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ADAPTIVE PASSIVITY-BASED CONTROL OF AVERAGE DC-TO-DC POWER CONVERTER MODELS

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SUMMARY

An adaptive feedback regulation scheme is proposed for the stabilization of average models of dc-to-dc power converters exhibiting unknown but constant resistive loads. The scheme is based on a dynamical feedback policy which suitably modifies the total energy of the closed-loop system while inducing appropriate damping injections on the desired stabilization error dynamics. The performance of the proposed adaptative regulators is tested through computer simulations including stochastic perturbation inputs.

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1. INTRODUCTION

Feedback regulation of dc-to-dc power supplies has been extensively treated in the literature. Conference proceedings, such as the IEEE Power Electronics Specialist Conference (PESC) Records, Multi-volume Series, edited over the years,[1,2] a growing list of text books[3–7] and edited collections of research articles[8] reflect both the theoretical and practical importance of this field. We remark that dc-to-dc power supplies and, more generally, the area of *Power Electronics*, which has been traditionally credited to the discipline of *Industrial Electronics*, enjoys a growing interest in the *Automatic Control* community.

A frequent assumption in the design of feedback regulators for dc-to-dc power supplies is that the converter loads and the parameters associated with the various circuit components are

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perfectly known. In practise, however lack of precise knowledge about these parameters arises from inescapable measurement errors, unavoidable ageing effects and imperfectly modelled loads. These facts motivate the adoption of an adaptive feedback approach for the design of regulation loops in dc-to-dc power supplies. Adaptive control of dc-to-dc power supplies has been treated, from an approximate linearization viewpoint, by Sanders and Verghese in Reference 9. Their approach relies on Lyapunov stability and passivity considerations for the linear feedback controller design. A full adaptive feedback input–output linearization viewpoint for dc-to-dc power supplies was proposed by Sira-Ramírez *et al.* Reference 10. An adaptive feedback design technique that suitably combines input–output linearization, through generalized observability canonical forms as developed by Fliess in Reference 11, and the backstepping design procedure, was recently presented by Sira-Ramírez *et al.* in Reference 12, 13.

In the last few years, a feedback control design methodology for non-linear systems that exploits the physical restrictions of the system, and, in particular, its energy properties, has been developed. The approach, known as *passivity-based* controller design, consists of an energy-shaping stage where the closed-loop total energy of the system is modified, and a damping injection stage where the required dissipation is added in order to achieve asymptotic stability. The approach has been successfully used in the regulation of Euler–Lagrange (EL) systems, such as rigid and flexible robotic manipulators (see the works of Takegaki and Arimoto[14] and the subsequent developments by Ortega and Spong,[15] and Brogliato *et al.*[16] The same technique has also been used, with the same degree of success, in the regulation of electro-mechanical energy conversion devices (see the work by Ortega and Espinosa[17] and a recent article by Ortega *et al.*[18] A non-adaptive passivity-based approach has also been recently developed for dc-to-dc power converters by the authors.[19]

The main motivation of this work is to extend the developments in Reference 19, and explore the viability of applying the described passivity-based controller design methodology for the *adaptive* stabilization of a class of average models of pulse-width-modulation (PWM) regulated dc-to-dc power converters. For the sake of completeness, we treat three types of switched power supplies. Namely, 'boost' (or 'step-up' converter), 'buck-boost' (or step up-down' converter) and the 'buck' (step down) types of converters. The encouraging results corresponding to the passivity-based approach for PWM dc-to-dc power converter regulation, developed in Reference 19, have motivated an actual experimental comparison of several feedback controllers, including the classical linear controller. The several controllers that have been compared correspond to: a linear controller, a feedback linearization controller and a passivity-based controller. The results and experimental data are fully reported in a recent article by Escobar *et al.* in Reference 20 (see also Reference 21). The bottom line of the experimental results is that the passivity-based controller outperforms, both, in simplicity of implementation and robustness with respect to external noises and modelling errors, the classical and the feedback linearization schemes.

This article is organized as follows. Section 2 presents the average PWM models of the three types of dc-to-dc power converters. To make the presentation self-contained, non-adaptive passivity-based feedback regulators, such as those developed in Reference 19, are briefly revisited in Section 2 for the treated converters. Section 3 assumes that the resistive loads of the converters under study is constant but, otherwise, totally unknown. We proceed to derive adaptive feedback regulation schemes based on passivity considerations. Section 4 contains simulations of the proposed passivity-based adaptive controllers. Section 5 contains the conclusions and suggestions for further research in this area.

2. PASSIVITY-BASED REGULATION OF AVERAGE DC-TO-DC POWER CONVERTER MODELS

Average models of PWM regulated dc-to-dc switched power converters have been justified from theoretical and practical grounds in the work of many authors. Average models of dc-to-dc power converters were first introduced by Middlebrook and Ćuk in References 22, 23, under the name of *state average* models. Their approach was based on a discretization viewpoint, based on simplifying approximations of the corresponding state transition matrices of the involved linear systems. The state average models were later generalized and refined, using the analytic theory of averaging of differential equations, in the work of Krein *et al.*,[24] where a wider variety of average models with improved approximation features were shown to be entirely feasible. A completely different viewpoint based on Filippov's geometric averaging, was pursued later Sira-Ramírez *et al.* in References 25, 26. These developments were motivated by definite mathematical connections between Sliding Mode Control and PWM regulation of non-linear systems. The approach obtained exactly the same average models initially proposed by Middlebrook and Ćuk. A physically motivated justification of the average PWM models to dc-to-dc power converters, based on an Euler–Lagrange formulation, has also been proposed b Sira-Ramírez and Delgado in Reference 27.

In this section, we use the average PWM models of dc-to-dc power converters without further justification. We proceed to find non-adaptive passivity-based feedback controllers for average models of dc-to-dc power converters without additional considerations about the nature of the approximation that such models imply when the corresponding duty ratio designs are used in actual (i.e. discontinuous) PWM feedback regulation loops. We simply point out that as the sampling frequency in the actual PWM regulation scheme is increased, the closed-loop state responses rapidly converge towards the corresponding closed-loop average state trajectories (see Reference 10).

We remark also that our controller schemes, at least for the 'boost' and 'buck-boost' converters, are of the *indirect* type. i.e. we deliberately seek to indirectly regulate the output capacitor voltage towards a feasible desired equilibrium value. For this we design a feedback controller which primarily accomplishes the asymptotic regulation of the input current towards the unique equilibrium value corresponding to the required constant output voltage. If the opposite policy is adopted, the resulting controllers are invariably *unstable* due to the well-known *non-minimum phase* character of the output voltage when taken as a converter output variable. We stress that for the average 'buck' converter case, direct, or indirect, feedback regulation policies are equally feasible and devoid of any non-minimum phase instabilities. Further details and mathematical justifications of these facts can be found in the articles by Sira-Ramírez and Lischinsky-Arenas[28] and Sira-Ramírez *et al.*[19]

2.1 A passivity-based controller for the 'boost' converter

Consider the average PWM model of a 'boost' converter circuit, shown in Figure 1 (see References [10, 19, 27]).

$$\dot{z}_1 = -(1-\mu)\frac{1}{L}z_2 + \frac{E}{L}$$
$$\dot{z}_2 = (1-\mu)\frac{1}{C}z_1 - \frac{1}{RC}z_2 \qquad (1)$$

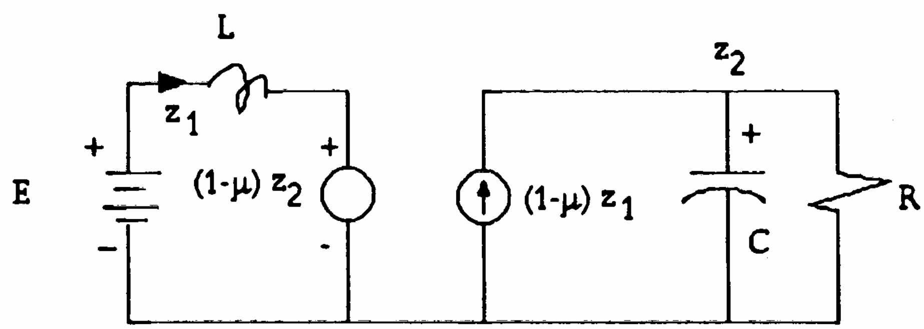

Figure 1. Average model of PWM regulated 'boost' converter

where z_1 and z_2 denote the *average input current* and the *average output capacitor voltage*, respectively. The scalar quantity μ stands for the *duty ratio* function which truly acts as the external control input to the average system model. The duty ratio is naturally constrained to take values in the interval [0, 1] of the real line.

For ease of reference we will be using the following, more compact, matrix representation of system (1):

$$\mathscr{D}_B \dot{z} + (1-\mu)\mathscr{J}_B z + \mathscr{R}_B z = \mathscr{E}_B \tag{2}$$

where

$$\mathscr{D}_B = \begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix}, \quad \mathscr{J}_B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad \mathscr{R}_B = \begin{bmatrix} 0 & 0 \\ 0 & 1/R \end{bmatrix}, \quad \mathscr{E}_B = \begin{bmatrix} E \\ 0 \end{bmatrix} \tag{3}$$

Suppose it is desired to indirectly regulate the average output capacitor voltage to a constant equilibrium value given by, $z_2 = V_d$. Corresponding to this objective for the average output voltage z_2, the corresponding required average input current may be *uniquely* computed from (1) as

$$z_1 = \frac{V_d^2}{\mathrm{RE}} \tag{4}$$

Consider then the error variables $\tilde{z}_1(t) = z_1(t) - z_{1d}(t)$ and $\tilde{z}_2(t) = z_2(t) - V_d$. We denote the average state error vector by $\tilde{z}^T = [\tilde{z}_1(t), \tilde{z}_2(t)]$. The average error vector dynamics is then given by

$$\mathscr{D}_B \dot{\tilde{z}} + (1-\mu)\mathscr{J}_B \tilde{z} + \mathscr{R}_B \tilde{z} = \mathscr{E}_B - (\mathscr{D}_B \dot{z}_d + (1-\mu)\mathscr{J}_B z_d + \mathscr{R}_B z_d) \tag{5}$$

One may perform a *damping injection* on (5) by considering the following desired error dissipation term:

$$\mathscr{R}_{Bd} \tilde{z} = (\mathscr{R}_B + \mathscr{R}_{1B})\tilde{z} \tag{6}$$

where,

$$\mathscr{R}_{1B} = \begin{bmatrix} R_1 & 0 \\ 0 & 0 \end{bmatrix}, \quad R_1 > 0 \tag{7}$$

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Adding to both sides of equation (5) the necessary expressions, we obtain

$$\mathscr{D}_B\dot{\tilde{z}} + (1-\mu)\mathscr{J}_B\tilde{z} + \mathscr{R}_{Bd}\tilde{z} = \mathscr{E}_B - (\mathscr{D}_B\dot{z}_d + (1-\mu)\mathscr{J}_B z_d + \mathscr{R}_B z_d - \mathscr{R}_{1B}\tilde{z}) \tag{8}$$

Suppose for a moment that the right-hand side of equation (8) is identically zero. Under these circumstances the stabilization error dynamics would satisfy

$$\mathscr{D}_B\dot{\tilde{z}} + (1-\mu)\mathscr{J}_B\tilde{z} + \mathscr{R}_{Bd}\tilde{z} = 0 \tag{9}$$

Motivated by the form of the total energy function of the average system model, given by $H = \frac{1}{2}z^T\mathscr{D}_B z$, we propose a desired energy function, denoted by H_d, associated with the average state error vector as

$$H_d = \tfrac{1}{2}\tilde{z}^T\mathscr{D}_B\tilde{z} > 0 \quad \forall\,\tilde{z} \neq 0 \tag{10}$$

Take the expression (10) as a Lyapunov function candidate for the error dynamics (9). The time derivative of $H_d(t)$, along the solutions of (9), results, for some strictly positive constant α, in the following expression:

$$\dot{H}_d(t) = -\tilde{z}\mathscr{R}_{Bd}\tilde{z} \leqslant -\alpha H_d(t) < 0 \quad \forall\,\tilde{z} \neq 0 \tag{11}$$

We conclude that if the error dynamics coincides with (9), then the stabilization error behaviour is asymptotically stable to zero.
Thus, in order to have (9) satisfied one must demand, from (8) that

$$\mathscr{D}_B\dot{z}_d + (1-\mu)\mathscr{J}_B z_d + \mathscr{R}_B z_d - \mathscr{R}_{1B}\tilde{z} = \mathscr{E}_B \tag{12}$$

These conditions are explicitly written as

$$L\dot{z}_{1d} + (1-\mu)z_{2d} - R_1(z_1 - z_{1d}) = E$$

$$C\dot{z}_{2d} - (1-\mu)z_{1d} + \frac{1}{R}z_{2d} = 0 \tag{13}$$

The problem, thus, consists in, given a desired constant value for the input current $z_{1d} = V_d^2/\mathrm{RE}$, finding a bounded function $z_{2d}(t)$, and a suitable duty ratio function $\mu(t)$, such that (13) is satisfied.

Suppose then that it is desired to regulate z_1 towards the constant value $z_{1d} = V_d^2/\mathrm{RE}$. In order to find a suitable feedback duty ratio function for this task, one proceeds to eliminate the input variable μ from the set of equations (13). Using the first equation of (13), the required μ is given by

$$\mu(t) = 1 - \frac{1}{z_{2d}(t)}\left[E + R_1\left(z_1 - \frac{V_d^2}{\mathrm{RE}}\right)\right] \tag{14}$$

Substituting (14) into the second equation of (13), one obtains, after some algebraic manipulations, the differential equation satisfied by the controller state, $z_{2d}(t)$,

$$\dot{z}_{2d} = -\frac{1}{\mathrm{RC}}\left\{z_{2d} - \frac{V_d^2}{Ez_{2d}}\left[E + R_1\left(z_1 - \frac{V_d^2}{\mathrm{RE}}\right)\right]\right\} \tag{15}$$

The 'remaining' dynamics associated with the controller (14) and (15), is obtained by letting z_1 coincide with its desired equilibrium value. Such a dynamics is given by

$$\dot{z}_{2d} = -\frac{1}{\mathrm{RC}}\left(z_{2d} - \frac{V_d^2}{z_{2d}}\right) \tag{16}$$

The 'zero dynamics' (16) has two asymptotically stable equilibrium points represented by $z_{2d} = V_d$ and $z_{2d} = -V_d$. It is easy to realize that $z_{2d} = V_d$ is asymptotically stable for all initial conditions of (16) satisfying $z_{2d}(0) > 0$. Similarly, the second equilibrium point is asymptotically stable for all initial conditions satisfying, $z_{2d}(0) < 0$.

We summarize the above result in the following proposition:

Proposition 2.1

Consider the averaged dynamics of the 'boost' converter,

$$\mathcal{D}_B \dot{z} + (1-\mu)\mathcal{J}_B z + \mathcal{R}_B z = \mathcal{E}_B \tag{17}$$

with $z^T = [z_1, z_2] \in \mathcal{R}^2$, z_1 being the average inductor current and z_2 the average capacitor voltage. The quantity $\mu \in [0, 1]$ is the duty ratio function.

Define a non-linear dynamic state feedback controller as

$$\dot{z}_{2d} = -\frac{1}{\mathrm{RC}}\left\{z_{2d} - \frac{V_d^2}{E z_{2d}}\left[E + R_1\left(z_1 - \frac{V_d^2}{\mathrm{RE}}\right)\right]\right\}$$

$$\mu(t) = 1 - \frac{1}{z_{2d}(t)}\left[E + R_1\left(z_1 - \frac{V_d^2}{\mathrm{RE}}\right)\right] \tag{18}$$

where the dynamical controller initial condition is chosen so that, $z_{2d}(0) > 0$ and the constant reference value for z_2, denoted by V_d is a strictly positive quantity. The quantity R_1 is a designer-chosen constant with the only restriction of being strictly positive. Under these conditions, the closed-loop system (17) and (18) has an equilibrium point given by,

$$(z_1, z_2, z_{2d}) = \left(\frac{V_d^2}{\mathrm{RE}}, V_d, V_d\right) \tag{19}$$

which is asymptotically stable.

2.2. *A passivity-based controller for the 'buck-boost' converter*

The following proposition summarizes the passivity-based controller for an average model of the buck-booster converter circuit

Proposition 2.2.

Consider the average dynamics of the 'buck-boost' converter circuit (see Figure 2),

$$\mathcal{D}_{BB} \dot{z} + (1-\mu)\mathcal{J}_{BB} z + \mathcal{R}_{BB} z = \mu \mathcal{E}_{BB} \tag{20}$$

with $z^T = [z_1, z_2] \in \mathcal{R}^2$, z_1 being the average inductor current and z_2 the average capacitor voltage, $\mu \in [0, 1]$ being the duty ratio function,

$$\mathcal{D}_{BB} = \begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix}, \quad \mathcal{J}_{BB} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad \mathcal{R}_{BB} = \begin{bmatrix} 0 & 0 \\ 0 & 1/R \end{bmatrix}, \quad \mathcal{E}_{BB} = \begin{bmatrix} E \\ 0 \end{bmatrix} \tag{21}$$

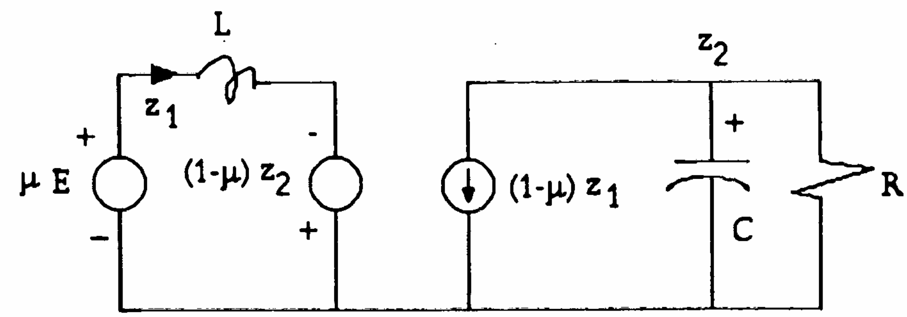

Figure 2. Average model of PWM regulated 'buck–boost' converter

Define a non-linear dynamic state feedback controller as

$$\dot{z}_{2d} = -\frac{1}{RC}\left\{z_{2d} + V_d\left(\frac{V_d}{E}+1\right)\left[\frac{E + R_1(z_1 - (v_d/R)(V_d/E+1))}{E - z_{2d}(t)}\right]\right\}$$

$$\mu(t) = \frac{z_{2d}(t) + R_1(z_1 - (v_d/R)(V_d/E+1))}{z_{2d}(t) - E} \tag{22}$$

where the controller initial condition $z_{2d}(0) < \mathrm{E}$ and where $-V_d$ is a constant reference value for z_2, with $V_d > 0$, and $R_1 > 0$ is a designer-chosen constant. Under these conditions, the closed-loop system (20–22) has an equilibrium point,

$$(z_1, z_2, z_{2d}) = \left(\frac{V_d}{R}\left(\frac{V_d}{E}+1\right), -V_d, -V_d\right) \tag{23}$$

which is asymptotically stable.

Proof. The fact that (23) is an equilibrium point for (20)–(22) follows from direct substitution. Define,

$$z_{1d} = \frac{V_d}{R}\left(\frac{V_d}{E}+1\right) \tag{24}$$

Note that z_{1d} and z_1 coincide at the equilibrium point. Let $\tilde{z}$ denote, as before, the error signal $z - z_d$. We can write (20) in terms of these error signals as

$$\mathscr{D}_{\mathrm{BB}}\dot{\tilde{z}} + (1-\mu)\mathscr{J}_{\mathrm{BB}}\tilde{z} + \mathscr{R}_{\mathrm{BB}d}\tilde{z} = \psi \tag{25}$$

where

$$\psi = \mu\mathscr{E}_{\mathrm{BB}} - \left[\mathscr{D}_{\mathrm{BB}}\dot{z}_d + (1-\mu)\mathscr{J}_{\mathrm{BB}}z_d + \mathscr{R}_{\mathrm{BB}}z_d - \begin{bmatrix} R_1\tilde{z}_1 \\ 0 \end{bmatrix}\right] \tag{26}$$

and $\mathscr{R}_{\mathrm{BB}d}$ is a positive-definite matrix given by

$$\mathscr{R}_{\mathrm{BB}d} = \begin{bmatrix} R_1 & 0 \\ 0 & 1/R \end{bmatrix}, \quad R_1 > 0 \tag{27}$$

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Expression (26) is explicitly written as

$$\psi_1 = -L\dot{z}_{1d} + (1-\mu)z_{2d} + R_1\tilde{z}_1 + \mu E$$
$$\psi_2 = -C\dot{z}_{2d} - (1-\mu)z_{1d} - \frac{1}{R}z_{2d} \tag{28}$$

Using (20)–(22) one has $\psi_1 = 0$ and $\psi_2 = 0$. The resulting stabilization error system is then given by the asymptotically stable dynamics,

$$\mathscr{D}_{BB}\dot{\tilde{z}} + (1-\mu)\mathscr{J}_{BB}\tilde{z} + \mathscr{R}_{BBd}\tilde{z} = 0 \tag{29}$$

Using as a Lyapunov function the total energy of the error system $H_d(t) = \frac{1}{2}z^T\mathscr{D}_{BB}\tilde{z} > 0$ one obtains, for some constant $\alpha > 0$, that along the trajectories of (29) the following relation is satisfied:

$$\dot{H}_d(t) = -\tilde{z}^T\mathscr{R}_{BBd}\tilde{z} \leqslant -\alpha\|\tilde{z}\|^2 \tag{30}$$

where α may be taken as $\min\{R_1, 1/R\}$. One concludes that $\tilde{z} \to 0$ asymptotically.

The zero dynamics associated with the proposed dynamical feedback controller is given by

$$\dot{z}_{2d} = -\frac{1}{RC}\left[z_{2d} + V_d\left(\frac{V_d}{E}+1\right)\frac{E}{E-z_{2d}}\right] \tag{31}$$

which has an asymptotic equilibrium point at $z_{2d} = -V_d$ for all initial conditions satisfying $z_{2d}(0) < E$ and it also has an equilibrium point located at $z_{2d} = E + V_d$ for all $z_{2d}(0) > E$. □

2.3. *A passivity-based controller for the 'buck' converter*

The following proposition summarizes the passivity-based controller for an average model of the 'buck' converter circuit shown in Figure 3. The proof of the result is left as an exercise for the interested reader.

Proposition 2.3.

Consider the averaged dynamics of the 'buck' converter,

$$\mathscr{D}_b\dot{z} + (\mathscr{J}_b + \mathscr{R}_b)z = \mu\mathscr{E}_b \tag{32}$$

with $z^T = [z_1, z_2] \in \mathscr{R}^2$, z_1 being the average inductor current and z_2 the average capacitor voltage, $\mu \in [0, 1]$ being the duty ratio function,

$$\mathscr{D}_b = \begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix};\quad \mathscr{J}_b = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix};\quad \mathscr{R}_b = \begin{bmatrix} 0 & 0 \\ 0 & 1/R \end{bmatrix};\quad \mathscr{E}_b = \begin{bmatrix} E \\ 0 \end{bmatrix} \tag{33}$$

Define a linear time-varying state feedback controller as

$$\dot{z}_{2d} = -\frac{1}{RC}\left(z_{2d} - V_d\right)$$
$$\mu(t) = \frac{z_{2d}(t) - R_1(z_1 - V_d/R)}{E} \tag{34}$$

where the controller initial condition, $z_{2d}(0)$, satisfies, $E > z_{2d}(0) > 0$, and where $V_d > 0$, is a constant reference value for z_2. The parameter, $R_1 > 0$, is a designer chosen constant. Under

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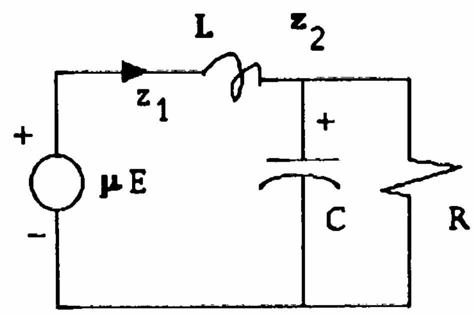

Figure 3. Average model of PWM regulated 'buck' converter

these conditions, the closed-loop system (32)–(34) has an equilibrium point,

$$(z_1, z_2, z_{2d}) = \left(\frac{V_d}{R}, V_d, V_d\right) \tag{35}$$

which is asymptotically stable.

3. PASSIVITY-BASED ADAPTIVE CONTROLLER DESIGN FOR DC-TO-DC POWER CONVERTERS

In this section we assume that the resistive loads, in the various converters, are known to be constant, but otherwise completely unspecified. This type of uncertain situation is commonly present in many practical applications of dc-to-dc power converters. To handle this type of uncertainty, adaptive versions of the previously developed feedback controllers are developed. We emphasize that the results here presented can be extended, with little difficulty, to the case of uncertainties in the rest of the circuit component parameters, L, C and E, of the dc-to-dc power converter circuit.

3.1. An adaptive controller for the 'boost' converter

Proposition 3.1.

Consider the averaged dynamics (17) of the 'boost' converter, where $C > 0$, $L > 0$, $E > 0$ are known constants representing the capacitance, inductance and external voltage, respectively, and $R > 0$ is the *unknown* load charge resistance. Define an *adaptive* non-linear dynamic state feedback controller as

$$\begin{aligned}
\dot{z}_{2d} &= -\frac{\hat{\theta}}{C}\left\{z_{2d} - \frac{V_d^2}{E z_{2d}}\left[E + R_1\left(z_1 - \hat{\theta}\frac{V_d^2}{E}\right) + L\frac{V_d^2}{E} z_{2d}(z_2 - z_{2d})\right]\right\}\\
\mu &= 1 - \frac{1}{z_{2d}}\left[\mathrm{E} + R_1\left(z_1 - \hat{\theta}\frac{V_d^2}{E}\right) + L\frac{V_d^2}{E} z_{2d}(z_2 - z_{2d})\right]\\
\dot{\hat{\theta}} &= -z_{2d}(z_2 - z_{2d})
\end{aligned} \tag{36}$$

where the dynamical controller initial condition is chosen so that, $z_{2d}(0) > 0$ and $\hat{\theta}(0) > 0$. The constant reference value for z_2, denoted by V_d, is a strictly positive quantity. The quantity $\hat{\theta}$ denotes the estimate of $1/R$. The parameter R_1 is a designer-chosen constant with the only restriction of being strictly positive. Under these conditions, it is always possible to choose the controller's initial state $z_{2d}(0)$ and $\hat{\theta}(0)$, such that the closed-loop system (17) and (36) has an equilibrium point given by

$$(z_1, z_2, z_{2d}, \hat{\theta}) = \left(\frac{1}{R}\frac{V_d^2}{E}, V_d, V_d, \frac{1}{R}\right) \tag{37}$$

which is *asymptotically stable*.

Proof. It can be verified, by direct substitution, that (37) represents an equilibrium point for the closed-loop system.
Define

$$z_{1d} = \hat{\theta}\frac{V_d^2}{E} \tag{38}$$

Note that z_{1d} and z_1 coincide at the equilibrium point. Let, again, $z - z_d$ stand for the error vector $\tilde{z}$.

In terms of the error signals, (17) is rewritten as

$$\mathscr{D}_B\dot{\tilde{z}} + (1-\mu)\mathscr{J}_B\tilde{z} + \mathscr{R}_{Bd}\tilde{z} = \psi \tag{39}$$

where

$$\psi = \mathscr{E}_B - \left[\mathscr{D}_B\dot{z}_d + (1-\mu)\mathscr{J}_B z_d + \mathscr{R}_B z_d - \begin{bmatrix} R_1\tilde{z}_1 \\ 0 \end{bmatrix}\right] \tag{40}$$

and $\mathscr{R}_{Bd}$ is a positive-definite matrix given by

$$\mathscr{R}_{Bd} = \begin{bmatrix} R_1 & 0 \\ 0 & 1/R \end{bmatrix}, \quad R_1 > 0 \tag{41}$$

Expression (40) is explicitly written as

$$\begin{aligned} \psi_1 &= -L\dot{z}_{1d} - (1-\mu)z_{2d} + R_1\tilde{z}_1 + E \\ \psi_2 &= -C\dot{z}_{2d} + (1-\mu)z_{1d} - \frac{1}{R}z_{2d} \end{aligned} \tag{42}$$

Using (38) and (36) one has $\psi_1 = 0$ and $\psi_2 = \tilde{\theta}z_{2d}$, where $\tilde{\theta} = \hat{\theta} - 1/R$.

The resulting stabilization error system is then given by the following perturbed dynamics:

$$\mathscr{D}_B\dot{\tilde{z}} + (1-\mu)\mathscr{J}_B\tilde{z} + \mathscr{R}_{Bd}\tilde{z} = \begin{bmatrix} 0 \\ \tilde{\theta}z_{2d} \end{bmatrix} \tag{43}$$

Using as a Lyapunov function the total energy of the stabilization error system plus the 'energy' associated with the parameter estimation error

$$H_d(t) = \tfrac{1}{2}\left[\tilde{z}^T\mathscr{D}_B\tilde{z} + \tilde{\theta}^2\right] \tag{44}$$

one verifies that, along the trajectories of (43), the following relation is satisfied:

$$\dot{H}_d(t) = -\tilde{z}^{\mathrm{T}}\mathscr{R}_{\mathrm{B}d}\tilde{z} + \tilde{\theta}\left[\dot{\tilde{\theta}} + z_{2d}(z_2 - z_{2d})\right] \tag{45}$$

Using the last equation in (36) and the fact that $\dot{\tilde{\theta}} = \dot{\hat{\theta}}$, one obtains

$$\dot{H}_d(t) = -\tilde{z}^{\mathrm{T}}\mathscr{R}_{\mathrm{B}d}\tilde{z} \leqslant -\alpha\|\tilde{z}^2\| \tag{46}$$

where α may be taken to be $\min\{R_1, 1/R\}$. One concludes that $\tilde{z}$ and $\tilde{\theta}$ *are bounded and that* $\tilde{z}$ is square integrable. To actually show that $\tilde{z} \to 0$ asymptotically, it must be verified that $\tilde{z}$ is uniformly continuous. For this, it suffices to show that $\dot{\tilde{z}}$ is bounded. From the perturbed error dynamics (43), and the established boundedness of $\tilde{\theta}$ and $\tilde{z}$, it follows that $\dot{\tilde{z}}$ is bounded if, and only if, z_{2d} is bounded. In order to prove that z_{2d} is bounded, note first that its associated zero dynamics, given by

$$\dot{z}_{2d} = -\frac{\hat{\theta}}{C}\left(z_{2d} - \frac{V_d^2}{z_{2d}}\right) \tag{47}$$

is asymptotically stable towards the equilibrium point located at $z_{2d} = V_d$, for all initial conditions satisfying $z_{2d}(0) > 0$, provided $\hat{\theta} > 0\ \forall t$ The dynamics (47) is also asymptotically stable towards a second equilibrium point, located at $z_{2d} = -V_d$, for all initial conditions satisfying $z_{2d}(0) < 0$, provided $\hat{\theta} > 0\ \forall t$.

Take as a Lyapunov function candidate for the controller dynamics, $V_2 = C/2\,(z_{2d} - V_d)^2$. The time derivative of V_2 along the trajectories of (36) results in the following expression:

$$\dot{V}_2 = -\hat{\theta}(z_{2d} - V_d)\left\{z_{2d} - \frac{V_d^2}{z_{2d}}\left[\left(1 + \frac{R_1}{E}\tilde{z}_1\right) + L\frac{V_d^2}{E}z_{2d}\tilde{z}_2\right]\right\} \tag{48}$$

Then, by virtue of the boundedness of $\tilde{z}_1$, $\tilde{z}_2$, and $\hat{\theta}$, and the fact that initial conditions for such variables can be entirely chosen at will and, also provided that $\hat{\theta} > 0\ \forall t$, it follows that given positive constants β_1 and β_2, with

$$0 < \beta_1 < \frac{E}{R_1}, \quad 0 < \beta_2 < \frac{2E^2}{LV_d^3}\sqrt{\frac{R_1}{E}\beta_1} \tag{49}$$

such that initial conditions for the error vector components satisfy, $|\tilde{z}_1| < \beta_1$; $|\tilde{z}_2| < \beta_2$, then, the time derivative of V_2, given by (48), is strictly negative *outside* the closed interval $[Z_m, Z_M]$ of the real line, containing in its interior the equilibrium point, V_d, for z_{2d}, where

$$Z_m = V_d\left\{\left[\sqrt{1 - \frac{R_1}{E}\beta_1 + \frac{L^2V_d^6}{4E^4}\beta_2^2}\right] - \frac{LV_d^3}{2E^2}\beta_2\right\}$$

$$Z_M = V_d\left\{\left[\sqrt{1 + \frac{R_1}{E}\beta_1 + \frac{L^2V_d^6}{4E^4}\beta_2^2}\right] - \frac{LV_d^3}{2E^2}\beta_2\right\} \tag{50}$$

We conclude that $\tilde{z}$ is absolutely continuous and hence $\lim_{t\to\infty}\tilde{z}(t) = 0$. Moreover, given that z_2 asymptotically converges to the same equilibrium point of z_{2d}, given by V_d, it follows z_1 converges to its corresponding equilibrium value, V_d^2/RE. Since z_1 and z_{1d} asymptotically converge to the same equilibrium point, then, it follows that, necessarily, $\hat{\theta} \to 1/R$. □

3.2. An adaptive controller for the 'buck–boost' converter

The following proposition summarizes the properties of a passivity-based non-linear adaptive dynamical controller for the 'buck–boost' converter. The proof follows similar arguments to those already used in the previous proposition.

Proposition 3.2.

Consider the averaged dynamics (20) and (21) of the 'buck–boost' converter circuit, where $C > 0$, $L > 0$, $E > 0$ are *known* constants representing the capacitance, inductance and external voltage, respectively, and $R > 0$ is the *unknown* load charge resistance.
Define an adaptive non-linear dynamic state feedback controller as

$$\dot{z}_{2d} = -\frac{\hat{\theta}}{C}\left\{z_{2d} + V_d\left(\frac{V_d}{E}+1\right)\left[\frac{E + LV_d(V_d/E+1)z_{2d}(z_2 - z_d) + R_1(z_1 - V_d(V_d/E+1)\hat{\theta})}{E - z_{2d}(t)}\right]\right\}$$

$$\mu(t) = \frac{z_{2d}(t) + LV_d(V_d/E+1)z_{2d}(z_2 - z_d) + R_1(z_1 - V_d(V_d/E+1)\hat{\theta})}{z_{2d}(t) - E} \tag{51}$$

where the dynamical controller initial condition is chosen so that, $z_{2d}(0) < E$ and $\hat{\theta}(0) > 0$. The constant reference value for z_2, denoted by $-V_d$, is a strictly negative quantity. The quantity $\hat{\theta}$ denotes the estimate of 1/R. The parameter R_1 is a designer-chosen constant with the only restriction of being strictly positive. Under these conditions, it is always possible to choose the controller's initial state $z_{2d}(0)$ and $\hat{\theta}(0)$, such that the closed-loop system (20) and (51) has an equilibrium point given by

$$z_1, z_2, z_{2d}, \hat{\theta}) = \left(\frac{V_d}{R}\left(\frac{V_d}{E}+1\right), -V_d, -V_d, \frac{1}{R}\right) \tag{52}$$

which is *asymptotically stable*.

3.3. An adaptive controller for the 'buck' converter

The following proposition summarizes the passivity-based adaptive controller for an average model of the 'buck' converter circuit. The proof of the result is left as an excercise for the interested reader.

Proposition 3.3.

Consider the averaged dynamics (32) and (33) of the 'buck' converter circuit, where $C > 0$, $L > 0$, $E > 0$ are *known* constants representing the capacitance, inductance and external voltage, respectively, and $R > 0$ is the *unknown* load charge resistance.
Define a linear adaptive time-varying state feedback controller as

$$\dot{z}_{2d} = -\frac{\hat{\theta}}{C}\left(z_{2d} - V_d\right)$$

$$\mu(t) = \frac{-LV_d z_{2d}(z_2 - z_{2d}) + z_{2d}(t) - R_1(z_1 - V_d\hat{\theta})}{E}$$

$$\dot{\hat{\theta}} = -z_{2d}(z_2 - z_{2d}) \tag{53}$$

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where the controller initial condition, $z_{2d}(0)$, satisfies, $E > z_{2d}(0) > 0$, $\hat{\theta}(0) > 0$ and where $V_d > 0$, is a constant reference value for z_2. The parameter, $R_1 > 0$, is a designer-chosen constant. Under these conditions, the closed-loop system (32) and (53) has an equilibrium point,

$$(z_1, z_2, z_{2d}, \hat{\theta}) = \left(\frac{V_d}{R}, V_d, V_d, \frac{1}{R}\right) \tag{54}$$

which is *asymptotically stable*.

4. SIMULATION RESULTS

Simulations of the closed-loop behaviour of the average boost converter and the passivity-based indirect adaptative feedback controller were performed on the following perturbed version of the 'boost' converter circuit:

$$\dot{z}_1 = -(1-\mu)\frac{1}{L}z_2 + \frac{E+\eta}{L}$$
$$\dot{z}_2 = (1-\mu)\frac{1}{C}z_1 - \frac{1}{RC}z_2 \tag{55}$$

where η represents an external stochastic perturbation input affecting the system behaviour directly through the external voltage source value. Note that this perturbation input is of the "unmatched" type, i.e., it enters the system equations through an input channel vector field, given by $[1/L\ 0]^T$ which is *not* in the range space of the control input channel, given by the vector field $[z_2/L \quad z_1/C]^T$. The magnitude of the noise was chosen to be, approximately, 5% of the value of E. The circuit parameter values were taken to be the following 'typical' values:

$$C = 20\ \mu\text{F}, \quad R = 30\ \Omega, \quad L = 20\ \text{mH}, \quad \text{E} = 15\ \text{V}$$

$z_{1d} = 3.125$ A, with a steady-state duty ratio of $U = 0.6$. This corresponds to a nomial average output voltage, $z_2 = V_d = 37.5$ V. Figure 4 shows the closed-loop state trajectories corresponding to the feasible adaptive duty ratio synthesizer derived for the 'boost' converter. This figure also presents the trajectory of the duty ratio function, the trajectory of the parameter estimation values and a realization of the computer-generated stochastic perturbation signal η/L, addressed to as the 'total perturbation noise'.

The simulation show that the proposed controller achieves the desired indirect stabilization of the output voltage around the desired equilibrium value while exhibiting a high degree of robustness with respect to the external stochastic perturbation input.

4.1. 'Bust–Boost' converter

Simulations were also carried for evaluating the closed-loop behaviour of an indirectly adaptively regulated, perturbed 'buck–boost' converter The converter parameter values were chosen to be identical to those of the previously considered 'boost' converter simulation example. The perturbed switch-regulated model used in the simulations was taken to be

$$\dot{z}_1 = -(1-u)\frac{1}{L}z_2 + u\frac{E+\eta}{L}$$
$$\dot{z}_2 = -(1-u)\frac{1}{C}z_1 - \frac{1}{RC}z_2 \tag{56}$$

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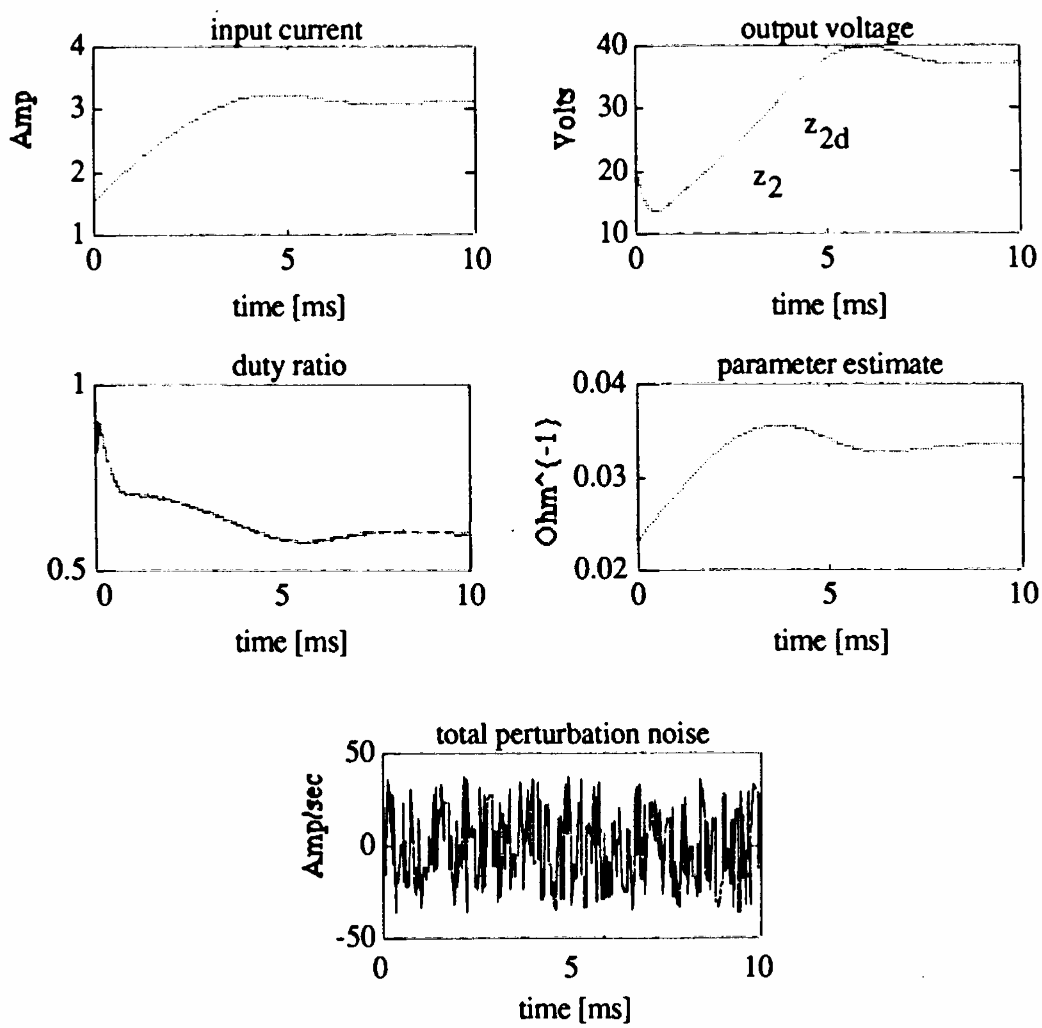

Figure 4. Simulation results for performance evaluation of the indirect adaptive controller in a perturbed average 'boost' converter

The desired average input inductor current was set to be $z_{1d} = I_d = 1.875$ A, with a steady-state duty ratio of $U = 0.6$. This corresponds to a nominal average output voltage, $z_2 = -V_d = -22.5$ V, Figure 5 shows the closed-loop state trajectories corresponding to the adaptive duty ratio synthesizer derived for the 'buck–boost' converter. This figure also presents the trajectory of the duty ratio function, the trajectory of the parameter estimation values and

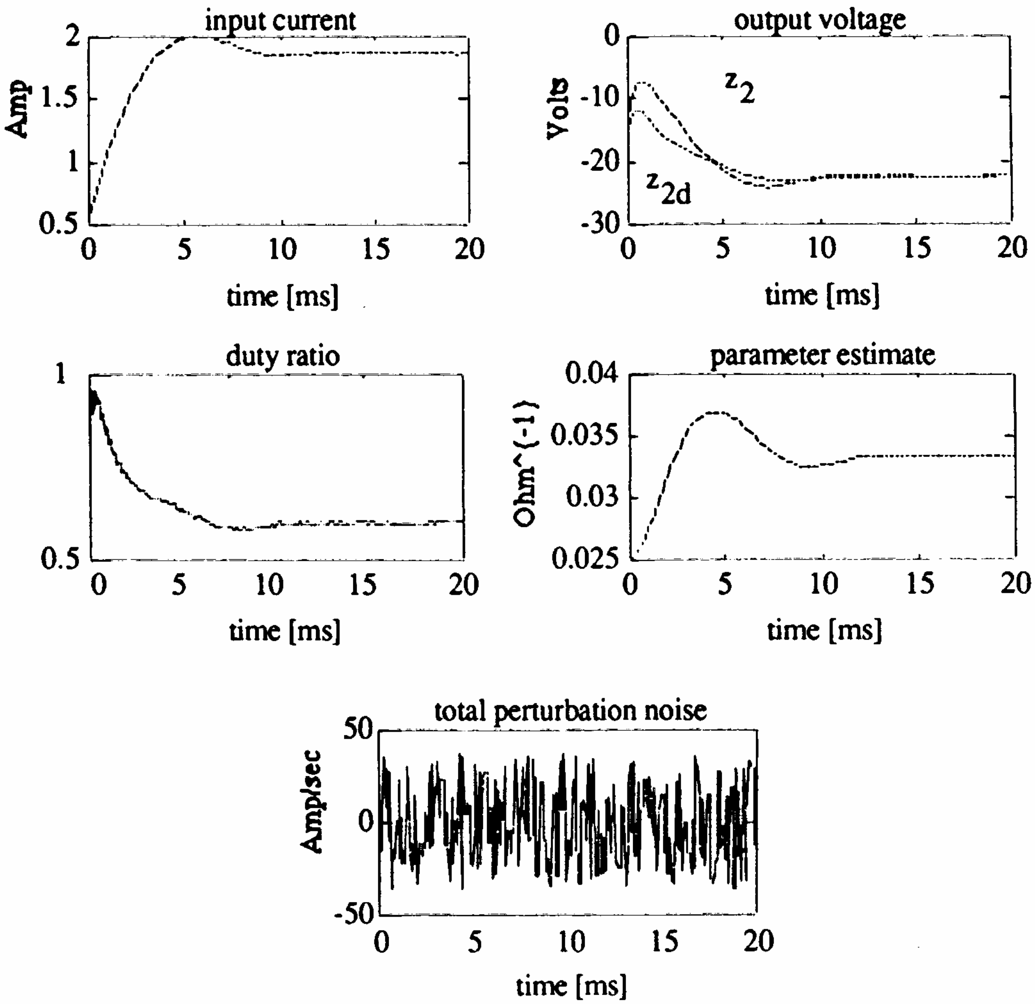

Figure 5. Simulation results for performance evaluation of the indirect adaptive PWM controller in a perturbed average 'buck-boost' converter

a realization of the total perturbation noise signal, η/L. The magnitude of the perturbation noise η was chosen to be, approximately, 5% of the value of E.

As it can be seen from the simulations, the proposed adaptive controller achieves the desired indirect stabilization of the output voltage around the desired equilibrium value while exhibiting a high degree of robustness with respect to the 'unmatched' external stochastic perturbation input.

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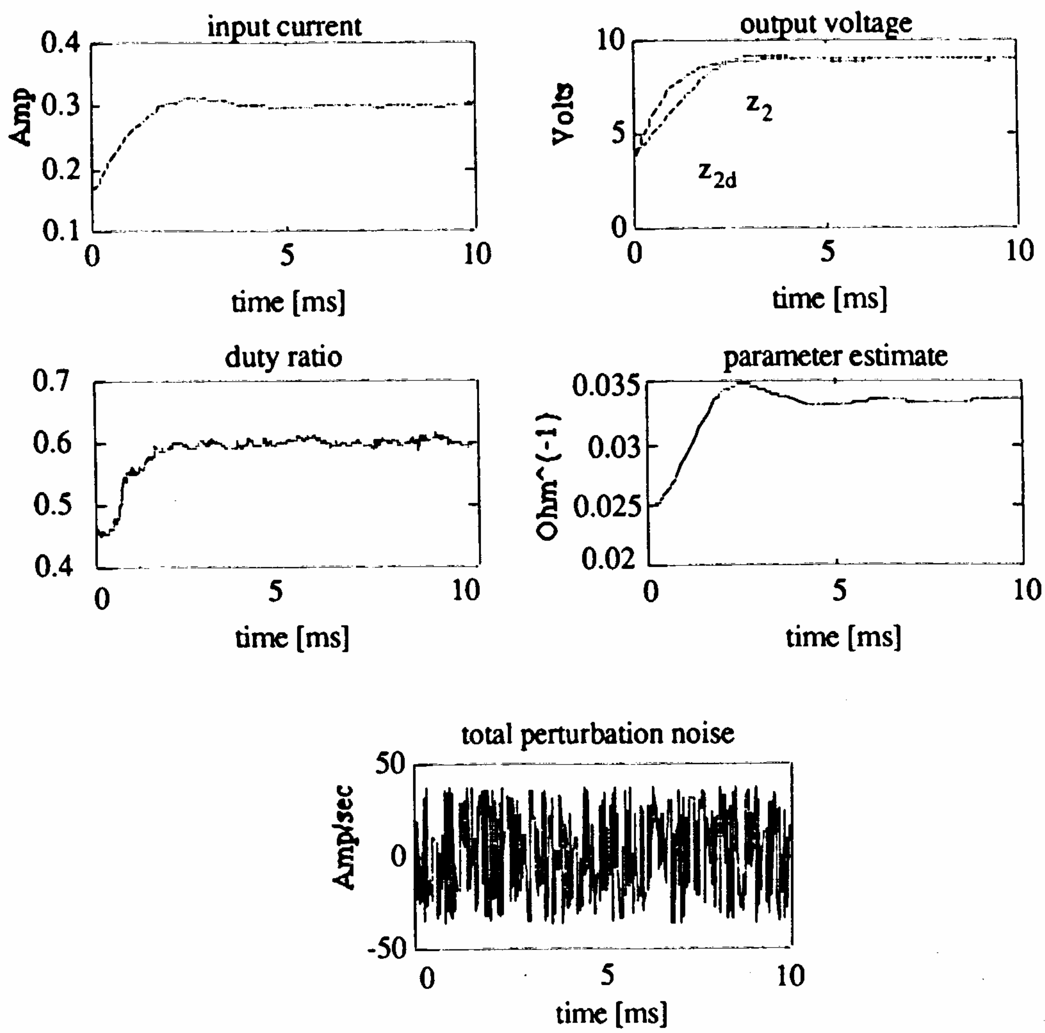

Figure 6. Simulation results for performance evaluation of the indirect adaptive controller in a perturbed 'buck' converter

4.2. *'Buck' converter*

Simulations were also carried for an indirect regulation scheme acting on a perturbed, 'buck' converter of the form,

$$\dot{z}_1 = -\frac{1}{L}z_2 + u\frac{E+\eta}{L}$$

$$\dot{z}_2 = \frac{1}{C}z_1 - \frac{1}{RC}z_2 \qquad (57)$$

The desired average input inductor current was set to be $z_{1d} = 0.3$ A, with a steady-state duty ratio of $V = 0.6$. This corresponds to a nominal average output voltage, $z_2 = V_d = 9$ V. Figure 6 shows the closed-loop state trajectories corresponding to the adaptive duty ratio synthesizer derived for the "buck" converter. This figure also presents the trajectory of the duty ratio function, the trajectory of the parameter estimation values and a realization of the total perturbation noise signal η/L. The magnitude of the perturbation noise η was chosen to be, approximately, 5% of the value of E.

5. CONCLUSIONS

In this article a passivity-based regulation scheme has been developed for the on-line feedback specification of the stabilizing duty ratio function in various kinds of dc-to-dc power converters. The controller designs were first tackled under the assumptions of perfectly known loads and then they were extended to handle, in an adaptative fashion, the more realistic case of uncertain resistive loads. The proposed approach is based on a combination of closed-loop energy shaping and appropriate stabilizing damping injection through dynamical state feedback. The proposed technique, which uses the total energy of the system as a Lyapunov function was shown to easily accommodate for parameteric uncertainties at the load. The results may also be extended to those cases where other important circuit parameters are also regarded to be constant but unknown. Based on the encouraging experimental results reported in Reference 20, 21, further work is in progress to implement, in a laboratory set-up, several non-linear adaptive regulation schemes including the one desired in this article, for some of the dc-to-dc power converters here described.

Average models dc-to-dc power converters have been known to be differentially flat (see Reference 29) i.e. all system variables are differential functions of the total energy of the system, which then qualifies as a linearizing output. As such, an interesting line of research can be proposed which exploits the differential flatness property of the system in connection with the possibilities of energy shaping and damping injecion, i.e. passivity, controller design techniques.

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