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ADAPTIVE DYNAMICAL INPUT–OUTPUT LINEARIZATION OF DC TO DC POWER CONVERTERS: A BACKSTEPPING APPROACH

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SUMMARY

Dynamical adaptive regulation of pulse-width-modulation (PWM) controlled power supplies is proposed using a suitable combination of average dynamical input–output linearization and the 'backstepping' controller design method. Nonlinear average models of dc to dc power supplies are not transformable to parametric pure nor parametric strict feedback canonical forms by means of parameter-independent state co-ordinate transformation. A more direct approach is therefore proposed for implementing the fundamental ideas related to the so called 'non-overparametrized' adaptive backstepping algorithm which avoids explicit transformations to the above mentioned canonical forms. Dynamical adaptive feedback controllers are developed for the regulation of the input-inductor current towards desirable constant values. The validity of the proposed approach, regarding control objectives and robustness with respect to unmodelled, yet unmatched, and bounded stochastic perturbation inputs, is tested through digital computer simulations.

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1. INTRODUCTION

Switchmode dc to dc power converters are customarily regulated by means of pulse-width-modulation (PWM) feedback control strategies. The feedback control stabilization and tracking problems associated with the operation of such dc power supplies involves a largely intractable discontinuous feedback control problem which is defined on the combination of piecewise linear system structures. In fact, only approximate strategies have been proposed in the last twenty years to deal with the PWM control of this class of nonlinear systems. Results on the regulation of dc to dc power supplies can be found in conference proceedings, such as the yearly Power Specialist Conference Records, the multi-volume series edited by Middlebrook and Cúk[1] and the collection of articles recently edited by Bose.[2] Also, background material may be found in specialized books such as Kassakian *et al.*,[3] Severns and Bloom,[4] Csaki *et al.*,[5] and many others.

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A simplifying assumption usually adopted in the regulation problem of a dc to dc power supply, is that the circuit parameters are perfectly known. However, this assumption may prove to be invalid in a number of practical situations. Most frequently one has only an approximate knowledge of the values of the converter circuit components and of the external voltage source. Commonly, ageing effects on such components substantially alter the known nominal value of the circuit parameters. These issues justify the need for an adaptive feedback control strategy for the feedback regulation of output capacitor voltages or, alternatively, input inductor currents for switch-controlled devices delivering constant power to loads. Nonlinear adaptive feedback control of PWM controlled dc to dc power supplies has been proposed recently by Sira–Ramírez *et al.* in Reference 6. The approach adopted in these contributions was based on an extension to discontinuous feedback systems of the results already proposed by Sastry and Isidori[7] for the stabilization of partially linearizable uncertain minimum phase, nonlinear systems. More recently, results dealing with the use of the overparametrized backstepping adaptive control algorithm have been derived and applied to the regulation of dc to dc power converters of the 'Boost' and 'Buck–Boost' types (see Reference 8). The approach relies on nominal transformations of the actual uncertain system to Fliess generalized observability canonical form (see Fliess[9]).

In this article, the 'non-over parametrized' adaptive feedback control strategy is adopted which is based on a combination of the *adaptive backstepping* controller design algorithm and input–output *dynamical* feedback linearization. The backstepping adaptation algorithm was developed in a series of remarkable contributions by Kanellakopoulos *et al.*,[10,11] and Kristic *et al.*[12] The fundamental objective in their work was to provide a systematic framework for the regulator design of uncertain systems. The backstepping adaptation algorithm was shown to be suitable for a large class of state linearizable nonlinear systems with constant, but unknown, parameters.

In this paper we assume that the converter circuit parameters are completely unknown. The synthesis of the feedback regulator, and of the associated parameter estimation update laws, are then carried out by resorting to a direct adaptive controller calculation procedure applied to the uncertain system model. We emphasize that the average PWM models of the various converters examined in this article are *not* transformable to parametric pure and strict parametric feeback canonical forms by means of parameter-independent state co-ordinate transformation. For this reason, our proposed direct approach to the backstepping procedure does not rely on explicit state co-ordinate transformations to parametric pure or strict parametric feedback canonical forms.[10,11,12] In fact, in our controller calculation approach, observability of the uncertain system is demanded while inputs are allowed to appear in the intermediate steps of the procedure. Moreover, control input derivatives are invariably present at the final stage of the proposed algorithm. As a result, *dynamical*, rather than static, feedback duty ratio synthesizers are obtained in an implicit manner, i.e. as the on-line solution of a nonlinear, time-varying, differential equation. These features set our method apart from the usually stringent condition demanded by the traditional backstepping algorithm whenever the regulated output relative degree is less than the state dimension. In essence, rather than attempting an input-to-state linearization, a dynamical input–output linearization strategy is developed. We derive dynamical adaptive feedback controllers for several types of dc to dc power converters with nonlinear observable average models. The proposed method requires a slight modification of the direct calculation procedure proposed by the authors in connection with the adaptive regulation of the Buck converter.[13]

The proposed adaptive controller calculation procedure, which is somewhat equivalent to the non-over parametrized backstepping algorithm but much easier to understand, establishes, as a first step, the need to impose a desirable algebraic relation involving an 'implicit' estimate of the output error derivative. The validity of this 'pseudo-controller' relation is shown to yield first order perturbed evolution of the output error signal. This in turn can result in an asymptotically stable behaviour for the output error, provided appropriate parameter update estimation laws were suitably devised. However, the update laws are not computed at this first step and the fact that the pseudo-controller expression is not valid is taken into account; setting the stage for the second step of the algorithm. The fundamental consideration at this point is a joint Lyapunov stability analysis for the output error and the parameter update estimation errors. The rest of the adaptive controller calculation procedure is devoted to guarnatee the asymptotic validity of the desired pseudo-controller expressions. A pseudo-controller error expression is next formulated and an estimate of its first order time derivative computed; based again on an 'implicit estimate' of the uncertain parameters. The estimated pseudo-controller error derivative is subsequently forced to satisfy a desirable linear relation involving the actual pseudo-controller error. As a result, a perturbed first order evolution of the pseudo-controller error is obtained, on which a separate Lyapunov stability analysis can be carried out to obtain the required parameter estimation update laws. Use of the computed update estimation laws on the desired linear algebraic expression, involving the estimated pseudo-controller error derivative, allows one to solve, in terms of a nonlinear, time-varying differential equation, for the required average feedback control, or duty ratio, actions. In fact, the same results are obtained if the time derivative of the duty ratio function is considered as a fictitious, or auxiliary, external control input.

In Section 2 the developments leading to dynamical adaptive PWM control strategies for dc to dc power supplies of the 'Boost' and 'Buck–Boost' types containing unknown parameter values, are presented. Computer simulations are presented in Section 3 for the assessment of the closed-loop performance of the derived dynamical adaptive controller strategies. The simulations clearly indicate the effectiveness and robustness of the proposed adaptive feedback regulation schemes with respect to unmodelled, and unmatched, external stochastic perturbation inputs of bounded nature. Section 4 contains the conclusions and suggestions for further work in this area.

2. A BACKSTEPPING ADAPTIVE CONTROLLER APPROACH FOR THE REGULATION OF DC TO DC POWER SUPPLIES

In this section we present the switchmode and the average PWM models of the various dc to dc power converters to be treated. An adaptive regulation approach is developed which seeks the stabilization of the average input inductor current to its desired equilibrium point by means of a dynamical adaptive controller. Comparisons of the derived controllers with their non-adaptive versions are also considered.

2.1. *The Boost Converter*

Consider the Boost converter circuit, shown in Figure 1. The system of differential equations, describing the input inductor current $I(t)$ and the output capacitor voltage $V(t)$, are given by

$$\dot{I}(t) = -\frac{1}{L}(1-u)V(t) + \frac{E}{L}$$

$$\dot{V}(t) = \frac{1}{C}(1-u)I(t) - \frac{1}{RC}V(t) \tag{1}$$

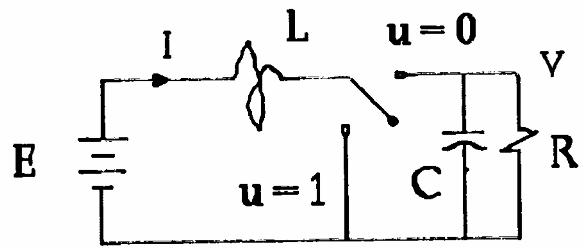

Figure 1. Boost converter circuit

where L, C and R are respectively the inductance, capacitance and resistance values of the circuit components. The quantity E represents the constant value of the external voltage source. The control input function u is the *switch position function* taking values in the discrete set $\{0, 1\}$. We shall be considering only one possibility for the regulated output function, here denoted by y; namely, the regulated output is considered to be the input inductor current $I(t)$. Thus, indirect feedback regulation of the output capacitor voltage is accomplished. It is well known that, for all except the Buck converter, if the output capacitor voltage $V(t)$ is taken as the regulated output, then the system is non-minimum phase.[14] In such a non-minimum phase case our proposed method leads to an *unstable* adaptive controlled (see References 15 and 16 for a reset piecewise unstable controller approach to this problem). As a result, regulation of the input inductor current is preferred. The approach leads to an adaptive controller which is *dynamical* in nature.

The steps leading to the adaptive dynamical controller escape the traditional considerations, associated with the parametric pure and parametric strict feedback canonical forms, and 'control' inputs and their first order time derivatives naturally appear in our proposed backstepping calculation procedures.

A PWM feedback strategy for the specification of the switch position function u, occurring at regularly sampled instants of time is usually specified as follows:

$$u = \begin{cases} 1 & \text{for } t_k \leqslant t < t_k + \mu_a(t_k)T \\ 0 & \text{for } t_k + \mu_a(t_k)T \leqslant t \leqslant t_k + T \end{cases}$$
$$t_k + T = t_{k+1}\text{: } k = 0, 1, \ldots \tag{2}$$

where $\mu_a(t_k)$ is the value of the *actual duty ratio* function at the sampling instant t_k. The actual duty ratio function is obtained from the (feedback) *computed duty ratio function*, denoted by μ, when restricted to taking values in the closed interval $[0, 1]$. The sampling period, T, is assumed to be constant.

A direct approach to the output stabilization problem of a system of the form (1), with a controller (2), requires an *exact discretisation* procedure, followed by a nonlinear discrete time duty ratio feedback design alternative. This program results in quite a complex controller expression, obtained from surprisingly complicated algebraic manipulations involving state transition matrices of the linear 'structures' associated with the switch regulated circuit. For these reasons, a simpler alternative is usually proposed which is based on a suitable average (i.e. infinite frequency) approximation of the converter model. In spite of the fact that the average approximation deteriorates for low sampling frequencies, its validity has been well established, both from a theoretical standpoint as well as from practical design considerations, in dc to dc power converters (see Middlebrook[1] and the article by Sira-Ramírez and Lischinsky-Arenas[14])

The *average model* for the PWM controlled Boost converter (1), (2), is obtained by formally replacing the switch position function u by the duty ratio function μ. Since the state and output variables have an average connotation we denote them differently from the original variables.

$$\dot{\zeta}_1 = -\frac{1}{L}(1-\mu)\zeta_2 + \frac{E}{L}$$

$$\dot{\zeta}_2 = \frac{1}{C}(1-\mu)\zeta_1 - \frac{1}{RC}\zeta_2 \tag{3}$$

$$\eta = \zeta_1$$

The Average PWM model (3) has the enormous advantage of reducing any output stabilization problem, defined on the converter model (1), to a standard *nonlinear feedback controller design problem* in which the duty ratio function μ plays the role of the required input variable. Once the computed duty ratio function, μ, is synthesized as a feedback function for the regulation of the average system, the actual duty ratio function, μ_a, is obtained from a bounding operation on the values of μ, to the unit interval.

We denote the values of the several parameters defining the the circuit equations as

$$\theta_1 = \frac{1}{L}; \quad \theta_2 = \frac{1}{C}; \quad \theta_3 = \frac{1}{RC}; \quad \theta_4 = \frac{E}{L} \tag{4}$$

The actual values of these parameters are assumed to be totally unknown, hence the need for controller adaptation.

Note that a slight degree of 'nonlinear overparametrization' is allowed in the definition of the components of the vector of unknown parameters θ. For instance, θ_4 would not be considered as a *new* parameter if, E, the value of the external voltage source, was perfectly known.

Under the assumption of a constant value of the duty ratio function $\mu = U$, with $0 < U < 1$, the equilibrium values of the average PWM converter models are readily obtained from (3) and (4) as

$$\zeta_1(U) = \frac{\theta_3\theta_4}{\theta_2\theta_1(1-U)^2}; \quad \zeta_2(U) = \frac{\theta_4}{\theta_1(1-U)} \tag{5}$$

The primary objective of our adaptive duty ratio synthesizer is in the feedback regulation of the average input inductor current $\zeta_1(t)$, towards a known, constant, equilibrium value, denoted by $X_1 = \zeta_1(U)$. This value corresponds to some constant value, U, of the (actual) duty ratio function. Average input inductor current regulation sidesteps the non-minimum phase problems associated with the direct regularization of the average output capacitor voltage, $\zeta_2(t)$, and at the same time provides an indirect method for the regulation of such a circuit variable.[14]

2.1.1. An Adaptive Controller for the Boost Converter. In this section we present a direct approach to the backstepping adaptive controller design. The adopted calculation procedure becomes equivalent to the traditional 'non-overparametrized' backstepping algorithm when the chosen output corresponds to the 'linearizing co-ordinate' of the system. However, in the particular cases treated here, the average models of the converters *are not* transformable to the canonical forms developed in References 10 and 11 nor do they satisfy the stringent structural and equilibrium conditions demanded for the regressor vectors.

Consider the average PWM Boost converter model

$$\begin{aligned}\dot{\zeta}_1 &= -\theta_1(1-\mu)\zeta_2 + \theta_4 \\ \dot{\zeta}_2 &= -\theta_2(1-\mu)\zeta_1 + \theta_3\zeta_2 \\ \eta &= \zeta_1\end{aligned} \tag{6}$$

where η is the average value of the output signal, in this case being the input inductor current $I(t)$. The parameters θ_i, $i = 1, \ldots, 4$ represent the *actual* values of the uncertain parameter, as given by (4). We denote an estimate of the unknown parameter values

$$\hat{\theta}_i,\ i = 1, 2, 3, 4 \tag{7}$$

In the following paragraphs we proceed to apply a modified version of the 'non-overparametrized' backstepping algorithm for the synthesis of an adaptive feedback controller for the average Boost converter model (7). Once the adaptive controller expressions are found, the average state variables ζ_1, ζ_2, appearing in the feedback controller, and the parameter adaptation laws, are substituted respectively by the actual (i.e. non-averaged) variables $I(t)$, $V(t)$. This procedure has been shown to be valid in many nonlinear regulation schemes proposed for dc to dc power supplies, including those which are based on an adaptive control alternative.[6,14]

Step 0

We let z_1 stand for the output variable error, defined as

$$z_1 = \eta - \zeta_1(U) = \zeta_1 - \zeta_1(U) \tag{8}$$

According to the average system model equations (6) the time derivative of the output error z_1, is of unknown nature and given by

$$\dot{z}_1 = -\theta_1(1-\mu)\zeta_2 + \theta_4 \tag{9}$$

An estimate of the time derivative of the error variable z_1 may be obtained directly from (9) by replacing the components of the unknown parameter vector θ by their estimated values $\hat{\theta}$.

$$\hat{\dot{z}}_1 = -\hat{\theta}_1(1-\mu)\zeta_2 + \hat{\theta}_4 \tag{10}$$

Adding to and substracting from the actual value of the parameters, θ_i, $i = 1, \ldots, 4$, their estimated values $\hat{\theta}_i$, $i = 1, \ldots, 4$, we can rewrite the expression (9) as

$$\begin{aligned}\dot{z}_1 &= -(\theta_1 - \hat{\theta}_1)(1-\mu)\zeta_2 + (\theta_4 - \hat{\theta}_4) - \hat{\theta}_1(1-\mu)\zeta_2 + \hat{\theta}_4 \\ &= \hat{\dot{z}}_1 + (\theta - \hat{\theta})^T \begin{bmatrix} -(1-\mu)\zeta_2 \\ 0 \\ 0 \\ 1 \end{bmatrix}\end{aligned} \tag{11}$$

Step 1

Let the estimated value of the output error derivative, $\hat{\dot{z}}_1$ satisfy

$$\hat{\dot{z}}_1 = -c_1 z_1 ; c_1 > 0 \tag{12}$$

where c_1 is a positively scalar design constant. We call the following form of expression (12), the 'pseudo-controller' equation.

$$-\hat{\theta}_1(1-\mu)\zeta_2+\hat{\theta}_4=-c_1(\zeta_1-\zeta_1(U)) \tag{13}$$

This equation represents a desired algebraic relation by which an effective stabilization of the output error would be possible when used in combination with a suitable estimation update law for the unknown parameters θ.

If the pseudo-controller relation (13) were valid, then from (12) and (11), the output error z_1 would satisfy

$$\dot{z}_1=-c_1z_1+(\theta-\hat{\theta})^T\begin{bmatrix}-(1-\mu)\zeta_2\\0\\0\\1\end{bmatrix} \tag{14}$$

Choosing $\boldsymbol{\Gamma}$ to be a positive definite diagonal matrix whose elements γ_{ii}, $i=1,2,3,4$, will be called *parameter adaptation gains*, we could consider next a scalar positive definite Lyapunov function of the form

$$V_1=\tfrac{1}{2}[z_1^2+(\theta-\hat{\theta})^T\boldsymbol{\Gamma}^{-1}(\theta-\hat{\theta})] \tag{15}$$

The time derivative of V_1 would result in

$$\dot{V}_1=-c_1z_1^2+(\theta-\hat{\theta})^T\boldsymbol{\Gamma}^{-1}\left(-\dot{\hat{\theta}}_1+z_1\boldsymbol{\Gamma}\begin{bmatrix}-(1-\mu)\zeta_2\\0\\0\\1\end{bmatrix}\right) \tag{16}$$

However, the pseudo-controller expression (13) is not generally valid from the outset. Thus a pseudo-controller error must be defined and our control effort in the second step of the algorithm should be geared towards forcing such an error to zero.

Consider then the pseudo-controller error

$$z_2=\hat{\dot{z}}_1+c_1z_1=-\hat{\theta}_1(1-\mu)\zeta_2+\hat{\theta}_4+c_1(\zeta_1-\zeta_1(U)) \tag{17}$$

Using (17), the actual expression for the output error derivative becomes

$$\dot{z}_1=-c_1z_1+z_2+(\theta-\hat{\theta})^T\begin{bmatrix}-(1-\mu)\zeta_2\\0\\0\\1\end{bmatrix} \tag{18}$$

As a consequence, the derivative of the Lyapunov function (15) is given by

$$\dot{V}_1 = -c_1 z_1^2 + z_1 z_2 + (\theta - \hat{\theta})^T \Gamma^{-1} \left(-\dot{\hat{\theta}}_1 + z_1 \Gamma \begin{bmatrix} -(1-\mu)\zeta_2 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right) \tag{19}$$

The second step recognizes that the desired algebraic relation defining the pseudo-controller (13) can be achieved asymptotically only by means of appropriate control actions combined with a suitable parameter estimation effort.

Step 2

Consider now the time derivative of the pseudo-controller error z_2. This quantity is also of known nature, as it depends on the actual parameter values θ, as well as on the time derivatives of the yet unspecified parameter estimates.

The time derivative of z_2 may then be written as

$$\begin{aligned} \dot{z}_2 = {} & \theta_1(-c_1(1-\mu)\zeta_2) + \theta_2(-\hat{\theta}_1(1-\mu)^2\zeta_1) + \theta_3(\hat{\theta}_1(1-\mu)\zeta_2) + \theta_4(c_1) \\ & - \dot{\hat{\theta}}_1(1-\mu)\zeta_2 + \hat{\theta}_1\dot{\mu}\zeta_2 + \dot{\hat{\theta}}_4 \end{aligned} \tag{20}$$

An estimate of the pseudo-controller error derivative (20) is then obtained as

$$\begin{aligned} \hat{\dot{z}}_2 = {} & \hat{\theta}_1(-c_1(1-\mu)\zeta_2) + \hat{\theta}_2(-\hat{\theta}_1(1-\mu)^2\zeta_1) + \theta_3(\hat{\theta}_1(1-\mu)\zeta_2) + \hat{\theta}_4(c_1) \\ & - \dot{\hat{\theta}}_1(1-\mu)\zeta_2 + \hat{\theta}_1\dot{\mu}\zeta_2 + \dot{\hat{\theta}}_4 \end{aligned} \tag{21}$$

Adding and substracting, in (20), the corresponding components of the estimates of the parameter vector, $\hat{\theta}$, from the components of the vector of actual parameter values, θ, one can easily rewrite the derivative of the pseudo-controller error z_2 as

$$\dot{z}_2 = \hat{\dot{z}}_2 + (\theta - \hat{\theta})^T \begin{bmatrix} -c_1(1-\mu)\zeta_2 \\ -\hat{\theta}_1(1-\mu)^2\zeta_1 \\ \hat{\theta}_1(1-\mu)\zeta_2 \\ c_1 \end{bmatrix} \tag{22}$$

We now let the estimate of the pseudo-controller error derivative satisfy

$$\hat{\dot{z}}_2 = -c_2 z_2 ; c_2 > 0 \tag{23}$$

If the time derivatives of the estimated parameters were already known, then expression (23) would constitute a *controller equation*. We proceed then to find the update laws for the estimation of the unknown parameters.

Substituting (23) in (22), the closed-loop behaviour of the pseudo-controller error z_2 is found to be governed by

$$\dot{z}_2 = -c_2 z_2 + (\theta - \hat{\theta})^T \begin{bmatrix} -c_1(1-\mu)\zeta_2 \\ -\hat{\theta}_1(1-\mu)^2\zeta_1 \\ \hat{\theta}_1(1-\mu)\zeta_2 \\ c_1 \end{bmatrix} \tag{24}$$

Note that the output error z_1 and the pseudo-controller error z_2 qualify as 'transformed states' of the average system since the following locally invertible, input-dependence, state co-ordinate transformation relates them to the original average state variables ζ_1 and ζ_2

$$\begin{aligned} z_1 &= \zeta_1 - \zeta_1(U) \\ z_2 &= -\hat{\theta}(1-\mu)\zeta_2 + \hat{\theta}_4 + c_1(\zeta_1 - \zeta_1(U)) \end{aligned} \tag{25}$$

The jacobian matrix of this transformation is obtained as

$$\frac{\partial z}{\partial \zeta} = \begin{bmatrix} 1 & 0 \\ c_1 & -\hat{\theta}_1(1-\mu) \end{bmatrix} \tag{26}$$

which is non-singular everywhere except at persistently saturated values of the duty ratio function $\mu = 1$. This condition represents an unstable open loop situation in the adopted PWM setup. For this reason, locally asymptotically stable behaviour can be guaranteed for the original state variables as long as asymptotically stable behaviour is devised for the transformed variables z_1 and z_2. The local non-singularity of the jacobian matrix is equivalent to the local observability of the average system (6).[14]

Using then a scalar Lyapunov function of the form

$$V_2 = \tfrac{1}{2}[z_1^2 + z_2^2 + (\theta - \hat{\theta})^T \mathbf{\Gamma}^{-1}(\theta - \hat{\theta})] = V_1 + \tfrac{1}{2}z_2^2 \tag{27}$$

one finds that the time derivative of V_2 satisfies

$$\begin{aligned} \dot{V}_2 = &- c_1 z_1^2 + z_1 z_2 - c_2 z_2^2 \\ &+ (\theta - \hat{\theta})^T \mathbf{\Gamma}^{-1} \left(-\dot{\hat{\theta}} + z_1 \mathbf{\Gamma} \begin{bmatrix} -(1-\mu)\zeta_2 \\ 0 \\ 0 \\ 1 \end{bmatrix} + z_2 \mathbf{\Gamma} \begin{bmatrix} -c_1(1-\mu)\zeta_2 \\ -\hat{\theta}_1(1-\mu)^2\zeta_1 \\ \hat{\theta}_1(1-\mu)\zeta_2 \\ c_1 \end{bmatrix} \right) \end{aligned} \tag{28}$$

If the update law for the estimated value of the parameters is now chosen as

$$\dot{\hat{\theta}} = z_1 \mathbf{\Gamma} \begin{bmatrix} -(1-\mu)\zeta_2 \\ 0 \\ 0 \\ 1 \end{bmatrix} + z_2 \mathbf{\Gamma} \begin{bmatrix} -c_1(1-\mu)\zeta_2 \\ -\hat{\theta}_1(1-\mu)^2\zeta_1 \\ \hat{\theta}_1(1-\mu)\zeta_2 \\ c_1 \end{bmatrix} \tag{29}$$

then an asymptotically stable behaviour to zero can be guaranteed for both the output error and the pseudo-controller error while achieving bounded evolution of the parameter estimates $\hat{\theta}$. Indeed, the output derivative of the Lyapunov function (28) is obtained as

$$\dot{V}_2 = \tfrac{1}{2}(-c_1 z_1^2 + z_1 z_2 - c_2 z_2^2) \tag{30}$$

which is a negative semi-definite function if the design parameter c_i; $i = 1, 2$ are chosen to satisfy $c_i > 2$.[10] Asymptotically stable behaviour of the transformed state variables z_1, z_2, as well as of the components of the vector of parameter estimation errors, follows from Barbalat's Lemman.[17] From (21), (23) and (29) the duty ratio function μ can be readily obtained in an implicit manner, as the solution of a nonlinear time-varying differential equation. One obtains, after some straightforward manipulations

$$\dot{\mu} = \frac{1}{\hat{\theta}_1 \zeta_2} \{ - c_1 c_2 (\zeta_1 - X_1) - (c_1 + c_2) [- \hat{\theta}_1 (1 - \mu) \zeta_2 + \hat{\theta}_4] + \hat{\theta}_1 (1 - \mu) [\hat{\theta}_2 (1 - \mu) \zeta_1 - \hat{\theta}_3 \zeta_2]$$
$$- [\gamma_{44} + \gamma_{11} (1 - \mu)^2 \zeta_2^2] [(\zeta_1 - X_1) + c_1 (- \hat{\theta}_1 (1 - \mu) \zeta_2 + \hat{\theta}_4 + c_1 (\zeta_1 - X_1))]\} \tag{31}$$

One can clearly see that the first two terms in the dynamical controller (31) are just trying to impose a second order linear behaviour on the output error with real eigenvalues located at $-c_1$ and $-c_2$. But instead of having the actual expression for the output error derivative in the desired second order dynamics, an estimate of this derivative is obtained. The third term of the controller arises from the need to annihilate the relevant part of the original system second order nonlinear average dynamics. Elimination is, however, being carried out again in an approximate manner through the estimated value of the relevant expression. The effect of the parameter uncertainty is seen throughout the controller expression but, especially, in the last nonlinear term which represents a quadratic weight (dependent upon the design gains chosen for the parameter adaptation law) on the output error and the pseudo-controller error. It is interesting to note that these nonlinear terms are constituted precisely by the squares of the variable coefficients of the unknown parameters appearing in the state equation corresponding to the chosen regulated output variable. This comparison becomes more striking when we write down the dynamical controller expression for the non-uncertain boost converter circuit, obtained from an average input–output linearization approach.[14]

$$\dot{\mu} = \frac{1}{\hat{\theta}_1 \zeta_2} \{ - c_1 c_2 (\zeta_1 - X_1) - (c_1 - c_2) [- \theta_1 (1 - \mu) \zeta_2 + \theta_4]$$
$$+ \theta_1 (1 - \mu) [\theta_2 (1 - \mu) \zeta_1 - \theta_3 \zeta_2]\} \tag{32}$$

It should be stressed that the output μ of the dynamical controller (31) constitutes the *computed* duty ratio function. The *actual* duty ratio function, denoted by μ_a, is obtained from the following bounding operation on μ,

$$\mu_a(t) = \begin{cases} 1 & \text{for } \mu(t) \geqslant 1 \\ \mu(t) & \text{for } 0 \leqslant \mu(t) < 1 \\ 0 & \text{for } \mu(t) < 0 \end{cases} \tag{33}$$

The above physical restriction on the values of the computed duty ratio function results in local stabilization of the average input inductor current. This is a well-known limitation of linear and nonlinear feedback control designs for dc to dc power supplies.[1,14]

Summarizing, the adaptive controller for the average system is given by (31), with the estimates of the parameters obtained from the integration of (29). The duty ratio synthesizer for the PWM regulated system is obtained by replacing the average state variables ζ_1, ζ_2, appearing in the controller expressions, respectively with the actual state variables $I(t)$ and $V(t)$.

2.1.2. Summary of Adaptive Dynamical Controller Expression. We next summarize the controller expressions in terms of the original state variables I and V. The computed duty ratio function is given by,

$$\begin{aligned}\dot{\mu} = \frac{1}{\hat{\theta}_1 V_2}\{ &- c_1 c_2 (I - X_1) - (c_1 + c_2)[-\hat{\theta}_1(1-\mu)V + \hat{\theta}_4] \\ &+ \hat{\theta}_1(1-\mu)[\hat{\theta}_2(1-\mu)I - \hat{\theta}_3 V] - [\gamma_{44} + \gamma_{11}(1-\mu)^2 V^2] \\ &\times [(I - X_1) + c_1(-\hat{\theta}_1(1-\mu)V + \hat{\theta}_4 + c_1(I - X_1))]\}\end{aligned} \tag{34}$$

where $X_1 = \zeta_1(U)$ is the desired average value of the input inductor current.

The actual duty ratio function is obtained by bounding the controller output responses to the closed interval [0, 1].

The parameter estimation update law is given by

$$\begin{bmatrix} \dot{\hat{\theta}}_1 \\ \dot{\hat{\theta}}_2 \\ \dot{\hat{\theta}}_3 \\ \dot{\hat{\theta}}_4 \end{bmatrix} = \Gamma \begin{bmatrix} -\{(I - X_1) + [-\hat{\theta}_1(1-\mu)V + \hat{\theta}_4 + c_1(I - X_1)]c_1\}(1-\mu)V \\ [-\hat{\theta}_1(1-\mu)V + \hat{\theta}_4 + c_1(I - X_1)]\hat{\theta}_1(1-\mu)^2 I \\ -[-\hat{\theta}_1(1-\mu)V + \hat{\theta}_4 + c_1(I - X_1)]\hat{\theta}_1(1-\mu)V \\ (I - X_1) + [-\hat{\theta}_1(1-\mu)V + \hat{\theta}_4 + c_1(1 - X_1)]c_1 \end{bmatrix} \tag{35}$$

The control input u is given by equation (2).

2.2. *The Buck–Boost Converter*

In this section we briefly summarize the steps leading to a dynamical adaptive duty ratio synthesizer for the Buck–Boost converter based on a modification of the 'non-overparametrized' backstepping algorithm. The circuit is depicted in Figure 2. In the following formulae X_1 stands for the desired value of the average input inductor current, i.e. $X_1 = \zeta_1(U)$. The diagonal matrix Γ is constituted by parameter adaptation gains which are design constants to be chosen.

Buck–Boost converter model

$$\begin{aligned}\dot{I}(t) &= \frac{1}{L}(1-u)V(t) + \frac{E}{L}u \\ \dot{V}(t) &= -\frac{1}{C}(1-u)I(t) - \frac{1}{RC}V(t) \\ y &= I(t)\end{aligned} \tag{36}$$

Definition of uncertain system parameter

$$\theta_1 = \frac{1}{L}; \quad \theta_2 = \frac{1}{C}; \quad \theta_3 = \frac{1}{RC}; \quad \theta_4 = \frac{E}{L} \tag{37}$$

Average model of Buck–Boost converter

$$\begin{aligned}\dot{\zeta}_1 &= \theta_1(1-\mu)\zeta_2 + \theta_4\mu \\ \dot{\zeta}_2 &= -\theta_2(1-\mu)\zeta_1 - \theta_3\zeta_2 \\ \eta &= \zeta_i\end{aligned} \tag{38}$$

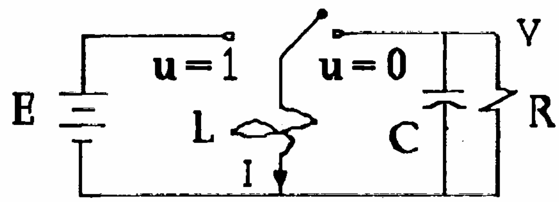

Figure 2. Buck–Boost converter circuit

Equilibrium points of average Buck–Boost converter state variables

$$\zeta_1(U) = \frac{\theta_3\theta_4 U}{\theta_1\theta_2(1-U)^2}; \quad \zeta_2(U) = -\frac{\theta_4 U}{\theta_1(1-U)} \tag{39}$$

Definition of output variable error

$$z_1 = \zeta_1 - X_1 \tag{40}$$

Estimate of first order time derivative of the output error

$$\hat{\dot{z}}_1 = \hat{\theta}_1(1-\mu)\zeta_2 + \hat{\theta}_4\mu \tag{41}$$

Pseudo-controller expression

$$\hat{\theta}_1(1-\mu)\zeta_2 + \hat{\theta}_4\mu = c_1(\zeta_1 - X_1) \tag{42}$$

Output error when pseudo-controller expression is valid

$$\dot{z}_1 = -c_1 z_1 + (\theta - \hat{\theta})^T \begin{bmatrix} (1-\mu)\zeta_2 \\ 0 \\ 0 \\ \mu \end{bmatrix} \tag{43}$$

Definition of pseudo-controller error

$$z_2 = \hat{\theta}_1(1-\mu)\zeta_2 + \hat{\theta}_4\mu + c_1(\zeta_1 - X_1) \tag{44}$$

Observability test

$$\frac{\partial z}{\partial \zeta} = \begin{bmatrix} 1 & 0 \\ c_1 & \hat{\theta}_1(1-\mu) \end{bmatrix} \tag{45}$$

First order time derivative of pseudo-controller error

$$\begin{aligned} \dot{z}_2 = {} & \theta_1(c_1(1-\mu)\zeta_2) + \theta_2(-\hat{\theta}_1(1-\mu)^2\zeta_1) - \theta_3(\hat{\theta}_1(1-\mu)\zeta_2) + \theta_4(c_1\mu) \\ & + \dot{\hat{\theta}}_1(1-\mu)\zeta_2 + (\hat{\theta}_4 - \hat{\theta}_1\zeta_2)\dot{\mu} + \dot{\hat{\theta}}_4\mu \end{aligned} \tag{46}$$

Estimated value of first order time derivative of the pseudo-controller error

$$\begin{aligned} \dot{z}_2 = {} & \hat{\theta}_1(c_1(1-\mu)\zeta_2) + \hat{\theta}_2(-\hat{\theta}_1(1-\mu)^2\zeta_1) - \hat{\theta}_3(\hat{\theta}_1(1-\mu)\zeta_2) + \hat{\theta}_4(c_1\mu) \\ & + \dot{\hat{\theta}}_1(1-\mu)\zeta_2 + (\hat{\theta}_4 - \hat{\theta}_1\zeta_2)\dot{\mu} + \dot{\hat{\theta}}_4\mu \end{aligned} \tag{47}$$

Output error when pseudo-controller expression is not valid

$$\dot{z}_1 = -c_1 z_1 + z_2 + (\theta - \hat{\theta})^T \begin{bmatrix} (1-\mu)\zeta_2 \\ 0 \\ 0 \\ \mu \end{bmatrix} \tag{48}$$

Parameter estimation update law

$$\dot{\hat{\theta}} = z_1 \Gamma \begin{bmatrix} (1-\mu)\zeta_2 \\ 0 \\ 0 \\ \mu \end{bmatrix} + z_2 \Gamma \begin{bmatrix} -c_1(1-\mu)\zeta_2 \\ -\hat{\theta}_1(1-\mu)^2\zeta_1 \\ \hat{\theta}_1(1-\mu)\zeta_2 \\ c_1\mu \end{bmatrix} \tag{49}$$

Adaptive computed duty ratio synthesizer

$$\begin{aligned} \dot{\mu} = \frac{1}{\hat{\theta}_4 - \hat{\theta}_1\zeta_2} \{ & -c_1c_2(\zeta_1 - X_1) - (c_1 + c_2)[\hat{\theta}_1(1-\mu)\zeta_2 + \hat{\theta}_4\mu] \\ & -\hat{\theta}_1(1-\mu)[-\hat{\theta}_2(1-\mu)\zeta_1 - \hat{\theta}_3\zeta_2] \\ & -[\gamma_{44}\mu^2 + \gamma_{11}(1-\mu)^2\zeta_2^2][(\zeta_1 - X_1) + c_1(\hat{\theta}_1(1-\mu)\zeta_2 + \hat{\theta}_4\mu + c_1(\zeta_1 - X_1)]\} \end{aligned} \tag{50}$$

Comparison with non-adaptive controller

$$\begin{aligned} \dot{\mu} = \frac{1}{\theta_4 - \theta_1\zeta_2} \{ & -c_1c_2(\zeta_1 - X_1) - (c_1 + c_2)[\theta_1(1-\mu)\zeta_2 + \theta_4\mu] \\ & -\theta_1(1-\mu)[-\theta_2(1-\mu)\zeta_1 - \theta_3\zeta_2]\} \end{aligned} \tag{51}$$

Summary of Adaptive Controller Expressions in Terms of Actual State Variables Adaptive computed duty ratio synthesizer

$$\begin{aligned} \dot{\mu} = \frac{1}{\hat{\theta}_4 - \hat{\theta}_1 V} \{ & -c_1c_2(I - X_1) - (c_1 + c_2)[\hat{\theta}_1(1-\mu)V + \hat{\theta}_4\mu] \\ & -\hat{\theta}_1(1-\mu)[-\hat{\theta}_2(1-\mu)I - \hat{\theta}_3 V] - [\gamma_{44}\mu^2 + \gamma_{11}(1-\mu)^2V^2] \\ & \times [(I - X_1) + c_1(\hat{\theta}_1(1-\mu)\mathrm{V} + \hat{\theta}_4\mu + c_1(I - X_1))]\} \end{aligned} \tag{52}$$

where the estimated parameters $\hat{\theta}_i$, $i = 1, 2, 3, 4$, are obtained from the following update laws started from arbitrary initial conditions,

$$\begin{bmatrix} \dot{\hat{\theta}}_1 \\ \dot{\hat{\theta}}_2 \\ \dot{\hat{\theta}}_3 \\ \dot{\hat{\theta}}_4 \end{bmatrix} = \Gamma \begin{bmatrix} -\{(I - X_1) + c_1[\hat{\theta}_1(1-\mu)V + \hat{\theta}_4\mu + c_1(I - X_1)]\}(1-\mu)V \\ -[\hat{\theta}_1(1-\mu)V + \hat{\theta}_4\mu + c_1(I - X_1)]\hat{\theta}_1(1-\mu)^2 I \\ -[\hat{\theta}_1(1-\mu)V + \hat{\theta}_4\mu + c_1(I - X_1)]\hat{\theta}_1(1-\mu)V \\ \{(1 - X_1) + c_1[\hat{\theta}_1(1-\mu)V + \hat{\theta}_4\mu + c_1(I - X_1)]\}\mu \end{bmatrix} \tag{53}$$

The components of the positive definite diagonal matrix Γ constitute design parameters to be chosen.

Actual duty ratio function

$$\mu_a(t) = \begin{cases} 1 & \text{for } \mu(t) \geqslant 1 \\ \mu(t) & \text{for } 0 < \mu(t) < 1 \\ 0 & \text{for } \mu(t) \leqslant 0 \end{cases} \tag{54}$$

Switch position function

$$u = \begin{cases} 1 & \text{for } t_k \leqslant t < t_k + \mu_a(t_k)T \\ 0 & \text{for } t_k + \mu_a(t_k)T \leqslant t \leqslant t_k + T \end{cases}$$
$$t_k + T = t_{k+1},\ k = 0, 1, \ldots \tag{55}$$

3. SIMULATION RESULTS

Simulations were carried out to assess the adaptively controlled behaviour of both converters. In order to test the robustness of the proposed regulation schemes with respect to external perturbation inputs, perturbed models of the actual PWM controlled converter were used in the simulations. The perturbed models included an external stochastic perturbation input, v, additively influencing the external source voltage represented by E. Thus, in the Boost converter case the adaptive dynamical controller was used on the perturbed model

$$\dot{I}(t) = -\frac{1}{L}(1-u)V(t) + \frac{E + v(t)}{L}u$$
$$\dot{V}(t) = \frac{1}{C}(1-u)I(t) - \frac{1}{RC}V(t)$$
$$y = I(t) \tag{56}$$

i.e. the external voltage source E was assumed to include an unmatched additive noisy input v affecting the behaviour of all converter during the 'on' and 'off' stages of the switchings.

In the Buck–Boost converter simulations the perturbated model was taken to be

$$\dot{I}(t) = \frac{1}{L}(1-u)V(t) + \frac{E + v(t)}{L}u$$
$$\dot{V}(t) = -\frac{1}{C}(1-u)I(t) - \frac{1}{RC}V(t)$$
$$y = I(t) \tag{57}$$

The following 'unknown' values of the circuit parameters, were used for simulation purposes

$$C = 181{\cdot}82\mu\text{F};\quad L = 0{\cdot}27\ \text{mH};\quad R = 2{\cdot}44\ \Omega;\quad E = 14{\cdot}667\ \text{Volts}$$

for both converters. These values of the circuit components yield the following actual values of the model parameters

$$\theta_1 = 3{\cdot}6 \times 10^3;\quad \theta_2 = 5{\cdot}5 \times 10^3;\quad \theta_3 = 2{\cdot}25 \times 10^3;\quad \theta_4 = 52{\cdot}8 \times 10^3$$

The sampling frequency was set to be 100 KHz and the random noise amplitude was set to be 2·44 Volts (16 per cent of the value of E).

Figure 3 depicts the dynamic adaptively regulated state responses of the Boost converter. The figure also shows the evolution of the estimated parameter values as obtained from the designed

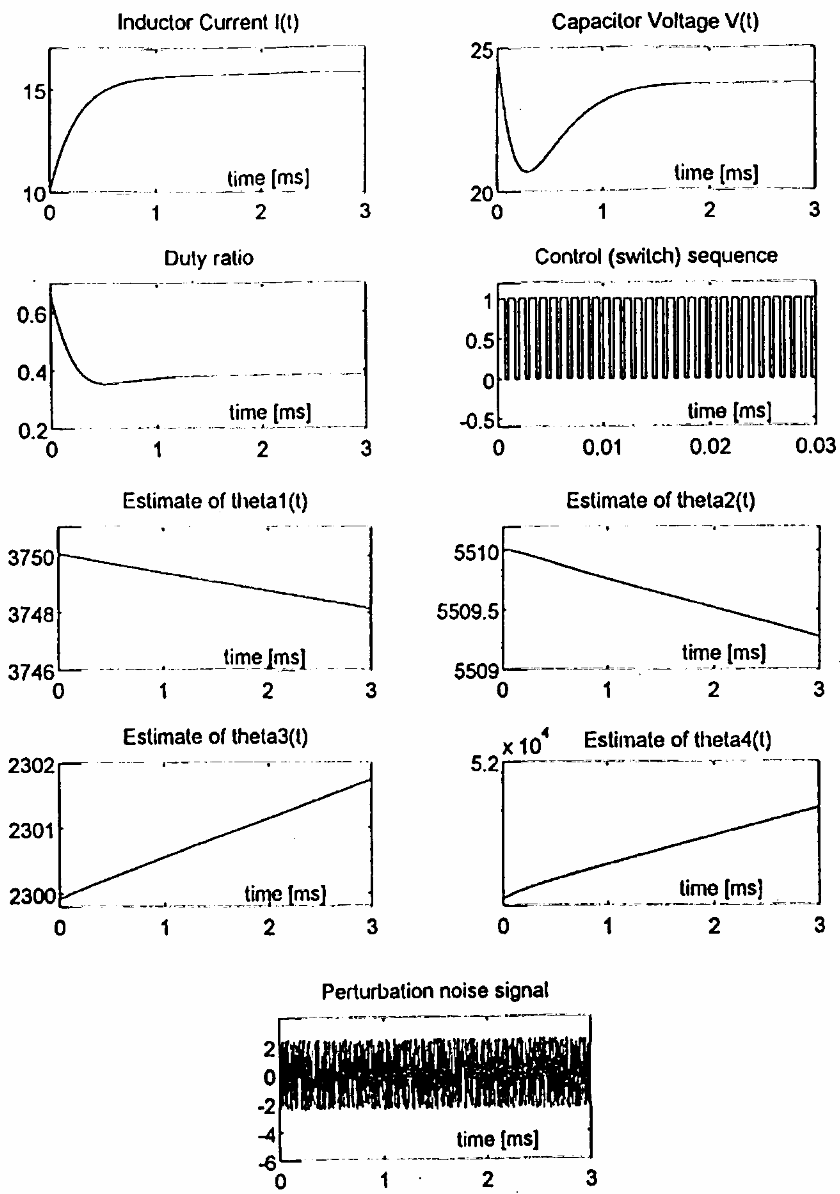

Figure 3. Dynamic adaptively controlled state trajectories of perturbed Boost converter, evolution of controller parameter estimates and perturbation noise signal

update law. The duty ratio function and a small portion of the switchings actions are also depicted in this figure. Finally, a sample of the perturbation noise input $v(t)$ is also shown.

The desired equilibrium value for the average input inductor current was set to be $I(t) = 15{\cdot}75$ amp. The obtained steady state equilibrium value for the average output capacitor

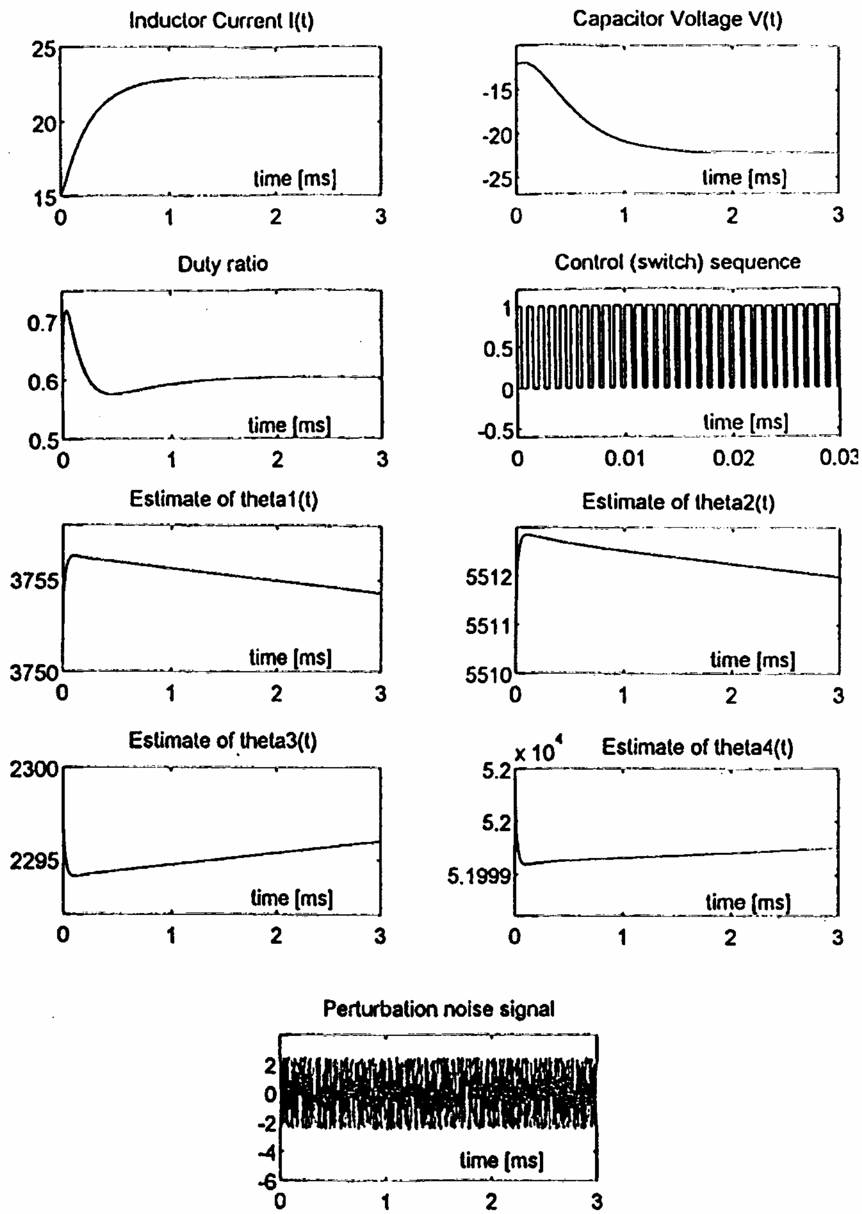

Figure 4. Dynamic adaptively controlled state trajectories of perturbed Buck–Boost converter, evolution of controller parameter estimates and perturbation signal

voltage was $V = 23{\cdot}77$ Volts. The duty ratio function corresponding to this equilibrium is $\mu = U = 0{\cdot}38$. The regulated output variable, $I(t)$, is seen to converge asymptotically towards the desired equilibrium value.

Figure 4 shows the dynamic adpatively regulated state responses of the Buck–Boost converter. The figure also shows the evolution of the estimated parameters arising from the update law. The duty ratio function and a small portion of the switching action are also depicted in this figure. A sample of the perturbation noise input $v(t)$ is also shown.

The chosen equilibrium value for the average input inductor current was $I(t) = 22{\cdot}5$ amp. The value of the average capacitor voltage resulted in $V(t) = -22$ Volts, with a duty ratio $\mu = U = 0{\cdot}6$. The regulated output variable $I(t)$ converges asymptotically towards the required equilibrium value.

4. CONCLUSIONS

An adaptive pulse-width-modulation feedback regulation approach, based on input–output linearization and a direct version of the backstepping algorithm, has been developed which results in the effective input inductor current stabilization for parameter uncertain dc to dc power converters. The approach resorts to the infinite frequency average model of the PWM regulated converter and requires no transformations to either parametric pure, or parametric strict, feedback canonical forms.

The control strategy presented was shown to be remarkably robust with respect to external, yet unmatched, perturbation input signals of stochastic but bounded nature. Computer simulations were carried out to access the robustness of the closed-loop behaviour of the perturbed circuit.

A comparison of the adaptive controllers expressions with those corresponding to the non-adaptive controllers, obtained by pole placement on the average nominal system, was carried out. This comparison reveals that, apart from use of the parameter estimates in the adaptive version of the controller, a nonlinear term invariably appears which contains a weighted sum of the output controller error and the pseudo-controller error multiplied by the sum of the squares of the variable coefficients associated with the unknown parameters in the first order time derivative of the regulated output variable.

A need has also arisen to overcome the non-physically appealing linear parametrization restriction, explicit in the model assumptions of the adaptive backstepping calculation procedure. In average models of dc to dc power supplies, a nonlinear parametrization is inherent which involves the lumped circuit component values R, C, L and E. This case may now be handled from the interesting general perspective proposed by Karsenti and Lamnabhi-Lagarrigue.[18]

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