A LYAPUNOV APPROACH FOR THE DESIGN OF VARIABLE STRUCTURE SYSTEMS

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Abstract This article presents a design method for the sliding mode creation in VSS [1] using a Lyapunov function approach.

Consider a linear dynamic system $\dot{\underline{x}} = A\underline{x}+\underline{b}u$ with $u = -\underline{f}^T\underline{x}$ and $(A,\underline{b})$ controllable. Let $V(\underline{x}) = \underline{x}^TP\underline{x}$ be a Lyapunov function candidate. Krasovski's sinthesis technique [2] defines a Variable Structure Controller (VSC) of the form $f_i = \beta_i$ for $sx_i < 0$ and $f_i = \alpha_i$ for $sx_i > 0$ where $\alpha_i > 0$ and $\beta_i < 0$ $(i=1,2,\ldots,k)$ are the bounds for the feedback gains and $s = \underline{b}^TP\underline{x} = 0$ is the switching surface. P is obtaied as a positive solution of $A^TP+PA=-Q$. This "modulated bang-bang" feedback control minimizes a standard quadratic performance index defined on the state trajectories alone (no control penalty) and results in an asymptotically stable closed loop system.

A sliding regime can be created on $s = 0$ if and only if one prescribes appropriate values for the bounds of the feedback gains f_i, which yield $s\, ds/dt < 0$. These are $\beta_i \leq m_i$; $\alpha_i \geq m_i$ with $m_i = [(\underline{b}^TPA)_i-(\underline{b}^TPA)_n(\underline{b}^TP)_i/(\underline{b}^TP)_n]/(\underline{b}^TP\underline{b})$; $i=1,2,\ldots,k$. The number of feedback loops k is determined from the fact that only n-k-1 equalities of the form: $(\underline{b}^TPA)_i/(\underline{b}^TP)_i = (\underline{b}^TPA)_n/(\underline{b}^TP)_n$ are to be satisfied.

The active switching of the VSC keeps the state on $s=0$ in a sort of jittering motion that slides towards its stable equilibrium. Ideally this motion satisfies $s=0$ and $ds/dt=0$ leading to the "equivalent control problem". This control would keep the state trajectory on $s=0$ if no perturbations were present. Under these circumstances, the ideal sliding dynamics is given by $d\underline{x}/dt = (A-\underline{b}\underline{b}^TPA/\underline{b}^TP\underline{b})\underline{x}$ on $s=0$. Except for a zero at the origin these eigenvalues coincide with the transmission zeros of the open loop system with output $y = \underline{b}^TP\underline{x}$. This establishes an affine relationship among the coefficients of the open loop characteristic polynomial and those of the ideal sliding. The setting of the latter yields the former which in turn are achievable by linear feedback on the original plant.

Consider the identity $V(\underline{x}) = \underline{x}^TP\underline{x} = \underline{x}^T(P-P\underline{b}\underline{b}^TP/\underline{b}^TP\underline{b})\underline{x}+s^2/\underline{b}^TP\underline{b}$. All of the ingredients for the sliding regime formation are present in this _ideal sliding dynamics based decomposition_ of $V(\underline{x})$. Under a suitable change of coordinate $\underline{z} = P_1\underline{x}$ with $P_1^TP_1 = P$ this decomposition is equivalent to the _geometrically based decomposition_ of $\underline{x}^T\underline{x}$ obtained by direct sum decomposition of the state space taking s as a projection subspace ($X = X_1 + X_2$) with X_1 being the _projection of X on s along $P\underline{b}$_ and X_2 the _projection of X on $P\underline{b}$ along s_. The time derivative of $V(\underline{x})$ contains two clearly identified terms: one dealing with the sliding regime creation process ($2s\,ds/dt/\underline{b}^TP\underline{b}$) and the other intimately related to the ideal sliding dynamics stability:

$$\underline{x}^T[(A-\underline{b}\underline{b}^TPA/\underline{b}^TP\underline{b})^TP+P(A-\underline{b}\underline{b}^TPA/\underline{b}^TP\underline{b})]\underline{x}$$

Design Example

Consider the dynamical system described by: $dx_1/dt = x_2$; $dx_2/dt = x_3$; $dx_3/dt = -0.0226x_1-2.262x_2-0.21x_3 + u$.

An infinite time performance index without control penalty defined by $Q = \text{diag}\ [10,1,1]$ and a surface equation $s = \underline{b}^T P \underline{x} = 20.0x_1 + 8.0x_2 + x_3 = 0$ yields a VSC with two feedback loops and magnitude bounds for the feedback gains given by $m_1 = 155.776$, $m_2 = 44.58$. The ideal sliding dynamics is assymptotically stable. Fig.1 shows a sample of the state trajcetory in the open loop case and Fig. 2 shows the corresponding VSC feedback response.

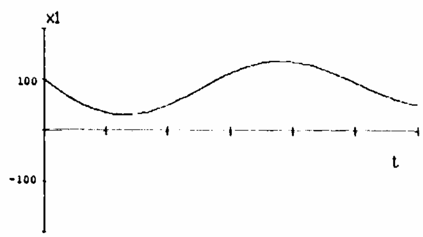

Fig.1

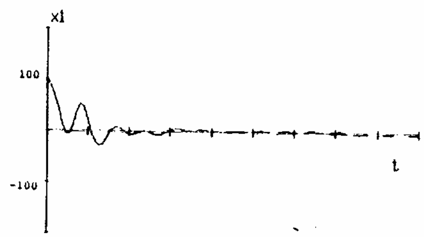

Fig.2

CONCLUSIONS

The relevance of Lyapunov's Second Method for the design of VSS for linear stationary plants has been established. A simple Lyapunov function decomposition ,based on geometric arguments induced by considering the switching surface as a projection subspace, leads to a separation of the sliding mode formation process and the ideal sliding dynamics stabilization. The approach was shown to have strong connections with optimal feedback design via Krasovskii's sinthesis procedure . A simple third order design example was also furnished.

A number of problems within the VSS area can be further analyzed from this viewpoint: Multivariable systems, time-varying systems, non-linear systems and the non-linear sliding manifold case [3].

REFERENCES

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[3] Sira-Ramirez,H., "Non-linear Sliding Manifolds for Linear and Bilinear Systems" 24th IEEE Conference on Decision and Control.Fort Lauderdale,Florida. December 1985 (to appear).