A BILINEAR OBSERVER APPROACH TO A CLASS OF NON-LINEAR STATE RECONSTRUCTION PROBLEMS

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ABSTRACT

A class of deterministic non-linear state reconstruction problems generated by a linear dynamic plant with non-linear state-output maps is considered. For a large class of output non-linearities it is shown that the problem can be reduced to a deterministic bilinear state reconstruction problem for which a number of results are already available [1]-[5].

PROBLEM FORMULATION AND MAIN RESULTS

Consider a deterministic state reconstruction problem defined on :

$$\dot{\underline{x}}(t) = A\,\underline{x}(t) + B\,\underline{u}(t) \tag{1}$$

with $\underline{x}\in R^n$, $\underline{u}\in R^m$, $B=[\underline{b}_1,\underline{b}_2,\ldots,\underline{b}_m]$ and a non-linear output of the form:

$$\underline{y} = \underline{g}(\underline{x}) = \tilde{G}_p\,\tilde{\underline{x}}^{[p]} \tag{2}$$

where $\tilde{\underline{x}}^{[p]}$ is the p-th homogeneous tensor power of $[{1 \atop x}]$ known as the p-th family of tensor powers of $\underline{x}$; $(\tilde{\underline{x}}^{[p]})'=[1,\underline{x}',(\underline{x}^{[2]})',\ldots,(\underline{x}^{[p]})']$. $\underline{x}^{[k]}$ has components constituted by the lexicographic listing of the $N(n,k)=\binom{n+k-1}{k}$ linearly independent forms $x_1^{k_1}x_2^{k_2}\ldots x_k^{k_n}$ with $\Sigma k_i = k$.

This problem requires, in general, a non-linear observer [1] with all the hardware and theoretical difficulties involved [2].

We consider the possibility of reducing the above problem to an equivalent bilinear state reconstruction problem for which a good deal of results are known. This possibility is easily established from the fact that the input-output behaviour of (1)-(2) is totally equivalent to that of :

$$\frac{d}{dt}\tilde{\underline{x}}^{[p]} = (\tilde{A}_{[p]} + \textstyle\sum_{i=1}^{m} u_i\,\tilde{B}_{i[p]})\,\tilde{\underline{x}}^{[p]} \tag{3}$$

$$\underline{y} = \tilde{G}_p\,\tilde{\underline{x}}^{[p]} \tag{4}$$

where u_i is the i-th component of $\underline{u}$, $\tilde{A}_{[p]}=\mathrm{diag}[0,A,A_{[2]},\ldots,A_{[p]}]$; $\tilde{B}_i=\begin{vmatrix}0 & 0\\ \underline{b}_i & 0\end{vmatrix}$ and $\tilde{B}_{i[p]}$ has its non-zero entries in blocks immediately below the main diagonal blocks with dimensions corresponding to the tensor power components of $\tilde{\underline{x}}^{[p]}$. $\underline{b}_i$ is the i-th column of B.

Proposition Let $\tilde{G}_p=[G_0,G_1,\ldots,G_p]$, then the pairs $(A_{[k]},G_k)$ are observable for all k if the pair $(\tilde{A}_{[p]},\tilde{G}_p)$ is observable.

It is also simple to show that the Lie Algebra generated by the structural matrices of (3) is *solvable* thus existing the possibility of upper-triangularization of the systems equations and a global representation for the solution as products of matrix exponentials.

An observer for (3)-(4) is readily given by :

$$\frac{d}{dt}\underline{z} = (\tilde{A}_{[p]} + \textstyle\sum_{i=1}^{m} u_i\tilde{B}_{i[p]})\,\underline{z} + H_0(\underline{y}-\underline{w}) + \sum_{i=1}^{m} u_iH_i(\underline{y}-\underline{w}) \tag{5}$$

$$\underline{w} = \tilde{G}_p\,\underline{z} \tag{6}$$

The estimation error $\underline{e}=\underline{z}-\tilde{\underline{x}}^{[p]}$ is governed by the dynamics:

$$\frac{d}{dt}\underline{e} = (\tilde{A}_{[p]}-H_0\tilde{G}_p)\,\underline{e} + \textstyle\sum_{i=1}^{m} u_i(\tilde{B}_{i[p]}-H_i\tilde{G}_p)\,\underline{e} \tag{7}$$

Theorem An assymptotic state observer for (3)-(4) (i.e (1)-(2)) exists if and only if for some given $Q = Q' > 0$ there exists a $P = P' > 0$ such that:

$$P(\tilde{A}_{[p]} - H_0\tilde{G}_p) + (\tilde{A}_{[p]} - H_0\tilde{G}_p)'P + Q = 0 \qquad (8)$$

$$P(\tilde{B}_{i[p]} - H_i\tilde{G}_p) + (\tilde{B}_{i[p]} - H_i\tilde{G}_p)'P = 0 \; ; \; i=1,2,\ldots,m \qquad (9)$$

If the pair $(\tilde{A}_{[p]},\tilde{G}_p)$ is observable, condition (8) is automatically satisfied for a suitable matrix H_0 which places the poles in the left halfplane. This requirement can also be weakened to have $(\tilde{A}_{[p]},\tilde{G}_p)$ detectable.

It is also easy to see that a necessary condition for the existence of a non-trivial solution to equations (9), $(\tilde{B}_{i[p]}-H_i\tilde{G}_p)$ must have either a zero eigenvalue or else a pair of opposite eigenvalues (w.r.t the origin)

CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

We have examined a class of non-linear state reconstruction problems defined on linear plants with non-linear multinomial outputs. A higher dimensional bilinear state estimation problem, requiring simpler hardware for analog simulation or simpler digital computer software, can be proposed for the solution of the original problem. A direct non-linear approach for the class of output maps considered requires an extremely large collection of multipliers and ,generally, sophisticated analog computer capabilities for implementation or design experiments. The conditions we propose for the existence of an assymptotic observer are represented by Lyapunov algebraic equations whose solution can be obtained by standard computer routines extensively available.

The ideas and results in [6] and [7] allow extension of these results to the case of non-linear analytic output maps expressible, in an approximate fashion, by a suitable linear transformation of a conveniently high degree family of homogeneous tensor powers of the state vector.

A Lie Algebraic setting for the bi-linear state reconstruction problem, and for the class of problems here presented, require special attention as a possible research area.

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