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pp.1–14 (col.fig.: NIL)
PROD.TYPE: COM
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ED: SHIVA/BALA
PAGN: DORTHY SCAN: MOULI

INTERNATIONAL JOURNAL OF ROBUST AND NONLINEAR CONTROL
Int. J. Robust Nonlinear Control 2002; **12**:1–14 (DOI: 10.1002/rnc.687)

Sliding mode control of DC-to-DC power converters using integral reconstructors

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SUMMARY

A sliding mode feedback controller, based on integral reconstructors is developed for the regulation of the 'boost' DC-to-DC power converter circuit conduction in continuous conduction mode. The feedback control scheme uses only output capacitor voltage measurements, as well as knowledge of the available input signal, represented by the switch positions. The robustness of the feedback scheme is tested with abusively large, unmodelled, sudden load resistance variations. Copyright © 2002 John Wiley & Sons, Ltd.

KEY WORDS: integral reconstructors; sliding modes; DC-to-DC power converters

1. INTRODUCTION

Sliding mode control of DC-to-DC Power Converters, in continuous conduction mode, has been extensively treated in the literature over the last 15 years. Initial steps were given by Venkatarramanan *et al.* [1] for the 'boost' converter circuit. The differential geometric aspects of sliding mode control were later exploited by Sira-Ramirez and Ilic-Spong [2], in order to obtain a systematic controller design procedure for a larger class of switched bi-linear circuits. The reader is referred to the several existing authoritative books on Power Electronics (see References [3–5]) for interesting practical operation details and available feedback control methods. A survey of important developments in the regulation of this ubiquitous class of power supplies can be found in the book edited by Bose [6]. For other control methods applicable to DC-to-DC Power Converters, the reader is referred to the book by Ortega *et al.* [7]

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Contract grant sponsor: CONACYT, México; contract/grant number: 32681-A

Received 5 November 2000
Revised 9 April 2001
Accepted 12 November 2001

and to the recent article by Escobar *et al.* [8] where comparisons of several nonlinear feedback control schemes are experimentally carried out. A recent book by Utkin *et al.* [9], contains an interesting, and rather complete, account of sliding mode control of these useful power electronics devices.

In this article, we propose a new sliding mode feedback control option for the 'boost' converter circuit. The approach is based on the recently introduced ideal of *Integral Reconstructors*, or, also called, *Generalized PI controllers* (see References [10–13]). This control technique side-steps the need for any asymptotic observers, or on-line calculations based on samplings and time-discretizations, in the feedback regulation of observable linear dynamic systems. The extension of the integral reconstructor-based feedback control technique to the nonlinear arena is here accomplished in the context of the switched regulation of a DC/DC power converter circuit, of the 'boost' type, operating in continuous conduction mode. The basic idea resides in obtaining a nonlinear integral input–output parameterization, or integral reconstructor, of the unmeasured system state variable and proceed to use it in the sliding surface synthesis. Due to a constant bias, arising from the unknown value of the initial state, the definitive sliding surface expression incorporates a compensating output error integral control action. The integral reconstructor-based sliding mode scheme is shown to exhibit the stabilizing features of the traditional sliding mode control, but it turns out to be vastly superior as far as robustness, with respect to unmodelled parameter variations, is concerned. The feedback control scheme is thus based only on the available output voltage and the applied input signal without need for asymptotic observers or on-line calculations, based on sampled values of measured signals. This gives traditional 'op-amps', and modern integrated analog circuits, a renewed importance in the feedback regulation of power electronics circuits.

Section 2 presents the 'boost' converter model and establishes the feedback control objectives. It also revisits the rationale behind the traditional sliding mode control approach for the regulation of the boost converter. In Section 3 we introduce the sliding mode controller based on integral reconstructors and proceed to analytically derive its asymptotically stabilizing properties. Section 4 presents some digital computer simulations illustrating the performance of the proposed feedback controller. In this section we also examine the robustness of the proposed feedback control scheme when the circuit is subject to unmodelled, sudden, large load variations (up to 500 per cent of its nominal value). The obtained results are highly encouraging for actual implementation. Section 5 is devoted to present the conclusions of this work and gives some suggestions for further research.

2. CONTROL OF THE BOOST CONVERTER USING INTEGRAL RECONSTRUCTORS

2.1. *The 'boost' converter mode*

Consider the 'boost' converter circuit, shown in Figure 1. The system is described by the set of equations

$$\begin{aligned} L\dot{z}_1 &= -uz_2 + E \\ C\dot{z}_2 &= uz_1 - \frac{z_2}{R} \end{aligned} \tag{1}$$

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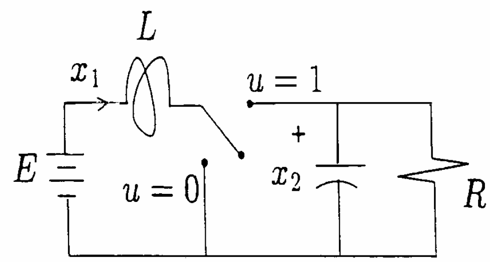

Figure 1. The 'boost' converter circuit.

where z_1 represents the inductor current and z_2 is the output capacitor voltage. The control input u, representing the switch position function, is a discrete-valued signal taking values in the set $\{0, 1\}$. The system parameters are constituted by: L, which is the inductance of the input circuit; C the capacitance of the output filter and R, the output load resistance. The external voltage source has the constant value E. We assume that the circuit is in continuous conduction mode, i.e. the average value of the inductor current never drops to zero, due to load variations.

We introduce the following state normalization and time scale transformation:

$$x_1 = \frac{z_1}{E}\sqrt{\frac{L}{C}}, \quad x_2 = \frac{z_2}{E}, \qquad \tau = \frac{t}{\sqrt{LC}} \tag{2}$$

The normalized model is thus given by

$$\begin{aligned} \dot{x}_1 &= -ux_2 + 1 \\ \dot{x}_2 &= ux_1 - \frac{x_2}{Q} \\ y &= x_2 \end{aligned} \tag{3}$$

where now, with an abuse of notation, the '.' represents derivation with respect to the normalized time, τ. The variable x_1 is the normalized inductor current, x_2 is the normalized output voltage and u, still represents the switch position function. The constant system parameters are all comprised now in the circuit 'quality' parameter, denoted by Q and given by the strictly positive quantity, $R\sqrt{C/L}$. It is assumed that the only system variable available for measurement is the output capacitor voltage x_2.

The operating normalized equilibrium point for the system can be computed in the following idealized manner: Assume that by means of an infinite frequency discontinuous control input a constant value, $\bar{x}_2 = V$, of the output capacitor voltage and of the inductor current are achieved. To this constant equilibrium state value, it corresponds a *constant equivalent control*, or *average control input*, denoted by u_{eq}, which is obtained from the first equation in (3), as $\bar{u}_{eq} = 1/V$. The corresponding normalized equilibrium value of the inductor current, according to the second equation of (3) is then given by $\bar{x}_1 = V^2/Q$. Note that since, $\bar{u}_{eq}$, must be bounded within the closed interval [0,1], then, necessarily, the achievable normalized constant voltage values, V, for x_2, are strictly greater than 1.

The normalized 'boost' circuit equations exhibit two important properties which should be remarked. We summarize these properties in the following proposition.

Proposition 2.1
In an average sense, the output capacitor voltage variable x_2 is a *non-minimum phase* output variable, while the input inductor current is a *minimum phase* output variable.

Proof
Take the average output capacitor voltage, x_2, as the system output, i.e. $y = x_2$. Imposing a constant value, V_d, on the output, y, results in an average control input of the form, $u(x_1) = V_d/(x_1 Q)$. Under normal, non-saturated, operating conditions, the control input satisfies: $0 < u(x_1) < 1$, and, hence, $x_1 > 0$. This feedback control input, then, induces a corresponding *zero dynamics* on the variable x_1, given by the unstable nonlinear dynamics

$$\dot{x}_1 = -V_d^2/(Qx_1) + 1$$

The instability of this dynamics can be readily verified by considering its tangent linearization around the equilibrium point, $\bar{x}_1 = V_d^2/Q$. This dynamics is given by $\dot{x}_\delta = x_\delta$, where $x_\delta = x_1 - \bar{x}_1$.

Take now the average inductor current, x_1, as the system output, $y = x_1$. Letting, $x_1 = \bar{x}_1 = V_d^2/Q$, the corresponding average control input is given by, $u(x_2) = 1/x_2$, which under, non-saturated, operating conditions satisfies $0 < u(x_2) < 1$ and, hence $x_2 > 1 > 0$. One is lead to the following asymptotically stable zero dynamics:

$$\dot{x}_2 = \frac{V_d^2}{Qx_2} - \frac{x_2}{Q}$$

Note that the quantity, $W(x) = (1/2)x_2^2$, satisfies the *linear* differential equation given by, $\dot{W} = -(2/Q)(W - \bar{W})$, with $\bar{W} = 1/2V_d^2$. Thus $W \to \bar{W}$ exponentially asymptotically. The physically significant corresponding equilibrium point of the variable, x_2, is located at the value, $\bar{x}_2 = V_d > 0$. This is, clearly, the only achievable equilibrium given the non-negative nature of the average control input values. □

The two facts, in the above proposition, have motivated *indirect* feedback control schemes based on inductor current regulation, or, alternatively, stored energy regulation (see References [14,15]).

2.2. Control objectives and traditional sliding mode control

The control objective consists in sustaining, by means of discontinuous feedback control, an average constant equilibrium value of the normalized output capacitor voltage, x_2, given by the desired value: $\bar{x}_2 = V_d$.

A traditional solution to the proposed control problem consists in adopting the following sliding mode control scheme (see Reference [2]). Consider a sliding surface co-ordinate function $\sigma(x)$ whose zero level set value ideally induces an average constant equilibrium value on the normalized inductor current, given by $\bar{x}_1 = V_d^2/Q$. The sliding surface may then be set to be of the form

$$S = \{x \in R^2 \mid \sigma(x) = x_1 - \bar{x}_1 = x_1 - V_d^2/Q = 0\} \tag{4}$$

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The ideal sliding dynamics corresponding to $\sigma(x) = 0$ is given, as pointed out above, by the following nonlinear asymptotically stable zero dynamics for the output capacitor voltage:

$$\dot{x}_2 = \frac{V_d^2}{Qx_2} - \frac{x_2}{Q} \tag{5}$$

The required sliding mode controller achieving finite time convergence to the switching line $\sigma(x) = 0$ is given by

$$u = \begin{cases} 1 & \text{for} \quad \sigma(x) > 0 \\ 0 & \text{for} \quad \sigma(x) < 0 \end{cases} \tag{6}$$

The discontinuous feedback controller (6) is easily derived from the general reaching condition: $\sigma\dot{\sigma} < 0$, (see Reference [16]) which, in our case, yields, $\sigma\dot{\sigma} = \sigma(-ux_2 + 1)$. Starting from zero initial conditions one has, $\sigma(x(0)) < 0$. Thus the choice $u = 0$ results in a growing normalized inductor current governed by $\dot{x}_1 = 1$ and a zero output voltage (governed by $\dot{x}_2 = -x_2/R$, $x_2(0) = 0$). Thus the current x_1 grows, while $\sigma(x)$ approaches zero and x_2 remains at zero. If $\sigma = 0$ is overshot, then, given the achieved positive value of x_1, u is set to 1 and the output voltage starts growing while the derivative of x_1 starts decreasing, i.e. σ decreases back to zero. The sustained condition $\sigma = 0$ causes the voltage x_2 to rapidly grow towards its equilibrium value, $V_d > 1$, according to the asymptotically stable zero dynamics (5). The sliding surface is then reachable from the zero initial conditions.

More generally, considering the product $\sigma\dot{\sigma} = \sigma(-ux_2 + 1)$, at any value of the state vector in a vicinity of the sliding surface, we see that when $\sigma < 0$, the choice $u = 0$ yields a positive second factor in the previous product. Therefore, $\sigma\dot{\sigma}$ is negative. Similarly, when σ is positive, the choice $u = 1$ yields a negative value for the factor $(-ux_2 + 1) = -x_2 + 1$, since now x_2 is larger than 1. The product $\sigma\dot{\sigma}$ is again negative.

We summarize the previous results in the following proposition.

Proposition 2.2

The discontinuous feedback policy (6), creates, in finite time, a *sliding regime* on the sliding surface (4). The discontinuously controlled motions induce an average constant value of the normalized inductor current, x_1, given by $\bar{x}_1 = V_d^2/Q$, and a corresponding ideal sliding dynamics for the output capacitor voltage, x_2, whose trajectories asymptotically reach the desired constant voltage value, $\bar{x}_2 = V_d$.

Figure 2 illustrates the traditional sliding mode controlled motions for a typical converter circuit with high frequency sampling rate.

Unfortunately, the above controller is based on a measurement of the inductor current, or, equivalently, of its normalized value, x_1. In practice, and within the domain of DC/DC power convertors, the inductor current, x_1, is known to be a hard signal to measure precisely. This is due to the high-frequency switching commanding the inductor current time derivative and the *high-pass filter* nature of the input circuit. For this reason, a scheme which is based on the non-minimum phase output variable, $y = x_2$, is usually preferable. How this can be achieved is explained in Section 3, by invoking the philosophy of integral reconstructors-based feedback control, recently developed within the linear systems context. In the next paragraphs, we examine the robustness, with respect to load resistance variations, of the traditional sliding mode controller in the *indirect* regulation of the output capacitor voltage.

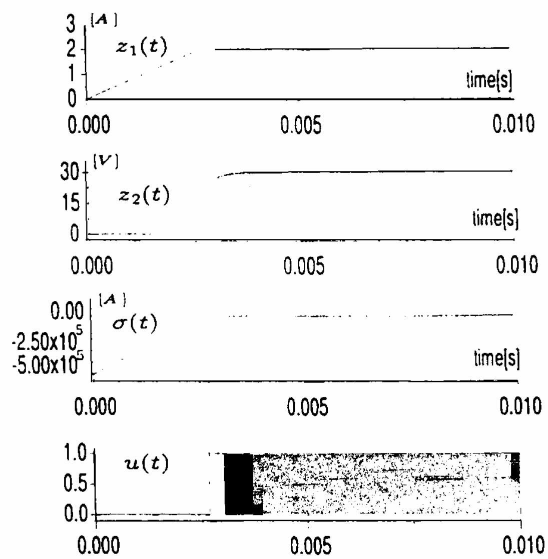

Figure 2. Traditional sliding mode controlled responses of the 'boost' converter circuit.

2.3. *Robustness of traditional sliding mode control to load variations*

Sliding mode control is known to be a robust feedback control technique with respect to unmodelled external uncertain signals and plant parameter variations. However, when used in an indirect scheme such as it is in our case, the invariance of the sliding motions with respect to the minimum phase variable may bring in undesirable feedback performance in the output capacitor voltage ideal sliding dynamics. It should be clear from the system equations (1) that if the inductor current is kept constant, a sudden, unmodelled variation of the parameter Q, due to a load resistance variation, will inevitably lead the ideal sliding motions towards an undesirable equilibrium value.

Figure 3 shows the undesirable performance of the sliding mode-based control scheme with respect to a sudden, and permanent, variation of the load resistance value in a typical 'boost' converter. As expected, the output voltage significantly varies when the load parameter is subject to a sudden unmodelled variation.

3. A SLIDING MODE CONTROLLER BASED ON INTEGRAL RECONSTRUCTORS

The normalized system (3) is observable, in an average sense, from the measured normalized output variable $y = x_2$. This is easily verified since the 'observability' matrix

$$\frac{\partial(y, \dot{y})}{\partial x} = \begin{bmatrix} 0 & 1 \\ u & -\dfrac{1}{Q} \end{bmatrix} \tag{7}$$

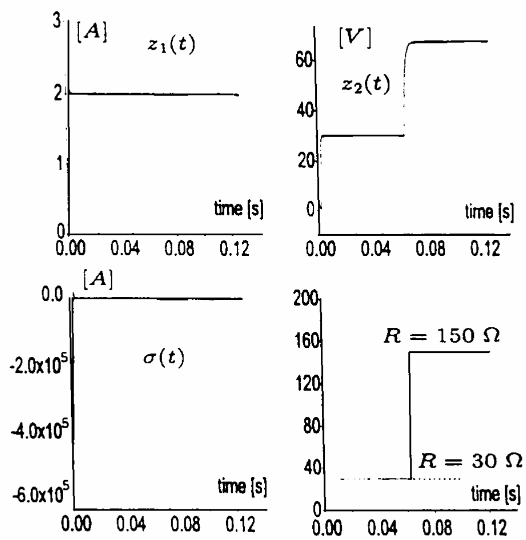

Figure 3. Sliding mode controlled 'boost' converter performance to unmodelled load variations.

is rank 2 for all average values of u which are not identically equal to zero. Since the average value of the input, under ideal sliding mode conditions, is $u_{eq} = 1/V_d > 0$, the observability condition is clearly met.

An integral input output parameterization, or an integral reconstructor, of the normalized inductor current, $x_1(\tau)$, is directly obtained from the first equation of (3),

$$\begin{aligned} x_1(\tau) &= \int_0^\tau (1 - u(\rho)y(\rho))\,d\rho \\ x_2(\tau) &= y(\tau) \end{aligned} \tag{8}$$

The integral reconstructor of x_1, in Equation (8), may be considered to be a 'open-loop estimate' of the normalized inductor current x_1 which is biased by an unknown constant value, represented by the initial condition $x_1(0)$. We denote by $\hat{x}_1(\tau)$, the integral reconstruction of x_1 in Equation (8), i.e.

$$x_1(\tau) = \int_0^\tau (1 - u(\rho)y(\rho))\,d\rho \tag{9}$$

It is clear that the relation linking the estimated value $\hat{x}_1$ of x_1 to its actual value, is just given by

$$x_1(\tau) = \hat{x}_1(\tau) + x_1(0) \tag{10}$$

We use estimate (9) of the inductor current, x_1, in the sliding surface definition, (4), and proceed to complement the expression with an integral control action, computed on the basis of the output voltage stabilization error, $y - V_d$.

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Consider then the following integral reconstructor-based sliding mode controller:

$$u = \begin{cases} 1 & \text{for} \quad \hat{\sigma}(y, u, \xi) > 0 \\ 0 & \text{for} \quad \hat{\sigma}(y, u, \xi) < 0 \end{cases} \tag{11}$$

$$\hat{\sigma}(y, u, \xi) = \int_0^t (1 - u(\rho)y(\rho))\,\mathrm{d}\rho - \frac{V_\mathrm{d}^2}{Q} + k_0\xi \tag{12}$$

$$\dot{\xi} = y(\tau) - V_\mathrm{d}, \quad \xi(0) = 0 \tag{13}$$

with k_0 a strictly positive design constant to be chosen later.

The modified sliding surface co-ordinate function, $\hat{\sigma}$, can also be equivalently written in terms of the, non-measured, actual state x_1 as

$$\hat{\sigma}(x_1, \xi) = x_1 - \frac{V_\mathrm{d}^2}{Q} - x_1(0) + k_0\xi \tag{14}$$

In spite of the unknown value of $x_1(0)$, expression (14) is found to be useful for our analysis purposes.

The time derivative of any of the two equivalent expressions of the modified sliding surface co-ordinate function (14), or (12), is given by

$$\dot{\hat{\sigma}}(y, u, \xi) = 1 - uy + k_0(y - V_\mathrm{d}) \tag{15}$$

Note that on $\hat{\sigma} = 0$ the inductor current, x_1, is given by the expression $x_1 = V_\mathrm{d}^2/Q + x_1(0) - k_0\xi$.

The equivalent control, corresponding to the modified sliding surface co-ordinate function is now given by

$$u_\mathrm{eq} = \frac{1 + k_0(y - V_\mathrm{d})}{y} \tag{16}$$

A sliding regime locally exists on $\hat{\sigma}(y, u, \xi) = 0$ whenever the following *intermediate condition*, $0 < u_\mathrm{eq} < 1$, is satisfied (see Reference [17]):

$$0 < 1 + k_0(y - V_\mathrm{d}) < y \tag{17}$$

which is equivalent to the following set of inequalities:

$$\begin{aligned} & V_\mathrm{d} - \frac{1}{k_0} < y < V_\mathrm{d} + \frac{1 - V_\mathrm{d}}{1 - k_0} \qquad \text{for } k_0 > 1 \\ & y > V_\mathrm{d} - \min\left\{\frac{1}{k_0}, \frac{\sqrt{d} - 1}{1 - k_0}\right\} \quad \text{for } 0 < k_0 < 1 \end{aligned} \tag{18}$$

Thus, the set of values for k_0 that guarantees a larger region of existence of a sliding regime corresponds to the condition, $k_0 \in (0, 1)$. The following choice of, k_0, as a strictly positive constant, within the interval:

$$0 < k_0 < \frac{1}{V_\mathrm{d}} < 1 \tag{19}$$

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clearly guarantees the non-empty character of the region of existence of sliding motions and it will prove to be the most convenient to assure reachability from the origin of the closed-loop system state space.

For the reachability of the sliding surface from the origin, suppose the system is initially resting at the zero state, $x_1(0) = 0$, $x_2(0) = 0$, $\xi(0) = 0$, then, the initial value of the modified sliding surface is negative, $\hat{\sigma}(x_1(0), 0) = \hat{\sigma}(0, 0) = -V_{\mathrm{d}}^2/Q < 0$, and the initial value of the product, $\hat{\sigma}\dot{\hat{\sigma}}$, is given by

$$\hat{\sigma}(0,0)\dot{\hat{\sigma}}(0,0) = -\frac{V_{\mathrm{d}}^2}{Q}(1 - k_0 V_{\mathrm{d}})$$

The modified sliding surface $\hat{\sigma}$, thus, starts increasing towards zero from the given zero initial condition, provided k_0 is chosen within the prescribed interval. The region of existence of a sliding motion, is, therefore, always reachable from the origin by the proposed switched control strategy (11).

The local reachability of the sliding surface, $\hat{\sigma} = 0$, from an arbitrary initial state value, is established by the well-known condition, $\hat{\sigma}\dot{\hat{\sigma}} < 0$, to be verified in a local neighbourhood of the modified sliding surface. As we have already seen, this neighbourhood can be made to include the origin of the state space, which is a common starting point for the DC/DC power converter operation.

Suppose, then, that (19) is valid. Let $\hat{\sigma} < 0$, then, according to (11), the control is set to $u = 0$. The time derivative of the modified sliding surface co-ordinate is given by $\dot{\hat{\sigma}} = 1 + k_0(y - V_{\mathrm{d}})$. Then for all $y > V_{\mathrm{d}} - l/k_0$, the time derivative, $\dot{\hat{\sigma}}$ is positive and the product $\hat{\sigma}\dot{\hat{\sigma}}$ is negative. Suppose now that $\hat{\sigma}$ is positive, then, the control input is given by $u = 1$. The time derivative of the sliding surface co-ordinate is $\dot{\hat{\sigma}} = 1 - y + k_0(y - V_{\mathrm{d}})$. Thus, for all $y > (1 - k_0 V_{\mathrm{d}})/(1 - k_0) = V_{\mathrm{d}} - (V_{\mathrm{d}} - 1)/(1 - k_0)$, the product $\hat{\sigma}\dot{\hat{\sigma}}$ is, again, negative. We conclude that a sliding regime exists on the modified sliding surface, $\hat{S} = \{(y, u, \xi) \mid \sigma(y, u, \xi) = 0\}$, which is locally reachable in finite time, by means of the proposed discontinuous control law (11).

The ideal sliding dynamics, obtained from the *invariance conditions*, $\hat{\sigma} = 0$, $\dot{\hat{\sigma}} = 0$, is now obtained as

$$\begin{aligned} \dot{y} &= \frac{1 + k_0(y - V_{\mathrm{d}})}{y}\left[\frac{V_{\mathrm{d}}^2}{Q} + x_1(0) - k_0\xi\right] - \frac{y}{Q} \\ \dot{\xi} &= y - V_{\mathrm{d}} \end{aligned} \tag{20}$$

where we stress that the output signal, y, satisfies the non-singularity condition, $y > 1 > 0$.

The only constant equilibrium point, $(\bar{y}, \bar{\xi})$, of the ideal closed-loop sliding dynamics (20) is given by

$$\bar{y} = V_{\mathrm{d}}, \qquad \bar{\xi} = \frac{1}{k_0}x_1(0) \tag{21}$$

It remains to be proved that the nature of the stability of the equilibrium point with respect to ideal sliding trajectories starting on the sliding surface $\hat{S}$. It may be verified that such an equilibrium point is not attractive from every point the sliding surface. We prove, thus, local asymptotic stability, which suffices for our purposes. By resorting to tangent linearization of the ideal sliding dynamics, we can indeed see that the equilibrium point (21) is locally asymptotically stable.

The tangent linearization of the ideal sliding dynamics (20) is given by

$$\begin{aligned} \dot{\xi}_\delta &= y_\delta \\ \dot{y}_\delta &= -\frac{k_0}{V_{\rm d}}\xi_\delta - \frac{2-k_0 V_{\rm d}}{Q} y_\delta \end{aligned} \tag{22}$$

where $\xi_\delta = \xi - x_1(0)/k_0$ and $y_\delta = y - V_{\rm d}$. Since $0 < k_0 < 1/V_{\rm d}$, the linearized system (22) is asymptotically stable to zero. The result follows.

Note that a small value of the design parameter, k_0, not only increases the damping in the linearized average version of the closed-loop system, but it also lowers the corresponding natural frequency. This results, generally speaking, in a slower convergence of the controlled motions towards the origin of the incremental variables and, hence, a slower convergence of the nonlinear controlled system output towards the desired constant equilibrium.

Figure 4 depicts the local asymptotically stable nature of the desired equilibrium point for the ideal sliding dynamics (22). The ideal sliding trajectories are shown in the local sliding surface co-ordinates ξ, y for a typical converter parameter Q and for the design values, $k_0, V_{\rm d}$, used below in the Simulation Results section.

We summarize the proven result in the following proposition.

Proposition 3.1

Consider a 'boost' converter, represented in normalized form by (3), in which it is desired to stabilize the measured output variable, $y = x_2$, towards the given constant value, $V_{\rm d} > 0$. Suppose that the control input, u, is also available for measurement. Then, the following integral reconstructor-based sliding mode controller, using only input–output, information:

$$\begin{aligned} u &= \begin{cases} 1 & \text{for} \quad \hat{\sigma}(y,u) > 0 \\ 0 & \text{for} \quad \hat{\sigma}(y,u) < 0 \end{cases} \\ \hat{\sigma}(y,u,\xi) &= \int_0^\tau (1 - u(\rho)y(\rho))\,{\rm d}\rho - \frac{V_{\rm d}^2}{Q} + k_0\xi \\ \dot{\xi} &= y(\tau) - V_{\rm d}, \quad \xi(0) = 0, \quad 0 < k_0 < \frac{1}{V_{\rm d}} \end{aligned} \tag{23}$$

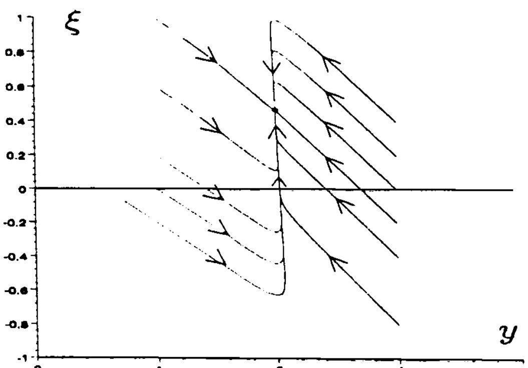

Figure 4. Local asymptotic stability of the ideal sliding dynamics towards the desired equilibrium point.

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yields a permanent sliding motion on the surface

$$\hat{S} = \{(y, u, \xi) \mid \hat{\sigma}(y, u, \xi) = 0\}$$

$$= \left\{(x, \xi) \,\middle|\, x_1 - \frac{V_d^2}{Q} - x_1(0) + k_0\xi = 0\right\} \tag{24}$$

which is reachable from the origin in finite time. The induced sliding motions on the sliding manifold, $\hat{S}$, ideally, locally asymptotically stabilize the trajectories of the circuit variables x_1, x_2 and ξ towards the equilibrium values

$$\bar{x}_1 = \frac{V_d^2}{Q}, \quad \bar{x}_2 = V_d, \quad \bar{\xi} = \frac{1}{k_0}x_1(0)$$

where $x_1(0)$ is the unknown initial state of the normalized inductor current variable x_1. The sliding motions exist on, $\hat{S}$, whenever the regulated values of the output, y, satisfy the inequality

$$y > V_d - \min\left\{\frac{1}{k_0}, \frac{V_d - 1}{1 - k_0}\right\} \tag{25}$$

Figure 5 depicts the integral reconstructor-based sliding mode feedback control scheme for the stabilization of the normalized 'boost' converter circuit.

4. SIMULATION RESULTS

Simulations were performed on a typical 'boost' converter circuit with parameter values given by

$$L = 20 \text{ mH}, \quad C = 20\ \mu\text{F}, \quad R = 30\ \Omega, \quad E = 15 \text{ V}$$

This parameter values yield a value of Q given by $Q = 0.9486$ and a time normalization factor given by $t = 6.32 \times 10^{-4}\tau$.

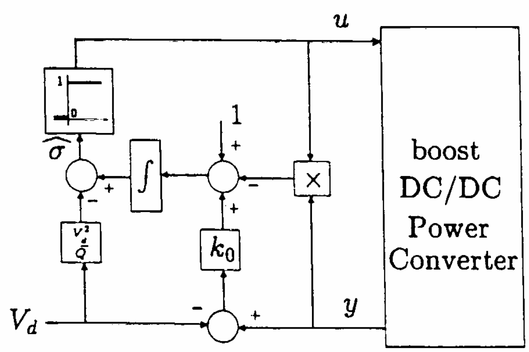

Figure 5. Integral reconstructor-based sliding mode control scheme for the stabilization of the 'boost' converter circuit.

It was desired to bring the 'boost' converter trajectories from unknown initial conditions (taken to be, for the simulation purposes, $x_1(0) = 0.5$ and $x_2(0) = 0.8$) towards the final desired value of $\bar{z}_2 = 30$ V, with corresponding $\bar{z}_1 = 2$ A. The simulations, shown in Figure 6, depict the performance of the proposed sliding mode plus integral reconstructor-based feedback control

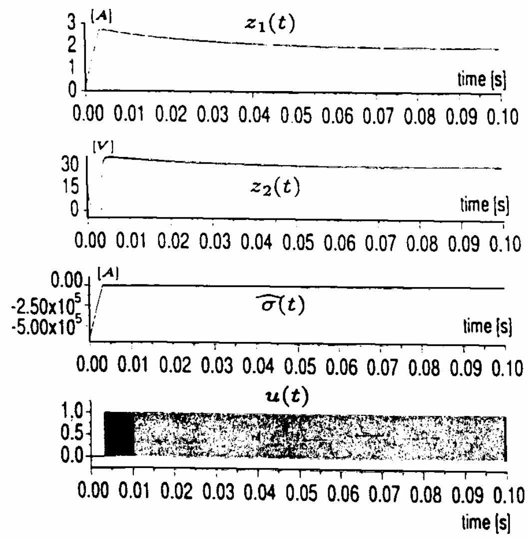

Figure 6. Controlled 'boost' converter performance.

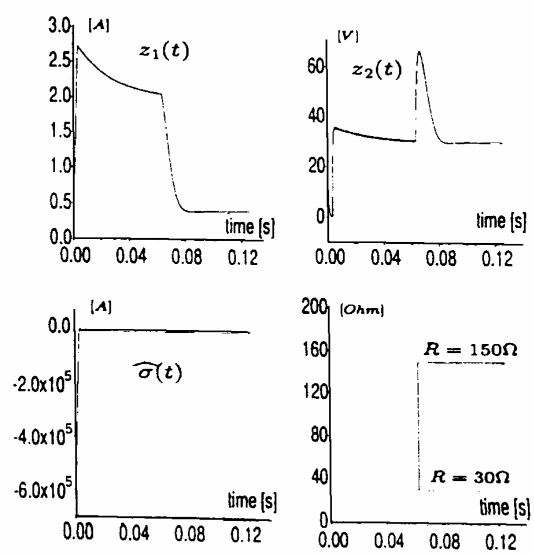

Figure 7. Controlled 'boost' converter performance to unmodelled load variations of 500 per cent.

scheme on the behaviour of the considered DC/DC 'boost' converter circuit. The underlying sampling frequency was set to be 158.22 KHz, and the value of the design constant k_0 was set to be $k_0 = 0.1 < 1/Vd = 0.5$.

4.1. Robustness to load variations

In order to test the robustness of the proposed GPI sliding mode control scheme, we let the load resistor R undergo a sudden unmodelled and *permanent* variation of 500 per cent of its nominal value of 30 Ω. This variation took place, approximately, at time, $t = 0.0633$ s, while the system was not yet stabilized to the desired voltage value. Figure 7 shows the excellent recovering features of the proposed controller to the imposed load variation.

5. CONCLUSIONS

In this article we have extended the integral reconstructor-based control technique to the realm of sliding mode control, within the context of a specific nonlinear physical example of wide interest in the Power Electronics area. We have proposed an asymptotically stabilizing sliding mode controller which only requires measurements of the non-minimum phase state variable of the converter, represented by the output capacitor voltage. The integral reconstructor-based controller is motivated by the usual indirect design of the traditional sliding surface co-ordinate function in terms of the normalized inductor current variable stabilization error. An integral reconstructor of the normalized inductor current variable, exhibiting a constant 'off-set' error, is synthesized in terms of an integral of a simple nonlinear function of the available input and the measured output signals. The sliding surface synthesis uses this 'open loop' estimate of the inductor current in combination with a suitable integral output stabilization error compensation term. The integral input–output parameterized sliding surface is shown to be locally reachable and, once a sliding regime is established, a locally asymptotically stable ideal sliding dynamics is obtained on the sliding manifold which converges to the desired equilibrium values for the normalized circuit variables.

Through computer simulations, the proposed control scheme was shown to be remarkably robust with respect to unusually large unmodelled load parameter variations of up to 500 per cent. Note that for an extremely large load it is possible that the inductor current drops to the zero value, thus saturating the controller action to yield a fixed switch position and a consequent temporary, or permanent, loss of feedback. Strategies to efficiently emerge from, and avoid, such situations, are known as operation in discontinuous conduction mode. These are the object of sustained studies in the current literature. Our approach, while being quite robust in this respect, is not devised to entirely avoid such possibility.

A needed extension of the results, here presented, requires the use of a more complete model including parasitic voltages in the diodes, as well as internal resistances in the inductor, in the transistors realizing the switch, and in the external voltage source. These imperfections should also include a non-ideally constant external voltage source.

The same integral reconstructor-based sliding mode control technique is applicable to 'Buck' converters, 'Buck-Boost' converters, un-interruptible power supplies and, possibly, to the 'Cúk' converter. An interesting topic for further study is represented by the integral

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reconstructor-based AC voltage generation problem using traditional DC/DC Power Converters (see Reference [18]).

ACKNOWLEDGEMENTS

The authors are grateful to anonymous reviewers for their kind observations and suggestions to improve the article.

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