_2 is the temperature in the reactor (C). x_3 is the jacket temperature (C), u is the controller output signal on the scale [0, 1] and it represents an equal percentage valve position function. F is the feed rate (m^3), V is the reactor volume (m^3), C_{Ai} is the concentration of the reactant in the feed (kgmole m^{-3}), k is the reaction rate coefficient (m^3 $kgmole^{-1}$ s^{-1}), T_i is the temperature of the feed (C), ΔH_R is the heat of the reaction, assumed to be constant (J $kgmole^{-1}$), ρ is the density of the reactor contents (kgmole m^{-3}), C_p is the heat capacity of the reactants (J $kgmole^{-1}$ s^{-1}), U is the overall heat transfer coefficient (J s^{-1} m^{-2} deg^{-1}C), A is the heat transfer area (m^2), V_C is the jacket volume (m^3), ρ_C is the density of the coolant (kg m^{-3}), C_{pC} is the specific heat of the coolant (J $kg^{-1}$$deg^{-1}$C), F_C is the coolant rate (m^3 s^{-1}), T_{Ci} is the coolant inlet temperature (C), k_0 is the Arrhenius frequency parameter (m^3 s^{-1} $kgmole^{-1}$), E is the activation energy of the reaction

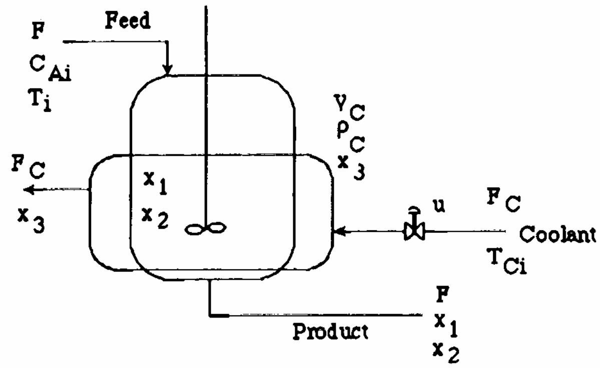

Figure 5. Continuously stirred tank reactor system.

(J kgmole^{-1}), R is the ideal gas law constant, 8314·39 J kgmole^{-1} K^{-1}, F_{Cmax} is the maximum flow through the control valve (m^3 s^{-1}), α is the valve rangeability parameter (see Fig. 5).

We let the auxiliary control ω be defined as $\omega = F_{\mathrm{Cmax}}\alpha^{-u}$. The operating region of the system is, again, constituted by the strict first orthant in R^3, where the concentration and the temperatures are all positive

$$\mathcal{X} = \{x \in R^3 \text{ s.t. } x_i > 0 \text{ for } i = 1, 2, 3\}$$

Consider, as in the previous examples, the storage function $V(x) = 1/2x^{\mathrm{T}}x$. The field $g(x)$ is given by

$$\begin{bmatrix} 0 \\ 0 \\ -\dfrac{1}{V_{\mathrm{C}}}(x_3 - T_{\mathrm{C_i}}) \end{bmatrix}$$

Hence, since $L_g v = -(x_3/V_{\mathrm{C}})(x_3 - T_{\mathrm{C_i}})$, the system can be passivified with respect to $V(x)$ in all of $\mathcal{X}$ except on the plane represented by $x_3 = T_{\mathrm{C_i}}$. Physically speaking, this forbidden plane is never visited by the state trajectory, since the jacket temperature will always be superior to the coolant inlet temperature $T_{\mathrm{C_i}}$.

Let $k(x)$ be defined as

$$k(x) = k_0 \exp\left\{ -\frac{E}{R(x_2 + 273{\cdot}16)} \right\}$$

The dissipative and non-dissipative components of $f(x)$ are given by

$$f_{\mathrm{d}}(x) = \begin{bmatrix} \dfrac{F}{V}x_1 - k(x)x_1^2 \\ -\left(\dfrac{E}{V} + \dfrac{UA}{V\rho C_{\mathrm{p}}}\right)x_2 \\ -\dfrac{UA}{V_{\mathrm{C}}\rho_{\mathrm{C}}C_{\mathrm{pC}}}x_3 \end{bmatrix}; \quad f_{\mathrm{nd}}(x) = \begin{bmatrix} \dfrac{F}{V}C_{\mathrm{A_i}} \\ \dfrac{FT_i}{V} - \dfrac{\Delta H_{\mathrm{R}}}{\rho C_{\mathrm{p}}}k(x)x_1^2 + \dfrac{UA}{V\rho C_{\mathrm{p}}}x_3 \\ \dfrac{UA}{V_{\mathrm{C}}\rho_{\mathrm{C}}C_{\mathrm{pC}}}x_2 \end{bmatrix}$$

In this case there are no invariant components of $f(x)$ with respect to $V(x)$.

The passivifying state dependent input coordinate transformation is given by

$$\omega = \frac{V_c}{x_3(x_3 - T_{C_i})}\left[\frac{F}{V}C_{A_i}x_1 + \frac{F}{V}T_i x_2 - \frac{\Delta H_R}{\rho C_p}k(x)x_1^2 x_2 \right.$$
$$\left. + UA\left(\frac{1}{V\rho C_p} + \frac{1}{V_C\rho_C C_{pC}}\right)x_2 x_3 - vx_1 + \gamma x_1^2\right]$$

with γ being a strictly positive constant. The closed loop system is obtained as

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix} -\left(\frac{F}{V} + k(x)x_1\right) & 0 & 0\\ 0 & -\left(\frac{F}{V} + \frac{UA}{V\rho C_p}\right) & 0\\ 0 & 0 & -\left(\frac{UA}{V_C\rho_C C_{pC}} + \gamma\frac{x_1^2}{x_3^2}\right)\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$
$$+ \begin{bmatrix} 0 & 0 & \frac{FC_{A_i}}{Vx_3}\\ 0 & 0 & J_{23}(x)\\ -\frac{FC_{A_i}}{Vx_3} & -J_{23}(x) & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \frac{x_1}{x_3}\end{bmatrix}v$$
$$y = x_1$$

where

$$J_{23}(x) = \left(\frac{UA}{V\rho C_p} + \frac{FT_i}{Vx_3} - \frac{\Delta H_R}{\rho C_p}\frac{k(x)x_1^2}{x_3}\right)$$

The time derivative of $V(x)$, along the system trajectories, satisfies

$$\dot{V}(x) = -\frac{F}{V}x_1^2 - k(x)x_1^3 - \left(\frac{F}{V} + \frac{UA}{V\rho C_p}\right)x_2^2$$
$$-\frac{UA}{V_C\rho_C C_{pC}}x_3^2 + x_1 v - \gamma x_1^2 \le yv - \gamma y^2 \le yv$$

and the system had been made strictly output passive and, hence, passive.

A dynamic passivity-based feedback controller for the system may be obtained by following the procedure outlined in §3.2. Such a controller is given by the following set of differential and algebraic equations

$$\begin{bmatrix}\dot{\xi}_1\\ \dot{\xi}_2\end{bmatrix} = \begin{bmatrix} -\left(\frac{F}{V} + k(x)x_1\right) & 0\\ 0 & -\left(\frac{F}{V} + \frac{UA}{V\rho C_p}\right)\end{bmatrix}\begin{bmatrix}\xi_1\\ \xi_2\end{bmatrix} + \begin{bmatrix}\frac{FC_{A_i}}{Vx_3}\\ J_{23}(x)\end{bmatrix}\bar{x}_3 + \begin{bmatrix}R_1(x_1 - \xi_1)\\ R_2(x_2 - \xi_2)\end{bmatrix}$$
$$v = \frac{x_3}{x_1}\left[\frac{FC_{A_i}}{Vx_3}\xi_1 + \left(\frac{UA}{V\rho C_p} + \frac{FT_i}{Vx_3} - \frac{\Delta H_R}{\rho C_p}\frac{k(x)}{x_3}\right)\xi_2 + \left(\frac{UA}{V_C\rho_C C_{pC}} + \gamma\frac{x_1^2}{x_3^2}\right)\bar{x}_3 \right.$$
$$\left. - R_3(x_3 - \bar{x}_3)\right]$$

Simulations of the closed loop system constituted by the plant and the above

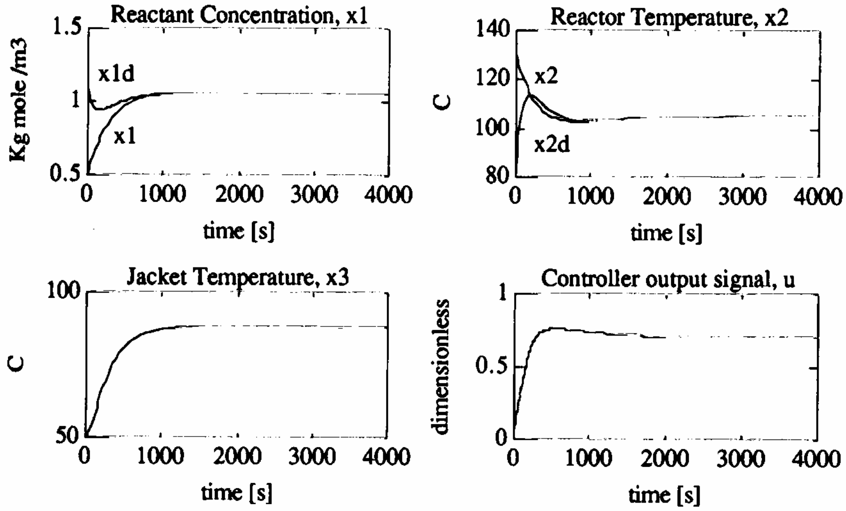

Figure 6. Simulation results of the passivity-based regulated CSTR system.

derived dynamical feedback compensator were performed for a CSTR with the following parameters (see Smith and Corripio 1985)

$$V = 7{\cdot}08\,\mathrm{m}^3;\quad \rho = 19{\cdot}2\,\mathrm{kgmole\,m^{-3}};\quad C_\mathrm{p}1{\cdot}8115\times10^5\,\mathrm{J\,kgmole\,deg^{-1}C};$$

$$A = 5{\cdot}40\,\mathrm{m}^2;\quad \rho_\mathrm{C} = 1000\,\mathrm{kg\,m^{-3}};\quad k_0 = 0{\cdot}0744\,\mathrm{m^3\,s^{-1}\,kgmole^{-1}};$$

$$C_\mathrm{pC} = 4184\,\mathrm{J\,kg^{-1}\,deg^{-1}C};\quad E = 1{\cdot}182\times10^7\,\mathrm{J\,kgmole^{-1}};\quad F_\mathrm{Cmax} = 0{\cdot}020\,\mathrm{m^3\,s^{-1}};$$

$$F = 7{\cdot}5\times10^{-3}\,\mathrm{m^3\,s^{-1}};\quad \alpha = 50;\quad \Delta H_\mathrm{R} = -9{\cdot}86\times10^7\,\mathrm{J\,kgmole^{-1}};$$

$$U = 3550\,\mathrm{J\,s^{-1}\,m^{-2}\,deg^{-1}C};\quad C_{\mathrm{A}_i} = 2{\cdot}88\,\mathrm{kgmoles\,m^{-3}};\quad T_{\mathrm{C}_i} = 27{\cdot}0\,\mathrm{C};\quad T_i = 66{\cdot}0\,\mathrm{C}$$

The desired equilibrium, corresponding to a constant value of u is obtained as

$$\bar{x}_1 = 1{\cdot}056\,\mathrm{kgmole\,m^{-3}};\quad \bar{x}_2 = 105\,\mathrm{C};\quad \bar{x}_3 = 88\,\mathrm{C};\quad \bar{u} = 0{\cdot}70$$

The damping coefficients R_1, R_2, R_3 and γ were set to be

$$R_1 = 0{\cdot}01;\quad R_2 = 0{\cdot}01;\quad R_3 = 0{\cdot}001;\quad \gamma = 0{\cdot}01$$

Figure 6 depicts the simulations of the CSTR system regulated by the dynamical feedback controller synthesized by passivity considerations.

4.4. *An isothermic CSTR system*

Consider the following model of a CSTR in which an isothermal, liquid-phase multicomponent chemical reaction takes place (the model presented here is taken from Kravaris and Palanki 1988). The chemical reaction system is

$$P \rightarrow Q \rightarrow R$$

with an unmodelled first-order side reaction from P. The reactants P and Q are highly acidic, while R is neutral. The control objective is to keep the total concentration of P and Q at a constant value by adjusting the molar feed rate of P. Figure 7 gives a schematic representation of the system, where V is the volume of

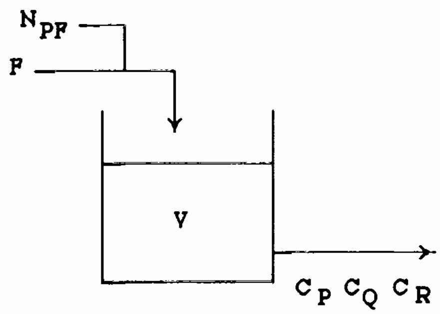

Figure 7. Isothermic continuously stirred tank reactor system.

the tank, F is the volumetric feed rate and N_{PF} is the molar feed rate of the species P, effectively acting as a control input to the system.

The differential equations describing the system in terms of dimensionless variables are

$$\dot{x}_1 = -(1 + D_{a1})x_1 + u$$
$$\dot{x}_2 = D_{a1}x_1 - x_2 - D_{a2}x_2^2$$
$$y = x_1 + x_2$$

where $x_1 = C_P/C_{P0}$ is the ratio of the concentration C_P of the species P, and the desired concentration, C_{p0}, of the species P and Q. The state variable $x_2 = C_Q/C_{P0}$ is the ratio of the concentration C_Q of the species Q and the desired concentration C_{P0}. The dimensionless control input is given by $u = N_{PF}/FC_{P0}$. The constants $D_{a1} = k_1V/F$ and $D_{a1} = k_2VC_{P0}/F$ are assumed to be perfectly known with k_1 and k_2 being, respectively the first and second-order rate constants.

For a constant feed rate $u = \bar{u}$, the corresponding equilibrium value of the state vector is given by

$$\bar{x}_1 = \frac{\bar{u}}{(1 + D_{a1})}; \quad \bar{x}_2 = -0{\cdot}5 + (0{\cdot}25 + D_{a1}D_{a2}\bar{x}_1)^{1/2}$$

i.e. the system has a non-zero constant equilibrium for a given constant input.

The operating region of the system is given by the following set

$$\mathcal{X} = \{x \in R^2 \text{ s.t. } x_i > 0; i = 1, 2\}$$

The vector fields $f(x)$ and $g(x)$ are readily found to be

$$f(x) = \begin{bmatrix} -(1 + D_{a1})x_1 \\ D_{a1}x_1 - x_2 - D_{a2}x_2^2 \end{bmatrix}; \quad g(x) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Consider first a simple change of coordinates which places the total concentration y as a state variable

$$\begin{bmatrix} y \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

The transformed system reads now as

$$\dot{y} = -y + u - D_{a2}x_2^2$$
$$\dot{x}_2 = D_{a1}y - (1 + D_{a1})x_2 - D_{a2}x_2^2$$

The transformed vector fields are given by

$$\tilde{f}(y, x_2) = \begin{bmatrix} -y - D_{a2}x_2^2 \\ D_{a1}y - (1 + D_{a1})x_2 - D_{a2}x_2^2 \end{bmatrix}; \quad \tilde{g}(x) = g(x) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

We consider the storage function given by

$$V(y, x_2) = \tfrac{1}{2}(y^2 + x_2^2)$$

The quantity $L_{\tilde{g}}V(y, x_2)$ is, in this case, simply given by

$$L_{\tilde{g}}V(y, x_2) = L_g V(y, x_2) = y = x_1 + x_2 \neq 0 \text{ in } \mathcal{X}$$

Hence, the system is passifiable by means of an affine state feedback control law.

The natural components of the transformed drift vector field $\tilde{f}(y, x_2)$ are given by

$$\tilde{f}_d(y, x_2) = \begin{bmatrix} -y - D_{a2}x_2^2 \\ -(1 + D_{a1})x_2 - D_{a2}x_2^2 \end{bmatrix}; \quad \tilde{f}_{nd}(y, x_2) = \begin{bmatrix} 0 \\ D_{a1}y \end{bmatrix}; \quad \tilde{f}_I(y, x_2) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The time derivative of the storage function $V(y, x_2)$ satisfies

$$\dot{V} = -y^2 - D_{a2}x_2^2 y - (1 + D_{a1})x_2^2 - D_{a2}x_2^3 + uy + D_{a1}yx_2 \leq y(u + D_{a1}x_2)$$

This immediately suggests the following change of input coordinates

$$v = u + D_{a1}x_2$$

The partially closed loop system, in passivity canonical form, results now in

$$\begin{bmatrix} \dot{y} \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -D_{a1} \\ D_{a1} & 0 \end{bmatrix} \begin{bmatrix} y \\ x_2 \end{bmatrix} + \begin{bmatrix} -\left(1 + \dfrac{D_{a2}x_2^2}{y}\right) & 0 \\ 0 & -(1 + D_{a1} + D_{a2}x_2) \end{bmatrix} \begin{bmatrix} y \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} v$$

Following the controller design procedure outlined in § 3.2, we obtain the following set of auxiliary controlled differential equations which guarantee, after some appropriate damping injections, the negativity of the derivative of the modified storage function

$$V_d(y, y_d, x_2, x_{2d}) = \tfrac{1}{2}[(y - y_d)2 + (x_2 - x_{2d})^2]$$

$$\begin{bmatrix} \dot{y}_d \\ \dot{x}_{2d} \end{bmatrix} = \begin{bmatrix} 0 & -D_{a1} \\ D_{a1} & 0 \end{bmatrix} \begin{bmatrix} y_d \\ x_{2d} \end{bmatrix} + \begin{bmatrix} -\left(1 + \dfrac{D_{a2}x_2^2}{y}\right) & 0 \\ 0 & -(1 + D_{a1} + D_{a2}x_2) \end{bmatrix} \begin{bmatrix} y_d \\ x_{2d} \end{bmatrix}$$
$$+ \begin{bmatrix} 1 \\ 0 \end{bmatrix} v + \begin{bmatrix} R_y & 0 \\ 0 & R_2 \end{bmatrix} \begin{bmatrix} (y - y_d) \\ (x_2 - x_{2d}) \end{bmatrix} \tag{4.7}$$

Letting $y_d = \bar{y} = \bar{x}_1 + \bar{x}_2$ and solving for the control v from the first equation, we obtain the following dynamical state feedback controller, where the variable ξ—the state of the controller—replaces the auxiliary variable x_{2d}.

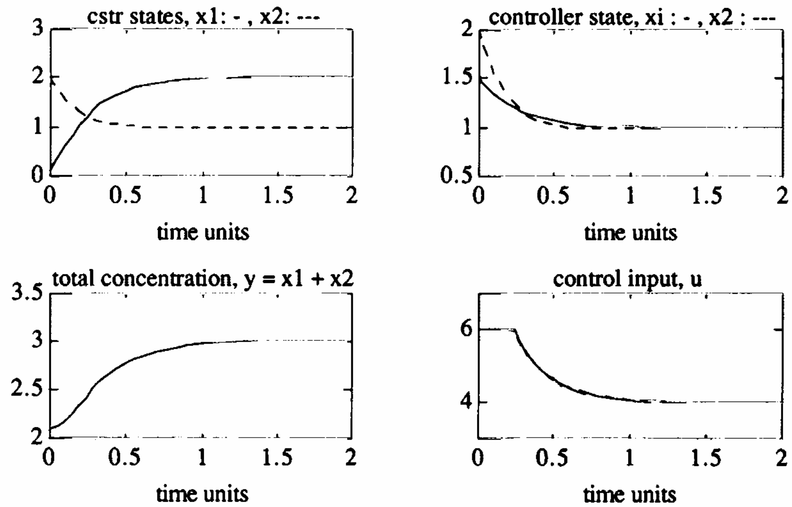

Figure 8. Simulation results of the passivity-based regulated isothermic CSTR system.

$$v = \left(1 + \frac{D_{a2}x_2^2}{y}\right)\bar{y} + D_{a1}\xi - R_y(y - \bar{y})$$

$$\dot{\xi} = -(1 + D_{a1} + D_{a2}x_2)\xi + D_{a1}\bar{y} + R_2(x_2 - \xi)$$

In terms of the original control input u, the controller is found to be

$$u = \left(1 + \frac{D_{a2}x_2^2}{y}\right)\bar{y} - D_{a1}(x_2 - \xi) - R_y(y - \bar{y})$$

$$\dot{\xi} = -(1 + D_{a1} + D_{a2}x_2)\xi + D_{a1}\bar{y} + R_2(x_2 - \xi)$$

Simulations were carried out to test the performance of the proposed dynamic feedback controller. As in Kravaris and Palanki (1988) we took the following values for the parameters

$$D_{a1} = 1; \quad D_{a2} = 1$$

The equilibrium points, corresponding to a steady-state value of u given by $\bar{u} = 4$, are readily found to be

$$\bar{x}_1 = 2; \quad \bar{x}_2 = 1; \quad \bar{y} = 3$$

The design constants representing the damping injections to the transformed system were set to be

$$R_y = 2; \quad R_2 = 2;$$

Figure 8 depicts the regulated responses of the closed loop system exhibiting nice stability features without overshoots as well as a substantially improved settling time. In order to test the performance with respect to input saturations we purposely limited the control input to take values in the closed interval [0, 6]. In spite of the effects caused by the initial control input saturation, the feedback controller manages to stabilize the system as expected.

5. Conclusions

In this article, we have proposed a passivity-based approach for the regulation of a large class of nonlinear systems, especially nonlinear chemical systems. A geometric interpretation was given to the possiblities of 'passifying', by means of affine feedback, an arbitrary nonlinear system describing a continuous process. For monovariable systems, passivization is achievable by means of control input space coordinates transformations, provided the energy storage function of the system is strictly relative degree one in the region of interest. This requirement does not seem to be very stringent, for a large class of monovariable nonlinear systems describing common industrial continuous processes.

Passivity-based controllers have been traditionally applied to the class of lagrangian systems. In particular, the approach has been applied to mechanical systems (such as robots), electro-mechanical systems (such as induction motors) and to purely electrical systems (such as DC-to-DC power converters, etc). It should be emphasized that for a large class of nonlinear systems, especially those representing biological and chemical process control systems, the concept of 'stored energy', is not as simple as in the area of lagrangian systems. This fact has prevented the advantageous application of passivity-based regulation schemes to such classes of systems. However, we have demonstrated that one half of the square of the norm of the state vector always qualifies as a suitable, and simple, positive definite energy storage function from which sensible, and efficient, controller designs can be systematically obtained. The results proposed here apply to *any* linear, or nonlinear system, independently of its output relative degree and of its minimum or non-minimum phase character. Of course, in the case of non-minimum phase systems, any stabilizing controller will result in unfeasible control actions, whether unstable or unbounded, as the desired equilibrium state is sustained. In general terms, the possibilities of 'passivization' of nonlinear systems by means of regular affine feedback have been shown to be equally valid for monovariable and multivariable cases (see Sira-Ramírez and Delgado 1997).

In this article, our aim was to bring to the attention of the reader, the simplicity and efficiency of the passivity based regulation scheme for a large class of nonlinear systems. Passivity-based regulation, as presented here, can be easily compared against linearization techniques in terms of the controller complexity and other practically oriented criteria such as robustness. Even if the system must be first passivified by an input coordinate transformation, the resulting passivity-based dynamical feedback controller is far simpler than the exact linearization controller.

Extensions of the above results to systems with delays, characteristic of so many industrial continuous processes, appear as an interesting challenge from the view-point of passivity. Another challenging area is that of combining the advantages of the passivity-based approach with discontinuous feedback strategies, such as sliding mode control.

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