Lecture Notes in Control and Information Sciences 222

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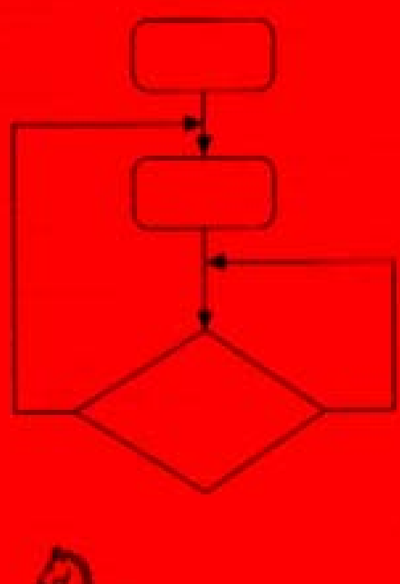
Control Using Logic-Based Switching



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Lagrangian Modeling and Control of Switch Regulated DC-to-DC Power Converters *

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1 Introduction

Modeling and regulation of switched dc-to-dc power converters was initiated by the pioneering work of Middlebrook and Ćuk [1] in the mid seventies. The area has undergone a wealth of practical and theoretical development as evidenced by the growing list of research monographs, and textbooks, devoted to the subject (see Severns and Bloom [2], and Kassakian *et al* [3]).

In this article, a Lagrangian dynamics approach is used for deriving a physically motivated model of the dc–to–dc power converters of the "Boost" type. The approach, however, is suitable to be applied to any kind of dc-to-dc power converter. The proposed modeling technique is based on a suitable parametriza-

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tion, in terms of the switch position parameter, of the Euler-Lagrange functions describing each intervening system and subsequent application of the Lagrangian formalism. The resulting system is also shown to be a Lagrangian system, hence, passivity based regulation is proposed as a natural controller design technique (see Ortega *et al*, [4] for details). The second part of the article designs and compares the performance of traditional (static) sliding mode controller and that of a dynamical passivity-based sliding mode controller. The comparison is carried out in terms of evaluating a physically motivated scalar performance index involving the total stored energy. It is shown that, depending on the choice of the controller's initial state, the passivity based controller might render a finite performance index while the traditional controller results in an unbounded index.

2 Modeling of Switched Euler–Lagrange Systems

An Euler–Lagrange system is classically characterized by the following set of nonlinear differential equations, known as *Lagrange equations*,

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) - \frac{\partial \mathcal{L}}{\partial q} = -\frac{\partial \mathcal{D}}{\partial \dot{q}} + \mathcal{F}_q \tag{2.1}$$

where q is the vector of *generalized positions*, assumed to have n components, represented by $q_1, \ldots, q_n$, and $\dot{q}$ is the vector of *generalized velocities*. The scalar function $\mathcal{L}$ is the *Lagrangian* of the system, defined as the difference between the *kinetic energy* of the system, denoted by $\mathcal{T}(\dot{q}, q)$, and the *potential energy* of the system, denoted by $\mathcal{V}(q)$, i.e,

$$\mathcal{L}(\dot{q}, q) = \mathcal{T}(\dot{q}, q) - \mathcal{V}(q) \tag{2.2}$$

The function $\mathcal{D}(\dot{q})$ is the *Rayleigh dissipation cofunction* of the system. The vector $\mathcal{F}_q = (\mathcal{F}_{q_1}, \ldots, \mathcal{F}_{q_n})$ represents the ordered components of the set of *generalized forcing functions* associated with each generalized coordinate.

We refer to the set of functions $(\mathcal{T}, \mathcal{V}, \mathcal{D}, \mathcal{F})$ as the *Euler–Lagrange functions* of the system and symply express a system Σ by the ordered cuadruple

$$\Sigma = (\mathcal{T}, \mathcal{V}, \mathcal{D}, \mathcal{F}) \tag{2.3}$$

We are particularly interested in dynamical systems containing a single *switch*, regarded as the only *control function* of the system. The switch position, denoted by the scalar u, is assumed to take values on a discrete set of the form $\{0, 1\}$. We assume that for each one of the switch position values, the resulting system is an Euler-Lagrange system (EL system for short) characterized by its corresponding EL parameters.

Definition 2.1 *We define a* switched EL function $\mathcal{M}_u$, *associated with the EL functions* $\mathcal{M}_0$ *and* $\mathcal{M}_1$, *as a function, parametrized by the switch position* u, *which is consistent with* $\mathcal{M}_0$ *and* $\mathcal{M}_1$, *for the corresponding values of the switch position parameter,* $u \in \{0,1\}$, *i.e.,*

$$\mathcal{M}_u|_{u=0} = \mathcal{M}_0 \ ; \ \mathcal{M}_u|_{u=1} = \mathcal{M}_1 \tag{2.4}$$

A system arsing from switchings among the EL systems Σ_0 and Σ_1 is said to be a switched EL system, whenever it is completely characterized by the set of consistent switched EL functions

$$\Sigma_u = (\mathcal{T}_u, \mathcal{V}_u, \mathcal{D}_u, \mathcal{F}_u) \tag{2.5}$$

Assume we are given two EL system models, Σ_0 and Σ_1, characterized by EL parameters, $(\mathcal{T}_0, \mathcal{V}_0, \mathcal{D}_0, \mathcal{F}_0)$ and $(\mathcal{T}_1, \mathcal{V}_1, \mathcal{D}_1, \mathcal{F}_1)$, respectively. Our basic modeling problem consists, generally speaking, in determining a consistent parametrization of the EL functions, $(\mathcal{T}_u, \mathcal{V}_u, \mathcal{D}_u, \mathcal{F}_u)$ in terms of the switch position parameter, u, with corresponding switched Lagrangian $\mathcal{L}_u$, such that the system model obtained by direct application of the EL equations (2.5) on $\mathcal{L}_u$, results in a parametrized model, Σ_u, which is consistent,in the sense described above, with the models Σ_0 and Σ_1 .

2.1 A Lagrangian Approach to the Modeling of a "Boost" Converter

Consider the switch–regulated "Boost" converter circuit of Figure 1. We consider, separately, the Lagrange dynamics formulation of the two circuits associated with each one of the two possible positions of the regulating switch. In order to use standard notation we refer to the input current x_1 in terms of the derivative of the circulating charge q_L, as $\dot{q}_L$. Also the capacitor voltage x_2 will be written as q_C/C where q_C is the electrical charge stored in the output capacitor. The switch position paramerter, u, is assumed to take values in the discrete set $\{0,1\}$

Consider then $u = 1$. In this case, two separate, or decoupled, circuits are clearly obtained and the corresponding Lagrange dynamics formulation can be carried out as follows.

Define $\mathcal{T}_1(\dot{q}_L)$ and $\mathcal{V}_1(q_C)$ as the kinetic and potential energies of the circuit, respectively. We denote by $\mathcal{D}_1(\dot{q}_C)$ the Rayleigh dissipation cofunction of the circuit. These quantities are readily found to be

$$\begin{aligned} \mathcal{T}_1(\dot{q}_L) &= \frac{1}{2}L\,(\dot{q}_L)^2 \\ \mathcal{V}_1(q_C) &= \frac{1}{2C}q_C^2 \\ \mathcal{D}_1(\dot{q}_C) &= \frac{1}{2}R\,(\dot{q}_C)^2 \\ \mathcal{F}^1_{q_L} &= E \ ; \ \mathcal{F}^1_{q_C} = 0 \end{aligned} \tag{2.6}$$

where $\mathcal{F}^1_{q_L}$ and $\mathcal{F}^1_{q_C}$ are the *generalized forcing* functions associated with the coordinates q_L and q_C, respectively.

Consider now the case $u = 0$. The corresponding Lagrange dynamics formulation is carried out in the next paragraphs.

Define $\mathcal{T}_0(\dot{q}_L)$ and $\mathcal{V}_0(q_C)$ as the kinetic and potential energies of the circuit, respectively. We denote by $\mathcal{D}_0(\dot{q}_L, \dot{q}_C)$ the Rayleigh dissipation function of the circuit. These quantities are readily found to be,

$$\begin{aligned} \mathcal{T}_0(\dot{q}_L) &= \frac{1}{2}L\,(\dot{q}_L)^2 \\ \mathcal{V}_0(q_C) &= \frac{1}{2C}q_C^2 \\ \mathcal{D}_0(\dot{q}_L, \dot{q}_C) &= \frac{1}{2}R\,(\dot{q}_L - \dot{q}_C)^2 \\ \mathcal{F}^0_{q_L} &= E \quad ; \quad \mathcal{F}^0_{q_C} = 0 \end{aligned} \tag{2.7}$$

where, $\mathcal{F}^0_{q_L}$ and $\mathcal{F}^0_{q_C}$ are the *generalized forcing* functions associated with the coordinates q_L and q_C, respectively.

The EL parameters of the two situations, generated by the different switch position values, result in identical kinetic and potential energies. The switching action merely changes the Rayleigh dissipation cofunction between the values $\mathcal{D}_0(\dot{q}_C)$ and $\mathcal{D}_1(\dot{q}_L, \dot{q}_C)$. Therefore, the *dissipation structure* of the system is the only one affected by the switch position.

$$\begin{aligned} \mathcal{T}_u(\dot{q}_L) &= \frac{1}{2}L\,(\dot{q}_L)^2 \\ \mathcal{V}_u(q_C) &= \frac{1}{2C}q_C^2 \\ \mathcal{D}_u(\dot{q}_L, \dot{q}_C) &= \frac{1}{2}R\,[(1-u)\dot{q}_L - \dot{q}_C]^2 \\ \mathcal{F}^u_{q_L} &= E \quad ; \quad \mathcal{F}^u_{q_C} = 0 \end{aligned} \tag{2.8}$$

The switched lagrangian function associated with the above defined EL parameters is given by

$$\mathcal{L}_u = \mathcal{T}_u(\dot{q}_L) - \mathcal{V}_u(q_C) = \frac{1}{2}L\,(\dot{q}_L)^2 - \frac{1}{2C}q_C^2 \tag{2.9}$$

One then proceeds, using the Lagrange equations (2.1), to formally obtain the parametrized differential equations defining the switch regulated system corresponding to (2.8). Such equations are given by,

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}_u}{\partial \dot{q}_L}\right) - \frac{\partial \mathcal{L}_u}{\partial q_L} &= -\frac{\partial \mathcal{D}_u}{\partial \dot{q}_L} + \mathcal{F}_{q_L} \\ \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}_u}{\partial \dot{q}_C}\right) - \frac{\partial \mathcal{L}_u}{\partial q_C} &= -\frac{\partial \mathcal{D}_u}{\partial \dot{q}_C} + \mathcal{F}_{q_C} \end{aligned} \tag{2.10}$$

Use of (2.9) on (2.10) results in the following set of differential equations

$$\begin{aligned} L\ddot{q}_L &= -(1-u)R[(1-u)\dot{q}_L - \dot{q}_C] + E \\ \frac{q_C}{C} &= R[(1-u)\dot{q}_L - \dot{q}_C] \end{aligned} \tag{2.11}$$

which can be rewritten, after substitution of the second equation into the first, as

$$\begin{aligned} \ddot{q}_L &= -(1-u)\frac{q_C}{LC} + \frac{E}{L} \\ \dot{q}_C &= -\frac{1}{RC}q_C + (1-u)\dot{q}_L \end{aligned} \tag{2.12}$$

Using $x_1 = \dot{q}_L$ and $x_2 = q_C/C$ one obtains

$$\begin{aligned} \dot{x}_1 &= -(1-u)\,\frac{1}{L}x_2 + \frac{E}{L} \\ \dot{x}_2 &= (1-u)\,\frac{1}{C}x_1 - \frac{1}{RC}x_2 \end{aligned} \tag{2.13}$$

The proposed switched dynamics (2.13) coincides with the classical state model developed in [1]. The fact that switched circuits, such as the above presented "Boost" converter, can be modeled using the Lagrangian formalism, implies that a *passivity-based approach* can be naturally attempted in the design of stabilizing feedback control policies. In order to establish suitable comparisons we first revisit the traditional static sliding mode controller and its main properties.

3 Regulation of the "Boost" Converter

3.1 Traditional Sliding Mode Control of the "Boost" Converter

Consider the "Boost" converter circuit, shown in Figure 1, described by the set of differential equations (2.13).

We shall denote by $\overline{x}_1$ and $\overline{x}_2$ the state variables of the system under *ideal sliding mode* conditions. In other words, $\overline{x}_1$ and $\overline{x}_2$, represent the "average" values of the state variables under sliding mode operation. The "equivalent control", denoted by u_{eq}, represents an ideal, i.e., a *virtual* feedback control action that smoothly keeps the controlled state trajectories of the system evolving on the sliding surface, provided motions are started, precisely, at the sliding surface itself (see Utkin [5] for definitions).

In order to avoid a well-knwon unstable closed loop behavior due to the non-minimum phase character of the converter we proceed, instead of regulating the output capacitor votage, x_2, to *indirectly* regulate such a variable as indicated in the following proposition.

Proposition 3.1 *Consider the switching line $s = x_1 - V_d^2/RE$, where $V_d > 0$ is a desired constant capacitor voltage value. The switching policy, given by*

$$u = 0.5\,[\,1 - \text{sign}\,(s)\,] = 0.5\,[\,1 - \text{sign}\,(x_1 - V_d^2/RE)\,] \tag{3.1}$$

locally creates a stable *sliding regime on the line $s = 0$ with ideal sliding dynamics characterized by*

$$\overline{x}_1 = \frac{V_d^2}{RE} \;\; ; \;\; \dot{\overline{x}}_2 = -\frac{1}{RC}\left[\overline{x}_2 - \frac{V_d^2}{\overline{x}_2}\right] \;\; ; \;\; u_{eq} = 1 - \frac{E}{\overline{x}_2} \tag{3.2}$$

Moreover, the ideal sliding dynamics behaviour of the capacitor voltage variable, described by (3.2), can be explicitly computed as

$$\overline{x}_2(t) = \sqrt{V_d^2 + \left[\overline{x}_2^2(t_h) - V_d^2\right] e^{-\frac{2}{RC}(t-t_h)}} \tag{3.3}$$

where t_h stands for the reaching instant of the sliding line $s = 0$ and $\overline{x}_2(t_h)$ is the capacitor voltage at time t_h.

Figure 2 depicts a typical "start up" state variables evolution, from zero initial conditions, in a current-mode controlled "Boost" converter such as that of Proposition 3.1.

A measure of the performance of the sliding mode controlled system, described above, is obtained by using the integral of the stored stabilization error energy. This quantity is given by

$$\mathcal{I}_B = \int_0^\infty H(\tau)\mathrm{d}\,\tau = \int_0^\infty \frac{1}{2}\left[L\left(x_1(\tau) - \frac{V_d^2}{RE}\right)^2 + C\,(x_2(\tau) - V_d)^2\right]\mathrm{d}\,\tau \tag{3.4}$$

Such a performance criterion can also be regarded as a weighted integral square state stabilization error for the state vector. We simply address such an index as the "WISSSE" index.

Proposition 3.2 *The WISSSE index, computed along the sliding mode controlled trajectories of the "Boost" converter, is unbounded, independently of the initial conditions of the converter.*

3.2 Passivity-Based Sliding "Current-Mode" Controller for the "Boost" Converter

In the following developments we introduce an auxiliary state vector, denoted by x_d. The basic idea is to take x_d as a "desired" vector trajectory for the converter state vector x. This auxiliary vector variable will be determined on the basis of energy shape considerations, and passivity, imposed on the evolution of the error vector $x - x_d$. The feedback regulation of the auxiliary state x_d, towards the desired constant equilibrium value of the state x, will in fact result in the specification of a *dynamical output feedback controller* for the original converter state. We will be using a sliding mode control viewpoint for the regulation of x_d towards the desired equilibrium value of x.

We rewrite the "Boost" converter equations (2.13) in matrix-vector form as

$$\mathcal{D}_B\dot{x} + (1-\mu)\mathcal{J}_B x + \mathcal{R}_B x = \mathcal{E}_B \tag{3.5}$$

where

$$\mathcal{D}_B = \begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix} \;;\; \mathcal{J}_B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \;;\; \mathcal{R}_B = \begin{bmatrix} 0 & 0 \\ 0 & 1/R \end{bmatrix} \;;\; \mathcal{E}_B = \begin{bmatrix} E \\ 0 \end{bmatrix} \tag{3.6}$$

Consider the stored stabilization error energy, H_d, of the state x with respect to the auxiliary state variable x_d,

$$H_d = \frac{1}{2}(x - x_d)^T \mathcal{D}_B (x - x_d) \tag{3.7}$$

Suppose x_d satisfies the following controlled differential equation

$$\mathcal{D}_B\dot{\tilde{x}} + (1-u)\mathcal{J}_B\tilde{x} + \mathcal{R}_{Bd}\tilde{x} = 0 \tag{3.8}$$

where $\mathcal{R}_{Bd} = \mathcal{R} + \mathcal{R}_B$, with $\mathcal{R} = \mathrm{diag}[R_1 \; 0]$. Then, the following proposition holds valid.

Proposition 3.3 *Given a desired state vector trajectory $x_d(t)$ for the converter state vector $x(t)$, the error vector $\tilde{x} = x - x_d$ asymptotically decays to zero, from any arbitrary initial condition $\tilde{x}(0)$, whenever $x_d(t)$ is obtained as a solution of the controlled differential equation:*

$$\mathcal{D}_B\dot{x}_d + (1-u)\mathcal{J}_B x_d + R_{Bd}x_d = \mathcal{E}_B$$

for any given control policy u which is equally applied to both the plant and the auxiliary system. Moreover, for some positive constants α, and β, which may be, respectively, taken as $\alpha = \min\{R_1, 1/R\}$ and $\beta = \max\{L, C\}$, the time derivative of the total stored error energy H_d satisfies

$$\dot{H}_d \;=\; -(x - x_d)^T\mathcal{R}_{Bd}(x - x_d) \;\leq\; -\frac{\alpha}{\beta}H_d \;\leq\; 0$$

Consider now the auxiliary system defining x_d, explicitly written as

$$\begin{aligned} L\dot{x}_{1d} + (1-u)x_{2d} - R_1(x_1 - x_{1d}) &= E \\ C\dot{x}_{2d} - (1-u)x_{1d} + \frac{1}{R}x_{2d} &= 0 \end{aligned} \tag{3.9}$$

The following proposition depicts the most important features of a sliding current-mode regulation policy of the auxiliary system (3.9) towards the desired constant equilibrium state $(x_{1d}(\infty), x_{2d}(\infty)) = (V_d^2/RE, V_d)$ of the "Boost" converter.

Proposition 3.4 *Consider the switching line* $s = x_{1d} - V_d^2/RE$, *where* $V_d > 0$ *is a desired constant capacitor voltage value for the auxiliary variable* x_{2d} *and for the converter's capacitor voltage* x_2. *The switching policy, given by*

$$u = 0.5[1 - \text{sign}\ (s)\ ] = 0.5[1 - \text{sign}\ (x_{1d} - V_d^2/RE)\ ] \qquad (3.10)$$

locally creates a sliding regime on the line $s = 0$. *Moreover, if the sliding-mode switching policy (3.10) is applied to* both *the converter and the auxiliary system, the converter state trajectory* $x(t)$ *converges towards the auxiliary state trajectory* $x_d(t)$ *and, in turn,* $x_d(t)$ *converges towards the desired equilibrium state. i.e.,*

$$(x_1, x_2) \rightarrow (x_{1d}, x_{2d}) \rightarrow \left(\frac{V_d^2}{RE}, V_d\right)$$

The ideal sliding dynamics is then characterized by

$$\overline{x}_{1d} = \frac{V_d^2}{RE}\ ;\ \dot{\overline{x}}_{2d} = -\frac{1}{RC}\left[\overline{x}_{2d} - \left(\frac{V_d^2}{E}\right)\frac{E + R_1(\overline{x}_1 - V_d^2/RE)}{\overline{x}_{2d}}\right]$$

$$u_{eq} = 1 - \frac{E + R_1(\overline{x}_1 - V_d^2/RE)}{\overline{x}_{2d}} \qquad (3.11)$$

where $\overline{x}_1$ *is the converter's inductor current under sliding mode conditions, primarily occurring in the controller's state space and induced, through the control input* u *on the controlled system state space.*

Figure 3 depicts simulations of a typical closed loop state behaviour for a passivity-based regulated converter system whose converter's initial state starte from several initial conditions.

We can prove the following result.

Theorem 3.5 *Consider the WISSSE performance index (3.4). The passivity-based sliding current-mode controller, described in Proposition 3.4, yields identically unbounded WISSSE index behaviour as the traditional sliding current-mode static controller of Proposition 3.1, provided initial conditions for the controller and the plant are chosen to be identical. If, on the other hand, initial conditions for the dynamical controller are chosen to be precisely at the required constant state equilibrium vector for the controlled system, then the WISSSE index is* finite. *This property holds true when the initial states of the converter are taken to be different from those of the controlled converter, provided it is guaranteed that* $\overline{x}_2(t) < V_d$ *for all* $t > t_h$.

References

[1] Middlebrook R D, Ćuk S. A General Unified Approach to Modeling Switching-Converter Power Stages. IEEE Power Electronics Specialists Conference (PESC) 1976; 18-34.

[2] Severns R P, Bloom G E. Modern DC–to–DC Switchmode Power Converter Circuits, Van Nostrand-Reinhold, New York 1983.

[3] Kassakian J G, Schlecht M, Verghese G C. Principles of Power Electronics, Addison-Wesley, Reading, Mass. 1991.

[4] Ortega R, Loría A, Kelly R, Praly L. On Passivity–Based Output Feedback Global Stabilization of Euler–Lagrange Systems. Int J of Robust and Nonl Contr 1995; 5: 313-324, .

[5] Utkin V I. Sliding Modes and Their Applications in Variable Structure Systems. MIR Editors, Moscow 1978.

FIGURES

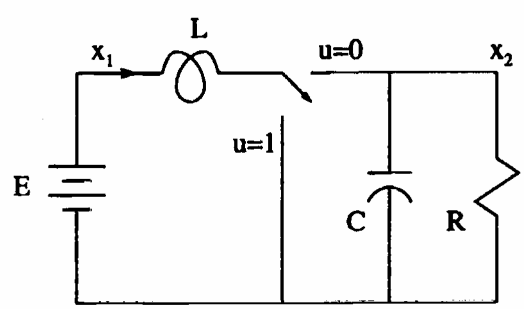

Figure 1: The "Boost" Converter Circuit.

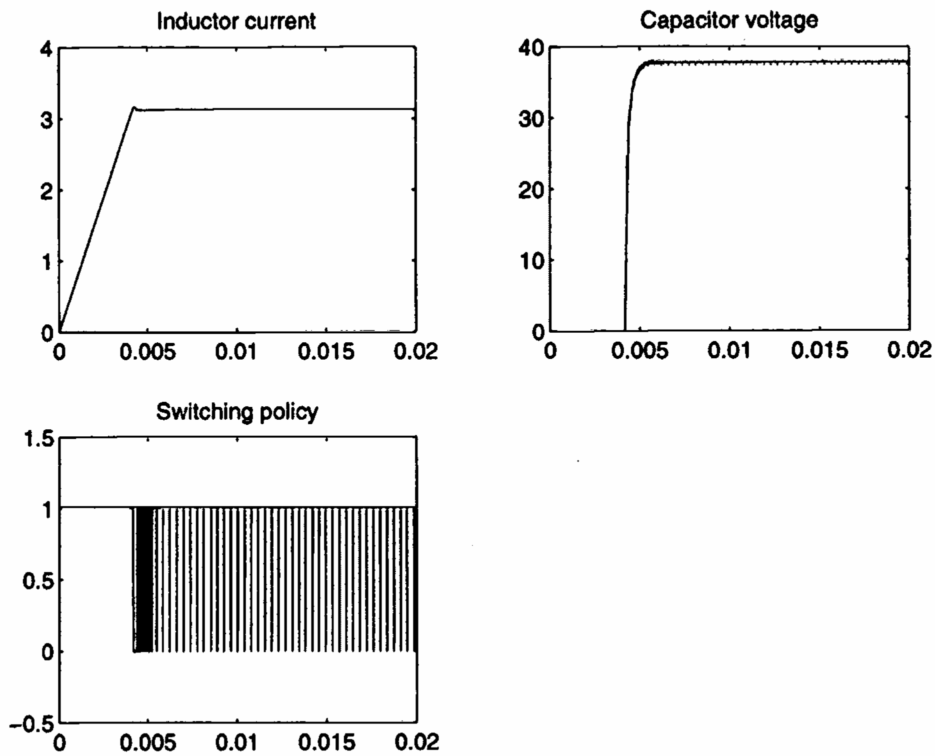

Figure 2: Typical Sliding "Current-Mode" Controlled State Responses for the "Boost" Converter.

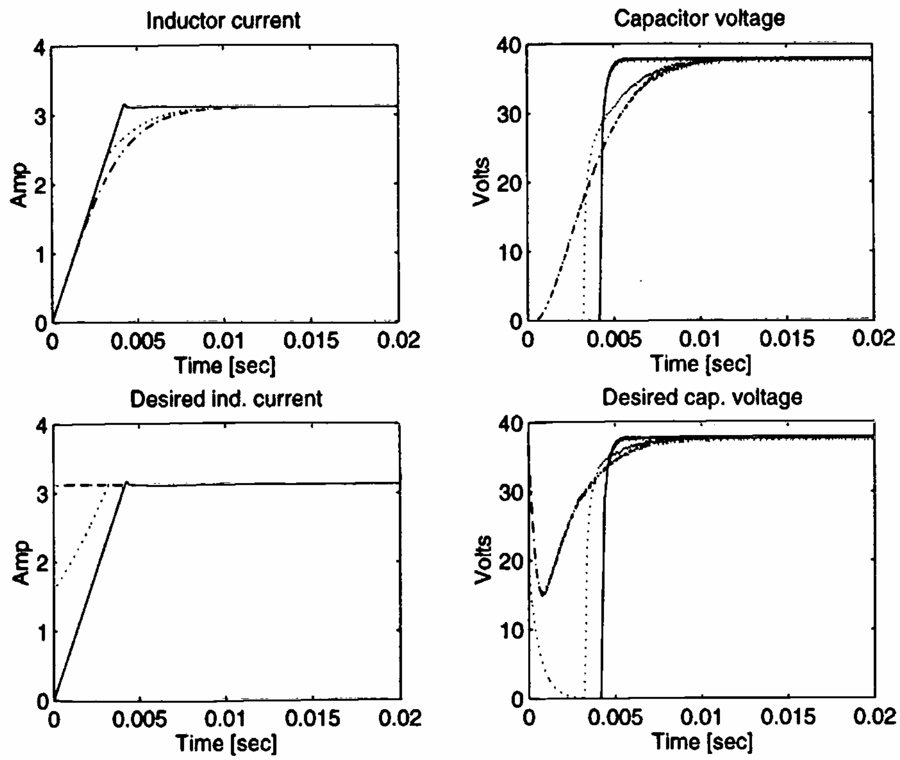

Figure 3: Controller and Plant State Resposes for Different Controller Initial Conditions, (—) $(x_{1d}(0), x_{2d}(0)) = (0,0)$; $(\cdots)$ $(x_{1d}(0), x_{2d}(0)) = (1.6, 19)$ (-·-·) $(x_{1d}(0), x_{2d}(0)) = (3.125, 37.5)$.