

# SPACECRAFT REORIENTATION MANEUVERS VIA LINEARIZING VARIABLE STRUCTURE CONTROL<sup>1</sup>

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## Abstract

A Variable Structure Control (VSC) approach is presented for multi-axial spacecraft attitude maneuvers. Nonlinear sliding surfaces are proposed which result in asymptotically stable, ideal linear decoupled sliding motions of Cayley-Rodrigues attitude parameters, as well as of angular velocities. The resulting control laws are interpreted as more robust and more easily implemented versions of those previously obtained by feedback linearization.

## 1. INTRODUCTION

Multi-axial large angle spacecraft attitude maneuvers generally pose a complex nonlinear dynamic control problem which must be solved on line with limited on-board computational capabilities. This is especially the case for rapid retargeting of space based directed energy beam pointing systems.

A number of approaches have been proposed for the adequate treatment of this problem. Linearization around nominal points (Breakwell<sup>1</sup>) or else a sequence of single axis maneuvers (Breakwell<sup>1</sup>, Hefner *et al.*<sup>2</sup>) eliminate the nonlinear nature of the problem, but, at the expense of slow response and underutilization of actuators. Nonlinear optimal control theory has also been applied to this class of problems (Junkins and Turner<sup>3</sup>) with various degrees of success. An immediate advantage is the possibility of considering multiaxial rotations without resorting to a single axis decomposition strategy. The computational burden is, however, significantly increased, and off line two point boundary value problems (TPBVP) have to be solved for each required maneuver. Other works (Vadali and Junkins<sup>4</sup>) address the same problem by using the method of particular solutions on the intrinsic TPBVP. Other direct solution methods involve a combination of optimal control theory and polynomial feedback control approximation (Dwyer<sup>5</sup>, Carrington and Junkins<sup>6</sup>).

Recently, exact feedback linearization (Hunt *et al.*<sup>7</sup>) has found a number of applications in spacecraft attitude maneuvers (Batten and Dwyer<sup>8</sup>, Dwyer<sup>9</sup>). In this approach, the nonlinear slewing problem is solved by formulating maneuvers based on an equivalent Brunovsky canonical version of the system obtained through nonlinear transformation of coordinates and nonlinear feedback (See also Dwyer<sup>10</sup>). The effect of elastic deformations can also be taken into account (Dwyer<sup>11</sup>, Monaco and Stornelli<sup>12</sup>), requiring, however, elastic deformation feedback for torque profile correction.

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In a recent article (Vadali<sup>13</sup>), the use of VSC for large angle rotational maneuvers has been proposed, especially when pulsed width, pulsed frequency modulation thrusters are available (Wie and Barba<sup>14</sup>). An elegant optimal control approach for the sliding surface synthesis problem was formulated by minimizing mean square quaternion error and mean square angular velocity, obtaining a closed form solution for the sliding surface. The sliding surface relating attitude quaternion and angular velocity was shown to be linear for each axis. The approach achieves a design simplicity which also results in robust controlled motions. However, the stability characteristics of the ideal sliding kinematics could not be independently prescribed for each orientation parameter, but rather, homogeneous asymptotic stability was achieved with a common preselected exponential rate of decay for a function of the Euler attitude parameters.

Motivated by the above referenced paper, in this article a more general VSC scheme is proposed, based on nonlinear sliding manifolds, defined in the spacecraft kinematic variables. Sliding regimes are found, that result in a controlled reduced system on which the relaxation time of each attitude coordinate can be independently chosen. As in (Vadali<sup>13</sup>), the resulting control laws require only an estimate of the computed torques, as well as a switching logic entirely determined by the desired kinematics, which is now freely chosen. In contrast, earlier work (e.g. Dwyer<sup>9,10,11</sup>) relied on the exact computation of the required torque profiles, manifestly less robust than the now proposed "overshoot and switch" scheme. Inasmuch as the effects of elastic distortion on line-of-sight motion can be accommodated by increasing VSC gains, only rigid motion will be modeled here.

In Section 2 of this paper the nonlinear equations of motion are presented for the control of a spacecraft undergoing multi-axial rotational maneuvers. Following Wang<sup>15</sup>, Cayley-Rodrigues attitude parameters are used instead of Euler angles or quaternions, for rationality, nonredundancy and non singularity. In Section 3, following Utkin<sup>16</sup>, a linearizing sliding surface and average control effort estimates are obtained, for the attitude maneuver problem. The use of well known results (Slotine and Sastry<sup>17</sup>) to avoid the chattering problem is also proposed for the synthesis of the actual commanded torque generation (See Batten and Dwyer<sup>8</sup>), i.e., saturating torque actuators can be assumed rather than ideal torque switchings. This is done at the expense of velocity of surface reachability. Section 4 contains the conclusions and suggestions for further research in this area.

## 2 BACKGROUND

### 2.1. Spacecraft attitude dynamics

A spacecraft driven by reaction wheels, aligned with the principal axes of inertia, is governed by the following kinematics and dynamics equations:

$$\frac{d}{dt}\xi = \frac{1}{2} [I + \xi\xi^T + \xi \times] w$$

$$[I^0 - I^a] \frac{d}{dt} w = h(\xi) \times w - \tau \quad (2.1)$$

where :

$$h(\xi) = C(\xi)C(-\xi(0))[I^0 w(0) + I^a \Omega(0)]$$

$$C(\xi) = 2(1 + \xi^T \xi)^{-1} [I + \xi\xi^T - \xi \times] - I \quad (2.2)$$

and  $\xi$  is the Gibbs vector of Cayley-Rodrigues parameters ( See Wertz 1978, Dwyer 1986a ) defined as :

$$\xi = e \tan\left(\frac{\phi}{2}\right) \quad (2.3)$$

denoting the result of a virtual rotation of  $\phi$  radians about a virtual unit axis vector  $e$ , with reference to a preselected inertial reference system.  $\Omega = \text{col}[\Omega_1, \Omega_2, \Omega_3]$  where  $\Omega_i$  is the angular velocity of the  $i$ -th reaction wheel, about the  $i$ -th principal axis of the spacecraft, measured in radians per second ( not used except for initialization of  $h$  ),  $w = \text{col}[w_1, w_2, w_3]$  is the vector of angular velocities of the spacecraft with respect to its principal axes of inertia.  $I^0$  denotes the corresponding system inertia matrix.  $I^a = \text{diag}[I^a_1, I^a_2, I^a_3]$  with  $I^a_i$  representing the axial moment of inertia of the  $i$ -th wheel, all measured in  $\text{Kg.m}^2$ ;  $\tau = \text{col}[\tau_1]$  where  $\tau_i$  is the reaction torque generated by the  $i$ -th wheel motor.  $I$  stands for the identity matrix in  $R^3$  and  $[\xi \times]$  denotes the matrix defining the vector "cross" product operation in spacecraft coordinates. The matrix  $C(\xi)$  represents the change of variables matrix taking the inertial reference coordinate system into the rotated ( body ) coordinates.

The kinematic equations of an externally controlled spacecraft using symmetric pair of thrusters with locked wheels are the same as before, but the dynamic equations are now :

$$I^0 \frac{d}{dt} w = (I^0 w) \times w + \tau^0 \quad (2.4)$$

where  $\tau^0$  now represents the externally applied torque, again resolved along the principal axes. The matrix  $I^0$  is, as before, the matrix of moments and products of inertia for the complete spacecraft system about its principal axes. These and other examples ( Carrington and Junkins<sup>6</sup>, Monaco and Stornelli<sup>12</sup> ) motivated the more general formulation of Sira and Dwyer<sup>18</sup>. The reader is referred to that article for generalities about VSC in the context to be used in this article.

### 3. VSC FOR SPACECRAFT REORIENTATION MANEUVERS

In this section the general results of Sira and Dwyer<sup>18</sup> are applied to reorientation maneuvers. The solution for the single axis problem provides a design avenue for the treatment of the multi-axis problem.

A **nonlinear sliding surface** is proposed which results in a **linear ideal sliding motion** for the kinematic variables. The ideal sliding motions, for each attitude parameter, can be prescribed as independent (decoupled) exponentially stable motions towards a desired final orientation.

#### 3.1. VSC of Spacecraft Reorientation Maneuvers

In this section the Variable Structure Control of spacecraft large angle reorientation maneuvers is presented. The relevant equations are presented in terms of the desired reduced order dynamic behavior, and the sliding surface that achieves such ideal behavior. Both the equivalent torque profile and the variable structure controller that achieves surface reachability and sliding motion (quasi-invariance) are explicitly computed.

##### 3.1.1. Single Axis Reorientation

The single axis reorientation maneuver control problem can be defined with the Cayley-Rodrigues kinematic description of the externally controlled spacecraft :

$$\frac{d}{dt}\xi = \frac{1}{2}(1 + \xi^2)w$$

$$\frac{d}{dt}w = \frac{1}{I^0}\tau^0 \quad (3.1)$$

In this case a linear system equations may be obtained by the alternative choice of the angular displacement as the orientation parameter ( in which case the double integrator system  $\dot{\theta} = w$ ;  $\dot{w} = \frac{1}{I^0}\tau^0$  is obtained ). However, (3.1) is used to stress the viability of applying the VSC approach directly to the nonlinear model, as well as to motivate the choice of control for the multivariable case. On the other hand, it can be shown that by using the linear model and a linear sliding surface, not only the globality of the sliding mode existence is sacrificed but one also loses the command over the rapidity of convergence towards the desired rest orientation.

A desirable sliding surface is :

$$S = \{ (\xi, w) : s = w - 2\frac{\lambda}{1+\xi^2}(\xi - \xi_d) = 0 \}$$

$$\lambda < 0, \xi_d = \text{constant} \quad (3.2)$$

With  $\xi_d$  being the final desired value of the orientation parameter. This nonlinear sliding surface is shown in Fig. 1a. Its obvious advantage is that it provides a **linear** reduced order ideal sliding dynamics, as can be easily seen from substitution of  $w$ , from (3.2), into the first part of (3.1) :

$$\frac{d}{dt}(\xi) = \lambda(\xi - \xi_d) \quad ; \quad K < 0 \quad (3.3)$$

i.e.,  $\xi = \xi_d$  is an asymptotically and exponentially stable equilibrium point.

Differentiating  $s$  with respect to  $t$  in (3.2) and using (3.1), one obtains :

$$\frac{ds}{dt} = \frac{[-\lambda(1-\xi^2+2\xi\xi_d)]}{(1+\xi^2)^2}s - \frac{2\lambda^2(\xi-\xi_d)(1-\xi^2+2\xi\xi_d)}{(1+\xi^2)^2} + \frac{\tau^0}{I^0} \quad (3.4)$$

The equivalent control<sup>16</sup> is obtained by directly enforcing the ideal sliding conditions  $s = 0$ ,  $\dot{s} = 0$  in (3.4). This results in :

$$\tau^0_{EQ} = \frac{2I^0\lambda^2(1-\xi^2+2\xi\xi_d)(\xi-\xi_d)}{(1+\xi^2)^2} \quad (3.5)$$

Roughly speaking, (3.5) establishes the fact that faster maneuvers require larger applied controlled torques. The equivalent control constitutes a reference level for the computation of the actual VSC feedback gains. Using the conditions for the existence of a sliding motion ( See Utkin<sup>16</sup> ),

$$\lim_{s \rightarrow 0^+} \dot{s} > 0, \quad \lim_{s \rightarrow 0^-} \dot{s} < 0 \quad (3.6)$$

these gains can be synthesized as :

$$\tau^0 = -k |\tau^0_{EQ}| \text{sign}(s) \quad ; \quad k > 1 \quad (3.7)$$

or, if a saturated controller is used, the "sign" function is simply replaced by the "saturation" function (See Slotine and Sasthy<sup>19</sup>) :

$$\tau^0 = -k |\tau^0_{EQ}| \text{sat}(s, \epsilon) \quad (3.8)$$

with

$$sat(s, \epsilon) = \begin{cases} sign s & \text{for } |s| > \epsilon \\ \frac{s}{\epsilon} & \text{for } |s| \leq \epsilon \end{cases} \quad (3.9)$$

#### Example 1 (Single-axis Rest-to-Rest Maneuver)

In the simulations shown in Fig. 1b-1d; the control law (3.8)-(3.9) has been used on a spacecraft with design parameters:  $I'' = 114.562 \text{ Kg.m}^2$ ;  $\xi_d = 0$ ;  $\lambda = -0.14 \text{ sec}^{-1}$ ;  $k = 1.2$ . These figures show the phase portrait, the linearized time response of the attitude parameter and the torque profile. The value of  $\epsilon$  in the saturated controller was chosen as 0.01 units of the surface coordinate. Simulations were run using the Simnon interactive simulation package developed for nonlinear systems analysis and design (Elmqvist 19). Due to the fact that Cayley-Rodrigues parameters yield a denominator bounded away from zero in the equivalent control (3.5), the feedback laws are devoid of singularities (such singularities may occur when Euler angles or quaternions are used to describe the kinematic equation).

#### 3.1.2. Multiaxial Reorientation

Multiaxial reorientation maneuvers are characterized by a desired orientation parameter vector  $\xi(t_f) = \xi_d$  and a boundary condition of the form  $w_i(t_f) = 0$ ;  $i = 1, 2, 3$  where  $t_f$  denotes the final maneuvering time and  $w_i$  represents the angular velocity about the  $i$ -th principal axis. A VSC approach to this problem starts by considering  $\xi_d$  as an equilibrium point for the reduced order ideal sliding motions taking place in the sliding surface  $s = w - m(\xi) = 0$ . The ideal sliding dynamics is to be governed by the vector field  $f_{1d}(\xi)$  which must have  $\xi = \xi_d$  as an equilibrium point, i.e.:  $f_{1d}(\xi_d) = 0$ .

If a linear behavior of the sliding motion is preferred then the desired vector field is of the form:

$$f_{1d}(\xi) = \Lambda(\xi - \xi_d) \quad (3.10)$$

where  $\Lambda$  is an arbitrary constant stable matrix. The desired ideal sliding kinematics will be governed by:

$$\frac{d}{dt} \xi = \Lambda(\xi - \xi_d) \quad (3.11)$$

thus obtaining an asymptotically stable motion towards the desired orientation parameter vector.

By virtue of (2.1), the sliding surface is represented by:

$$S = \{(\xi, w) : s = w - 2(1 + \xi^T \xi)^{-1} [I - \xi \times \xi] \Lambda(\xi - \xi_d) = 0\} \quad (3.12)$$

It then follows that:

$$m(\xi) = 2(1 + \xi^T \xi)^{-1} [\Lambda(\xi - \xi_d) - \xi \times \Lambda(\xi - \xi_d)] \quad (3.13)$$

If  $\Lambda$  is diagonal ( $\Lambda = \text{diag}[\lambda_1, \lambda_2, \lambda_3]$ ;  $\lambda_i < 0$  for all  $i$ ), the sliding motions are decoupled, and arbitrary time constants of exponential convergence can be individually imposed on the controlled kinematic motions towards the desired final orientation.

Thus, for the externally controlled spacecraft, the equivalent torque is:

$$\tau_{EQ}^0 = I'' \frac{\partial}{\partial \xi} [F^{-1}(\xi) \Lambda(\xi - \xi_d)] \Lambda(\xi - \xi_d) - [(I'' F^{-1}(\xi) \Lambda(\xi - \xi_d)) \times (F^{-1}(\xi) \Lambda(\xi - \xi_d))] \quad (3.14)$$

while for control with orthogonal reaction wheels it is:

$$\begin{aligned} -\tau_{EQ} &= [I'' - I''] \left\{ \frac{\partial}{\partial \xi} [F^{-1} \Lambda(\xi - \xi_d)] \Lambda(\xi - \xi_d) \right. \\ &\quad \left. - h(\xi) \times (F^{-1} \Lambda(\xi - \xi_d)) \right\} \end{aligned} \quad (3.15)$$

Evaluation of the gradients yields the same computed torques obtained in Dwyer<sup>11</sup>. The VS Controller can now be synthesized by any of the previous methods.

With  $\Lambda$  diagonal, the linearizing sliding surfaces are explicitly given by:

$$\begin{aligned} s_1 &= \frac{2}{1 + \|\xi\|^2} [\lambda_1(\xi_1 - \xi_{1d}) + \lambda_2 \xi_3(\xi_2 - \xi_{2d}) - \lambda_3 \xi_2(\xi_3 - \xi_{3d})] + w_1 \\ s_2 &= \frac{2}{1 + \|\xi\|^2} [-\lambda_1 \xi_3(\xi_1 - \xi_{1d}) + \lambda_2(\xi_2 - \xi_{2d}) + \lambda_3 \xi_1(\xi_3 - \xi_{3d})] + w_2 \\ s_3 &= \frac{2}{1 + \|\xi\|^2} [\lambda_1 \xi_2(\xi_1 - \xi_{1d}) - \lambda_2 \xi_1(\xi_2 - \xi_{2d}) + \lambda_3(\xi_3 - \xi_{3d})] + w_3 \end{aligned} \quad (3.16)$$

with  $\|\xi\|^2 = \xi_1^2 + \xi_2^2 + \xi_3^2$

The equivalent torque equations are, however, complex to implement. A crude estimate of the equivalent control is obtained instead, as if the maneuver were of the single axis type. This idea, although rather "ad-hoc", is in the same spirit as the method of the hierarchy of controls (See Young<sup>20</sup>). Thus, using the above expressions for the linearizing sliding surfaces the following control laws are proposed for externally controlled spacecraft:

$$\tau_i^0 = -k_i |\hat{\tau}_{i, EQ}| \text{sign}(s_i) ; k_i > 1, i = 1, 2, 3 \quad (3.17)$$

where

$$\hat{\tau}_{i, EQ}^0 = \frac{2I'' \lambda_i \lambda_j^2 (1 - \xi_i^2 + 2\xi_i \xi_j \xi_d)(\xi_j - \xi_{jd})}{(1 + \xi_i^2)^2} ; i = 1, 2, 3 \quad (3.18)$$

can be considered as a crude estimate of the actual equivalent control. Similar estimates hold for reaction wheel control torques. The ideal sliding motions of the controlled attitude parameters, evolve according to decoupled linearized dynamics given by (3.11) when  $\Lambda = \text{diag}(\lambda_i)$ .

To avoid high frequency firing of thrusters and their associated chattering problem in the sliding dynamics, a saturated controller can be used, as before.

$$\tau_i = -k_i |\hat{\tau}_{i, EQ}| \text{sat}(s_i, \epsilon_i) \quad (3.19)$$

where  $\text{sat}(s_i, \epsilon_i)$  is the saturation function previously defined in (3.9) with the obvious substitutions. This alternative is especially useful if reaction wheels or control moment gyros are used as actuators. The variable structure torque given by (3.19) guarantees reachability of the sliding surface for any trajectory reasonably close to the intersection of the sliding submanifolds (to account for the approximate nature of Eq. (3.18)). Example 2 below demonstrates the validity of this controller for the creation of sliding motions.

#### Example 2 (Multiaxial Rest-to-Rest Maneuver)

Consider the spacecraft model of (2.4), controlled by external thrusters, characterized by the following inertia matrix taken from Vadali and Junkins<sup>4</sup> and Carrington and Junkins<sup>6</sup>:

$$I^0 = \begin{bmatrix} 114.562 & 0. & 0. \\ 0. & 86.067 & 0. \\ 0. & 0. & 87.212 \end{bmatrix}$$

In this case a rest-to-rest maneuver is attempted using a VSC for each symmetric pair of thrusters as if a single axis maneuver were to be performed, i.e., the control law, (3.19) is used for the VSC.

The desired attitude parameter values are all zero. Each attitude parameter ideal evolution equation is forced to be linear with an independent time constant. In this case we have chosen  $\lambda_1 = -0.15 \text{ sec}^{-1}$ ,  $\lambda_2 = -0.20 \text{ sec}^{-1}$ ,  $\lambda_3 = -0.16 \text{ sec}^{-1}$ . Sufficient magnification gains for the equivalent control estimates were found to be given by:  $k_1 = 1.5$ ,  $k_2 = 1.4$ ,  $k_3 = 1.7$ . The value of  $\epsilon$  in the saturated controller was taken as 0.01 units of the surface value.

Figs. 2a-2b depict the different phase portraits of state variables corresponding to each axis as well as the time evolution of the control torques.

#### 4 CONCLUSIONS AND SUGGESTIONS FOR RESEARCH

A general method of Variable Structure Control with nonlinear switching surfaces has been applied to multiaxial spacecraft reorientation maneuvers. Under the resulting modulated bang-bang control law, appropriate spacecraft attitude parameters were shown to follow linear, decoupled, exponentially stable motions towards their target values, with independently chosen time constants. More generally, any of the several previously published Attitude maneuvers, based on optimal control coupled with feedback linearization, can also be implemented, but with greatly reduced on-line computational complexity and improved robustness (Dwyer<sup>10</sup>, Vadali<sup>13</sup>). Cayley-Rodrigues attitude parameters were used to permit the singularity-free construction of exponentially decaying ideal motions for the specific attitude maneuvers examined in this paper.

The presence of structural deformations can be accounted for by higher VSC gains thereby not affecting line of sight pointing accuracy since the switching functions depend only on the kinematic variables. Nevertheless, structural force actuators can be used together with VSC maneuvering torque generation such as for optical figure control during slews (Dwyer<sup>11</sup>), and will be discussed elsewhere.

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FIG 1  
SINGLE AXIS MANEUVER

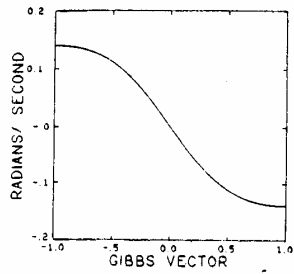


Fig. 1a Linearizing sliding surface

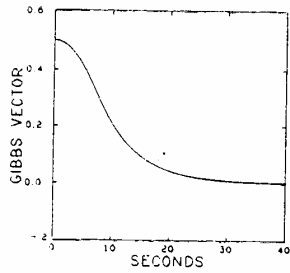


Fig. 1b Attitude Parameter  $\xi$  vs time

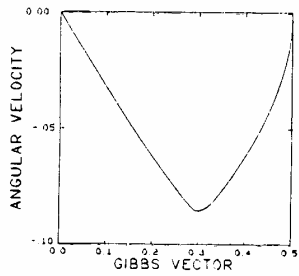


Fig. 1c Phase portrait  $w$  vs  $\xi$

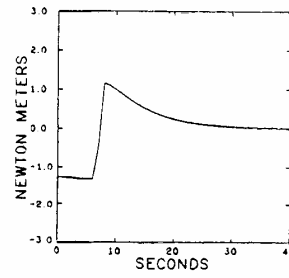


Fig. 1d Applied Torque  $\tau$  vs time

FIG 2  
MULTI-AXIS REST TO REST MANEUVER

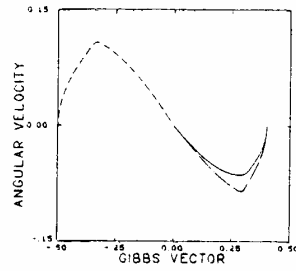


Fig. 2a Phase Portrait  $w_i$  vs  $\xi_i$ ,  $i = 1,2,3$

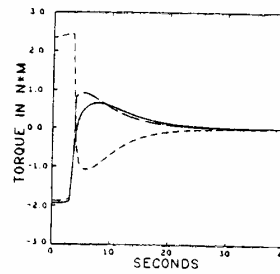


Fig. 2b Torque  $\tau_i$  vs time;  $i = 1,2,3$



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