Pergamon

PII: S0005-1098(96)00207-5

Automatica, Vol. 33, No. 4, pp. 499–513, 1997
© 1997 Elsevier Science Ltd. All rights reserved
Printed in Great Britain
0005-1098/97 $17.00 + 0.00

(9)

Passivity-Based Controllers for the Stabilization of DC-to-DC Power Converters*

H. SIRA-RAMIREZ,† R. A. PEREZ-MORENO,† R. ORTEGA‡ and M. GARCIA-ESTEBAN

Dynamical feedback controllers providing the synthesis of the duty ratio function in pulse-width-modulation (PWM) feedback controlled DC-to-DC power converters are derived using passivity-based considerations. The approach is shown to be naturally suited for the average-based regulation of several power supplies due to the Lagrangian nature of their average PWM models.

Key Words—Power supplies; passivity-based compensation; pulse-width modulation; Euler–Lagrange systems.

Abstract—Passivity-based feedback controllers are derived for the indirect stabilization of the average output voltage in pulse-width-modulation (PWM) controlled DC-to-DC power converters of the 'boost', 'buck–boost', and 'buck' types. The controller design is carried out on the basis of well-known average PWM models of such circuits. The average models are first shown to be Euler–Lagrange systems corresponding to a suitable set of average Euler–Lagrange parameters. The proposed regulators are based on an 'energy shaping plus damping injection' scheme, achievable through nonlinear dynamical feedback. The performance of the proposed passivity-based duty ratio synthesis policies is tested for the 'boost' converter case. The regulating feedback law, derived on the basis of a 'boost' model composed of ideal switches and ideal circuit components, is assessed, via computer simulations, on a realistic stochastically perturbed switched converter model, including parasitic resistances and parasitic voltage sources. © 1997 Elsevier Science Ltd.

1. INTRODUCTION

The feedback regulation of DC-to-DC power supplies is, broadly speaking, accomplished through either pulse-width-modulation (PWM) feedback strategies, or by inducing appropriate stabilizing sliding regimes. PWM control of these devices is treated in several books, among them those by Severns and Bloom (1982), Kassakian *et al.* (1991), and Rashid (1992). The topic has also been extensively treated by, among many other authors, Sira-Ramirez and Lischinsky-Arenas (1991). Sliding mode control of switched power supplies was first treated by Venkataramanan *et al.* (1985), and more recently by Sabanovic *et al.* (1993) in the context of motion control systems.

Both feedback controller design approaches entirely overlook the energy-related physical properties of either the original converter circuit or of its closed-loop structure. The controller design philosophy primarily insists on a mathematically motivated average closed-loop linearization, which is geared to solve the stabilization, or tracking, task.

In the context of PWM feedback policies, *state average models* of DC-to-DC power converters have been developed by Middlebrook and Ćuk (1976) from an approximate discretization viewpoint. An interesting refinement and generalization of this class of average PWM models has been proposed by Krein *et al.* (1990) using the analytical theory of averaging of differential equations. The same average models, known as *infinite switching frequency average models,* have been derived by Sira-Ramirez (1989), using as a justification Filippov's (geometric) averaging viewpoint (see Filippov, 1988).

All the average PWM models so far developed have been justified from a purely mathematical viewpoint, without due regard to their possible physical significance. It was one of our objectives in this study to demonstrate that average PWM models of power converters can be derived from the energy properties of the switched electrical circuit. The main advantage of underscoring the often overlooked physical properties of DC-to-DC power converters is that, in this way, we can advantageously exploit these properties at the

* Received 10 March 1995; received in final form 1 April 1996. This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor Henk Nijmeijer under the direction of Editor Tamer Başar. *Corresponding author* Professor Hebertt Sira-Ramirez. Tel. +58 74-442341; Fax +58 „Hebertt Sira-Ramirez. *Tel.* +58 74-442341; Fax +58 74-402847; E-mail isira@ing.ula.ve.

† Avenida Las Américas, Edifico Residencias El Roble, Piso 6, Apartamento F-6, Mérida 5101, Venezuela.

‡ Université de Compiègne, URA C.N.R.S. 817, BP 649, 60206 Compiègne, France.

§ Centro de Instrumentos, Universidad Nacional Autónoma de México, México, D.F., México. On temporary leave from the Instituto de Cibernética, Universidad Politécnica de Cataluña, Spain.

feedback controller design stage. In particular, we would like to explore the relevance and implications of a 'passivity-based' approach in the feedback duty ratio synthesis problem (for background on the passivity-based methodology for controller design see Takegaki and Arimoto (1981), and for the subsequent developments see Ortega and Spong (1989), Berghuis and Nijmeijer (1993) and Brogliato *et al.* (1995).

A passivity-based controller design technique would be directly, and most naturally, applicable to the average-based PWM regulation of DC-to-DC power converters, provided one can demonstrate that such idealized, mathematically motivated, models actually correspond to systems derivable from classical Euler–Lagrange (EL) dynamics considerations.

In this paper, an EL dynamics modeling approach, which establishes the relevant physical characteristics of well-known average models of DC-to-DC power converters, is first presented. In particular, we prove that the traditional average PWM models of switched converters of the 'boost', 'buck–boost', and 'buck' types are indeed EL systems. The approach consists in establishing a suitable set of average EL *parameters* modulated by the duty ratio function. This average set of parameters is derived on the basis of, both a 'consistency' and an 'intermediacy' requirement with respect to the EL parameters of the two intervening electrical circuit topologies. Interestingly, the derived average PWM models entirely coincide with the well-known *state average models* of DC-to-DC power converters, introduced by Middlebrook and Ćuk (1976), and they also coincide with the *infinite switching frequency* models, derived in Sira-Ramirez (1989) and unified by Amran *et al.* (1991).

Due to the nonminimum phase nature of the average output voltage variable, a direct application of the passivity-based design method, aimed primarily at output-voltage regulation, leads to an unstable dynamical feedback controller. This is due to an underlying partial inversion of the average system model, carried out at the controller design stage. For this reason, an *indirect* approach, consisting of output-voltage regulation through inductor current stabilization is undertaken. Indirect controller design for nonminimum phase systems has been justified, for nonlinear systems, in the work of Benvenuti *et al.* (1992) and, in the context of DC-to-DC power converters, in the work of Sira-Ramirez and Lischinsky-Arenas (1991). The indirect control technique also naturally arises, from module-theoretic results, in sliding mode control of linear multivariable nonminimum phase systems, as inferred from the work of Fliess and Sira-Ramirez (1993).

The performance of the derived, indirect, dynamical state feedback controllers was successfully tested, via computer simulations, for the 'boost' converter example. The model used for the switched boost converter included an unmodeled stochastic perturbation input, directly affecting the external voltage source, as well as unmodelled parasitic resistances attaches to each one of the circuit elements. The model for the switching arrangement, usually consisting of a transistor and a diode, was taken, as proposed by Czarkowsi and Kazimierczuk (1993), to be an ideal switch, combined with lumped forward (i.e. ON) resistances and a parasitic voltage source, associated with the conducting state of the diode.

This paper is organized as follows. Section 2 presents an EL dynamics-based derivation of the average PWM models of the 'boost', 'buck–boost', and 'buck' converters. An ideal equivalent circuit realization is also provided for the three kinds of converter. Section 3 develops the passivity-based feedback controllers and demonstrates, for the 'boost' and 'buck–boost' converter cases, the nonminimum phase character of direct output voltage regulation options. The 'buck' converter case does not exhibit a nonminimum phase character and, therefore, direct and indirect output voltage regulation schemes are seen to be equally feasible. The simulation results are presented in Section 4. Section 5 contains the conclusions, and suggestions for further research in this area.

2. AVERAGE MODELS OF DC-to-DC POWER CONVERTERS AS EL SYSTEMS

The results of this section, regarding the EL nature of average PWM DC-to-DC power converters, extend the work found in Sira-Ramirez and Delgado de Nieto (1995), where only the 'boost' converter case is treated.

2.1. *Generalities about EL electric circuits*

The EL dynamics of an electric circuit, containing no magnetic couplings between its different branches, is classically characterized by the following set of nonlinear differential equations (see Meisel 1966):

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathscr{L}}{\partial \dot{q}}\right) - \frac{\partial \mathscr{L}}{\partial q} = -\frac{\partial \mathscr{D}}{\partial \dot{q}} + \mathscr{F}_q, \qquad (1)$$

where $\dot{q}$ is the vector of flowing currents and q represents their time integrals, or *electric* *charges*. The vector of electric charges constitutes the *generalized coordinates* describing the circuit. This vector is assumed to have n components, represented by $q_1, \ldots, q_n$. The scalar function $\mathscr{L}$ is the *Lagrangian* of the system, defined as the difference between the *magnetic co-energy* of the circuit, denoted by $\mathscr{T}(\dot{q}, q)$, and the *electric field energy* of the circuit, denoted by $\mathscr{V}(q)$, i.e.

$$\mathscr{L}(\dot{q}, q) = \mathscr{T}(\dot{q}, q) - \mathscr{V}(q). \quad (2)$$

The function $\mathscr{D}(\dot{q})$ is the *Rayleigh dissipation cofunction* of the system. The vector $\mathscr{F}_g = (\mathscr{F}_{q_1}, \ldots, \mathscr{F}_{q_n})$ represents the ordered components of the set of *generalized forcing functions*, or *voltage sources*, associated with the generalized coordinates.

EL circuits are thus generally represented by the set of equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathscr{T}}{\partial \dot{q}}\right) - \frac{\partial \mathscr{T}}{\partial q} + \frac{\partial \mathscr{V}}{\partial q} = -\frac{\partial \mathscr{D}}{\partial \dot{q}} + \mathscr{F}_q \quad (3)$$

Following Ortega *et al.* (1995), we refer to the set of functions ($\mathscr{T}$, $\mathscr{V}$, $\mathscr{D}$, $\mathscr{F}$) as the *EL parameters* of the circuit, and simply express a circuit Σ by means of the ordered quadruple:

$$\Sigma = (\mathscr{T}, \mathscr{V}, \mathscr{D}, \mathscr{F}). \quad (4)$$

2.2. *The 'boost' converter*

2.2.1. *The switch-regulated model for the 'boost' converter.* Consider the switch-regulated 'boost' converter circuit in Fig. 1. The differential equations describing the circuit are

$$\begin{aligned} \dot{x}_1 &= -(1-u)\frac{1}{L}x_2 + \frac{E}{L}, \\ \dot{x}_1 &= (1-u)\frac{1}{C}x_1 - \frac{1}{RC}x_2, \end{aligned} \quad (5)$$

where x_1 and x_2 represent the input inductor current and the output capacitor voltage variables, respectively. The positive quantity E represents the constant-voltage value of the external voltage source. The variable u denotes the switch position function, acting as a control input. Such a control input takes place in the discrete set $\{0, 1\}$.

A PWM policy regulating the switch position function u may be specified as follows:

$$u(t) = \begin{cases} 1, & \text{for } t_k \le t < t_k + \mu(t_k)T \\ 0, & \text{for } t_k + \mu(t_k)T \le t < t_k + T \end{cases}$$
$$t_{k+1} = t_k + T;\ k = 0, 1, \ldots, \quad (6)$$

where t_k represents a sampling instant; the parameter T is the fixed sampling period, also called the *duty cycle*; the sampled values of the state vector $x(t)$ of the converter are denoted by $x(t_k)$. The function $\mu(\cdot)$ is the *duty ratio function,* truly acting as an external control input to the average PWM model of the converter (see Sira-Ramirez, 1989). The value of the duty ratio function $\mu(t_k)$ determines, at every sampling instant t_k, the width of the upcoming ON pulse as $\mu(t_k)T$ (during this period the switch is fixed at the position represented by $u = 1$). The actual duty ratio function $\mu(\cdot)$ is evidently a function limited to take values on the closed interval $[0, 1]$ of the real line.

2.2.2. *A Lagrangian formulation of the average PWM model.* We consider separately the Lagrangian dynamics formulation of the two circuits associated with each of the two possible positions of the regulating switch. Of course, the aim of carrying out this formulation is not to rederive the differential equations governing the circuit at each switch position. These may be trivially found from (5) itself. Our purpose is to gain some insight on the physical effects of the switching action in terms of the EL parameters of the two circuit topologies. In order to use standard notation, we rewrite the input current x_1 in terms of the derivative of the circulating electric charge q_{L}, as $\dot{q}_{\mathrm{L}}$. Also the capacitor voltage x_2 will be written as q_{C}/C where q_{C} is the electrical charge stored in the output capacitor.

Consider then $u = 1$. The resulting circuit is as shown in Fig. 2. In this case, two separate, or decoupled, circuits are clearly obtained, and the corresponding Lagrangian dynamics formulation can be carried out as follows.

Define $\mathscr{T}_1(\dot{q}_{\mathrm{L}})$ and $\mathscr{V}_1(q_{\mathrm{C}})$ as the magnetic co-energy and electric field energy of the circuit, respectively. We denote by $\mathscr{D}_1(\dot{q}_{\mathrm{C}})$ the Rayleigh

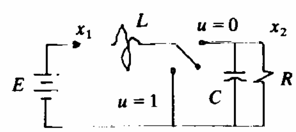

Fig. 1. Boost converter circuit.

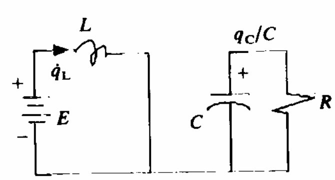

Fig. 2. Boost converter circuit ($u = 1$).

dissipation cofunction of the circuit. These quantities are readily found to be

$$\mathcal{T}_1(\dot{q}_L) = \tfrac{1}{2}L\dot{q}_L^2; \quad \mathcal{V}_1(g_C) = \frac{1}{2C}q_C^2,$$
$$\mathcal{D}_1(\dot{q}_C) = \tfrac{1}{2}R\dot{q}_C^2; \quad \mathcal{F}_{q_L} = E, \quad \mathcal{F}_{q_C} = 0, \tag{7}$$

where $\mathcal{F}_{q_L}$ and $\mathcal{F}_{q_C}$ are the *generalized forcing* functions associated with the coordinates q_L and q_C, respectively.

Evidently, the EL equations associated with these definitions immediately rederive (5), with $u = 1$, as can be verified by direct use of the general equation (2), or (3), on the set of EL parameters given in (7).

Consider now the case $u = 0$. The resulting circuit is as shown in Fig. 3. The corresponding Lagrangian dynamics formulation is carried out similarly to the case $u = 1$. That is, define $\mathcal{T}_0(\dot{q}_L)$ and $\mathcal{V}_0(q_L)$ as the magnetic co-energy and the electric field energy of the circuit, respectively. We denote by $\mathcal{D}_0(\dot{q}_L, \dot{q}_C)$ the Rayleigh dissipation cofunction of the circuit. These quantities are readily found to be

$$\mathcal{T}_0(\dot{q}_L) = \tfrac{1}{2}L\dot{q}_L^2; \quad \mathcal{V}_0(q_C) = \frac{2}{2C}q_C^2,$$
$$\mathcal{D}_0(\dot{q}_L, \dot{q}_C) = \tfrac{1}{2}R(\dot{q}_C - \dot{q}_L)^2, \tag{8}$$
$$\mathcal{F}_{q_L} = E, \quad \mathcal{F}_{q_C} = 0,$$

where, as before, $\mathcal{F}_{q_L}$ and $\mathcal{F}_{q_L}$ are the *generalized forcing* functions associated with the coordinates q_L and q_C, respectively.

Evidently, application of the general EL equations (3)–(8) immediately rederives (5), with $u = 0$, as can be easily verified.

The EL parameters of the two circuits, generated by the different switch-position values, result in identical magnetic co-energies, electric field energies, and forcing functions. The switching action merely changes the Rayleigh dissipation cofunction between the values $\mathcal{D}_0(\dot{q}_L)$ and $\mathcal{D}_1(\dot{q}_L, \dot{q}_C)$. Therefore, the *dissipation structure* of the system is the only one directly affected by the switch position function u.

Note that, according to the PWM switching policy (6), on every sampling interval of period T, the Rayleigh dissipation cofunction $\mathcal{D}_1(\dot{q}_L)$ is valid over only a fraction of the sampling period given by $\mu(t_k)$, while the Rayleigh dissipation cofunction $\mathcal{D}_1(\dot{q}_L, \dot{q}_L)$ is valid a fraction of the sampling period equal to $(1 - \mu(t_k))$.

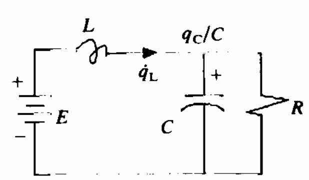

Fig. 3. Boost converter circuit ($u = 0$).

There, are of course, a variety of ways in which one could reasonably propose an average value of the Rayleigh dissipation cofunction for a circuit of the form (5), undergoing a switching policy of the form (6). One possible way is to propose the following set of EL parameters:

$$\mathcal{T}_\mu(\dot{q}_L) = \tfrac{1}{2}L\dot{q}_L^2; \quad \mathcal{V}_\mu(q_C) = \frac{1}{2C}q_C^2$$
$$\mathcal{D}_\mu(\dot{q}_L, \dot{q}_C) = \tfrac{1}{2}R[\dot{q}_C - (1-\mu)\dot{q}_L]^2; \tag{9}$$
$$\mathcal{F}^\mu_{q_L} = E, \quad \mathcal{F}^\mu_{q_C} = 0.$$

Note that in the cases where μ takes the extreme saturation values $\mu = 1$ or 0, one recovers, respectively, the dissipation cofunctions $\mathcal{D}_1(\dot{q}_C)$ in (7) and $\mathcal{D}_0(\dot{q}_L, \dot{q}_C)$ in (2.8) from the proposed average dissipation cofunction, $\mathcal{D}_\mu(\dot{q}_L, \dot{q}_L\mathrm{C})$, of equation (9). Indeed, such a 'consistency' condition is verified by noting that

$$\mathcal{D}_\mu(\dot{q}_L, \dot{q}_C)|_{\mu=0} = \mathcal{D}_0(\dot{q}_L, \dot{q}_C),$$
$$\mathcal{D}_\mu(\dot{q}_L, \dot{q}_C)|_{\mu=1} = \mathcal{D}_1(\dot{q}_C).$$

Also, it is easy to see that the proposed average Rayleigh dissipation cofunction satisfies an important 'intermediacy' condition of the form

$$\min\{\mathcal{D}_0(\dot{q}_L, \dot{q}_C), \mathcal{D}_1(\dot{q}_C)\} < \mathcal{D}_\mu(\dot{q}_L, \dot{q}_C) < \max\{\mathcal{D}_0(\dot{q}_L, \dot{q}_C), \mathcal{D}_1(\dot{q}_C)\}$$

for any μ lying in the open interval $(0, 1)$.

We note that the Lagrangian function associated with the above-defined average EL parameters is actually *invariant* with respect to the switch position function. Nevertheless, to keep the notation consistent, we denote it by

$$\mathcal{L}_\mu = \mathcal{T}_\mu(\dot{q}_L) - \mathcal{V}_\mu(q_C) = \tfrac{1}{2}L\dot{q}_L^2 - \frac{1}{2C}q_C^2 \tag{10}$$

One then proceeds, using the EL equations (3), to obtain the differential equations defining the average PWM model which corresponds to the proposed average EL parameters (9). Such equations are

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}_\mu}{\partial \dot{q}_L}\right) - \frac{\partial \mathcal{L}_\mu}{\partial q_L} = -\frac{\partial \mathcal{D}_\mu}{\partial \dot{q}_L} + \mathcal{F}^\mu_{q_L},$$
$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}_\mu}{\partial \dot{q}_C}\right) - \frac{\partial \mathcal{L}_\mu}{\partial q_L} = -\frac{\partial \mathcal{D}_\mu}{\partial \dot{q}_C} + \mathcal{F}^\mu_{q_C}. \tag{11}$$

Evaluation of (11) results in the following system of differential equations:

$$
\begin{aligned}
L\ddot{q}_{\mathrm{L}} &= (1-\mu)R[\dot{q}_{\mathrm{C}} - (1-\mu)\dot{q}_{\mathrm{L}}] + E, \\
\frac{q_{\mathrm{C}}}{C} &= -R[\dot{q}_C - (1-\mu)\dot{q}_{\mathrm{L}}],
\end{aligned} \tag{12}
$$

which can be rewritten, after substitution of the second equation of (12) into the first, as

$$
\begin{aligned}
\ddot{q}_{\mathrm{L}} &= -(1-\mu)\frac{q_{\mathrm{C}}}{LC} + \frac{E}{L}, \\
\dot{q}_{\mathrm{C}} &= -\frac{1}{RC}q_{\mathrm{C}} + (1-\mu)\dot{q}_{\mathrm{L}}.
\end{aligned} \tag{13}
$$

Using $z_1 = \dot{q}_{\mathrm{L}}$ and $z_2 = q_{\mathrm{C}}/C$ one obtains

$$
\begin{aligned}
\dot{z}_1 &= -(1-\mu)\frac{1}{L}z_2 + \frac{E}{L}, \\
\dot{z}_2 &= (1-\mu)\frac{1}{C}z_1 - \frac{1}{RC}z_2,
\end{aligned} \tag{14}
$$

where we denote by z_1 and z_2 the *average input current* and the *average output capacitor voltage*, respectively, of the PWM regulated 'boost' converter. We establish this distinction with the nonaveraged variables x_1 and x_2 so that the state variables associated with the average PWM model are not mistakenly confused with the *actual* PWM regulated circuit variables.

Note that the proposed average dynamics (14) coincides with the state average model developed by Middlebrook and Ćuk (1976), and with the infinite switching frequency model, or Filippov average model, found in Sira-Ramirez (1989) and Amran *et al.* (1991). To obtain the average model (14), one simply replaces the switch position function u in (5) by the duty ratio function μ, and the actual state variables x_1, x_2 by their averaged values z_1, z_2.

We have thus proven the following proposition.

Proposition 2.1. The state average model of the 'boost' converter (see Middlebrook and Ćuk, 1976), given by (14) is an EL system corresponding to the set of average EL parameters given by (9). These parameters are, in turn, obtained by suitable modulation, through the duty ratio function μ of the EL parameters, given by (7) and (8), which are associated to each one of the intervening circuit topologies arising from a particular value of the switch position function.

For ease of reference we will be using the following, more compact, matrix representation of (14):

$$
\mathscr{D}_{\mathrm{B}}\dot{z} + (1-\mu)\mathscr{J}_{\mathrm{B}}z + \mathscr{R}_{\mathrm{B}}z = \mathscr{E}_{\mathrm{B}}, \tag{15}
$$

where

$$
\begin{aligned}
\mathscr{D}_{\mathrm{B}} &= \begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix}; \quad \mathscr{J}_{\mathrm{B}} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}; \\
\mathscr{R}_{\mathrm{B}} &= \begin{bmatrix} 0 & 0 \\ 0 & 1/R \end{bmatrix}; \quad \mathscr{E}_{\mathrm{B}} = \begin{bmatrix} E \\ 0 \end{bmatrix}.
\end{aligned} \tag{16}
$$

2.2.3. *An ideal circuit realization for the average PWM model.* It is easy to realize that the average model (14) has a circuit-theoretic interpretation by letting the quantity $(1-\mu)z_2$ in the first equation represent a *controlled voltage source,* while also letting the quantity $(1-\mu)z_1$ in the second equation represent a *controlled input current source.* Figure 4 depicts the ideal equivalent circuit describing the average PWM model. In such a circuit, a quadripole connects the 'input' and 'output' circuits, which effectively replaces, in an average sense, the actual switching device.

Consider the isolated quadripole constituted by the ideal controlled sources, as shown in Fig. 5. Note that the (average) input power to the quadripole, expressed as the product of the average input current z_1 times the (reflected) average input voltage $(1-\mu)z_2$ is given by

$$
P_{\mathrm{in}} = \overbrace{z_1}^{\text{Input current}} \; \overbrace{(1-\mu)z_2}^{\text{Input voltage}}. \tag{17}
$$

On the other hand, the (average) output power delivered by the quadripole, expressed as the product of the average output current $(1-\mu)z_1$ times the output voltage z_2 is given by

$$
P_{\mathrm{out}} = \overbrace{(1-\mu)z_1}^{\text{Output current}} \; \overbrace{z_2}^{\text{Output voltage}}. \tag{18}
$$

In other words, the quadripole is a lossless, ideal

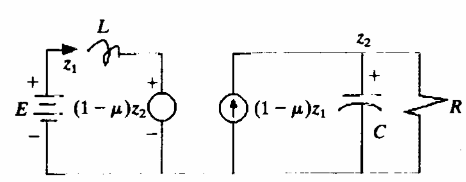

Fig. 4. Equivalent circuit of the average PWM model of the 'boost' converter circuit.

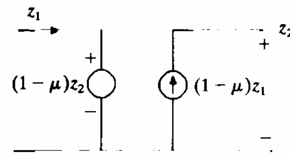

Fig. 5. Ideal transformer representing the average PWM switch position function.

(average) power transfering device satisfying

$$P_{\text{in}} = P_{\text{out}}. \tag{19}$$

The average input voltage to the quadripole $(1-\mu)z_2$ is amplified to the value z_2 at the output, while the input current to the quadripole z_1 is attenuated to the value $(1-\mu)z_1$ at the output. The switching element has thus been effectively replaced by an *ideal transformer* with *turn ratio* parameter given by $(1-\mu)$.

2.2.4. *Input–output and internal stability properties.* Given the results of Ortega *et al.* (1995) concerning EL systems, it is expected that the averaged circuit dynamics (15) satisfies the following energy balance equation:

$$\underbrace{H(t)-H(0)}_{\text{Stored energy}} + \underbrace{\frac{1}{RC^2}\int_0^t q_C^2(\tau)\,\mathrm{d}\tau}_{\text{Dissipated energy}} = \underbrace{\int_0^t E\dot{q}_L(\tau)\,\mathrm{d}\tau}_{\text{Supplied energy}},$$

where $H(t)=\frac{1}{2}z^{\mathrm{T}}\mathcal{D}_{\mathrm{B}}z=\mathcal{T}_\mu+\mathcal{V}_\mu$ is the *total energy* of the average circuit model. This follows trivially by taking the time derivative of $H(t)$ along the trajectories of (15) and noting the skew symmetry of $\mathcal{J}_{\mathrm{B}}$. The energy balance equation above also reveals that the forces $(1-\mu)\mathcal{J}_{\mathrm{B}}z$ appearing in (15) are *workless*, and proves the passivity of the operator $E\rightarrow\dot{q}_L$.

We proceed to establish the relationship between the equilibria of the average output voltage and the average input current. To this end assume a constant duty ratio function $\mu=U$. It easily follows from the average PWM model equations (14) that the corresponding stable equilibrium values for the average input current, denoted by I_d, and the average output voltage, denoted by V_d, are given by

$$I_d=\frac{E}{(1-U)^2R};\quad V_d=\frac{E}{1-U}. \tag{20}$$

Henceforth, given a desired equilibrium value V_d for the output voltage, which corresponds to a constant value of the duty ratio function $\mu=U=1-E/V_d$, the *unique* corresponding equilibrium value for the average input current is given by

$$I_d=\frac{V_d}{R(1-U)}=\frac{1}{RE}V_d^2 \tag{21}$$

This means that if we desire to regulate z_2 towards an equilibrium value V_d which is known to correspond to a steady state value U of the duty ratio function μ, then such a regulation can be *indirectly* accomplished by stabilizing the average input current z_1 towards the corresponding equilibrium value I_d computed from (21).

Now, consider the case where the average output capacitor voltage z_2 is regarded as the output of the average PWM model (14). A straightforward elimination of z_1 from the set of differential equations (14) leads to the following nonlinear input–output differential representation:

$$\ddot{z}_2+\left(\frac{1}{RC}+\frac{\dot{\mu}}{1-\mu}\right)\dot{z}_2+\frac{1}{LC}\left[(1-\mu^2)+\frac{L}{R}\frac{\dot{\mu}}{1-\mu}\right]z_2 = (1-\mu)\frac{E}{LC}. \tag{22}$$

The 'zero dynamics' at an equilibrium point $z_2=V_d$ associated with this input–output representation is obtained by letting $\dot{z}_2=0$ and $\ddot{z}_2=0$ (see Fliess, 1990). The resulting differential equation describing the 'remaining dynamics' of the duty ratio function μ is simply obtained as

$$\dot{\mu}=\frac{R(1-\mu)^2}{LV_d}[E-(1-\mu)V_d]. \tag{23}$$

The equilibrium points of (23) are given by

$$\mu=1;\qquad \mu=1-\frac{E}{V_d}. \tag{24}$$

The equilibrium value $\mu=U=1-(E/V_d)$ has physical significance, provided $V_d>E$. This fact confirms the 'amplifying' features of the 'boost' converter. However, the phase-plane diagram of equation (23), shown in Fig. 6, readily reveals that this equilibrium point is unstable. We conclude that the average PWM model of the 'boost' converter, with output represented by the average capacitor voltage z_2, is actually a *nonminimum phase* system.

Consider now the output of the circuit to be represented by the average input current z_1. One

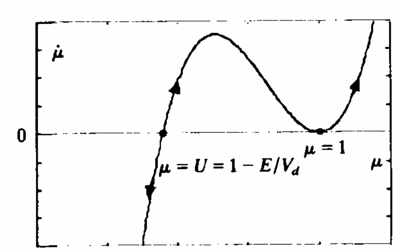

Fig. 6. Zero dynamics of 'boost' converter corresponding to average output voltage.

obtains the following differential input–output representation for the average system:

$$\ddot{z}_1 + \left(\frac{1}{RC} + \frac{\dot{\mu}}{1-\mu}\right)\dot{z}_1 + \left[(1-\mu)^2 \frac{1}{LC}\right] z_1 = \frac{E}{L}\left(\frac{1}{RC} + \frac{\dot{\mu}}{1-\mu}\right). \quad (25)$$

The 'zero dynamics' at an equilibrium point $z_1 = I_d$, associated with the input–output representation (25), is obtained as

$$\dot{\mu} = \frac{1-\mu}{RCE}[(1-\mu)^2 RI_d - E]. \quad (26)$$

The equilibrium points of (23) are given by

$$\mu = 1; \quad \mu = 1 - \sqrt{\frac{E}{RI_d}}; \qquad \mu = 1 + \sqrt{\frac{E}{RI_d}}. \quad (27)$$

The equilibrium value, $\mu = U = 1 - \sqrt{E/RI_d}$, has physical significance provided that RI_d, the average steady state voltage across the load resistor, satisfies $RI_d > E$. This fact confirms, once more, the 'amplifying' character of the 'boost' converter. The phase-plane diagram of equation (26), shown in Fig. 7, reveals that this equilibrium point is now locally stable. We conclude that the average PWM model of the 'boost' converter, with output represented by the average input inductor current $y = z_1$, is a *minimum phase* system.

2.3. *The 'buck–boost' converter circuit*

Consider then the switch-regulated 'buck–boost' converter circuit shown in Fig. 8. The differential equations describing the circuit are given by

$$\dot{x}_1 = (1-u)\frac{1}{L}x_2 + u\frac{E}{L},$$
$$\dot{x}_2 = -(1-u)\frac{1}{C}x_1 - \frac{1}{RC}x_2, \quad (28)$$

where x_1 and x_2 represent the input inductor current and the output capacitor voltage variables, respectively. The positive quantity E is

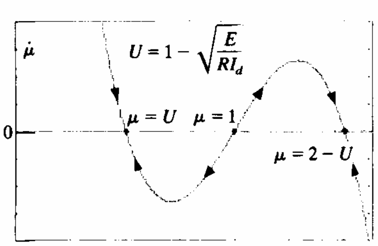

Fig. 7. Zero dynamics of 'boost' converter corresponding to average output voltage.

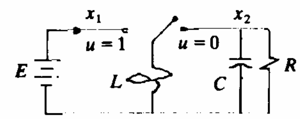

Fig. 8. The 'buck–boost' converter circuit.

the constant value of the external voltage source. The variable u is the switch position function, acting as a control input, taking values in the discrete set $\{0, 1\}$. It is assumed that a PWM regulation policy of the form (6) is available for the determination of the switch position function, as a function of time.

We summarize in the following proposition the developments demonstrating that the average PWM model of a 'buck–boost' converter is a EL system for a suitable set of average EL parameters.

Proposition 2.2. The state average model of the 'buck–boost' converter (see Middlebrook and Ćuk, 1976) given by

$$\dot{z}_1 = (1-\mu)\frac{1}{L}z_2 + \mu\frac{E}{L}$$
$$\dot{z}_2 = -(1-\mu)\frac{1}{C}z_1 - \frac{1}{RC}z_2 \quad (29)$$

is a EL system corresponding to the following set of average EL parameters:

$$\mathcal{T}_\mu(\dot{q}_L) = \tfrac{1}{2}L\dot{q}_L^2; \quad \mathcal{V}_\mu(q_C) = \frac{1}{2C}q_C^2,$$
$$\mathcal{D}_\mu(\dot{q}_L, \dot{q}_C) = \tfrac{1}{2}R[\dot{q}_C + (1-\mu)\dot{q}_L]^2; \quad (30)$$
$$\mathcal{F}^\mu_{q_L} = \mu E; \quad \mathcal{F}^\mu_{q_C} = 0,$$

obtained by suitable modulation, through the duty ratio function μ, of the EL parameters associated with each one of the circuits arising from a particular value of the switch position function $u \in \{0, 1\}$.

$u = 1$:

$$\mathcal{T}_1(\dot{q}_L) = \tfrac{1}{2}L\dot{q}_L^2; \quad \mathcal{V}_1(q_C) = \frac{1}{2C}q_C^2, \quad (31)$$
$$\mathcal{D}_1(\dot{q}_L) = \tfrac{1}{2}R\dot{q}_C^2; \quad \mathcal{F}^1_{q_L} = E; \quad \mathcal{F}^1_{q_C} = 0.$$

$u = 0$:

$$\mathcal{T}_0(\dot{q}_L) = \tfrac{1}{2}L\dot{q}_L^2; \quad \mathcal{V}_0(q_C) = \frac{1}{2C}q_C^2$$
$$\mathcal{D}_0(\dot{q}_L, \dot{q}_C) = \tfrac{1}{2}R(\dot{q}_L + \dot{q}_C)^2; \quad (32)$$
$$\mathcal{F}^0_{q_L} = 0; \quad \mathcal{F}^0_{q_C} = 0.$$

Figure 9 depicts the equivalent circuit of the average PWM regulated dynamics for the 'buck–boost' converter circuit.

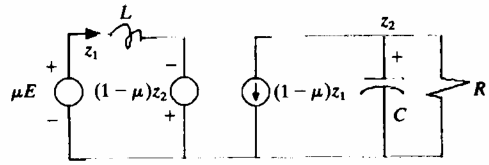

Fig. 9. Equivalent circuit of the average PWM model of the 'buck–boost' converter circuit.

We will be using the following matrix representation of (29):

$$\mathcal{D}_{BB}\dot{z} + (1-\mu)\mathcal{J}_{BB}z + \mathcal{R}_{BB}z = \mu\mathcal{E}_{BB}, \quad (33)$$

where

$$\mathcal{D}_{BB} = \begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix}; \quad \mathcal{J}_{BB} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix};$$
$$\mathcal{R}_{BB} = \begin{bmatrix} 0 & 0 \\ 0 & 1/R \end{bmatrix}; \quad \mathcal{E}_{BB} = \begin{bmatrix} E \\ 0 \end{bmatrix}. \quad (34)$$

2.3.1. *Some additional facts.* Similarly to the 'boost' converter case, one can easily establish the nonminimum phase character of the average model of the PWM regulated 'buck–boost' converter system when the output of the system is taken as the average capacitor voltage z_2. When the output of the system is taken to be the average input inductor current z_1, the resulting input–output system is seen to be locally minimum phase (see Sira-Ramirez and Lischinsky-Arenas, 1991).

Given a constant duty ratio function $\mu = U$, it easily follows from the average PWM equations (29) that the corresponding stable equilibrium values for the average input current, denoted by L_d, and the average output voltage, denoted by V_d, are given by

$$I_d = \left[-\frac{U}{(1-U)^2}\right]\frac{E}{R}; \quad V_d = -\left(\frac{U}{1-U}\right)E. \quad (35)$$

This means that, depending on the particular value of the steady state duty ratio function U, the 'buck–boost' converter can accomplish, in steady state, either source voltage 'amplification' or 'attenuation', modulo a polarity inversion, at the load.

It follows from (35) that, given a desired equilibrium value V_d for the output voltage, which corresponds to a constant value U of the duty ratio function μ, then the *unique* corresponding equilibrium value for the average input current I_d is given by

$$I_d = -\frac{V_d}{R(1-U)} = \left(\frac{V_d}{RE} - \frac{1}{E}\right)V_d. \quad (36)$$

Hence, if we want to regulate z_2 towards an equilibrium value V_d, which corresponds to a steady state value $U = V_d/(V_d - E)$ of the duty ratio function μ, then, such a regulation can be *indirectly* accomplished by stabilizing the average input current z_1 towards the corresponding equilibrium value I_d computed from (36).

2.4. *The 'buck' converter circuit*

The 'buck' converter model is described by the following set of differential equations, with variables defined as before:

$$\dot{x}_1 = -\frac{1}{L}x_2 + u\frac{E}{L},$$
$$\dot{x}_2 = \frac{1}{C}x_1 - \frac{1}{RC}x_2. \quad (37)$$

The following proposition summarizes the EL formulation of the average 'buck' converter model.

Proposition 2.3. The state average model of the 'buck' converter (see Middlebrook and Ćuk, 1976), given by

$$\dot{z}_1 = -\frac{1}{L}z_2 + \mu\frac{E}{L},$$
$$\dot{z}_2 = \frac{1}{C}z_1 - \frac{1}{RC}z_2, \quad (38)$$

is an EL system corresponding to the following set of average EL parameters:

$$\mathcal{T}_\mu(\dot{q}_L) = \tfrac{1}{2}L\dot{q}_L^2; \quad \mathcal{V}_\mu(q_C) = \frac{1}{2C}q_C^2,$$
$$\mathcal{D}_\mu(\dot{q}_L, \dot{q}_C) = \tfrac{1}{2}R[\dot{q}_C - \dot{q}_L]^2; \quad (39)$$
$$\mathcal{F}^\mu_{q_L} = \mu E; \quad \mathcal{F}^\mu_{q_C} = 0,$$

obtained by suitable modulation, through the duty ratio function μ of the EL parameters associated to each one of the circuits arising from a particular value of the switch position function $u \in \{0, 1\}$.

$u = 1$:

$$\mathcal{T}_1(\dot{q}_L) = \tfrac{1}{2}L\dot{q}_L^2; \quad \mathcal{V}_1(q_C) = \frac{1}{2C}q_C^2,$$
$$\mathcal{D}_1(\dot{q}_C) = \tfrac{1}{2}R(\dot{q}_C - \dot{q}_L)^2; \quad (40)$$
$$\mathcal{F}^1_{q_L} = E; \quad \mathcal{F}^1_{q_C} = 0.$$

$u = 0$:

$$\mathcal{T}_0(\dot{q}_L) = \tfrac{1}{2}L\dot{q}_L^2; \quad \mathcal{V}_0(q_C) = \frac{1}{2C}q_C^2,$$
$$\mathcal{D}_0(\dot{q}_L, \dot{q}_C) = \tfrac{1}{2}R(\dot{q}_C - \dot{q}_L)^2; \quad (41)$$
$$\mathcal{F}^0_{q_L} = 0; \quad \mathcal{F}^0_{q_C} = 0.$$

A matrix form for the average model of the 'buck' converter is given by

$$\mathscr{D}_b \dot{z} + (\mathscr{J}_b + \mathscr{R}_b) z = \mu \mathscr{E}_b, \tag{42}$$

where

$$\mathscr{D}_b = \begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix}; \quad \mathscr{J}_b = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix};$$
$$\mathscr{R}_b = \begin{bmatrix} 0 & 0 \\ 0 & 1/R \end{bmatrix}; \quad \mathscr{E}_b = \begin{bmatrix} E \\ 0 \end{bmatrix}. \tag{43}$$

If the output of the converter is taken as the average capacitor voltage, the input–output representation exhibits no zero dynamics and it is given by

$$\ddot{z}_2 + \frac{1}{RC}\dot{z}_2 + \frac{1}{LC} z_2 = \mu \frac{E}{LC}. \tag{44}$$

The constant equilibrium points for a constant duty ratio $\mu = U$ are found to be

$$\mu = U; \quad z_2 = V_d = UE. \tag{45}$$

If, on the other hand, the average inductor current is taken as the output of the system, the input–output representation results in

$$\ddot{z}_1 + \frac{1}{RC}\dot{z}_1 + \frac{1}{LC} z_1 = \frac{E}{L}\left(\dot{\mu} + \frac{1}{RC}\mu\right), \tag{46}$$

which exhibits the following equilibrium point, for a constant duty ratio of value U:

$$\mu = U; \quad z_1 = I_d = U\frac{E}{R}. \tag{47}$$

In this case, the zero dynamics turns out to be asymptotically stable towards the unique equilibrium point:

$$\dot{\mu} = -\frac{1}{RC}\left(\mu - \frac{RI_d}{E}\right). \tag{48}$$

3. PASSIVITY BASED PWM CONTROLLERS FOR DC-to-DC POWER CONVERTERS

3.1. *Controller design: the 'boost' converter*

3.1.1. *Direct output voltage regulation.* Suppose it is desired to regulate directly the output capacitor voltage to a constant value $z_{2d} = V_d$. Corresponding to this objective for the output voltage z_2, the required input current may be represented by a function $z_{1d}(t)$, to be determined later.

Consider then the error variables $\tilde{z}_1(t) = z_1(t) - z_{1d}(t)$ and $\tilde{z}_2(t) = z_2(t) - V_d$. We denote the average state error vector by $\tilde{z}$. Following the passivity-based methodology, we want to shape the closed-loop energy function to a desired energy function:

$$H_d = \tfrac{1}{2}\tilde{z}^T \mathscr{D}_B \tilde{z} \tag{49}$$

This choice is, as usual, motivated by the form of the total energy function of the average system model, which, as shown before, is given by $H = \tfrac{1}{2} z^T \mathscr{D}_B z$.

The average error vector dynamics is given by

$$\mathscr{D}_B(\dot{\tilde{z}} + (1-\mu)\mathscr{J}_B \tilde{z} + \mathscr{R}_B \tilde{z} = \mathscr{E}_B - (\mathscr{D}_B \dot{z}_d + (1-\mu)\mathscr{J}_B z_d + \mathscr{R}_B z_d). \tag{50}$$

To ensure asymptotic stability, we also perform a *damping injection* on (50) by defining the following desired Rayleigh error dissipation term

$$\mathscr{D}_d = \tfrac{1}{2}\tilde{z}^T \mathscr{R}_{Bd} \tilde{z} = \tfrac{1}{2}\tilde{z}^T(\mathscr{R}_B + \mathscr{R}_{1B})\tilde{z}, \tag{51}$$

where

$$\mathscr{R}_{1B} = \begin{bmatrix} R_1 & 0 \\ 0 & 0 \end{bmatrix}; \quad R_1 > 0. \tag{52}$$

Adding the necessary expressions to both sides of (50) we obtain

$$\mathscr{D}_B \dot{\tilde{z}} + (1-\mu)\mathscr{J}_B \tilde{z} + \mathscr{R}_{Bd}\tilde{z} = \Psi, \tag{53}$$

where

$$\Psi := \mathscr{E}_B - (\mathscr{D}_B \dot{z}_d + (1-\mu)\mathscr{J}_B z_d + \mathscr{R}_B z_d - \mathscr{R}_{1B}\tilde{z}). \tag{54}$$

The energy shaping plus damping injection will be achieved if we can set $\Psi = 0$. In this case, the stabilization error dynamics would satisfy

$$\mathscr{D}_B \dot{\tilde{z}} + (1-\mu)\mathscr{J}_B \tilde{z} + \mathscr{R}_{Bd}\tilde{z} = 0. \tag{55}$$

To explain the rationale of the approach, consider the behavior of the desired total energy H_d, the time derivative of which along the solution of (55) results, for some strictly positive constant α, in

$$\dot{H}_d = -\tilde{z}^T \mathscr{R}_{Bd}\tilde{z} \leq -\frac{\alpha}{\beta} H_d < 0 \quad \forall \tilde{z} \neq 0, \tag{56}$$

where α may be taken to be $\alpha = \min\{R_1, 1/R\}$ and $\beta = \max\{L, C\}$.

We conclude that, if the error dynamics coincides with (55), the stabilization error behavior is asymptotically stable to zero, i.e. $\tilde{z} \to 0$ independently of μ.

Thus, in order to satisfy (55), one must demand from (54) that

$$\mathscr{D}_B \dot{z}_d + (1-\mu)\mathscr{J}_B z_d + \mathscr{R}_B z_d - \mathscr{R}_{1B}\tilde{z} = \mathscr{E}_B. \tag{57}$$

These conditions are explicitly written as

$$L\dot{z}_{1d} + (1-\mu)z_{2d} - (z_1 - z_{1d})R_1 = E,$$
$$C\dot{z}_{2d} - (1-\mu)z_{1d} + \frac{1}{R} z_{2d} = 0. \tag{58}$$

The problem thus consists of, given a desired constant output voltage value $z_{2d} = V_d$, finding a bounded function $z_{1d}(t)$ and a suitable duty ratio function μ such that (58) is satisfied. We proceed to eliminate the variable $z_1(t)$ from (58) as follows. From the second equation in (58), one obtains

$$z_{1d}(t) = \frac{V_d}{R(1 - \mu(t))}. \tag{59}$$

Substituting this expression into the first equation in (58), one obtains, after some algebraic manipulations, an expression for the dynamical feedback duty ratio synthesizer of the form

$$\dot{\mu} = \frac{R(1-\mu)^2}{LV_d} \times \left[E - (1-\mu)V_d + R_1\left(z_1 - \frac{V_d}{R(1-\mu)}\right)\right] \tag{60}$$

This controller stabilizes z_1 and z_2 towards their desired values z_{1d} and z_{2d}, respectively. However, controller (60) is, unfortunately, not feasible due to its lack of stability. Indeed the 'remaining', or zero, dynamics associated with the above controller results in

$$\dot{\mu} = \frac{R(1-\mu)^2}{LV_d}[E - (1-\mu)V_d], \tag{61}$$

which coincides with the zero dynamics already found in (23) and shown to be unstable around its only physically meaningful equilibrium point.

3.1.2 *Indirect output voltage regulation.* The previous section has shown that a *direct* output voltage control scheme is unfeasible. In this section we provide a feasible regulation alternative based on an *indirect* output capacitor voltage control, achievable through the regulation of the input current. Note that some other possible alternatives include proposing a different error energy function for the system. In this instance, we have just chosen to explore the implications of using the *most natural* energy function for the system.

Suppose it is desired to regulate z_1 towards a

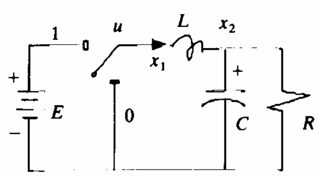

Fig. 10. 'Buck' converter circuit.

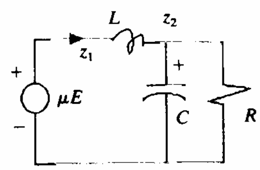

Fig. 11. Equivalent circuit of the average PWM model of the 'buck' converter circuit.

constant value $z_{1d} = I_d$. In order to find a suitable feedback controller for this task, one proceeds now to eliminate the variable z_{2d} from the set of equations (58). Using the first equation in (58), $z_{2d}(t)$ is given by

$$z_{2d}(t) = \frac{E + (z_1 - I_d)R_1}{(1 - \mu(t))}. \tag{62}$$

Substituting (62) in the second equation of (58), one obtains, after some algebraic manipulations,

$$\dot{\mu} = \frac{(1-\mu)}{C[E + (z_1 - I_d)R_1]}\left\{(1-\mu)^2 I_d - \frac{E + (z_1 - I_d)R_1}{R} - \frac{R_1 C}{L}[E - (1-\mu)z_2]\right\}. \tag{63}$$

The 'remaining' dynamics associated with controller (63) is obtained by letting z_1 and z_2 coincide with their corresponding desired values. Such dynamics is given by

$$\dot{\mu} = \frac{1-\mu}{RCE}[(1-\mu)^2 RI_d - E]. \tag{64}$$

The zero dynamics (64) coincides with the zero dynamics derived in (26), which was shown to be locally stable around the only physically meaningful equilibrium point. The indirect controller (63) is, therefore, feasible.

We will now complete the proof that the equilibrium point $(z_1, z_2, \mu) = (I_d, V_d, U)$ of the overall system, (14) and (63), is locally asymptotically stable. To this end, we introduce the following auxiliary variable:

$$\xi = \frac{1}{2}\left(\frac{E + (z_1 - I_d)R_1}{1-\mu}\right)^2 - \frac{V_d^2}{2}, \tag{65}$$

which is well defined for μ in a neighborhood of the equilibrium point $\mu = U \neq 1$. It is easy to show that ξ satisfies the following *linear* differential equation:

$$\dot{\xi} = -\frac{2}{RC}\xi + \frac{V_d^2 R_1}{RCE}(z_1 - I_d). \tag{66}$$

Recalling that $\tilde{z}_1 = z_1 - I_d \to 0$, and is exponentially fast, we conclude that $\xi \to 0$ as well. This implies that $z_{2d} \to V_d$ locally, and in turn implies that $\mu \to U$.

We have thus proven the following proposition.

Proposition 3.1. Given a desired constant value $V_d > E$ for the output capacitor voltage of a 'boost' converter, the dynamically generated duty ratio function (63), with I_d given by (21), locally asymptotically stabilizes the state trajectories of the average PWM model (14) towards the desired equilibrium point (I_d, V_d, U), with μ converging to a constant value $\bar{\mu} = U = 1 - E/V_d$.

3.1.3. *Further remarks*. The passivity-based dynamical duty ratio synthesizer design is carried out under the assumption that the average PWM model (14) of the converter captures the essential behavior of the actual switch-regulated circuit described by (5). This assumption has been shown to be only approximately valid due to the fact that, in practice, infinite sampling frequency and corresponding infinitely fast switchings are impossible to achieve. However, for *sufficiently high* sampling frequencies, feedback controllers designed on the basis of average models can indeed be used to regulate the actual switched converter, with rather satisfactory results (see Kassakian *et al.*, 1991). The scheme, shown in Fig. 12 is based in this philosophy. The underlying approach has been extensively used for similar nonlinear dynamical feedback controllers, and its validity has been justified both from a theoretical viewpoint and through extensive computer simulation results (see e.g. Sira-Ramirez and Lischinsky-Arenas, 1991, and references cited therein).

Two additional remarks are in order, regarding the use of a feedback PWM scheme such as the one shown in Fig. 12:

(i) The average-based duty ratio synthesizer produces a *computed* duty ratio function. As such, it is entirely possible that these computed values exceed the physical bounds of the required *actual* duty ratio function, which is necessarily limited, to the closed interval [0, 1]. For this reason, a *hard limiter* must be used in conjunction with the derived dynamical feedback regulator, as shown in Fig. 12. As a consequence of this limitation, only *local asymptotic stability* of the closed-loop system may actually be guaranteed. Large initial-state deviations may induce destabilizing saturation effects, which have not been accounted for in the previous developments.

(ii) The duty ratio synthesizer (63) requires the on-line values of the average PWM circuit states z_1 and z_2. These average states can be approximately obtained by *low-pass filtering* of the actual circuit states x_1 and x_2. Note, however, that in the scheme presented in Fig. 12 the *actual* circuit states x_1 and x_2 are used for feedback, rather than their averaged, or filtered, versions z_1 and z_2. It should be pointed out then that, again, for large sampling frequencies the difference between using one or the other set of states is entirely negligible, due to the underlying low-pass filtering effects of the system itself.

3.2. *Controller design: the 'buck–boost' converter*

Following exactly the same procedure as in the previous case, one concludes that for the 'buck–boost' converter a direct regulation policy of the output voltage is unfeasible due to nonminimum phase phenomena. We thus summarize in a proposition the dynamical feedback regulation scheme for achieving indirect output capacitor voltage regulation, towards a given desired equilibrium value V_d, through input current stabilization towards a desired constant value I_d, computable in terms of V_d, as given by (36).

Proposition 3.2. Given a desired constant value V_d for the output capacitor voltage of a

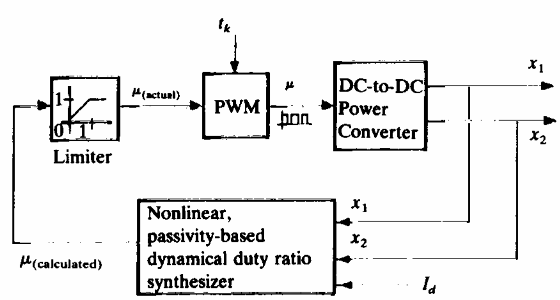

Fig. 12. PWM feedback control scheme for indirect, passivity-based, output voltage regulation for DC-to-DC power converters.

'buck–boost' converter, the dynamically generated duty ratio function given by

$$\dot{\mu} = \frac{1-\mu}{C[E + (z_1 - I_d)R_1]}\left\{(1-\mu)^2 I_d - \frac{\mu E + (z_1 - I_d)R_1}{R} - \frac{R_1 C}{L}[\mu E + z_2(1-\mu)]\right\} \tag{67}$$

locally asymptotically stabilizes the state trajectories of the average PWM model (29) towards the desired equilibrium point (I_d, V_d), with μ converging to a constant value given by $\mu = U = 1 - \sqrt{E/RI_d}$, with I_d obtained from V_d from (36).

Note that the zero dynamics associated with the controller (67) is given by

$$\dot{\mu} = \frac{(1-\mu)}{RCE}\{(1-\mu)^2 RI_d - \mu E\}, \tag{68}$$

which has three equilibrium points given by

$$\mu = 1; \qquad \mu = 1 + \frac{E}{2RI_d} \pm \sqrt{\left(\frac{E}{2RI_d}\right)^2 + \frac{E}{RI_d}}. \tag{69}$$

Two of the equilibrium points ($\mu = 1$, and the one corresponding to the plus sign of the square root) are unstable, while the remaining one, which is the only physically significant one is locally asymptotically stable.

3.3. *Controller design*: *the 'buck' converter*

The developments leading to a duty ratio synthesizer for the 'buck' converter are similar to those presented for the other converters. We only point out that, unlike in the previous cases, a direct output voltage regulation scheme is feasible due to the fact that the 'buck' converter exhibits no zero dynamics for such an output. In this case, the duty ratio synthesizer turns out to be *static* rather than dynamic. If, as before, we denote by V_d the desired output capacitor voltage, the passivity-based controller may be described as in the following proposition.

Proposition 3.3. Given a desired constant value $V_d < E$ for the output capacitor voltage of a 'buck' converter, the statically generated duty ratio function given by

$$\mu = \frac{V_d}{E} - \frac{1}{ER_1}\left(z_1 - \frac{V_d}{R}\right) \tag{70}$$

locally asymptotically stabilizes the state trajecotries of the average PWM model (38) towards the desired equilibrium point (I_d, V_d), with I_d given by $I_d = V_d/R$ while μ converges to a constant value given by $\mu = U = V_d/E$.

It is easy to see, by substituting z_1 in terms of z_2 and $\dot{z}_2$, that this controller is just a classical 'proportional derivative' controller.

The indirect output voltage regulation scheme results in a dynamic controller with asymptotically stable zero dynamics. The details are not presented.

4. SIMULATION RESULTS

Simulations were performed for the closed-loop behavior of a 'boost' circuit regulated by means of the passivity-based indirect PWM controller (63). In order to test the effectiveness and robustness of the proposed feedback controller with respect to unmodeled parasitic resistances and unmodeled realistic switching devices, the following stochastically perturbed version of a 'boost' converter circuit, taken from the work of Czarkowski and Kazimierczuk (1993), was used for the simulations:

$$\begin{aligned} \dot{x}_1 &= -\frac{1}{L}r(u) - (1-u)\frac{R}{L(R+r_C)}x_2 \\ &\quad + \frac{E+\eta}{L} - (1-u)\frac{V_F}{L}, \\ \dot{x}_2 &= (1-u)\frac{R}{(R+r_C)C}x_1 - \frac{1}{(R+r_C)C}x_2, \end{aligned} \tag{71}$$

where $r(u) = r_L + ur_{DS} + (1-u)(R_F + r_C \parallel R)$ r_L is the resistance associated with the inductor; r_{DS} is the resistance associated with the ON state of the transistor used in the realization of the switching element constituted by a transistor–diode arrangement; R_F is the forward resistance of the diode; r_C is the resistance associated with the output capacitor; and $r_C \parallel R$ denotes the resistance of a parallel arrangement of r_C and R. The voltage V_F represents a small constant voltage drop associated with the conducting phase of the diode. The signal η, added to the external source voltage, represents an external stochastic perturbation input affecting the system behavior.

Note that the perturbation input η is of the 'unmatched' type, i.e. it enters the system equations through an input channel vector field given by $[1/L \quad 0]^T$ which is *not* in the range space of the control input channel, given by the vector field

$$\begin{bmatrix} -\frac{x_{DS} + R_F + r_C \parallel R}{L}x_1 + \frac{R}{L(R+r_C)}x_2 - \frac{V_F}{L} \\ \frac{R}{(R+r_C)C}x_1 \end{bmatrix}$$

The peak-to-peak magnitude of the noise was chosen to be approximately 20% of the value of E. The circuit parameter values were taken to be the following 'typical' values: $C = 20\,\mu F$, $R = 30\,\Omega$, $L = 20\,mH$, $E = 15\,V$, $r_L = 0.05\,\Omega$, $r_C = 0.2\,\Omega$, $r_{DS} = 0.1\,\Omega$, $R_F = 0.05\,\Omega$, $V_F = 0.7\,V$. The sampling frequency for the PWM policy was set at 5 kHz. The duty ratio function is obtained from a sampling process carried out on the output $\mu(t)$ of the smooth dynamical duty ratio synthesizer (63). To avoid the use of low-pass filters, instead of using the averaged state variables z_1 and z_2 for feedback on the duty ratio synthesizer, we used, as it is customarily done, the actual PWM controlled states x_1 and x_2 on the controller expressions. The desired ideal average input inductor current was set to be $I_d = 3.125\,A$, with a steady-state duty ratio of $U = 0.6$. This corresponds to an ideal average output voltage of $z_2 = V_d = 37.5\,V$. Figure 13 shows the closed-loop state trajectories as well as the duty ratio function and a realization of the computer-generated stochastic perturbation signal η.

As can be seen from the simulations, the proposed dynamical feedback controller (63) achieves the desired indirect stabilization of the output voltage for the nonideal stochastically perturbed model around the desired equilibrium value. The average steady-state errors, with respect to the desired equilibrium values, range from approximately 2.5% in the average inductor current variable to 2.6% in the average capacitor voltage variable. The ideal duty ratio is achieved within less than 0.5% error. The controller performance also exhibits a high degree of robustness with respect to the external stochastic perturbation inputs.

Unknown load resistance variations generally affect the behavior of the closed-loop performance of the controlled converter. Simulations, shown in Fig. 14, were performed to depict the sensitivity of the regulated input current, the output capacitor voltage, and the duty ratio with respect to abrupt, but temporary, unmodeled changes in the load resistance R. An unmodeled sudden change in the load resistance was set to 80% of its nominal value. As can be seen from the figures, the controller manages rapidly to restore the desired steady-state conditions immediately after the load perturbation disappears. As expected, the state variable most affected by such a perturbation is the output voltage. Conversely, the duty ratio function is barely affected by such sudden load changes.

An extension of the above presented controller design method, which is also capable of handling unknown but constant loads, has been undertaken by the authors within the context of nonlinear *adaptive regulation*. The reader can find details in Sira-Ramirez *et al.* (1995).

5. CONCLUSIONS

Traditional state average models, or infinite switching frequency models, of DC-to-DC power converters were shown to be EL systems for a suitable set of average EL parameters. The derived average PWM models were also shown to be interpretable in terms of ideal circuit realizations, including internal controlled sources and modulated external inputs.

Physically motivated dynamic feedback duty ratio synthesizers were derived for the indirect average output voltage stabilization of DC-to-

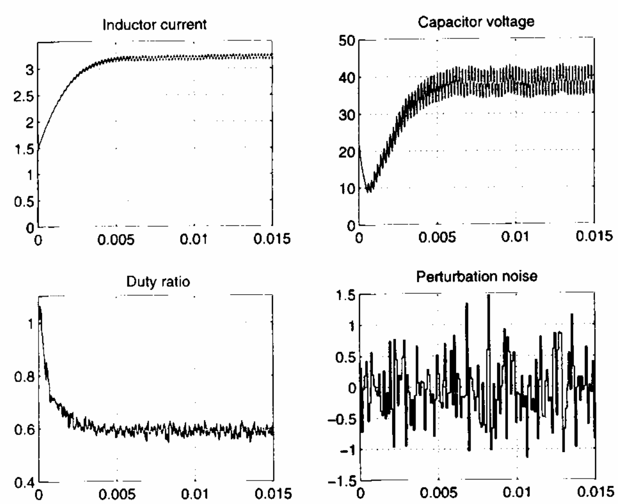

Fig. 13. Simulation results for performance evaluation of the indirect PWM controller in a realistic perturbed 'boost' converter.

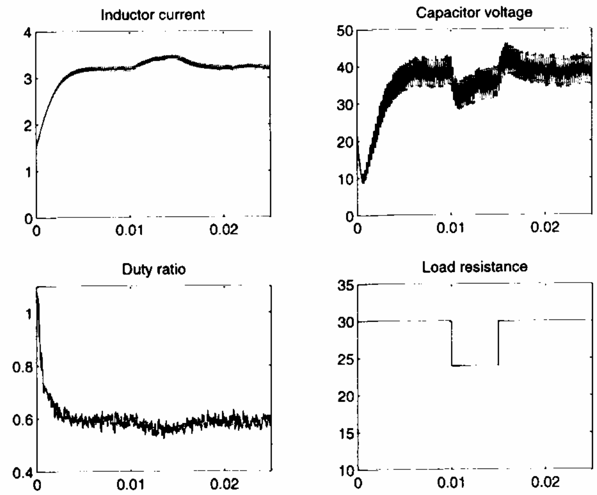

Fig. 14. Robustness evaluation of the indirect PWM controller in a realistic perturbed 'boost' converter.

DC power converters of the 'boost', 'buck–boost', and 'buck' types. The dynamic feedback controllers are based on the modification of the total energy function of the average converter circuit model. This procedure, together with the possibilities of enhancing the dissipation structure of the average models through suitable 'damping injections', were shown to yield asymptotically stable closed-loop behavior with feasible locally asymptotically stable controllers.

Other useful connections of passivity-based controllers with *differential flatness* (see Fliess *et al.*, 1992), associated with the average PWM models of DC-to-DC power converters, remain to be explored. Similarly, sliding-mode controllers, based on passivity considerations, remain to be developed for DC-to-DC power converters.

Acknowledgements—This work was supported by the Consejo de Desarrollo Cientifico, Humanistico and Tecnológico of the Universidad de Los Andes, under research grant I-456-94, and by The Commission of European Communities under contract ERB CHRX CT 93-0380. The authors are indebted to Mr Gerardo Escobar of Heudiasyc, Université de Téchnologie de Compiegne (France), for his invaluable help in performing the simulations appearing in this article. R. Ortega would also like to express his gratitude to Professor Sam Ben-Yaakov of Ben-Gurion University for introducing him to this topic.

REFERENCES

Amran, Y., F. Huliehlel and S. Ben-Yaacov (1991). Unified SPICE compatible average model of PWM converters. *IEEE Trans. Power Electron.*, **6**, 585–594.

Benvenuti, L., M. D. DiBenedetto and J. W. Grizzle (1992). Approximate output tracking for nonlinear non-minimum phase systems with applications to flight control. *Michigan Control Group Report CGR-92-20*, University of Michigan, Ann Arbor MI.

Berghuis, H. and H. Nijmeijer (1993). A passivity approach to controller–observer design for robots. *IEEE Trans. Robot. Automat.*, **TRA-9**, 740–754.

Brodliato, B., R. Ortega and R. Lozano (1995). Global tracking controllers for flexible joint manipulators: a comparative study. *Automatica*, **31**, 941–956.

Czarkowski, D. and M. K. Kazimierczuk (1993). Energy-conservation approach to modeling PWM DC-DC converters. *IEEE Trans. Aerospace Electron. Syst.*, **29**, 1059–1063.

Filippov, A. F. (1988). *Differential Equations with Discontinuous Right-Hand Sides*. Kluwer, Dordrecht.

Fliess, M. (1990). Generalized controller canonical forms for linear and nonlinear dynamics. *IEEE Trans. Automatic Control*, **AC-35**, 994–1001.

Fliess, M. and H. Sira-Ramirez (1993). Regimes glissants, structures variables linéaires et modules. *C.R. Acad. Sci. Paris, Ser. I*, vol. 317, *Automatique*, 703–706.

Kassakian, J. G., M. Schlecht and G. C. Verghese (1991). *Principles of Power Electronics*. Addison-Wesley, Reading, MA.

Krein, Ph., J. Bentsman, R. Bass and B. Lesieutre (1990). On the use of averaging for the analysis of power electronic systems. *IEEE Trans. Power Electron.*, **5**, 182–190.

Meisel, J. (1966). *Principles of Electromechanical Energy Conversion*. McGraw-Hill, New York.

Middlebrook, R. D. and Cúk, S. (1976). A general unified approach to modelling switching—converter power stages. In *IEEE Power Electronics Specialists Conference* (*PESC*), pp. 18–34.

Ortega, R. and G. Espinosa (1993). Torque Regulation of Induction Motors. *Automatica*, **29**, 621–633.

Ortega, R. and M. Spong (1989). Adaptive motion control of rigid robots: a tutorial. *Automatica*, **25**, 877–888.

Ortega, R., A. Loria, R. Kelly and L. Praly (1995). On passivity-based output feedback global stabilization of Euler–Lagrange systems. *Int. J. Robust Nonlinear Control*, **5**, 313–324.

Rashid, M. (1992). *Power Electronics, Circuits, Devices and Applications*. Prentice Hall, London.

Sabanovic, A., N. Sabanovic and K. Ohnishi (1993). Sliding modes in power converters and motion control systems. *Int. J. Control*, **57**, 1237–1259.

Severns, R. P. and G. E. Bloom (1982). *Modern DC-to-DC Switchmode Power Converter Circuits*. Van Nostrand-Reinhold, New York.

Sira-Ramirez, H. (1989). A geometric approach to pulse-width-modulated control in nonlinear dynamical systems. *IEEE Trans. Automatic Control*, **AC-34**, 184–187.

Sira-Ramirez, H. and M. Delgado de Nieto (1995). A Lagrangian approach to average modeling of pulse-width-modulation controlled DC-to-DC power converters. *IEEE Trans. Circuits Syst., Part I*, vol. 43, No. 5, pp. 427–430.

Sira-Ramírez, H. G., R. Ortega and M. García-Esteban (1995). Adaptive passivity-based control of average DC-to-DC power converter models. *Int. J. Adaptive Control Signal. Proc.*, (submitted).

Takegaki, M., and Arimoto, S. (1981). A new feedback method for dynamic control of manipulators. *ASME J. Dynamic Syst. Measurement Control*, **102,** 119–125.

Venkataramanan. V., A. Sabanovic and S. Cúk (1985). Sliding mode control of DC-to-DC converters. In *Proc. IECON'85*, pp. 251–258.