A PASSIVITY PLUS FLATNESS CONTROLLER FOR THE PERMANENT MAGNET STEPPER MOTOR

Hebertt Sira-Ramírez

ABSTRACT

A passivity based controller, in suitable combination with the flatness property of the system, is proposed for the effective feedback equilibrium to equilibrium regulation, via planned trajectory tracking, of the angular position in a permanent magnet (PM) stepper motor. The control scheme is shown to be easily modifiable as to include traditional proportional-integral-derivative (PID) feedback control actions which efficiently account for unmodeled load torque perturbations.

KeyWords: Passivity based control, differential flatness.

I. INTRODUCTION

Feedback controller design for nonlinear systems is generally faced with a hard-to-resolve compromise: How to modify the system structure by means of suitable feedback without altering the *beneficial nonlinear properties* of the system, which are known to be helpful in a given stabilization task, while imposing, on the closed-loop behavior, some of the intuitively simple and conceptually appealing features of a *linear* behavior? Moreover, how to *neutralize*, rather than completely eliminate, by means of the same feedback, those locally de-stabilizing forces in the system which work against the pre-specified control objective and, necessarily, increase the feedback control authority?.

By a simple examination of the "energy managing structure" of the system, it has been shown in [13] that a nonlinear feedback control design strategy can be proposed which is based on "neutralization and enhancement", rather than "elimination and imposition". Generally speaking, "neutralization" demands less effort than complete elimination. This is particularly important in regards to the locally de-stabilizing fields of a given open-loop nonlinear system. On the other hand, "enhancement" of the beneficial forces, namely; the stabilizing, or dissipative, forces of the system is far simpler and less demanding than the corresponding replacement, or forceful "imposition", in favor of a linear closed loop behavior. *Feedback linearization*, however intuitively appealing and conceptually effective, is recognized to be entirely based on "elimination and imposition".

However, at the same time, the conceptually appealing "hidden linear controllable" features of a given nonlinear system should not be under-estimated, or entirely neglected, as they are a valuable asset in feedback controller design and system analysis. Some of the linear features provide irreplaceable conceptual characteristics and design options, such as: simplicity, robustness with respect to internal instability problems, design flexibility, as well as -a usually neglected- "inverse physics" as "seen" from the system's internal properties and limitations towards the designer demands. All these features, and possibilities, have been elegantly bestowed into a single and ubiquitous property: differential flatness (see the far reaching theoretical contributions, and interesting applications examples, developed by Prof. M. Fliess and his colleagues in [4-6]).

In this article, a nonlinear feedback controller is proposed which effectively combines the natural energy dissipation properties of the PM stepper motor system with its differential flatness property. These two important structural properties of the system can be combined in the context of a dynamic passivity based feedback controller. The controller naturally arises from energy modification and damping injection considerations achievable on the basis of identifying, and exploiting, the natural "conservative and dissipation" structure of the nonlinear system dynamics. The passivity based controller translates into an efficient control scheme which allows for an equilibrium-to-equilibrium stabilization task, based on off-line planned trajectories prescriptions and on-line feedback trajectory tracking.

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Section 2 discusses the natural "energy managing structure" of the nonlinear multi-variable system and proceeds to identify, by means of a simple partial state feedback, the Generalized Hamiltonian nature of the PM stepper motor and its energy dissipation characteristics. An energy shaping plus damping injection based dynamic feedback controller is also synthesized which requires knowledge of the passive outputs reference trajectories. The flatness property of the stepper motor system, already established in [8], is further discussed in the context of passivity (see the seminal article by Byrnes *et al*, [2]). The passive outputs trajectories, which, due to flatness, are parameterized in terms of flat outputs trajectories, are then substituted in the feedback controller expression. The proposed dynamic feedback controller, computed on the basis of energy considerations, is now placed in terms of the off-line planned trajectories for the flat outputs. Section 3 presents the simulation results and proposes a variation of the developed controller. In order to account for constant load torque perturbations, a simple modification of the controller is made to include a traditional *outer loop* PID supplementary controller. Section 4 is devoted to some conclusions.

II. A PASSIVITY PLUS FLATNESS BASED CONTROLLER FOR THE PM STEPPER MOTOR

The PM stepper motor model used in this article is directly taken from the work of Zribi and Chiasson [14]. Further developments of nonlinear state and output feedback control techniques can be found in the articles by Bodson *et al.* [1] and Chiasson *et al.* [3]. An actual experimental sliding mode control implementation of a design, based on flatness considerations, was reported in an article by Zribi *et al.* [15].

2.1. A nonlinear model for the permanent magnet stepper motor

Consider a nonlinear model of a permanent magnet (PM) stepper motor (see Fig. 1)

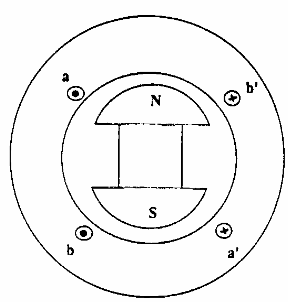

Fig. 1. PM Stepper motor ($n_p = 2$).

$$\frac{di_a}{dt} = \frac{1}{L}(\upsilon_a - Ri_a + K_m\omega\sin(N_r\theta))$$

$$\frac{di_b}{dt} = \frac{1}{L}(\upsilon_b - Ri_a - K_m\omega\cos(N_r\theta))$$

$$\frac{d\omega}{dt} = \frac{1}{J}(-K_m i_a\sin(N_r\theta) + K_m i_b\cos(N_r\theta) - B\omega - \tau)$$

$$\frac{d\theta}{dt} = \omega \tag{2.1}$$

where i_a represents the current in phase A of the motor, i_b is the current in the phase B of the motor, θ is the angular displacement of the shaft of the motor, υ_a and υ_b, stand, respectively, for the voltage applied on the windings of the phase A and phase B. The parameters R and L, the resistance and self inductances in each of the phase windings, are constant and assumed to be perfectly known. Similarly the number of rotor teeth N_r, the torque coefficient of the motor K_m, the rotor load inertia J and the viscous friction B are assumed to be known and constant. Magnetizing characteristics of typical stepper motors are nonlinear. Thus, reallistically speaking, K_m is not a constant, as assumed here. This fact, however, does not significantly change the design procedure proposed in this article. The load torque perturbation, denoted by τ, is, for all analysis purposes, assumed to be zero.

2.2 The simpler D-Q nonlinear model of the PM stepper motor

The nonlinear model 2.1 is deemed as inconvenient due to its inherent complexity, given, perhaps, by the transcendental nature of the trigonometric functions entering the description of the system. This fact has lead to the, so called, $D - Q$ (direct-to-quadrature) transformation, which is a partial state coordinate transformation in combination with an input coordinate transformation. This transformation, which gets rid of the trigonometric functions is given by

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} \cos(N_r\theta) & \sin(N_r\theta) \\ -\sin(N_r\theta) & \cos(N_r\theta) \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix}$$

$$\begin{bmatrix} \upsilon_d \\ \upsilon_q \end{bmatrix} = \begin{bmatrix} \cos(N_r\theta) & \sin(N_r\theta) \\ -\sin(N_r\theta) & \cos(N_r\theta) \end{bmatrix} \begin{bmatrix} \upsilon_a \\ \upsilon_b \end{bmatrix} \tag{2.2}$$

The current i_d is the *direct current* and i_q is the *quadrature* current. Also, υ_d and υ_q are addressed as the *direct* and *quadrature voltages*, respectively. They act as the new control inputs to the system.

The transformed system is given by

$$\frac{di_d}{dt} = \frac{1}{L}(\upsilon_d - Ri_d + N_r\omega Li_q)$$

$$\frac{di_q}{dt}=\frac{1}{L}(\upsilon_q-Ri_q-N_r\omega Li_d-K_m\omega)$$

$$\frac{d\omega}{dt}=\frac{1}{J}(K_m i_q-B\omega)$$

$$\frac{d\theta}{dt}=\omega \tag{2.3}$$

2.3 The control problem

The control objective is to drive the system from a given initial equilibrium value towards a final equilibrium value achieving, as a result, a desired final value for the position variable θ.

The equilibrium point $(\bar{i}_d, \bar{i}_q, \bar{\omega}, \bar{\theta})$ of the transformed system, for a given constant value of the direct voltage, $\upsilon_d=\bar{\upsilon}_d$ and $\upsilon_q=\bar{\upsilon}_q=0$, is given by

$$\bar{i}_d=\frac{\bar{\upsilon}_d}{R},\ \bar{i}_q=0,\ \bar{\omega}=0,\ \bar{\theta}=\text{arbitrary constant}$$

We assume that the equilibrium value of i_d is not zero. In fact, we will keep i_d away from zero throughout the equilibrium point transfer maneuver. As it will be shown, this is quite easy to guarantee.

Suppose, for a moment, that the vector relative degree {1, 1} outputs: i_d and i_q are held *constant* at some value $(i_d, i_q)=(\bar{i}_d, 0)$[1]. Then the *zero dynamics* corresponding to this set of values is given by the linear system

$$\frac{d\theta}{dt}=\omega;\ \frac{d\omega}{dt}=-\frac{B}{J}\omega \tag{2.4}$$

which exhibits two eigenvalues; one located at the origin, and the other located in the left half portion of the complex plane, at the point $(-B/J, 0)$. The system outputs, (i_d, i_q), are then *weakly minimum-phase* [2].

2.4 A passivity canonical model of the PM stepper motor

Consider the following positive definite (Lyapunov) *storage function*

$$V(i_d, i_q, \omega, \theta)=\frac{1}{2}[L(i_d^2+i_q^2)+J\omega^2+\gamma\theta^2] \tag{2.5}$$

where γ is a strictly positive constant.

The time derivative, along the controlled motions of the system, of the storage function satisfies

$$\dot{V}=-[R(i_d^2+i_q^2)+B\omega^2]+\upsilon_d i_d+\upsilon_q i_d+\gamma\theta\omega$$

$$\leq \upsilon_d i_d+\upsilon_q i_q+\gamma\theta\omega=i_d\left(\upsilon_d+\gamma\frac{\theta\omega}{i_d}\right)+\upsilon_q i_q \tag{2.6}$$

This expression, plus the weakly minimum-phase characteristics of the outputs i_d and i_q, reveals that the system is *passive* between the modified inputs $(\vartheta_d, \vartheta_q)=(\upsilon_d+\gamma\omega/i_d, \upsilon_q)$ and the original outputs (i_d, i_q). This justifies the following additional input coordinate transformation

$$\vartheta_d=\upsilon_d+\gamma\frac{\theta\omega}{i_d};\ \vartheta_q=\upsilon_q \tag{2.7}$$

which allows one to write down the system, in matrix form, as

$$\begin{bmatrix} L & 0 & 0 & 0\\ 0 & L & 0 & 0\\ 0 & 0 & J & 0\\ 0 & 0 & 0 & \gamma\end{bmatrix}\begin{bmatrix}\frac{di_d}{dt}\\ \frac{di_q}{dt}\\ \frac{d\omega}{dt}\\ \frac{d\theta}{dt}\end{bmatrix}=\begin{bmatrix} 0 & N_rL\omega & 0 & -\gamma\frac{\omega}{i_d}\\ -N_rL\omega & 0 & -K_m & 0\\ 0 & K_m & 0 & 0\\ \gamma\frac{\omega}{i_d} & 0 & 0 & 0\end{bmatrix}\begin{bmatrix} i_d\\ i_q\\ \omega\\ \theta\end{bmatrix}$$

$$+\begin{bmatrix} -R & 0 & 0 & 0\\ 0 & -R & 0 & 0\\ 0 & 0 & -B & 0\\ 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix} i_d\\ i_q\\ \omega\\ \theta\end{bmatrix}$$

$$+\begin{bmatrix} 1 & 0\\ 0 & 1\\ 0 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\vartheta_d\\ \vartheta_q\end{bmatrix} \tag{2.8}$$

The outputs of the system are taken to be $y_1=i_d$ and $y_2=i_q$, the direct and the quadrature currents. These are expressed, in matrix form, as

$$y=\begin{bmatrix} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}\begin{bmatrix} i_d\\ i_q\\ \omega\\ \theta\end{bmatrix} \tag{2.9}$$

The obtained model, clearly exhibits the *conservative* and the *dissipative* structure of the system since it is of the following special form

$$\mathcal{M}\dot{x}=\mathcal{J}(x)x+\mathcal{R}x+\mathcal{G}\vartheta$$

$$y=\mathcal{G}^T x \tag{2.10}$$

with

$$\mathcal{M}=\mathcal{M}^T>0,\ \mathcal{J}(x)+\mathcal{J}^T(x)=0,\ \mathcal{R}=\mathcal{R}^T\leq 0$$

[1] For a definition of vector relative degree, the reader is referred to the book by Isidori, [7] pp. 220.

In fact, it is easy to see the relation of the above "canonical form" with the traditional Generalized Hamiltonian systems. Let $V(x)$ be expressed as $V(x) = \frac{1}{2}x^T\mathcal{M}x$, with $\mathcal{M}$ evidently being positive definite and symmetric. The gradient *vector* of $V(x)$, given by $(\partial V/\partial x^T)^T$, and denoted by $\frac{\partial V}{\partial x}$ is simply found to be $\mathcal{M}x$. We rewrite the system (2.10) in the form

$$\dot{x} = [\mathcal{M}^{-1}\mathcal{J}(x)\mathcal{M}^{-1}]\frac{\partial V}{\partial x} + [\mathcal{M}^{-1}\mathcal{R}\mathcal{M}^{-1}]\frac{\partial V}{\partial x} + \mathcal{M}^{-1}\mathcal{G}(x)\vartheta$$

$$y = \mathcal{G}^T(x)\mathcal{M}^{-1}\frac{\partial V}{\partial x} = (\mathcal{M}^{-1}\mathcal{G})^T\frac{\partial V}{\partial x} \tag{2.11}$$

i.e., the system is in generalized Hamiltonian form (including dissipation terms)

$$\dot{x} = J(x)\frac{\partial V}{\partial x} + R\frac{\partial V}{\partial x} + G(x)\vartheta$$

$$y = G^T(x)\frac{\partial V}{\partial x} \tag{2.12}$$

2.5 A controller based on "energy shaping plus damping injection"

The "energy shaping plus damping injection" dynamic feedback controller design method, extensively treated in [9], yields, the following dynamic feedback controller specification,

$$\vartheta_d = L\frac{d}{dt}i_d^*(t) - N_pL\omega i_q^*(t) + \gamma\frac{\omega}{i_d}\zeta_2 + Ri_d^*(t)$$

$$\vartheta_q = L\frac{d}{dt}i_q^*(t) + N_pL\omega i_d^*(t) + K_m\zeta_1 + Ri_q^*(t) \tag{2.13}$$

with ζ_1 and ζ_2, satisfying,

$$J\dot{\zeta}_1 = K_m i_q^*(t) - B\zeta_1 + R_B(\omega - \zeta_1)$$

$$\gamma\dot{\zeta}_2 = \gamma\frac{\omega}{i_d}i_d^*(t) + R_\theta(\theta - \zeta_2) \tag{2.14}$$

with R_B, R_θ being positive design constants.

The original control inputs to the system are determined from the equalities,

$$\upsilon_d = \vartheta_d - \gamma\frac{\theta\omega}{i_d};\ \upsilon_q = \vartheta_q \tag{2.15}$$

Note that under equilibrium conditions, we have that $\overline{\upsilon}_d = \overline{\vartheta}_d$ and $\overline{\upsilon}_q = \overline{\vartheta}_q$. Furthermore, if $i_q^*(t)$ is made to converge to zero and ω converges to zero, it follows that ζ_1 also converges to zero. Under these circumstances θ and ζ_2 converge to a common limit.

We state the tracking error stabilization properties of the feedback controller (2.13), (2.14), as follows

Proposition 2.1 The passivity based dynamic feedback controller yields a state vector tracking error dynamics, described by the vector, $e = [i_d - i_d^*(t), i_q - i_q^*(t), \omega - \zeta_1, \theta - \zeta_2]$, which is globally exponentially asymptotically stable to zero.

Proof. Substituting the control input expressions, given in (2.13), into the d-q system model (2.3), we obtain, using the following definitions of the state tracking error variables: $e_1 = i_d - i_d^*(t)$ and $e_2 = i_q - i_q^*(t)$, $e_3 = \omega - \zeta_1$ and $e_4 = \theta - \zeta_1$

$$\begin{bmatrix} L & 0 & 0 & 0 \\ 0 & L & 0 & 0 \\ 0 & 0 & J & 0 \\ 0 & 0 & 0 & \gamma \end{bmatrix}\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \\ \dot{e}_4 \end{bmatrix} = \begin{bmatrix} 0 & N_pL\omega & 0 & -\gamma\frac{\omega}{i_d} \\ -N_pL\omega & 0 & -K_m & 0 \\ 0 & K_m & 0 & 0 \\ \gamma\frac{\omega}{i_d} & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix}$$

$$+\begin{bmatrix} -R & 0 & 0 & 0 \\ 0 & -R & 0 & 0 \\ 0 & 0 & -B-R_B & 0 \\ 0 & 0 & 0 & -R_\theta \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} \tag{2.16}$$

Using the modified energy function $V(e) = \frac{1}{2}(Le_1^2 + Le_2^2 + Je_3^2 + \gamma e_4^2)$, one establishes that, $\dot{V}(e) \leq \alpha V(e)$, with, $\alpha < 0$, being a constant dependent on the system parameters L, J, R and B and the design parameters, R_B and R_θ, given by,

$$\alpha = -\frac{\min\{R, B + R_B, R_\theta\}}{\max\{L, J, \gamma\}}$$

As a consequence of this fact, it is clear that the speed of convergence is inherently limited by the system constants and cannot be made arbitrarily large. The tracking error is then globally exponentially asymptotically stable to zero, i.e.

$$i_d \to i_d^*(t),\ i_q \to i_q^*(t),\ \omega \to \zeta_1,\ \theta \to \zeta_2 \tag{2.17}$$

In the absence of load perturbations, the desired current $i_q^*(t)$ is made to converge to zero and, then, i_q also converges to zero. The planned flat current $i_d^*(t)$ is made to converge to a nonzero constant. Then, i_d converges to the same value. The outputs i_d and i_q are passive, hence, ω and ζ_1 converge to zero. The angle θ, and ζ_2, both converge to a constant to be established. The flatness property, makes the final value of θ to be completely determined at will, as shown in the following section.

2.6 Differential flatness of the system

The PM stepper motor is easily seen to be differentially flat, since all variables in the system can be completely parameterized in terms of differential functions of the independent variables constituted by the direct current i_d and the motor shaft angular position θ (see [14,8] and [15]). The flat outputs, denoted by $F = (F_1, F_2) = (i_d, \theta)$, yield,

$$i_d = F_1,\ \theta = F_2,\ \omega = \dot{F}_2,\ i_q = \frac{J}{K_m}\ddot{F}_2 + \frac{B}{K_m}\dot{F}_2,$$

$$\vartheta_d = L\dot{F}_1 + RF_1 - N_p L\dot{F}_2\left(\frac{J}{K_m}\ddot{F}_2 + \frac{B}{K_m}\dot{F}_2\right)$$

$$\vartheta_q = \frac{LJ}{K_m}F_2^{(3)} + \frac{LB}{K_m}\ddot{F}_2 + R\left(\frac{J}{K_m}\ddot{F}_2 + \frac{B}{K_m}\dot{F}_2\right) + N_p L\dot{F}_2 F_1 + K_m\dot{F}_2 \qquad (2.18)$$

All system properties, in particular those concerning the ones needed for passivity based controller design, are already reflected in the above complete differential parameterization, as it can be easily verified.

Indeed, from (2.18) it is readily seen that i_d and i_q are *passive* outputs. For this, let i_d and i_q be arbitrary constants, say, $\bar{i}_d$, $\bar{i}_q$. Then, it follows, that $F_1 = \bar{i}_d$ is a constant (i.e., it has no zero dynamics). The differential equation $\frac{J}{K_m}\ddot{F}_2 + \frac{B}{K_m}\dot{F}_2 = \bar{i}_q$ yields an exponentially asymptotically stable to zero angular velocity $\dot{F}_2$. The flat output angular position, F_2, thus exponentially asymptotically converges to a constant value. The outputs $y = (i_d, i_q)$ are, thus, *weakly minimum-phase*. Since they are also vector relative degree equals to {1, 1}, they are, according to the results in [2], *passive*.

2.7 A dynamic controller combining passivity and flatness

The idea of combining passivity based control and differential flatness arises from the fact that the passivity based controller requires pre-specified passive outputs trajectories $i^*(t)$ and $i_q^*(t)$. Instead of directly specifying those trajectories, it is proposed to specify them in terms of the flat outputs. i.e., we take advantage of the fact that the passive outputs are differentially related to the flat outputs (which, incidentally, are devoid of zero dynamics). The (off-line) specification of such flat outputs already determines the rest of the system variables, including the passive outputs themselves.

Hence, we propose to plan the trajectories for the flat outputs and use them on the designed passivity based controller by substituting on it the differential relations linking the passive outputs with the flat outputs. The passivity based controller, exploiting the flatness property of the system, is then given by,

$$\vartheta_d = L\dot{F}_1^*(t) - N_p L\omega\left[\frac{J}{K_m}\ddot{F}_2^*(t) + \frac{B}{K_m}\dot{F}_2^*(t)\right] - \gamma\frac{\omega}{i_d}\zeta_2 + RF_1^*(t)$$

$$\vartheta_q = L\left[\frac{J}{K_m}(F_2^*(t))^{(3)} + \frac{B}{K_m}\ddot{F}_2^*(t)\right] + N_p L\omega F_1^*(t) + K_m\zeta_1 + R\left[\frac{J}{K_m}\ddot{F}_2^*(t) + \frac{B}{K_m}\dot{F}_2^*(t)\right] \qquad (2.19)$$

with ζ_1 and ζ_2, satisfying,

$$J\dot{\zeta}_1 = K_m\left[\frac{J}{K_m}\ddot{F}_2^*(t) + \frac{B}{K_m}\dot{F}_2^*(t)\right] - B\zeta_1 + R_B(\omega - \zeta_1)$$

$$\gamma\dot{\zeta}_2 = \gamma\frac{\omega}{i_d}F_1^*(t) + R_\theta(\theta - \zeta_2) \qquad (2.20)$$

with R_B, R_θ being, as before, positive design constants.

The advantages of this combination are manifold. First, if a passivity based controller has been designed, on the basis of physical energy dissipation considerations, the controller actions tend to take advantage of the beneficial nonlinearities by enhancing their dissipation properties while neutralizing the locally de-stabilizing fields. This yields a controller which requires less authority to achieve stabilization or trajectory tracking. Secondly, the flat outputs are fundamental system outputs which are devoid of internal dynamics and correspond to the linear controllability properties of the system. Hence, indirectly forcing these outputs to track pre-specified trajectories does not, *per se*, yield any internal stability problems. Note that the only troublesome aspect of the passive outputs associated zero dynamics, describing the constrained angular position evolution, lies in the fact that these passive outputs are only *weakly minimum-phase* with a potential towards instability. The prescribed passive outputs trajectories already contemplates that the corresponding angular position be forced to adopt a final constant value with corresponding zero angular velocity. The fact that the position output is a flat output circumvents any undesirable closed loop behavior arising from the possible adverse effects of the *weakly minimum-phase* characteristic of the constrained evolution of the passive outputs.

III. SIMULATION RESULTS

We conside. . PM stepper motor with the following parameters

$$R = 8.4\ \Omega\ \ L = 0.010\ \text{H},\ \ K_m = 0.05\ \text{V}-\text{s/rad}$$

$J = 3.6 \times 10^{-6}$ N – m – s^2/rad.

$B = 1 \times 10^{-4}$ N – m – s/rad. $N_r = 50$

It was desired to transfer the angular position θ from the initial value of $\theta_0 = 0$ rad., towards the final value $\theta_F = 0.03$ rad., following a trajectory specified by means of an interpolating time polynomial of the form $\psi(t, t_0, t_f)$ satisfying

$$\psi(t_0, t_0, t_f) = 0, \quad \psi(t_f, t_0, t_f) = 1 \tag{3.1}$$

Thus,

$$\theta^*(t) = \theta_0 + \psi(t, t_0, t_f)[\theta_F - \theta_0] \tag{3.2}$$

One such possible expression, including a particular interpolating polynomial $\psi(t, t_0, t_f)$, is given by

$$\theta^*(t) = \theta_0 + \left(\frac{t-t_0}{t_f-t_0}\right)^5 \left[r_1 - r_2\left(\frac{t-t_0}{t_f-t_0}\right) + r_3\left(\frac{t-t_0}{t_f-t_0}\right)^2 - \cdots - r_6\left(\frac{t-t_0}{t_f-t_0}\right)^5 \right](\theta_F - \theta_0) \tag{3.3}$$

with

$r_1 = 252$, $r_2 = 1050$, $r_3 = 1800$,

$r_4 = 1575$, $r_5 = 700$, $r_6 = 126$

and $t_0 = 0.01$ s, $t_f = 0.02$ s.

The flat output variable, i_d, was also made to follow a similar time trajectory $i_d^*(t)$, taking the d-current coordinate from the value $i_d(t_0) = i_{d0} = 0.3$ A, towards the final value $i_d(t_f) = i_{df} = 0.5$ A, during the same previous time interval $[t_0, t_f]$. In other words, we specified $i_d^*(t)$ as

$$i_d^*(t) = i_{d0} + \psi(t, t_0, t_f)(i_{df} - i_{d0}) \tag{3.4}$$

The passivity based feedback controller, proposed in the previous section, was used with the passive outputs reference trajectories given by (3.4) and

$$i_q^*(t) = \frac{J}{K_m}\ddot{\theta}^*(t) + \frac{B}{K_m}\dot{\theta}^*(t) \tag{3.5}$$

The design constants R_B and R_θ and γ, were set to be

$R_B = 0.05$, $R_\theta = 2$, $\gamma = 1$

Figure 2 shows the simulations of the closed-loop performance of the stepper motor mechanical and transformed electrical variables, in the $d - q$ coordinates, commanded by the designed passivity based controller with passive outputs reference trajectories planned in terms of the flat outputs. Figure 3 depicts the electrical variables in the $a - b$ coordinates. The load torque was set to zero in these simulations.

In order to account for unmodeled constant load torque perturbations, entering the angular velocity dynamics as τ, we used an *outer loop* proportional-

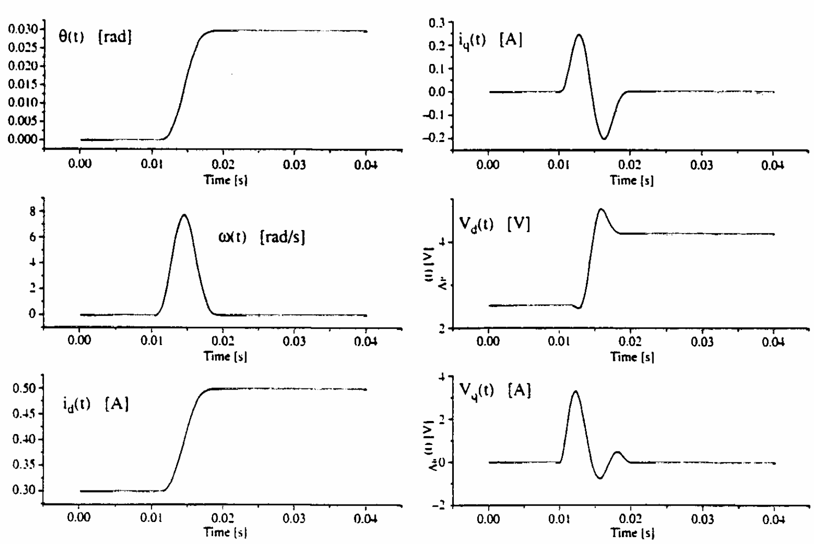

Fig. 2. PM Stepper motor closed loop response to Passivity + Flatness based controller (transformed d-q variables).

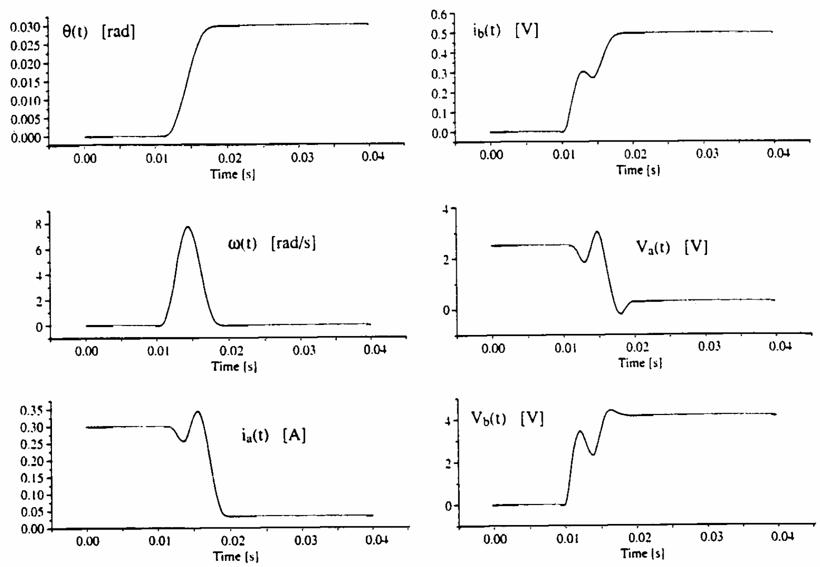

Fig. 3. PM Stepper motor closed loop response to Passivity + Flatness based controller (a-b variables).

integral-derivative (PID) controller feeding back the *dynamic controller* angular velocity tracking error $\epsilon(t) = \zeta_1 - \dot{\theta}^*(t)$. Although the design approach we followed was straightforward and intuitive, systematic methods for multivariable PID design do exist in the literature. The reader is referred to the book by Sinha [12] (chapter 7) and, also, to the works of Seraji [10,11]. The proposed PID controller guarantees, in this case, that ζ_1 actually tracks $\dot{\theta}^*(t)$, in spite of the perturbation load torque. Since ω is guaranteed to track ζ_1, by the previous arguments, the net result is that ω tracks $\dot{\theta}^*(t)$ in spite of the unknown but constant perturbations. The integral action of the PID controller corrects the angular position deviations.

The modified controller was set to be

$$\vartheta_d = L\frac{d}{dt}i_d^*(t) - N_r L\omega i_q^*(t) + \gamma\frac{\omega}{i_d}\zeta_2$$

$$+ Ri_d^* - k_{Pd}\epsilon + k_{Id}\eta + k_{Dd}\dot{\epsilon}$$

$$\vartheta_q = L\frac{d}{dt}i_q^*(t) + N_r L\omega i_d^*(t) + K_m\zeta_1 + Ri_q^*$$

$$k_{Pq}\epsilon - k_{Iq}\eta - k_{Dq}\dot{\epsilon} \qquad (3.6)$$

$$\dot{\eta} = \epsilon$$

Figure 4 shows the performance of the modified passivity based controller in the presence of constant but unknown load torque perturbations. We used $k_{Pd} = k_{Pq} = 0.01$, $k_{Id} = k_{Iq} = 60$ and $k_{Dd} = k_{Dq} = 0.001$. The load torque amplitude was taken to be 10^{-4} N-m.

IV. CONCLUSIONS

In this article, we have proposed a combination of "passivity and flatness" for the feedback regulation of a (nontrivial) nonlinear multi-variable system constituted by the PM stepper motor. The passivity based considerations lead to a natural feedback controller that takes advantage of the dissipation structure of the system, enhancing its stabilizing features, while creating suitable feedback that relegates to the conservative structure of the system the natural de-stabilizing fields of the open-loop dynamics. As a result, a dynamic feedback controller which requires less control effort, or authority, is obtained as compared, for instance, with a feedback linearizing controller. The manifold conceptual advantages of differential flatness were combined with those of passivity in order to provide natural equilibrium-to-equilibrium state trajectories for the passive outputs, in terms of the flat outputs of the system. The "stabilization-via-trajectory tracking" scheme is, generally speaking, a more demanding control objective which results in smoother transient performances for the motor currents and applied voltages. The controlled system comfortably tracks these trajectories thanks to their intimate relation with the hidden linear controllability properties of the system. The control scheme can be easily modified to include a traditional *outer loop* PID controller which effectively accounts for the unmodeled presence of constant, but unknown, load torque perturbations.

In the dynamic feedback controller proposed in this article, speed and position measurements are required. The detected speed may be replaced by a "filtered deriva-

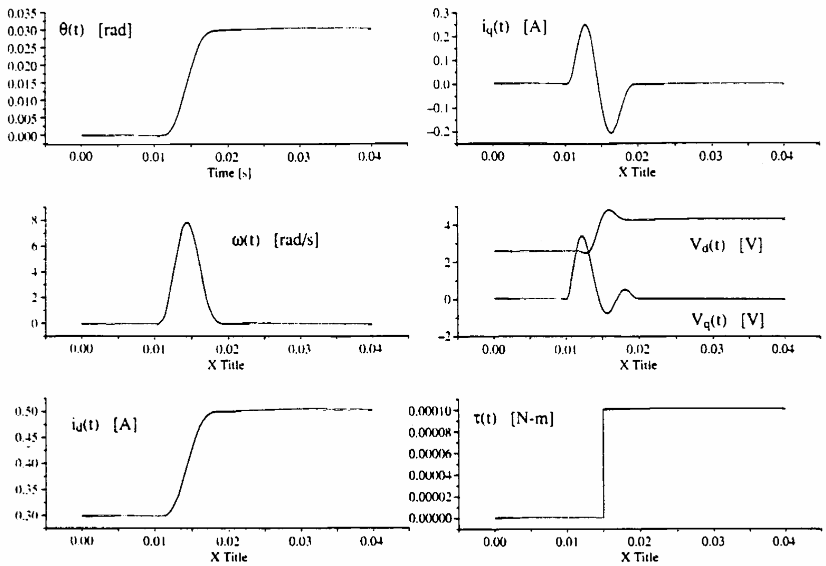

Fig. 4. PM Stepper motor closed loop response to Passivity + Flatness based controller including perturbation torque.

tive" of the position. This requires demonstrating the overall closed loop system stability. Another challenging problem includes the complete removal of all mechanical sensors. An important practical issue is constituted by the time-varying, or otherwise uncertain, characteristic of some of the stepper motor parameters. This requires extending the proposed control scheme to include robust and adaptive feedback control schemes. These topics constitute research alternatives which need to be further developed in the future.

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