Letters

Output-Feedback Global Stabilization of a Nonlinear Benchmark System Using a Saturated Passivity-Based Controller

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***Abstract*—We address the problem of regulation of the benchmark rotational/translational proof mass actuator using a passivity-based controller. We show that a (relatively straightforward) modification of the output feedback saturated input controller proposed by the authors in a previous paper provides a simple and robust solution to the global asymptotic stabilization problem. The design technique is based on the practically appealing principles of energy shaping and damping injection. Computer simulations show that the performance is comparable, and in some respects better, than the one obtained with a far more complicated full-state unsaturated feedback controller.**

***Index Terms*—Nonlinear systems, output feedback, passivity, saturated control.**

I. Introduction

Controlling a translational oscillator with an attached eccentric rotational proof mass actuator (TORA)[1] was recently proposed as a benchmark problem for nonlinear system design [1]. Several solutions, based on full-state unsaturated feedback, have been proposed in the literature. In [2] they present two control laws, one of them a cascade controller and the other one a feedback passivating controller. In [3] they propose a control law based in $\mathcal{L}_2$ disturbance attenuation approach. See also [4] for an interesting experimental comparison.

The TORA is a simple underactuated Euler-Lagrange (EL) system, that is a system described by the EL equations of motion. The problem of controlling this class of systems—for which one can profitably exploit the Lagrangian structure—has been systematically studied in [5], [6] and [7], see also the recent book [8], and [9] for a related approach. The main objective of this brief note is to show how the stabilization algorithms developed in those papers apply *mutatis mutandi* to this problem, providing simple and robust solutions. In particular, we prove that a (relatively straightforward) modification to the output-feedback saturated passivity-based controller (PBC) proposed in [7] ensures global asymptotic stability. The modification is needed because in [7] we consider only fully actuated[2] EL systems while the TORA example is underactuated. Computer simulations show that the transient behavior is comparable, and sometimes better, than the one obtained with a full-state unsaturated feedback controller.

Manuscript received January 18, 1997. Recommended by Associate Editor, M. Jankovic. This work was supported in part by the Comission of European Communities under contract ERB CHRX CT 93-0380 and by the Consejo Nacional de Ciencia y Tecnología of Mexico.

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Publisher Item Identifier S 1063-6536(99)01610-3.

[1] Also called RTAC in [1].

[2] An EL system is said to be fully actuated if the number of control actions equals the number of degrees of freedom, otherwise it is underactuated.

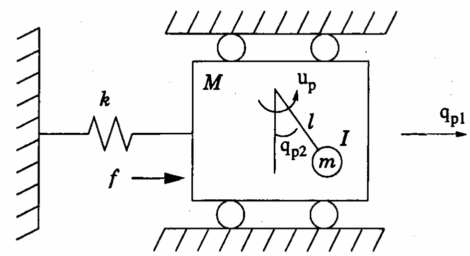

Fig. 1. Rotational/translational proof mass actuator.

II. Problem Formulation

The TORA system shown in Fig. 1 consists of a cart of mass M connected by a linear spring with stiffness k to a fixed wall.[3] The cart has only one-dimensional motion parallel to the spring axis. The proof mass actuator attached to the cart has mass m and moment of inertia I around its center of mass. The latter is located at a distance l from its rotational axis. The gravitational forces are neglected because the motion occurs in an horizontal plane. The control torque applied to the proof mass and the disturbance force on the cart are denoted by u_p and f, respectively. Similarly to the references quoted above, in the sequel we will concentrate on the behavior of the unperturbed system, therefore we will assume $f = 0$.

Let q_{p1} be the translational position of the cart and q_{p2} the angular position of the proof mass, where $q_{p2} = 0$ is perpendicular to the motion of the cart, and $q_{p2} = 90°$ is aligned with the positive q_{p1} direction.

The equations that describe the mechanical behavior of the system can be obtained applying the EL equations to the systems Lagrangian

$$\mathcal{L}(q_p, \dot{q}_p) = \mathcal{T}_p(q_p, \dot{q}_p) - \mathcal{V}_p(q_p) \tag{1}$$

where $q_p = [q_{p1}, q_{p2}] \in \mathbb{R}^2$ are the generalized coordinates. The first and second terms in (1) are the kinetic and potential energy functions, respectively, and they are defined for the TORA as

$$\begin{aligned}\mathcal{T}_p(q_p, \dot{q}_p) &= \frac{1}{2}\,\dot{q}_p^T D_p(q_p)\dot{q}_p \\ &= \dot{q}_p^T \begin{bmatrix} M+m & -ml\cos(q_{p2}) \\ -ml\cos(q_{p2}) & I+ml^2 \end{bmatrix} \dot{q}_p \\ \mathcal{V}_p(q_p) &= \frac{1}{2}\,kq_{p1}^2.\end{aligned}$$

This yields the model

$$D_p(q_p)\ddot{q}_p + C_p(q_p, \dot{q}_p)\dot{q}_p + \frac{\partial \mathcal{V}_p(q_p)}{\partial q_p} = M_p u_p \tag{2}$$

where $M_p = [0, 1]^T$ and (following the procedure of passivity-based control first articulated in [10]) we have factored the workless forces via the definition of

$$C_p(q_p, \dot{q}_p) = \begin{bmatrix} 0 & -ml\dot{q}_{p2}\sin(q_{p2}) \\ 0 & 0 \end{bmatrix}.$$

Hence, enforcing the property that

$$\dot{D}_p(q_p) = C_p(q_p, \dot{q}_p) + C_p^T(q_p, \dot{q}_p)$$

[3] See [1] for further details.

which is instrumental for the design of tracking PBC. Notice that the TORA has no natural damping.

For ease of reference we give the full systems equations as

$$\begin{bmatrix} M+m & -ml\cos(q_{p2}) \\ -ml\cos(q_{p2}) & I+ml^2 \end{bmatrix}\begin{bmatrix}\ddot{q}_{p1}\\ \ddot{q}_{p2}\end{bmatrix} + \begin{bmatrix} 0 & -ml\dot{q}_{p2}\sin(q_{p2}) \\ 0 & 0\end{bmatrix}\begin{bmatrix}\dot{q}_{p1}\\ \dot{q}_{p2}\end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & 0\end{bmatrix}\begin{bmatrix}q_{p1}\\ q_{p2}\end{bmatrix} = \begin{bmatrix} f \\ u_p\end{bmatrix}. \tag{3}$$

This is a very simple EL system that can be globally asymptotically stabilized (GAS) at any given constant equilibrium point, in particular at zero, using the energy-shaping plus damping injection principles introduced in control by Takegaki and Arimoto [11]. In its most elementary variation this technique leads to a proportional plus derivative (PD) controller

$$u_p = \underbrace{-k_p q_{p2}}_{u_{\rm ES}} - \underbrace{k_d \dot{q}_{p2}}_{u_{\rm D1}}$$

where $k_p, k_d > 0$, and $u_{\rm ES}$, $u_{\rm D1}$ are the energy-shaping and damping injection components of the control, respectively. The proof of GAS is easily established using the total energy

$$H = \mathcal{T}_p(q_p, \dot{q}_p) + \mathcal{V}_p(q_p) + \tfrac{1}{2}\, k_p q_{p2}^2$$

as Lyapunov function and applying LaSalle's principle.

In this paper we pose the more practically interesting problem of making the zero equilibrium GAS assuming that *only* q_{p2} is available for measurement. Furthermore, we assume the input is subject to a *saturation constraint*, i.e,

$$|u_p| \leq u_{\max}. \tag{4}$$

III. Solution

To solve the problem we follow the approach proposed in [6], which consists of designing an EL PBC with an interconnection constraint to ensure the closed-loop is still an EL system. In this way, the energy and dissipation functions of the closed-loop are simply the sum of the corresponding functions for the plant and the controller, respectively. Since these functions fully characterize the behavior of EL systems, the choice of the PBC boils down to a suitable selection of its energy and dissipation functions. This procedure was later extended in [7] to the case of saturated inputs. Notice, however, that in [7] we consider only fully actuated systems, hence a slight modification is needed to handle the TORA example. We refer the reader to those papers (and to [5], [8]) for further details on the PBC methodology. In particular Section II of [7] contains all the background material required for the understanding of this paper.

We propose an EL controller with scalar generalized coordinate $q_c \in \mathbb{R}$, zero kinetic energy, potential energy $\mathcal{V}_c(q_{p2}, q_c)$ and Rayleigh dissipation function $\mathcal{F}_c(\dot{q}_c)$ chosen as

$$\mathcal{V}_c(q_{p2}, q_c) = \frac{1}{b}\int_0^{q_c + bq_{p2}} \sigma_2(s)\, ds + \int_0^{q_{p2}} \sigma_1(s)\, ds$$

$$\mathcal{F}_c(\dot{q}_c) = \frac{1}{2ab}\, \dot{q}_c^2$$

where a, b are positive constants, and $\sigma_i(s) = k_i$ sat(s), $k_i > 0$, $i = 1, 2$, with sat(s): $\mathbb{R} \to \mathbb{R}$, a saturation function. We can take, for instance, sat$(s) = \tanh(s)$ or sat$(s) = 2\pi^{-1}\tan^{-1}(s)$, which, as pointed out in [7], are strictly increasing functions satisfying the following properties:

1) sat$(s) = 0 \Leftrightarrow s = 0$;
2) $s[\text{sat}(s)] \geq 0\ \forall s$;
3) sat$(-s) = -$sat(s).

Notice that the potential energy of the controller depends on the measurable output q_{p2}; this allows us to carry out the energy shaping step. The damping will be propagated via the controller dynamics. As explained in [6], to preserve the EL structure in closed-loop we impose the interconnection constraint

$$u_p = -\frac{\partial V_c(q_c, q_{p2})}{\partial q_{p2}}.$$

The controller dynamics is then given by

$$u_p = -\sigma_2(q_c + bq_{p2}) - \sigma_1(q_{p2}) \tag{5}$$

$$\dot{q}_c = -a\sigma_2(q_c + bq_{p2}). \tag{6}$$

The rationale behind this controller may be found in [6] and [7]. It may be roughly summarized as follows. From [6] we know that to ensure uniqueness and stability of the zero equilibrium the overall potential energy

$$\mathcal{V}(q_p, q_c) = \mathcal{V}_p(q_p) + \mathcal{V}_c(q_c, q_{p2})$$

must have a global minimum in zero. This can be easily verified here because $\mathcal{V}_p(q_p)$ is a quadratic function of q_{p1} and $\mathcal{V}_c(q_c, q_{p2})$ is the sum of two integrals of strictly increasing saturation functions, with global minimum at $q_c + bq_{p2} = 0$ and $q_{p2} = 0$, respectively.

Now, as thoroughly discussed in [6], due to the inability to inject damping directly (which stems from the fact that we do not measure velocities), to ensure the equilibrium is asymptotically stable we must verify a dissipation propagation condition. In our case, this condition (i.e., Assumption A.1 of Theorem 3.1 of [6]) may be stated as follows: If in the system dynamics (3) we fix u_p and q_{p2} to constant values then q_{p1} is (or converges to) a constant value.

Again, it is easy to see that this condition is verified, since $q_c + bq_{p2} = 0$, $q_{p2} = 0 \Rightarrow q_c = 0$. Now by direct substitution of u_p = const, $\dot{q}_{p2} = 0$ in the model (3) and after some manipulations we obtain that these conditions imply q_{p1} = const. It is interesting to recall that, as pointed out in [6, Remark 3.2], the dissipation propagation assumption is strictly weaker than the standard zero-state detectability condition from q_{p2}.

Finally, it follows from the proof of Proposition 7 in [7], that to enforce the input constraint (4) a restriction on the saturation gains must be also imposed.

The proposition below follows immediately from Theorem 3.1 of [6], Proposition 7 in [7] and the derivations above.

Theorem 1: Consider the TORA system described by (3), (4) in closed loop with the passivity based controller given in (5) and (6), with a, b positive constants. Define the state vector $x = [q_{p1}, \dot{q}_{p1}, q_{p2}, \dot{q}_{p2}, q_c]^T$. If

$$k_1 + k_2 \leq u_{\max} \tag{7}$$

then $x = \underline{0}$ is a globally asymptotically stable equilibrium of the closed-loop system. Consequently, for any set of initial conditions $x(0) = x_0$ we have

$$\lim_{t\to\infty} x(t) = \underline{0}.$$

Remark 2.1: Note that the control law (5) can be seen as a proportional plus derivative saturated controller where the derivative part is implemented using a dirty derivative estimation of the angular velocity error. The interest of using the saturations is to prevent the peaking phenomena as clearly pointed out in [12]. However, as discussed in [5], it makes the rate of convergence slower. We would like to stress the simplicity of our control scheme with respect to schemes reported in [3] and [2].

TABLE I

Description	Parameter	Value	Units
Cart mass	M	1.3608	Kg
Arm mass	m	0.096	Kg
Arm eccentricity	l	0.0592	m
Arm inertia	I	0.0002175	Kg/m^2
Spring stiffness	k	186.3	N/m

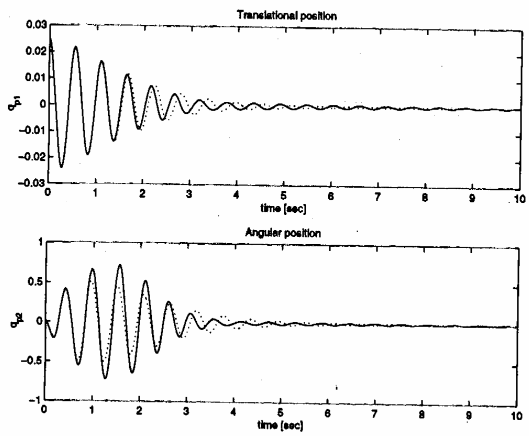

Fig. 2. Transient behavior for translational and angular positions.

Remark 2.2: After a preliminary version of this paper was accepted for publication in [13] we became aware of [14], where a dirty derivative (similar to the one used in robot control and in this paper) is added to the controller of [2] to relax the assumption of measurable velocity. We should point out that in the present work we have additionally considered the presence of input saturation.

Remark 2.3: Commissioning of this controller boils down to the selection of the positive coefficients a, b, k_1, k_2. Notice that the only constraint imposed by the theory is (7). However, transient performance is considerably affected by this choice. Some guidelines for the selection of these coefficients may be found in [5].

IV. Simulation Results

Computer simulations have been carried out to show the performance of the proposed controller. We use the TORA model (3) with the parameters values shown in Table I with the physical constraints $|q_{p1}| \leq 0.025$ m and $|u_p| \leq 0.100$ Nm given in [1].

All initial conditions are set to zero except the initial translational position which is set at its extreme value $q_{p1}(0) = 0.025$. We selected $\text{sat}(s) = \tanh(s)$ for the control law and after a few iterations in simulation to get the best transient behavior we chose the following parameters:

$$a = 550, \quad b = 4.5, \quad k_1 = 0.035, \quad k_2 = 0.018.$$

Notice that $k_1 + k_2$ is much smaller than the allowable bound (7). We observed, however, that transient performance was actually degraded for larger values of these gains.

A typical response of the system in closed loop with the proposed controller is shown in Figs. 2 and 3 (continuous trace). As we see the system exhibits good settling behavior, it stabilizes around 3 s, with a control effort, $|u_p| \leq 0.04$ N-m, well below its admissible upper bound.

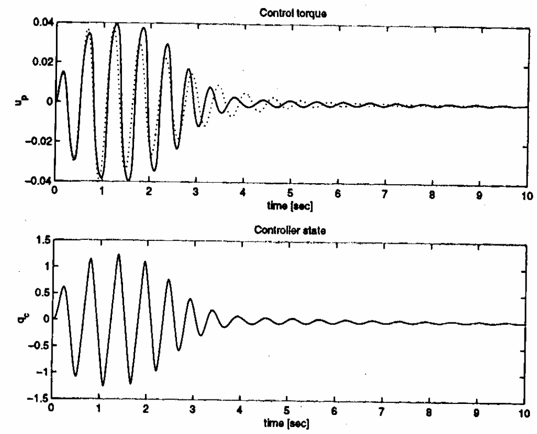

Fig. 3. Applied control and controller state.

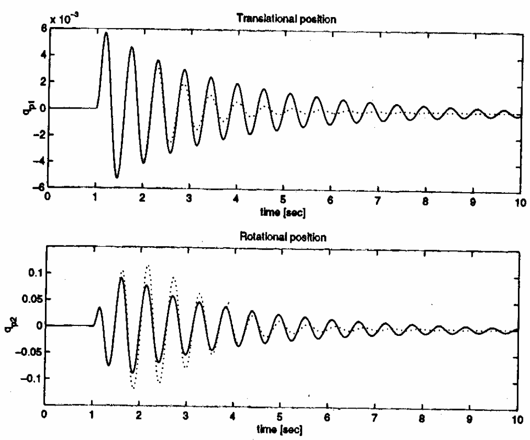

Fig. 4. Translational and rotational responses to an external disturbance.

It is interesting to compare our results with the ones obtained with the *full state feedback unsaturated* controllers reported in [2]. First of all, notice that our results pertain to the original system, while those given in [2] are carried out for its scaled version, (1) in [2] where the arbitrary value 0.1 is given for the scale coupling factor $\epsilon = ml/\sqrt{(I + ml^2)(M + m)}$. This factor equals 0.2 for the benchmark problem of [1], hence the plots are not directly comparable. Second, as pointed out in [2] the best results were obtained with the controller P3, which is a passifying controller consisting of a standard PD plus a nonlinear term that enforces the passivity property. Thus is very similar, at least in spirit, to the controller presented in this paper, although our controller is saturated and uses only output feedback. It is quite clear from the figures of [2] that the behavior of the backstepping-based controllers that did not exploit passivity properties, that is P1 and P2, is significantly inferior.

In any case, to carry out a comparison we have rewritten the controller P3 in [2] in the original variables of the benchmark problem. We propose for this controller the parameter values $k_0 = 0.01$, $k_1 = 1$, and $k_2 = 0.03$ which give a good transient response. Both controllers were simulated under the same initial conditions, and in order to make this comparison more clear we have superimposed

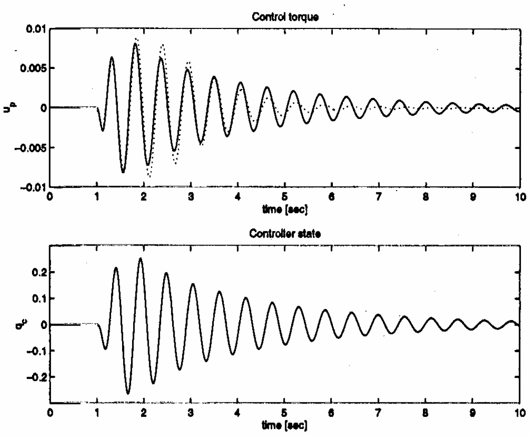

Fig. 5. Applied control and controller state for an external disturbance.

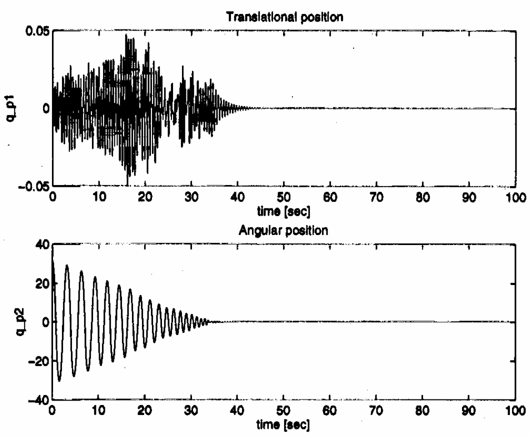

Fig. 6. Translational and rotational responses for $q_{p2}(0) = 10\pi$.

in all our figures the responses obtained with P3 of [2] (dotted line) to our responses. Notice from Figs. 2 and 3 that in spite of the fact that our controller is saturated and uses only output feedback the settling time is sometimes smaller for our controller.

In order to evaluate the robustness of the closed loop system with respect to the external disturbance f, we apply a pulse of amplitude 1 N, and duration 0.1 s to the system once it has reached its equilibrium point. A plot of this response is shown in Figs. 4, and 5. Notice that in this simulation the settling time for the controller P3 is slightly smaller than in the controller we propose.

To illustrate the global nature of our controller we present a simulation where we want to "unwind" the arm from an initial value of $q_{p2}(0) = 10\pi$ to the zero position, with all other initial conditions equal to zero.[4] In Figs. 6, and 7, we show the transient behavior. Remark from Fig. 7 that the controller actually saturates but global convergence is preserved as predicted by the theory.

[4]Notice that we are looking at the evolution of the system in Euclidean space, hence the points $q_{p2} = n\pi, n = \cdots, -1, 0, 1, \cdots$ are different. This is done just for the purposes of illustration.

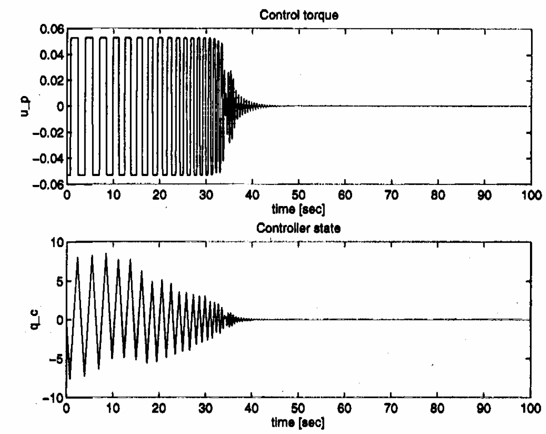

Fig. 7. Applied control and controller state for $q_{p2}(0) = 10\pi$.

V. Conclusion

The main (quite obvious, but unfortunately often overlooked) message that we wanted to convey with this paper is that when controlling physical systems it is useful to exploit its *physical* structure. In particular, for EL systems all the information on the dynamic behavior is contained in the energy and dissipation functions, hence it seems sensible to concentrate on these function for the controller design. This is the basic principle of PBC. The simplicity of the controller presented in this paper should be contrasted with the derivations reported in [2] (see also [15]). In the latter paper the first step is to make a coordinate change that transforms the system into the cascaded structure required by the backstepping technique. Unfortunately, since this coordinate transformation destroys the physical structure of the system, the controller design is now far from transparent—though in some sense systematic.

Acknowledgment

The authors are deeply grateful to the Associate Editor M. Jankovic for his careful reading of the paper and several clarifications concerning [2]. They also thank an anonymous reviewer for his constructive comments.

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