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DC-to-AC power conversion on a 'boost' converter

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SUMMARY

In this article, we provide an approximate sliding mode control-based solution to the DC–AC power conversion problem on a 'boost' converter. The approach uses the flatness property of the system as a pivot for generating a sequence of minimum phase output reference trajectory candidates. The generated candidates are obtained as differential parameterizations of the minimum phase inductor current variable in terms of the non-minimum phase desired output capacitor voltage. The associated residual dynamics of the ideal sliding motions is shown to reasonably approximate the desired biased sinusoidal output capacitor voltage signal. Copyright © 2001 John Wiley & Sons, Ltd.

KEY WORDS: DC-to-AC conversion; non-minimum phase output tracking

1. INTRODUCTION

DC-to-AC power conversion using the traditional switched DC-to-DC power converter topologies constitutes a relatively recent sub-area of the Power Electronics field and it has proven to constitute a challenging area from the nonlinear feedback controller design viewpoint. This is specially so for DC-to-AC conversion schemes using converters other than the step down 'buck' converter where both state variables constitute minimum phase outputs. For the 'boost', the 'buck-boost' and the 'Cúk' converters, the output capacitor voltage variables turn out to be non-minimum phase outputs, while the input inductor current variables are, indeed, minimum phase outputs. Tracking of a biased sinusoidal signal on the part of the capacitor voltage is, in spite of the systems simplicity, a surprisingly non-trivial problem. Our work is motivated by that of Cáceres and Barbi [1] where a sliding mode

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controller is proposed for a set of coupled boost converters viewing each converters AC output capacitor voltage as a bounded, unknown, perturbation for the other converters AC signal tracking task. Recently, an article was published by Zinober *et al.* [2] dealing with the one-stage problem. In Reference [2] two solution approaches are proposed. The first one reduces the AC generation problem to the tracking of a Fourier series solution of an Abel type of differential equation. The second approach proposes a *backstepping* controller for the tracking task.

In this article, we propose a procedure for devising an approximate *sliding mode* control based solution to the DC-to-AC power conversion problem using a 'boost' converter. Part of our considerations are based on the *differential flatness* property of the system (see the work of Fliess *et al.* [3]). We propose a functional iterative, off-line, computational scheme which pivots on the flat output definition. The iterations are shown to yield *finite differential parameterizations* of the minimum phase inductor current reference trajectory in terms of the desired non-minimum phase output capacitor voltage AC reference signal. The off-line computed candidates for the inductor current reference signal are then used to devise time-varying sliding surfaces on which the sliding mode existence conditions are readily inspected while the ideal output signal tracking errors are also assessed in an exact fashion. The frequency and amplitude limitations for the desired AC output voltage signal naturally emerge as a consequence of the well-known sliding mode existence conditions (see Utkin [4] and Sira-Ramírez [5]) and the nature of the approximating scheme.

Our *indirect* sliding mode control approach is in the same spirit of the general procedure proposed by Benvenuti *et al.* [6] for the solution of output tracking and output stabilization problems is non-minimum phase systems through minimum phase output stabilization and tracking. An alternative general solution method, proposed by Fliess *et al.* in Reference [7], resorts to the differential parameterization provided by the flat outputs, which are devoid of any zero dynamics, and translates the non-minimum phase output stabilization or tracking problem into an equivalent flat output stabilization or tracking problem. However, the previous approach cannot be applied to our particular output trajectory trakcing problem. A well-founded interesting procedure has been recently proposed in the work of Devasia and Paden [8] for the approximate solution of a fairly large class of non-minimum phase output trajectory tracking problems. Under some mild technical restrictions, their approach also entitles an iterative approximation scheme, based on locally induced contractions, performed on a bounded nonlinear operator. The procedure, nevertheless, includes finding stable solutions of (potentially unstable) perturbed linear time-varying differential equations related to the ideal closed-loop residual dynamics.

Section 2 revisits the feasibility of an indirect sliding mode control scheme based on current signal tracking as opposed to voltage signal tracking. In this section we also revisit the differential flatness of the 'boost' converter and proceed to develop a scheme for the generation of a suitable inductor current reference signal based on a sequential flat output elimination from the differential parameterization provided by flatness. A sliding mode feedback tracking strategy can then be proposed for the generation of AC signals at the DC converter's output voltage. Section 3 presents various simulation results which depict the feasibility of the proposed scheme and assesses the precision with which a desired AC output voltage signal is tracked, or generated, by the proposed indirect sliding mode control scheme. Section 4 is devoted to present the conclusions and suggestions for further research.

2. DC-TO-AC POWER CONVERSION USING A BOOST CONVERTER

The switched model of a ‘boost’ DC-to-DC power converter is given by

$$L\frac{\mathrm{d}}{\mathrm{d}t}x_1 = -ux_2 + E$$
$$C\frac{\mathrm{d}}{\mathrm{d}t}x_2 = ux_1 - \frac{x_2}{R} \tag{1}$$

where x_1 represents the (input) inductor current and x_2 is the (output) capacitor voltage. The switch position is represented by the control variable u and it takes values on the discrete set $\mathscr{U} = \{0, 1\}$. The circuit parameters L, C and R are assumed to be perfectly known.

Following Reference [2], we will be using throughout a ‘normalized’ model of the ‘boost’ converter, written in terms of a base voltage value V_b, a base current value, I_b, and a time basis t_b. The base voltage value is taken as, $V_b = E$; the base current is given by $I_b = E\sqrt{C/L}$ and the normalized time scale is set to be $\tau = t/t_b$ with $t_b = \sqrt{LC}$. This normalization yields the following simplified model for the ‘boost’ converter (see Figure 1)

$$\dot{z}_1 = -uz_2 + 1 \tag{2}$$

$$\dot{z}_2 = uz_1 - \frac{z_2}{Q} \tag{3}$$

where z_1 is the normalized inductor current, z_2 is the normalized capacitor voltage and Q represents the circuit ‘quality’ given by $Q = R\sqrt{C/L}$. Time derivation, represented by the ‘dot’ notation, is understood to be carried with respect to the normalized time variable τ. With the adopted normalization, the basis value for the total energy $\mathscr{E}_b$ turns out to be $\mathscr{E}_b = CE^2$. This means that the normalized total stored energy, here denoted by F, is given by

$$F = \tfrac{1}{2}(z_1^2 + z_2^2) \tag{4}$$

2.1. Problem formulation

It is desired to devise a discontinuous feedback control law for u, such that the normalized capacitor voltage, z_2, tracks a given desired voltage signal $z_2^*(\tau)$. This desired signal is assumed to be bounded, smooth and bounded away from zero (biased). Specifically, we are interested in

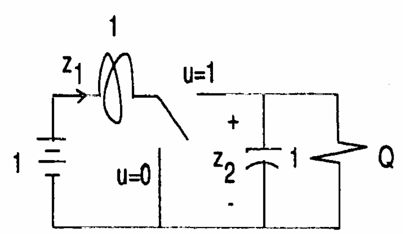

Figure 1. Normalized ‘boost’ converter circuit.

generating a normalized output voltage of the form $z_2(t) = A_n + (B_n/2)\sin\omega_n\tau$ with $A_n > B_n/2, > 0, \omega_n > 0$ with A_n, B_n being constants. The actual desired capacitor voltage, $x_2(t)$, is obtained as $x_2^*(t) = A + (B/2)\sin\omega t$, with $A = EA_n$ $B = EB_n$ and $\omega = \omega_n/\sqrt{LC}$.

2.2. Some of the difficulties

The normalized 'boost' converter state variable are known to exhibit the following properties (see Reference [9]).

1. The normalized capacitor voltage z_2 is a *non-minimum phase* output variable. This means that if a particular value for z_2, say $z_2^*(\tau)$ is imposed on the system behaviour, the resulting inductor current is characterized by an unstable 'residual dynamics'.
2. The normalized inductor current z_1 is a *minimum phase* output variable. Then, if a particular value for z_1, say $z_1^*(\tau)$ is perfectly tracked, the resulting capacitor voltage is represented as the trajectory of a globally stable 'residual dynamics'. The difficulty associated with the indirect AC signal generation problem is that it is not clear how to generate the suitable inductor current reference signal which has as a 'residual dynamics' solution, precisely, the desired output capacitor reference signal $z_2^*(\tau)$.
3. The 'boost' converter is known to be *differentially flat*, (see Reference [10]), which in the single input case at hand means that the converter dynamics are *exactly linearizable* by means of static state feedback. The flat output, which is, by definition, a variable devoid of zero dynamics, represents the *total stored energy of the circuit*. However, due to the discrete nature of the control input values, such an exact linearization is not feasible except in an average sense and certainly, not achievable by means of static sliding mode control. Furthermore, in order to determine a suitable reference signal for the flat output which solves the AC conversion problem, it is required to have a previous knowledge of the inductor current reference trajectory which exactly corresponds with the desired output AC capacitor voltage signal. Unfortunately, this reference trajectory is unknown. The hopeless circular argument is clear.

The above properties joined to the control acquisition structure of the converter equations and the discrete valued nature of the control input make it especially difficult for the synthesis of a switching feedback control law which results in a stable AC capacitor voltage reference signal tracking scheme.

2.3. A feasible indirect tracking approach

An approach which resorts to indirectly generate a desired capacitor voltage signal, $z_2^*(\tau)$, on the basis of tracking a suitable corresponding inductor current signal $z_1^*(\tau)$ clearly avoids the underlying non-minimum phase internal stability problems. As demonstrated below, this is an entirely feasible option. The difficulty, however, still resides in finding such a suitable inductor current reference signal $z_1^*(\tau)$. An approximate scheme to resolve this task will be presented in Section 2.5.

2.3.1. A sliding mode controller

Assume that a suitable smooth inductor current reference signal is given as $z_1^*(\tau) > 0$, whose time derivative is also bounded. A discontinuous sliding mode feedback controller which reaches, and sustains, a sliding motion on the time-varying sliding surface, $\sigma(t) = z_1 - z_1^*(\tau) = 0$, is given by

$$u = \begin{cases} 1 & \text{for } \sigma(t) > 0 \\ 0 & \text{for } \sigma(t) \leqslant 0 \end{cases} \tag{5}$$

2.3.2. Reachability of the sliding surface

Starting from zero initial conditions for z_1 and z_2, we have that since, initially, $z_1(\tau_0) < z_1^*(\tau_0)$ (i.e. $\sigma(\tau_0) < 0$). The switching strategy (5) sets $u = 0$ and, as it can be seen from (2), the normalized inductor current z_1 grows with slope equals to 1, while z_2 remains stationary at zero. The sliding surface reaching condition is thus satisfied from 'below', provided the reference signal $z_1^*(\tau)$ is designed with a time derivative which is bounded above by 1. Clearly, under such assumptions, the quantity $\sigma\dot{\sigma}$ is negative and given by $\sigma(1 - \dot{z}_1^*(\tau)) < 0$. When the sliding surface is reached and slightly overshot, the controller (5) starts to inject large positive current pulses to the output *RC* filter by intermittently letting $u = 1$. As a consequence, z_2 immediately starts to grow from zero, rapidly reaching the converters 'amplifying mode', $z_2 > 1$. Thus, while σ is positive, its time derivative, $\dot{\sigma} = -z_2 + 1$, becomes negative. Hence, the sliding surface reaching condition $\sigma\dot{\sigma} < 0$ is also satisfied from 'above' after the circuit is found in its amplifying mode.

2.3.3. The equivalent control, the ideal sliding dynamics and existence of a sliding motion

The 'equivalent control', corresponding to the ideal *invariance conditions* $\sigma = \dot{\sigma} = 0$ (see [4]) is obtained as

$$u_{EQ} = \frac{1 - \dot{z}_1^*(\tau)}{z_2} \tag{6}$$

The necessary and sufficient conditions for the existence of a sliding regime on $\sigma = 0$ are given, according to the results in Reference [5], by

$$0 < u_{EQ} < 1 \tag{7}$$

These conditions imply that, at each instant, the following set of inequalities must be satisfied for all τ after the sliding mode behaviour is reached:

$$0 < 1 - \dot{z}_1^*(\tau) < z_2(\tau) \tag{8}$$

The restrictions $\dot{z}_1^*(\tau) < 1$ and $\dot{z}_1^*(\tau) > 1 - z_2(\tau)$ imply, roughly speaking, limitations on the amplitude and frequency of the desired reference signal. Specific tracking limitations of the sliding mode control approach have to be worked out, in detail, for each particular given inductor current reference signal waveform, $z_1^*(\tau)$.

The *ideal sliding dynamics* corresponding to the sliding surface $\sigma = z_1 - z_1^*(\tau)$ is then given by the following stable time-varying nonlinear dynamics:

$$\dot{z}_2 = \left(\frac{1 - \dot{z}_1^*(\tau)}{z_2}\right) z_1^*(\tau) - \frac{z_2}{Q} \tag{9}$$

In order to establish the stability of (9) we define the Lyapunov function candidate, $\rho = (1/2)z_2^2$ which is easily seen to satisfy the following stable *linear* differential equation subject to bounded perturbations input signals:

$$\dot{\rho} = -\frac{1}{Q}[\rho - Q(1 - \dot{z}_1^*(\tau))z_1^*(\tau)] \tag{10}$$

The ideal sliding dynamics is thus stable for each bounded signal $z_1^*(\tau)$ with a first-order bounded time derivative. Note that, using the method of separation of variables in (9), the exact solution for z_2, provided that $z_1 = z_1^*(\tau)$, is obtained as

$$z_2(\tau) = \sqrt{Qz_1(\tau)(1 - \dot{z}_1(\tau)) - e^{-2/Q(\tau-\tau_0)}[Qz_1(\tau_0)(1 - \dot{z}_1(\tau_0)) - z_2^2(\tau_0)]} \tag{11}$$

Thus, the *steady-state* solution for $z_2(\tau)$, is given by

$$z_2(\tau) = \sqrt{Qz_1^*(\tau)(1 - \dot{z}_1^*(\tau))} \tag{12}$$

2.4. *Differential flatness of the 'boost' converter*

As already demonstrated in Reference [10], the boost converter is *differentially flat*. This implies the existence of a *differential function* of the state, termed the *flat output*, which completely parameterizes, all system variables (i.e. states, outputs, as well as the input). In other words, all system variables are expressible as functions of the flat output and a finite number of its time derivatives. The flat output for the 'boost' converter is the *total stored energy* of the circuit. For the normalized circuit considered here, the flat output is the normalized stored energy F, given by (4),

$$F = \tfrac{1}{2}(z_1^2 + z_2^2) \tag{13}$$

We remark that we are not particularly interested in the *feedback linearization* aspects provided by the differential flatness of the 'boost' converter circuit. In fact, this aspect is of little or no help in the sinusoidal signal generation task for the given converter, as it can be verified.

It should be clear that in order to obtain a suitable reference trajectory $z_1^*(\tau)$ for z_1, given that z_2 is of a particular form $z_2^*(\tau)$, one should proceed to *eliminate* the flat output F^* from the set of relations

$$F^* = \tfrac{1}{2}((z_1^*(\tau))^2 + (z_2^*(\tau))^2), \quad \dot{F}^* = z_1^*(\tau) - \frac{(z_2^*(\tau))^2}{Q} \tag{14}$$

However, such an elimination yields, as expected, an unstable *differential equation* relating $z_1 = z_1^*(\tau)$ and $z_2^*(\tau)$,

$$\dot{z}_1 = 1 - \frac{z_2^*(\tau)}{z_1}\left(\dot{z}_2^*(\tau) + \frac{z_2^*(\tau)}{Q}\right)$$

We shall still exploit the flat output elimination idea, in a different manner, in order to generate an approximating sequence of *static differential algebraic relationships* yielding normalized input inductor current reference signals z_1^*, exclusively in terms of the output capacitor voltage reference trajectory z_2^* and a *finite* number of its time derivatives. The *finite differential parameterizations* of z_1 in terms of z_2 will allow for the indirect sliding mode control-based generation of a large class of bounded AC output capacitor voltage profiles which are sufficiently differentiable.

2.5. An iterative procedure for generating a suitable inductor current reference

In order to simplify the notation we will temporarily suppress the asterisks and the time argument in the developments of this subsection. Consider then the set of relations (14). Those relations can be alternatively viewed as an *unbounded nonlinear differential operator*, defined on a Banach space, of the form $z_1 = H(z_1, z_2)$ with $z_2 = z_2^*(\tau)$ being available data. For this, we rewrite (14) in the following manner:

$$z_1 = \frac{z_2^2}{Q} + \dot{F}, \quad F = \tfrac{1}{2}(z_1^2 + z_2^2) \tag{15}$$

A traditional method to study, and approximately solve, differential operator equations of the above type is constituted by the well established method of 'functional iterations' or 'approximate functional iterative solutions'. This method has been the subject of sustained work by mathematicians, starting with the work of Liouville around 1836. The method of functional iterations was brilliantly formalized by Banach in Reference [11] and, later extensively used for the solution of integro-differential equations by Chaplygin [12]. The extension to deal with unbounded differential operators is due to Baluev [13]. Rather complete introductory references are constituted by the books by Kurpel' [14], that by Den Heijer [15] and the recent work by Chen *et al.*Chen [16], where the reader is referred for further details.

In our problem setting, the functional iterative process producing approximations of the underlying implicit relation between z_1 and the given z_2 is simply devised as

$$\begin{cases} z_{1,k} = \dfrac{z_2^2}{Q} + \dot{F}_k \\ F_{k+1} = \tfrac{1}{2}(z_{1,k}^2 + z_2^2) \end{cases} \tag{16}$$

The described functional iteration sequentially yields static relationships between z_1 and z_2 which only involve polynomial expression of z_2 and of its time derivatives. The algorithm, of course, should be 'initialized' by an arbitrary, but reasonable, trajectory $F_0(\tau)$ for the flat output F.

It is easy to see that initializing the above calculations from the natural equilibrium condition, $F_0(\tau) = constant$, one immediately obtains the actual static relationship between the equilibria of z_1 and z_2 for constant equivalent controls. Proceedings with the iterations, one obtains the following sequence of differential parameterizations for the normalized inductor current reference trajectory z_1,

$$z_{1,0} = \frac{z_2^2}{Q} \Rightarrow F_1 = \frac{1}{2}\frac{z_2^4}{Q^2} + \frac{1}{2}z_2^2 \tag{17}$$

$$z_{1,1} = \frac{z_2^2}{Q} + z_2\dot{z}_2\left(1 + \frac{2}{Q^2}z_2^2\right) \Rightarrow F_2 = \frac{1}{2}\left[\frac{z_2^2}{Q} + z_2\dot{z}_2\left(1 + \frac{2}{Q^2}z_2^2\right)\right]^2 + \frac{1}{2}z_2^2 \tag{18}$$

$$z_{1,2} = \frac{z_2^2}{Q} + \left(\frac{z_2^2}{Q} + z_2\dot{z}_2 + \frac{2z_2^3}{Q^2}\dot{z}_2\right)\left(\frac{2}{Q}z_2\dot{z}_2 + (\dot{z}_2)^2 + z_2\ddot{z}_2 + \frac{6z_2^2}{Q^2}(\dot{z}_2)^2 + \frac{2z_2^3}{Q^2}\ddot{z}_2\right) + z_2\dot{z}_2 \tag{19}$$

$$\vdots$$

$$z_{1,\infty} = \psi(z_2, \dot{z}_2, \ddot{z}_2, \ldots, z_2^{(k)}, \ldots) \tag{20}$$

Rather than studying the convergence of these iterations, we are interested in assessing the approximating features and the performance of the ideal sliding mode controlled feedback scheme which takes the first few generated differentially parameterized reference trajectory candidates, say $z_{1,0}(\tau)$ and $z_{1,1}(\tau)$, to constitute the minimum phase time-varying sliding surface, $\sigma = z_1 - z^*(\tau) = z_1 - z_{1,k}(\tau) = 0$, $k = 0, 1$. This assessment, under steady-state conditions, can be carried out in an *exact* manner thanks to (12).

Off-line generated differential parameterizations of the form (20) have been successfully used in the trajectory planning aspects of some non-minimum phase output trajectory tracking problems. They have been shown to provide efficient approximating reference trajectories for non-affine, non-feedback linearizable systems, such as the 'variable length pendulum' (see Reference [17]) and also for multivariable, non-minimum phase, nonlinear systems such as the PVTOL system [18]. In all these examples, the proper normalization of the system dynamics seems to be an advisable procedure.

2.5.1. *Output tracking error assessment under ideal sliding conditions*

Under ideal sliding conditions, $\sigma = \dot{\sigma} = 0$, the steady-state solution for $z_2(\tau)$ is given by expression (12) provided $z_1^*(\tau)$ is given. We can then compare the steady-state trajectory for $z_2(\tau)$ and the desired $z_2^*(\tau)$, for the reference trajectory candidates arising from (20).

For $z_1^*(\tau) = z_{1,k}(\tau)$ one obtains directly from (12), and using (16), the exact steady-state relation between $z_2(\tau)$ and $z_2^*(\tau)$ (recall $z_2^*(\tau)$ is bounded away from zero)

$$z_2(\tau) = \sqrt{Q z_{1,k}(1 - \dot{z}_{1,k})} = z_2^*(\tau) \sqrt{\left(1 + \frac{Q\dot{F}_k}{z_2^*(\tau)}\right)\left(1 - \frac{2}{Q} z_2^*(\tau)\dot{z}_2^*(\tau) - \ddot{F}_k\right)} \tag{21}$$

We define as a measure of the tracking error, $e_k(\tau)$, the quantity $[z_2(\tau)/z_2^*(\tau)]^2 - 1$, evaluated for each $z_{1,k}$. For instance, for $k = 0$, expression (21) and the tracking error take the form

$$z_2 = z_2^* \sqrt{1 - \frac{2}{Q} z_2^* \dot{z}_2^*}, \quad e_0 = -\frac{2}{Q} z_2^* \dot{z}_2^* \tag{22}$$

while for $k = 1$, the steady-state relation is readily computed as

$$z_2 = z_2^* \sqrt{\left[1 + \dot{z}_2^*\left(\frac{Q}{z_2^*} + \frac{2}{Q} z_2^*\right)\right]\left[1 - \dot{z}_2^*\left(\frac{2}{Q} z_2^* + \dot{z}_2^* + \frac{6}{Q^2}(z_2^*)^2 \dot{z}_2\right) - \ddot{z}_2^* z_2^*\left(1 + \frac{2}{Q^2} z_2^*\right)\right]} \tag{23}$$

Thus, as long as the time derivative of $z_2^*(\tau)$ is small in magnitude, the normalized voltage response, $z_2(\tau)$, and the desired voltage trajectory, $z_2^*(\tau)$, are close to each other. For periodical time functions this can, evidently, be achieved with a sufficiently small normalized frequency and reasonable amplitudes.

Note that it is quite difficult to establish the convergence of the above sequence of steady-state relations existing between the actual converter normalized response $z_2(\tau)$ and its desired value $z_2^*(\tau)$. In the simulations presented below, we show, for a typical converter, how the previously defined tracking error measures uniformly approach zero.

3. SIMULATION RESULTS

A typical 'boost' converter was chosen, with circuit parameters $L = 20$ mH, $C = 1$ μF, $R = 50\,\Omega$ and $E = 15$ V. For the normalized boost converter dynamics, the dimensionless circuit quality turned out to be $Q = 0.3535$. We took as sliding surfaces the candidates $\sigma = z_1 - z^*_{1,k}(\tau)$ for $k = 0, 1$, with $z^*_{1,0}(\tau)$ and $z^*_{1,1}(\tau)$ as given by (17) and (18), respectively. As a desired normalized output capacitor voltage signal, we chose, $z^*_2(\tau) = A_n + B_n/2 \sin \omega_n \tau$. The constants A_n, B_n and ω_n were adjusted so that the sliding mode existence conditions (7) were satisfied. These were set to be $A_n = 1.5$, $B_n = 0.8$, $\omega_n = 0.02$, which correspond with an actual sinusoidal voltage of the form $x^*_2(t) = 22.5 + 6.0 \sin(141.42t)$ V. Figure 2 shows the ideal steady-state defined errors, $[z_2/z^*_2]^2 - 1$, for $k = 0, 1, 2$. Clearly these errors become smaller as k grows. To also obtain a small error, due to chattering, the underlying sampling period was set to be of 0.1 normalized time units, which corresponds to an actual sampling frequency of about 70.71 kHz.

Figures 3 and 4 show the closed-loop actual output voltage and actual input current responses of the proposed sliding mode tracking controller corresponding, respectively, with the first, and second, sliding surface candidates. The simulated output voltage responses are shown along with the output capacitor voltage tracking error signal $x_2(t) - x^*_2(t)$. As it can be seen, we have, in each case, an good agreement between the generated sinusoidal signal $x_2(t)$ and the desired reference signal $x^*_2(t)$. The equivalent control trajectories, also shown in these figures, are bounded signals which, after sliding starts, uniformly remain bounded by the closed interval [0, 1]. This fact indicates that sliding motions exist throughout the entire closed-loop tracking process.

If a 100 per cent increase in the normalized frequency, ω_n, is demanded for the desired normalized reference signal, $z^*_2(\tau)$, (say, $\omega_n = 0.04$), then, in the first iteration approximation ($k = 0$), the resulting output voltage trajectory $x_2(t)$ is still very close to the desired normalized reference voltage $x^*_2(t)$, as it is depicted in Figure 5. The sliding mode existence conditions (7), are also clearly satisfied. The normalized inductor current reference trajectory obtained for the second iteration approximation ($k = 1$) also yields quite good tracking characteristics. The closed-loop behaviour is shown in Figure 6.

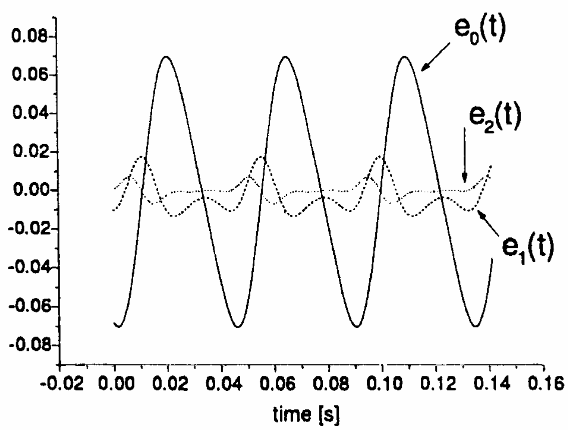

Figure 2. A measure of the steady-state output trajectory tracking errors.

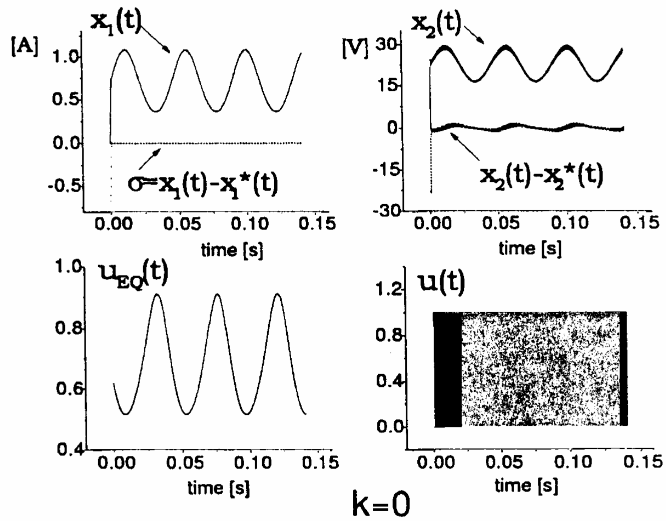

Figure 3. Closed-loop sliding mode controlled response of the 'boost' converter for AC output signal generation using an inductor current reference trajectory ($k = 0$, $A = 22.5$ V, $B = 12$ V, $\omega = 141.42$ rad/s).

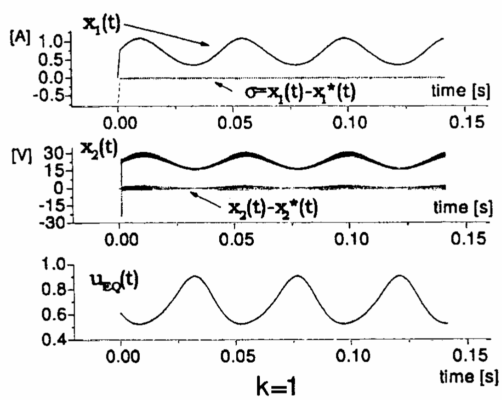

Figure 4. Closed-loop sliding mode controlled response of the 'boost' converter for AC output signal generation using an inductor current reference trajectory ($k = 1$, $A = 22.5$ V, $B = 12$ V, $\omega = 141.42$ rad/s).

4. CONCLUSIONS

In this article we have proposed a new approach for the approximate generation of biased sinusoidal AC voltage signals in the output of a DC-to-DC power converter circuit of the 'boost'

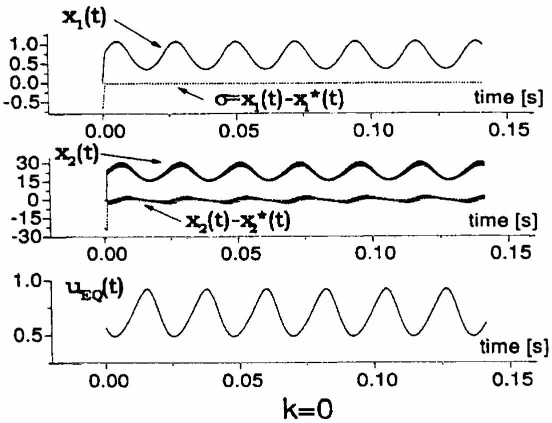

Figure 5. Closed-loop sliding mode controlled response of the 'boost' converter for AC output signal generation using an inductor current reference trajectory ($k = 0$, $A = 22.5$ V, $B = 12$ V, $\omega = 282.84$ rad/s).

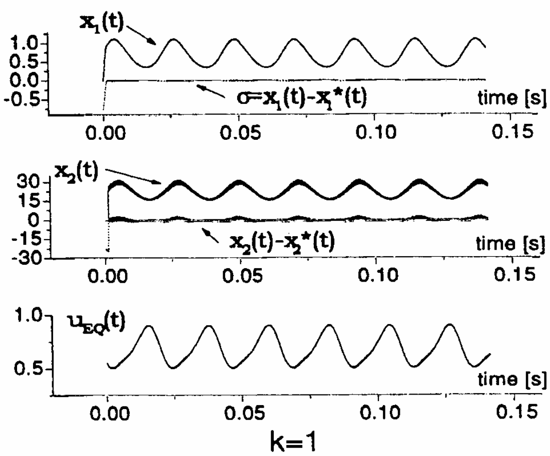

Figure 6. Closed-loop sliding mode controlled response of the 'boost' converter for AC output signal generation using an inductor current reference trajectory ($k = 1$, $A = 22.5$ V, $B = 12$ V, $\omega = 282.84$ rad/s).

type. The proposed approach is based on an indirect sliding mode reference inductor current trajectory tracking task. The reference inductor current signal is obtained in an off-line fashion from a simple iterative recursive algorithm which yields finite differential parameterizations of the inductor current in terms of the capacitor voltage. Such an off-line procedure is facilitated thanks to the differential flatness property of the circuit. Only a few iterations (1 or 2) are needed in order to obtain a suitable inductor current reference signal. The control scheme has been extended without any particular difficulties to generate biased sinusoidal signals for the 'buck-boost' and the 'Cuk' converter circuits. These developments will be reported elsewhere along with recently obtained real-time experimental results.

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REFERENCES

1. Cáceres R, Barbi I. Sliding mode controller for the boost inverter. *IEEE International Power Electronics Congress*, Cuernavaca, México, 14–17 October 1996; 247-252.
2. Zinober A, Fossas-Colet E, Scarrat JC, Biel D. Two sliding mode approaches to the control of a Buck-Boost system. *The 5th International Workshop on Variable Structure Systems*, Longboat Key, FL, 11–13 December 1998.
3. Fliess M, Lévine J, Martin Ph, Rouchon P. Flatness and defect of nonlinear systems: introductory theory and examples. *International Journal of Control* 1995; **61**:1327–1361.
4. Utkin VI. *Sliding Regimes in the Theory of Variable Structure Systems*. MIR: Moscow, 1977.
5. Sira-Ramirez H. Differential geometric methods in variable structure control. *International Journal of Control* 1988;**48**(4): 1359–1391.
6. Benvenuti L, di Benedetto MD, Grizzle JW. Approximate output tracking for nonlinear non-minimum phase systems with applications to flight control. *Journal of Robust and Nonlinear Control* 1991; **4**(1):397–414.
7. Fliess M, Sira-Ramírez H, Márquez R. Regulation of non-minimum phase outputs: a flatness based approach. In *Perspectives in Control*, Normand-Cyrot D (Ed.). Springer: Berlin, 1998.
8. Devasia S, Paden B. Stable inversion for nonlinear nonminimum phase time-varying systems. *IEEE Transactions on Automatic Control* 1998; **AC-43**(2):283–288.
9. Sira-Ramírez H, Lischinsky-Arenas P. The differential algebraic approach in nonlinear dynamical compensator design for DC-to-DC power converters. *International Journal of Control* 1991; **54**(1):111–134.
10. Sira-Ramírez H Ilic M. Exact linearization in switch mode DC-to-DC power converters. *International Journal of Control* 1989; **50**(2):511–524.
11. Banach S. Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales. *Fundamenta Mathematicae* 1922; 3:133–181.
12. Chaplygin SA. *A New Method for Approximate Integration of Differential Equations*. GITTL: Moscow, 1950.
13. Baluev AN. On a method by Chaplygin. *Vestnik, Leningrad University*. 1956; **11**(13):27–42.
14. Kurpel' NS. *Projection-Iterative Methods for Solution of Operator Equations*, vol. 46 (Translations of Mathematical Monographs). AMS: Providence, RI, 1976.
15. Den Haijer C. *The Numerical Solutions of Nonlinear Operator Equations by Imbedding Methods*, Mathematics Centre Tracts, vol. 107, Amsterdam, 1979.
16. Chen MJ, Chen ZY, Chen GR. *Approximate Solutions of Operator Equations*. World Scientific: Singapore, 1997.
17. Sira-Ramirez H. On the control of the variable length pendulum. *38th IEEE Conference on Decision and Control*, Phoenix, AZ, December 1999; 1188–1189.
18. Sira-Ramírez H. Non-minimum phase output reference trajectory tracking for a PVTOL aircraft, submitted for publication.

