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VARIABLE STRUCTURE CONTROL OF SPACECRAFT REORIENTATION MANEUVERS [1]

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Abstract

A Variable Structure Control (VSC) approach is presented for multi-axial spacecraft reorientation maneuvers. A nonlinear sliding surface is proposed which results in an asymptotically stable. ideal linear sliding motion of Cayley-Rodrigues attitude parameters. By imposing a desired equivalent dynamics on the attitude parameters. the approach is devoid of optimal control considerations. The single axis case provides a design scheme for the multiple axes design problem. Illustrative examples are presented.

INTRODUCTION

Multi-axial large angle maneuvers exercised for spacecraft reorientation generally pose a complex nonlinear dynamic control problem which must be solved on line with limited onboard computational capabilities.

Over the years. a number of approaches have been proposed for the adequate treatment of this problem. Classical linearization techniques around a nominal point were extended to sucessive operating point linearization to handle multi-axial large angle maneuvers following a prescribed nominal path (Breakwell 1981. Hefner et al 1980). This scheme. although rather accurate. is also time consuming. Typically a sequence of single axis maneuvers eliminates the nonlinear nature of the problem at the expense of underutilization of actuators.

Nonlinear optimal control theory has also been applied to this class of problems (Junkins and Turner 1980) with various degrees of success. An immediate advantage is the possibility of considering multi-axis rotations without resorting to a single axis decomposition strategy. The computational burden is however significantly increased and off line two point boundary value problems (TPBVP) have to be solved for each required maneuver. Other works (Vadali and Junkins 1982) address the same problem by using the method of particular solutions on the intrinsic TPBVP.

Other direct solution methods involve a combination of optimal control theory and polynomic feedback control approximation (Dwyer 1982. Carrington and Junkins 1983).

Recently. the so called exact feedback linearization approach (Hunt et al 1983) has found a number of extensions and applications in the spacecraft attitude control problem (Batten and Dwyer 1985 . Dwyer 1986a). In this approach the nonlinear slewing problem is solved by setting exact nominal commands based on an equivalent Brunovsky canonical version of the system obtained through nonlinear state space coordinates transformation and nonlinear feedback (See also Dwyer 1984). The effect of elastic deformations can also be taken into account (Monaco and Stornelli 1985. Dwyer 1986b) but only rigid body motion will be considered here.

In a recent article (Vadali 1986) the use of Variable Structure Controllers (VSC) for large angle rotational maneuvers has been proposed. specially when pulsed width. pulse frequency modulation thrusters are available (Wie and Barba 1984). Using an optimal control approach for the sliding surface synthesis problem. the resulting surfaces were shown to be linear. thus achieving a design simplicity which also resulted in a robust controlled motion. However. the stability characteristics of the ideal sliding kinematics can not be independently prescribed for each orientation parameter but rather. an homogeneous asymptotic stability is achieved with a common preselected exponential rate of decay for the attitude parameters evolution.

In this article we propose a general VSC scheme based on nonlinear sliding manifolds. defined in the spacecraft full state space coordinate functions comprising the kinematic and dynamic variables. The design problem for the ideal sliding regime is formulated in terms of a desirable reduced order dynamic behaviour of the controlled system.

We show the possibility of using sliding regimes resulting in a controlled reduced system on which the rapidity of the maneuver can be totally chosen at will. in an independent fashion for each attitude parameter. The design freedom is enhanced to a point where rest to rest maneuvers. detumbling. or arbitrary spinning (periodic tumbling or nutation) regimes can be imposed using the same basic conceptual framework. In this instance. however. we shall only present the reoritentation problem.

In section II of this paper we present the state space model for the control of a spacecraft undergoing multi axial rotational maneuvers. A sliding surface is considered on which the controlled ideal dynamics can be forced to exhibit independent exponential rate of asymptotic stability for each orientation parameter. A nonlinear static torque feedback profile is generated. through the equivalent control problem (Utkin 1978). which serves as a reference level for the gain specification problem leading to sliding surface reachability.

The use of well known results (Slotine and Sastry 1983) to avoid the chattering problem is proposed for the synthesis of the actual commanded torque generation through variable thrusters (See Batten and Dwyer 1985).i.e. saturating torque effectors are assumed rather than ideal torque switchings. This is done at the expense of velocity of surface reachability.

Section III contains two application examples demonstrating the design flexibility achievable through the use of Variable Structure Control.

The last section contains the conclusions and suggestions for further research in this area. The appendix summarizes some useful formulas closely related to the manipulations that are usually carried out when designing VSC for spacecraft whose attitude parameter evolution equations are described in terms of the Gibbs vector of Cayley-Rodriguez parameters.

II BACKGROUND AND MAIN RESULTS

2.1. Models for momentum transfer reaction wheels and external symmetric thrusters controlled spacecraft

A spacecraft driven by reaction wheels. aligned with the principal axes of inertia. is governed by the following kinematics and dynamics equations :

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$$\frac{d}{dt}\xi = \frac{1}{2}[I + \xi\xi^T + \xi x] w$$

$$[I^0 - I^a]\frac{d}{dt}w = h(\xi) x w - \tau \qquad (2.1)$$

where :

$$h(\xi) = C(\xi)C(-\xi(0))[I^0 w(0) + I^a \Omega(0)]$$

$$C(\xi) = 2(1 + \xi^T\xi)^{-1}[I + \xi\xi^T - \xi x] - I \qquad (2.2)$$

and ξ is the Gibbs vector (See Wertz 1978. Dwyer 1986a) defined as :

$$\xi = e \tan(\frac{\phi}{2}) \qquad (2.3)$$

denoting the result of a virtual rotation of ϕ radians about a virtual unit axis vector e . with reference to a preselected inertial reference system. $\Omega = col[\Omega_1, \Omega_2, \Omega_3]$ where Ω_i is the angular velocity of the i-th reaction wheel. about the i-th principal axis of the spacecraft. measured in radians per second. $w = col[w_1, w_2, w_3]$ is the vector of angular velocities of the spacecraft with respect to its inertia axis. I^0 denotes the system inertia. $I^a = diag[I^a{}_1, I^a{}_2, I^a{}_3]$ with $I^a{}_i$ representing the axial moment of inertia of the i-th wheel. measured in $Kg.m^2$: $\tau = col(\tau_i)$ where τ_i is the reaction torque generated by the i-th wheel motor. I stands for the identity matrix in R^3 and $[\xi x]$ denotes the matrix defining the vector " cross " product operation in spacecraft coordinates.

The matrix $C(\xi)$ represents the change of variables matrix taking the inertial coordinate system into the rotated (body) coordinates.

On the other hand. the kinematic equations of an externally controlled spacecraft using symmetric jet thrusters with locked wheels are the same as before. The complete model is then given by :

$$\frac{d}{dt}\xi = \frac{1}{2}[I + \xi\xi^T + \xi x] w$$

$$I^0 \frac{d}{dt}w = (I^0 w) x w + \tau \qquad (2.4)$$

where τ represents now the externally applied torque. The matrix I^0 . is . as before. the matrix of moments and products of inetia for the complete spacecraft system about its principal axes.

We shall restrict our developments to the external symmetric jet thruster controlled case. Due to the structural similarity of both models. the results derived for this case are readily extendable to the reaction momentum wheels control case.

2.2. Variable Structure Control on Nonlinear Systems

The theory of Variable Structure Systems (VSS) and their associated **sliding mode** behavior (Emelyanov 1967) has undergone extensive development in the last quarter of a century. Scientists from both. the Soviet Union and the United States have contributed with a wide embracing range of applications that cover from aerospace design problems (Utkin 1968. Calise 1984. Vadali 1986) to power systems control (Young and Kwatny 1982. Bengiamin and Chan 1982. Chan and Hsu 1983). robot manipulators (Young 1981 . Slotine and Sastry 1983) and hydropower generation (Ershler et al 1974). Survey articles (Utkin 1977. Utkin 1983). and several books (Itkis 1976 . Utkin 1978. Utkin 1981). contain a detailed account on the state of the art and the potentials for the future of this simple and yet powerful design methodology.

A surface or manifold in the state space represents static relationships among the different state variables describing the behavior of the system. If these relationships are enforced on the dynamic description of the system. the resulting reduced order dynamics may contain highly desirable features. The idea is to specify a feedback control action. of variable structure switching nature. which guarantees reachability of the prescribed manifold and. once the manifold conditions are met. it proceeds to mantain the system's motion constrained to this sliding or switching surface. The task is usually accomplished by oportune drastic changes in the structure of the feedback controller which induce velocity vector fields invariably directed towards the sliding manifold in its immediate vecinity.

Consider the nonlinear dynamic system. which captures the essential features of the spacecraft models (2.1).(2.4) :

$$\frac{dx_1}{dt} = F(x_1)x_2$$

$$\frac{dx_2}{dt} = f(x_1, x_2) + G(x_1, x_2) u \qquad (2.5)$$

where x_i (i = 1 . 2) are locally smooth coordinate systems defined on open sets of R^n respectively. $F(x_1)$ is an invertible smooth nxn matrix . G is also an invertible n x n matrix. f is a smooth n-dimensional vector field. We usually denote the pair of vectors (x_1, x_2) by x.

The control function $u : R^n \rightarrow R^n$ is a discontinuous function whose components are of the form :

$$u_i(x) = \begin{cases} u_i^+(x) & for \quad s_i(x) > 0 \\ u_i^-(x) & for \quad s_i(x) < 0 \end{cases} \qquad (2.6)$$

where $s_i : R^{2n} \rightarrow R$ is a smooth function for which the set $S_i = \{ x \in R^{2n} : s_i(x) = 0 \}$ defines a smooth $2n-1$ -dimensional smooth submanifold. S_i is called the **i-th sliding submanifold** . S is defined as the n-1-dimensional intersection of the sliding submanifolds : $S = \bigcap_{i=1}^{n} S_i$. We refer to this manifold as the **sliding manifold** .

The collection of surface coordinates $\{s_i\}$ is represented as a vector of functions $s = col(s_1, s_2, ..., s_n) : R^{2n} \rightarrow R^n$

The specification of a smooth feedback control function which makes S into an **invariant manifold** is known as the **equivalent control problem** .

It is assumed. without loss of essential generality (Lukyanov and Utkin 1981. Sira 1986). that the sliding manifold is of the form :

$$\{ x \in R^{2n} : s = x_2 - m(x_1) = 0 \} \qquad (2.7)$$

The equivalent control function describes the motion of the system in an average sense. The ideal invariant motion resulting from the equivalent control is the **ideal sliding dynamics** or the **equivalent reduced dynamics**

The actual motion of the system. when constrained to S . under the persistent action of the VSC is called the **sliding regime.**

We formulate the sliding regime creation problem as follows :

A smooth sliding manifold S and a VSC of the form (2.6) are desired so that the integral curves of the system locally approach S and remain constrained to this surface thanks to the active switchings of the feedback controller. The average. invariant. motions in S are deemed desirable in the following sense : An asymptotically stable motion is obtained on the manifold which converges towards a prespecified equilibrium point located in S.

In the context of spacecraft maneuvering problems. typically. the class of problems which fall into this category. are reorientation maneuvers and detumbling.

The effects of the control input functions on the reachability of S. as well as the explicit expression for the equivalent control. are better assesed from replacing the x_2 coordinate functions by the " surface coordinate functions " s . From (2.5) and (2.7) we obtain :

$$\frac{d}{dt}x_1 = F(x_1)(s+m(x_1)) \tag{2.8}$$

$$\frac{d}{dt}s = \hat{f}(x_1, s) + \hat{G}(x_1, s)u$$

where

$$\hat{f}(x_1, s) = \frac{\partial s}{\partial x_1}F(x_1)(s+m(x_1))+f(x_1, s+m(x_1))$$

$$= -\frac{\partial m}{\partial x_1}F(x_1)(s+m(x_1))+f(x_1, s+m(x_1))$$

$$\hat{G}(x_1, s) = G(x_1, s+m(x_1)) \tag{2.9}$$

The ideal sliding motion corresponds to having null components of the drift fields along directions not lying in the tangent space to the sliding manifold

Clearly. the sliding manifold S will be locally invariant if and only if the following pair of conditions are satisfied :

1) $s = 0$

2) $\frac{ds}{dt} = 0$ (2.10)

The role of the equivalent control is to anihilate the components of the drift vector field along directions which are transversal to the sliding surface. Using conditions (2.10) in the second part of (2.8) we obtain. after using (2.9). the equivalent control as :

$$u_{EQ}(x_1) = -\hat{G}^{-1}(x_1, 0)\hat{f}(x_1, 0)$$

$$= -G^{-1}(x_1, m(x_1))[-\frac{\partial m}{\partial x_1}F(x_1)m(x_1)+f(x_1, m(x_1))] \tag{2.11}$$

The existence and uniqueness of the equivalent control is a crucial factor in the determination of the necessary gains that achieve sliding conditions.

Using the first of conditions (2.10) on (2.8) we see that the ideal sliding motion is governed by :

$$\frac{d}{dt}x_1 = F(x_1)m(x_1) \tag{2.12}$$

The design problem is particularly simple if we realize that a desirable induced ideal dynamics:

$$\frac{d}{dt}x_1 = f_d(x_1) \tag{2.13}$$

can be obtained by letting the desired vector field substitute the reduced dynamics in (2.12) i.e:

$$m(x_1) = F^{-1}(x_1)f_d(x_1) \tag{2.14}$$

The design options are various. at this point. If stabilization of the system is the main objective. then f_d should have a stable equilibrium point in the desired location.
A second possibility is the creation of an arbitrary periodic response in the attitude parameters serving a specific design purpose. For instance. a periodic (limit cycle) behaviour of the state variables x_1 may be desirable in certain scanning maneuvers. This is known as induced nutation.(See Weiss et al 1974). Tracking and detumbling are also within the scope of the method with minor modifications. We shall present these design options elswhere.

Reachability Conditions For a sliding mode to exist in the intersection of the discontinuity (sliding) surfaces $s_i = 0$, it is necessary that the equivalent control exists and each of its components satisfies the condition (Utkin 1978) :

$$min\ (u_i^+, u_i^-) < u_{i\,EQ} < max\ (u_i^+, u_i^-) \tag{2.15}$$

The problem of determining the domain on which the trajectories converge towards the sliding manifold is equivalent to the problem of assessing the stability domain of the nonlinear system represented by the evolution equation for s in (2.8).

The specification of the variable structure gains (2.6) which achieve reachability of the sliding surface can be handled in a relatively simple manner if one does not insist upon having a sliding regime on each of the sliding submanifolds comprising the sliding surface S . To this end. it then suffices to consider the relative stability problem of (2.8) with a Lyapunov function of the form :

$$V(s) = s^T P s \tag{2.16}$$

where $P = P^T > 0$ is a positive definite symmetric matrix.

Several design schemes have been proposed for the sliding surface reachability in linear multivariable systems (Utkin and Young (1978). While for nonlinear systems Utkin (1978). provides a good account of the available methods.

In the single input case. the reachability conditions are well known (Itkis 1976. Utkin 1978) and are conceptually clearer . These conditions. in its most rigorous form are given by :

$$\lim_{s\to 0^+}\frac{ds}{dt} < 0 \quad : \quad \lim_{s\to 0^-}\frac{ds}{dt} > 0 \tag{2.17}$$

Geometrically. this means that on each side of the sliding surface the definig vector field has a projection sign onto the normal to the surface. such that the trajectories point. locally. towards the manifold. A sliding motion is thus guaranteed to exist.

A second interpretation exists in terms of the Lyapunov function $V(s) = s^2$ which is only positive semidefinite when expressed as a function of (x_1, x_2) . If the time derivative of this Lyapunov function is made into a negative quantity (i.e $s\frac{ds}{dt} < 0$) then reachability of the surface is guaranteed possibly in an asymptotic sense. This approach is known as the "relative stability approach ".

For multivariable systems. the exisiting methods generally group into two broad categories: **Diagonalization methods**, and the **Hierarchical methods** (Matheus *et al.*).The diagonalization methods. in turn. belong to one of two sub-categories. Input space diagonalization by input coordinates transformation or diagonalization by surface coordinates transformation. The reader is refered to Utkin (1978). Utkin and Young (1978) and Matheus *et al.* for more details on the available methods.

The hierarchy of controls method (Utkin 1978. Young 1978) is based on the specification of individual control laws that achieve submanifold reachability under the assumption that a hierarchical order has been established a priori among the control inputs. The occurrence of the sliding motions taking place first on those submanifolds corresponding to controllers higher in the hierarchy.

The diagonalization methods concern is to solve the interaction among the inputs from the input to surface map viepoint or else from the input to state map viewpoint. The "invariance principle " (Itkis 1976) guarantees that the ideal sliding motion is immune to these coordinate transformations.

In the diagonalization method one considers the Lyapunov function (2.16) and obtains the time derivative of this function using (2.8)-(2.9) :

$$\frac{d}{dt}V(s) = 2s^T P[-\frac{\partial m}{\partial x_1}F(x_1)s - \frac{\partial m}{\partial x_1}F(x_1)m(x_1)$$

$$+ f(x_1,s+m(x_1)) + G(x_1,s+m(x_1))u] \qquad (2.18)$$

Let $sign(s)$ denote $col[sign(s_i)]$ then, the choice :

$$u = -G^{-1}(x_1,s+m(x_1))[f(x_1,s+m(x_1)) - \frac{\partial m}{\partial x_1}F(x_1)m(x_1)$$

$$-\frac{\partial m}{\partial x_1}F(x_1)s + P^{-1}K\ sign(s)] \qquad (2.19)$$

results in a negative definite time derivative in (2.18), whenever K is a positive definite, diagonal, matrix.

The above controller is, however, quite maneuver-oriented (via the gradient of m , given by Eq. (2.14)). For this reason, ad hoc approaches are preferred, especially when the controller is to be computed with limited resources and, generally, on line.

If the region of attraction to the sliding manifold is not required to be large, a different option may be considered by eliminating part of the required surface coordinates feedback and using instead the equivalent control plus additional switching feedback. This option is simply characterized by letting the value of s be close to zero.

$$u = -G^{-1}(x_1, m(x_1))[f(x_1, m(x_1)) - \frac{\partial m}{\partial x_1}F(x_1)\dot{m}(x_1) +$$

$$P^{-1}K\ sign(s)]$$

$$= u_{EQ} - G^{-1}(x_1, m(x_1)P^{-1}K\ sign(s)$$

A simple scheme, borrowed from the single input case, tries to create a sliding regime on the basis of using a controller of the form :

$$u_i = -k_i |u_{i\,EQ}|\ sign(s_i) \qquad (2.20)$$

In this case, the parameters k_i are adjusted to create a sliding regime on either $s_i = 0$ *or* $s = 0$

2.3. VSC of Spacecraft Reorientation Maneuvers

In this section the Variable Structure Control of spacecraft large angle reorientation maneuvers is presented in the context of externally controlled vehicles using symmetric jet thrusters. We present the relevant problem equations in terms of the desired reduced order dynamic behaviour, and the sliding surface that achieves such ideal behaviour. Both the equivalent torque profile and the variable structure controller that achieves surface reachability and sliding motion (quasi-invariance) are computed using the general formulation given in section 2.2.

Reorientation Maneuvers

Reorientation maneuvers are characterized by a desired orientation parameter vector $\xi(t_f) = \xi_d$ and a boundary condition of the form $w_i(t_f) = 0$; $i = 1,2,3$ where t_f denotes the final maneuvering time and w_i represents the angular velocity about the i-th principal axis.

A VSC approach to this problem starts by considering ξ_d as an equilibrium point for the reduced order ideal sliding motions taking place in the sliding surface $s = w - m(\xi) = 0$; Identifying ξ with x_1 , using (2.14), and the nonsingularity of $F(\xi)$ it follows that $m(\xi_d) = 0$.

The ideal sliding dynamics is governed by the vector field $f_d(\xi)$ which must have $\xi = \xi_d$ as an equilibrium point, i.e: $f_d(\xi_d) = 0$.

If a **linear** behavior of the sliding motion is prefered then the desired vector field is of the form :

$$f_d(\xi) = \Lambda(\xi - \xi_d) \qquad (2.21)$$

where Λ is an arbitrary constant stable matrix. The desired ideal sliding kinematics will be governed by :

$$\frac{d}{dt}\xi = \Lambda(\xi - \xi_d) \qquad (2.22)$$

thus obtaining an asymptotically stable motion towards the desired orientation parameter vector.

By virtue of (2.14), and according to (2.7), the sliding surface is represented by :

$$S = \{(\xi, w) : s = w - F^{-1}(\xi)\Lambda(\xi - \xi_d)\}$$

$$= \{(\xi, w) : s = w - 2(1+\xi^T\xi)^{-1}[I - \xi x]\Lambda(\xi - \xi_d)\} \qquad (2.23)$$

where Eq. (A.2) of the appendix was used. It then follows that :

$$m(\xi) = 2(1+\xi^T\xi)^{-1}[\Lambda(\xi-\xi_d) - \xi\ x\ \Lambda(\xi - \xi_d)] \qquad (2.24)$$

If Λ is **diagonal** ($\Lambda = diag[\lambda_1, \lambda_2, \lambda_3]$: $\lambda_i < 0$ for all i), the sliding motions are decoupled and arbitrary speed of exponential convergence can be individually imposed on the controlled kinematic motions towards the desired final orientation.

The equivalent control is obtained using (2.24) in (2.11) with the appropriate values of G , F and f , in accordance with (2.1) or (2.4). Thus, for the externally controlled spacecraft, the equivalent torque is :

$$\tau_{EQ} = -I^0\{\frac{\partial}{\partial\xi}[F^{-1}(\xi)\Lambda(\xi-\xi_d)]\Lambda(\xi-\xi_d)$$

$$+(I^0)^{-1}[(I^0F^{-1}(\xi)\Lambda(\xi-\xi_d))\ x\ (F^{-1}(\xi)\Lambda(\xi-\xi_d))]\} \qquad (2.25)$$

The VS Controller can now be sinthesized by any of the previous methods.

For the sake of reference we summarize below some of the equations that make possible the consideration of a multiaxial slewing maneuver with ideal linear sliding dynamics, independently chosen for each attitude parameter.

The kinematic and dynamic equations for the model (2.4) are given explicitly by :

$$\frac{d}{dt}\xi_1 = \frac{1}{2}[(1+\xi_1^2)w_1 + (\xi_1\xi_2 - \xi_3)w_2 + (\xi_1\xi_3 + \xi_2)w_3]$$

$$\frac{d}{dt}\xi_2 = \frac{1}{2}[(\xi_2\xi_1 + \xi_3)w_1 + (1+\xi_2^2)w_2 + (\xi_2\xi_3 - \xi_1)w_3]$$

$$\frac{d}{dt}\xi_3 = \frac{1}{2}[(\xi_3\xi_1 - \xi_2)w_1 + (\xi_3\xi_2 + \xi_1)w_2 + (1+\xi_3^2)w_3]$$

$$\frac{d}{dt}w_1 = \frac{[I^0_2 - I^0_3]}{I^0_1}w_2w_3 + \frac{\tau_1}{I^0_1}$$

$$\frac{d}{dt}w_2 = \frac{[I^0_3 - I^0_1]}{I^0_2}w_1w_3 + \frac{\tau_2}{I^0_2}$$

$$\frac{d}{dt}w_3 = \frac{[I^0_1 - I^0_2]}{I^0_3}w_1w_2 + \frac{\tau_3}{I^0_3} \qquad (2.26)$$

With Λ diagonal, the linearizing sliding surfaces (2.23) are explicitly given by :

$$s_1 = \frac{2}{1+\|\xi\|^2}[\lambda_1(\xi_1 - \xi_{1d}) + \lambda_2\xi_3(\xi_2 - \xi_{2d}) - \lambda_3\xi_2(\xi_3 - \xi_{3d})] + w_1$$

$$s_2 = \frac{2}{1+\|\xi\|^2}[-\lambda_1\xi_3(\xi_1-\xi_{1d})+\lambda_2(\xi_2-\xi_{2d})+\lambda_3\xi_1(\xi_3-\xi_{3d})] + w_2$$

$$s_3 = \frac{2}{1+\|\xi\|^2}[\lambda_1\xi_2(\xi_1-\xi_{1d})-\lambda_2\xi_1(\xi_2-\xi_{2d})+\lambda_3(\xi_3-\xi_{3d})] + w_3 \tag{2.27}$$

with $\|\xi\|^2 = \xi_1^2+\xi_2^2+\xi_3^2$

The equivalent torque equations are, in this case, complex and impractical expressions, on which the design can not be generally based. For this reason we shall rather propose a crude estimate of the equivalent control obtained as if the maneuver were of the single axis type. This idea, although rather ad-hoc, is in the same spirit as the method of the hierarchy of controls. Thus, using the above expressions for the linearizing sliding surfaces we propose the following controllers: (the single axis rest-to-rest maneuver equivalent torque is derived in Example 3.1)

$$\tau_i = -k_i|\hat{\tau}_{i\,EQ}|sign(s_i) \quad ; k_i > 1\,, \quad i = 1,2,3 \tag{2.28}$$

where

$$\hat{\tau}_{i\,EQ} = \frac{2I^0_i\lambda_i^2(1-\xi_i^2+2\xi_i\xi_{i\,d})(\xi_i-\xi_{i\,d})}{(1+\xi_i^2)^2} \quad ; \ i = 1,2,3 \tag{2.29}$$

can be considered as a crude estimate of the actual equivalent control (2.25).

The ideal sliding motions of the controlled attitude parameters, evolve according to the decoupled linearized dynamics :

$$\frac{d}{dt}\xi_i = \lambda_i\,\xi_i \quad ; \lambda_i < 0 \quad ; i = 1,2,3$$

In order to avoid high frequency firing of thrusters and their associated chattering problem in the sliding dynamics, a saturated controller is proposed (See Slotine and Sastry 1983).

$$\tau_i = -k_i|\hat{\tau}_{i\,EQ}|sat(s_i\,,\epsilon_i) \tag{2.30}$$

where $sat(s_i,\epsilon_i)$ is the saturation function defined as :

$$sat(s_i\,,\epsilon_i) = \begin{cases} sign\,s_i & \text{for } |s_i| > \epsilon_i \\ +\dfrac{s_i}{\epsilon_i} & \text{for } |s_i| \le \epsilon_i \end{cases} \tag{2.31}$$

ϵ_i is a small positive parameter

This VSC guarantees reachability of the sliding surface for any trajectory reasonably close to the intersection of the sliding submanifolds to account for the approximate nature of Eq. (2.29). Example (3.2) demonstrates the validity of this controller for the creation of sliding motions.

III DESIGN EXAMPLES OF VSC SPACECRAFT MANEUVERS

Example 3.1 (Single Axis Rest-to-Rest Maneuver)

Consider the single axis rest to rest maneuver control problem defined on the Cayley-Rodriguez kinematic description of the externally controlled spacecraft :

$$\frac{d}{dt}\xi = \frac{1}{2}(1+\xi^2)w$$

$$\frac{d}{dt}w = \frac{1}{I^0}\tau \tag{3.1}$$

In this case a linear system may obtained by choosing the angular displacement as the orientation parameterer (in which case the doble integrator system $\dot{\theta} = w$; $\dot{w} = \frac{1}{I^0}\tau$ is obtained).

However, we shall use (3.1) to stress the viability of using the VSC approach directly to the nonlinear model. On the other hand, it can be shown that by using the linear model and a linear sliding surface, not only the globality of the sliding mode existence is sacrified but also, by resorting to a nonlinear sliding surface, which guarantees globality, one also loses the comand over the rapidity of convergence towards the desired rest orientation.

In synthesis, addressing the nonlinear model from the outset provides us with a definite advantageas far as the design problem is concerned.

Consider as a sliding surface :

$$S = \{ (\xi,w) : s = w - 2\frac{\lambda}{1+\xi^2}(\xi-\xi_d) = 0 \ ; \ \lambda < 0\,,\ \xi_d = constant \} \tag{3.2}$$

With ξ_d being the final desired value of the orientation parameter. In this case, it is clear that all singularity problems are conveniently avoided. The nonlinear sliding surface is shown in Fig. 1a. Its obvious advantage is that it provides a **linear** reduced order ideal sliding dynamics, as can be easily seen from substitution of w, from (3.2), into the first of (3.1) :

$$\frac{d}{dt}(\xi) = \lambda(\xi-\xi_d) \quad ; \ K < 0 \tag{3.3}$$

i.e., $\xi = \xi_d$ is an asymptotically stable equilibrium point.

Using (2.8)-(2.9), we obtain, in this case:

$$\frac{ds}{dt} = \frac{[-\lambda(1-\xi^2+2\xi\xi_d)]}{(1+\xi^2)^2}s - \frac{2\lambda^2(\xi-\xi_d)(1-\xi^2+2\xi\xi_d)}{(1+\xi^2)^2} + \frac{\tau}{I^0} \tag{3.4}$$

The equivalent control is obtained either using (2.11) with $m(\xi) = \frac{2\lambda(\xi-\xi_d)}{1+\xi^2}$, or directly enforcing the ideal sliding conditions (2.10) on (3.4) :

$$\tau_{EQ} = \frac{2I^0\lambda^2(1-\xi^2+2\xi\xi_d)(\xi-\xi_d)}{(1+\xi^2)^2} \tag{3.5}$$

which, roughly speaking, establishes the fact that faster maneuvers require larger applied controlled torques. The equivalent control consitutes a reference level for the computation of the actual VSC feedback gains, as it is readily seen from (2.15). Using the reachability conditions (2.17), these gains can be synthesized as :

$$\tau = -k\,|\tau_{EQ}|sign(s) \ ; \ k > 1 \tag{3.6}$$

or, if a saturated controller is used, the " sign " function is simply replaced by the " saturation " function :

$$\tau = -k\,|\tau_{EQ}|\,sat(s,\epsilon) \tag{3.7}$$

Using the expression (2.19), or solving from (3.4), we obtain an alternative expression for the controller :

$$\tau = \tau_{EQ} + r(\xi)s - k\ sign(s) \ ; \ k > 0 \tag{3.8}$$

with

$$r(\xi) = \frac{\lambda I^0(1-\xi^2+2\xi\xi_d)}{(1+\xi^2)^2}$$

This results in the stable surface dynamics $\frac{ds}{dt} = -k\,sign(s)$ for which the reachbility condition $s\frac{ds}{dt}$ $(= -k|s|) < 0$ is always satisfied. We prefer, however, the controller (3.7) for its inherent simplicity.

In the simulations shown in Fig. 1b-1e : we have used controller (3.9a) on a spacecraft with design parameters : $I^0 = 114.562\ Kg.m^2$: $\xi_d = 0$; $\lambda = -0.14 sec^{-1}$; $k = 1.2$. These figures show the phase portrait. the time responses of the state variables. the surface values evolution in time and the torque profile. The value of ϵ in the saturated controller was chosen as 0.01 units of the surface coordinate. Simulations were run using the **Simnon** interactive simulation package developed for nonlinear systems analysis and design (Elmquist 1975)

Due to the fact that we have a denominator bounded away from zero in the equivalent control (3.5) . we obtain feedback gains devoid of singularities which may cause infinite torque values for finite values of the orientation parameters (this is a common feature when Euler angles or quaternions are used to describe the kinemtic equation).

Example 3.2 (Multi-axis Rest-to-Rest Maneuvers)

Consider the spacecraft model of (2.4). controlled by external jet thrusters. characterized by the following parameters :

$$I^0 = \begin{vmatrix} 114.562 & 0. & 0. \\ 0. & 86.067 & 0. \\ 0. & 0. & 87.212 \end{vmatrix} \qquad (3.11)$$

In this case a rest-to-rest maneuver is attempted using a VSC for each symmetric pair of jets. as if a single axis maneuver were to be performed. i.e.. the control law (2.33) is used for the VSC.

The desired attitude parameters values are all zero. Each attitude parameter ideal evolution equation is forced to be linear with an independent time constant. In this case we have chosen $\lambda_1 = -0.15\ sec^{-1}$;$\lambda_2 = -0.20\ sec^{-1}$;$\lambda_3 = -0.16\ sec^{-1}$ and also the magnification gains for the " equivalent control estimates " are chosen as : $k_1 = 1.5$; $k_2 = 1.4$; $k_3 = 1.7$. The value of ϵ in the saturated controller was taken as 0.01 units of the surface value.

Fig 2a-2f depict the different phase portraits of state variables corresponding to each axis as well as the time evolution of the control torques.

CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

A Variable Structure Control approach for the multi-axial rest to rest reorientation maneuvers in externally controlled spacecraft has been presented. which allows for a decoupled linearization of evolution equation for the attitude parameters. Each attitude parameter can be forced to evolve according to a different asymptoticaly stable linear dynamics. with time constant entirely chosen at will. The simplicity of the approach makes it attractive for on board computer control command generation options. A saturated torque effector with variable thruster capabilities was assumed to avoid the high frequency firing of jet thrusters. The latter technique would also be especially suitable for internally controled spacecraft using momentum transfer reaction wheels.

The multiple axes reorientation maneuver was shown to be solvable using a "decentralized " approach. in which each pair of controlled symmetric thrusters carries a control program based on its single axis maneuver equivalent control law. This greately simplifies the design demands. while possibly restricting the range of angular maneuvers.

The approach enjoys robustness properties with respect to parameter variations or uncertainty and external disturbances. The computational complexity is well within the capability of modern on board computer control generation options. Indeed. only a bound is needed for the equivalent control torque. and only a sign comparison (with single bit assesment per input channel) is required to implement the switching function. This compares favorably with the otherwise required exact computed torques previously proposed.

Other interesting applications are also possible within this approach. Among them : detumbling. constant spinning. path tracking and induced nutation programs.

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APPENDIX

Some basic formulas

Let ξ denote the Gibbs vector representation of the Cayley-Rodriguez attitude parameters and :

$$F(\xi) = \frac{1}{2}[I + \xi\xi^T + \xi\times] \qquad (A.1)$$

where $\times$ stands for the vector " cross " product operator and the superscript T stands for vector transposition. then :

$$F^{-1}(\xi) = 2(1+\xi^T\xi)^{-1}[I - \xi\times] \qquad (A.2)$$

The following formulas are useful in carrying out some of the details involved in the application of Variable Structure Systems theory to the control of spacecraft maneuvering problems whose attitude Gibbs vector is described in terms of Cayley-Rodriguez parameters. Similar formulas for the Euler-quaternion parameters are given in Dwyer (1986b) :

$$\frac{\partial}{\partial w}F(\xi)w = F(\xi) \qquad (A.3)$$

$$\frac{\partial}{\partial \xi}[F(\xi)w] = \frac{1}{2}[\xi^T w\, I + \xi w^T - w\times] \qquad (A.4)$$

$$\xi^T F(\xi)w = \frac{1}{2}\xi^T w(1+\xi^T\xi) \qquad (A.5)$$

$$w\times[F(\xi)w] = \frac{1}{2}[-w^T\xi(I+\xi\times)w + w^T w\,\xi] \qquad (A.6)$$

$$F^{-1}(\xi)\,\xi = 2(1+\xi^T\xi)^{-1}[I - \xi\times] = 2(1+\xi^T\xi)^{-1}\xi \qquad (A.7)$$

SINGLE AXIS MANEUVER

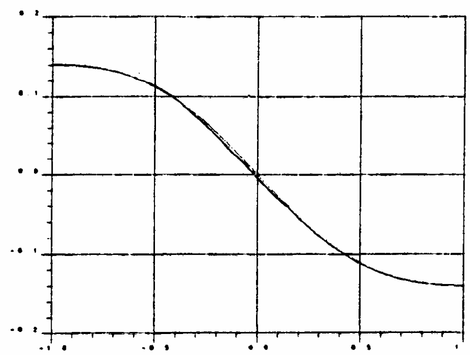

Fig. 1a Linearizing sliding surface

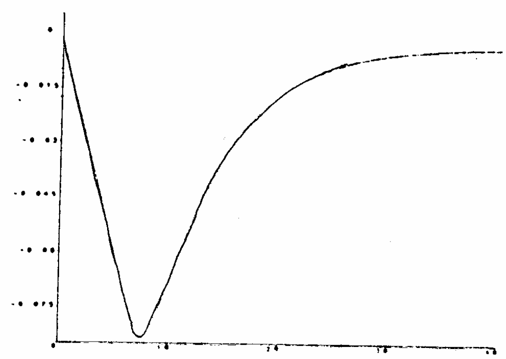

Fig. 1b Angular velocity w vs time

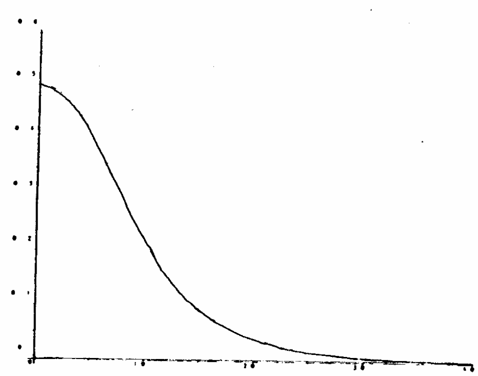

Fig. 1c Attitude Par meter ξ vs time

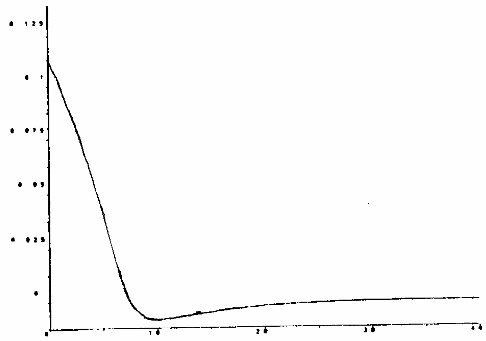

Fig. 1d Surface Coordinate value s vs time

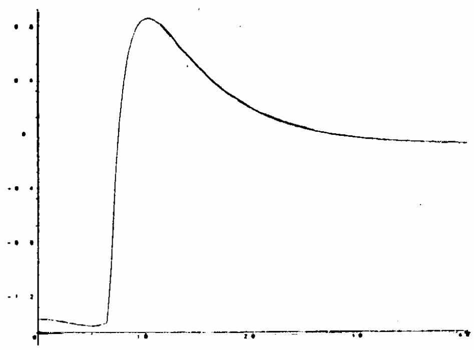

Fig. 1e Applied Torque τ vs time

MULTI-AXIS REST TO REST MANEUVER

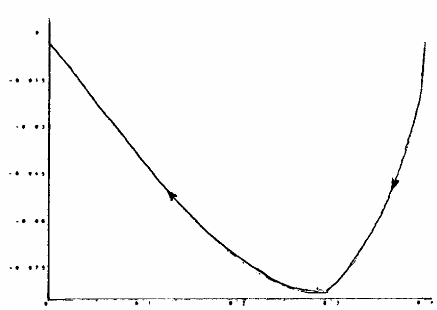

Fig. 2a Phase Portrait w_1 vs ξ_1

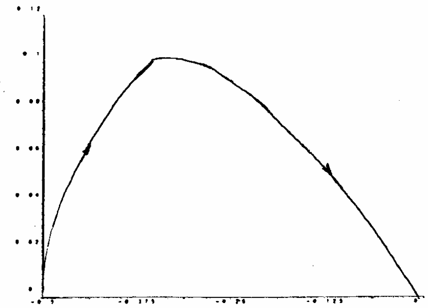

Fig. 2b Phase Portrait w_2 vs ξ_2

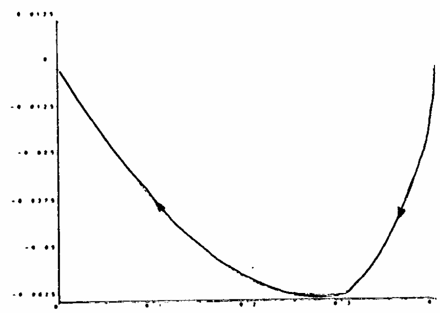

Fig. 2c Phase Portrait w_3 vs ξ_3

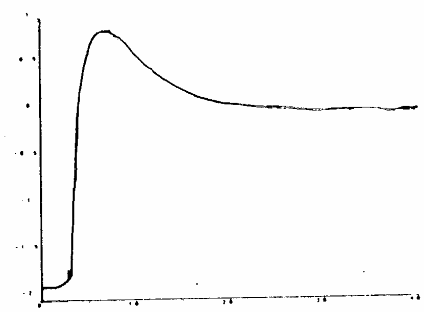

Fig. 2d Torque τ_1 vs time

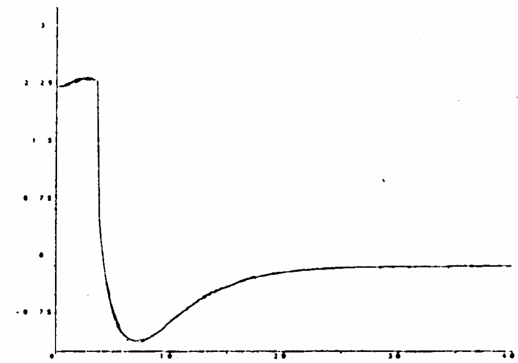

Fig. 2e Torque τ_2 vs time

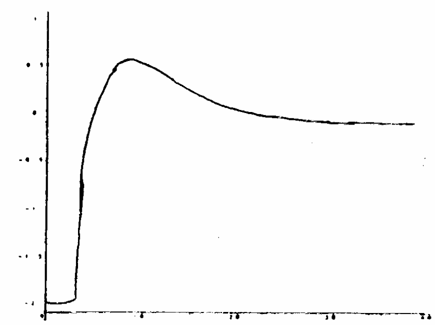

Fig. 2f Torque τ_3 vs time

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