# ON THE DESIGN OF VARIABLE STRUCTURE CONTROL FOR NONLINEAR SYSTEMS[1]

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## ABSTRACT

This article presents a series of examples on the differential geometric approach for the design of Variable Structure Controllers (VSC) leading to **sliding regimes** (Emelyanov [1]) in nonlinear smooth dynamical systems. New properties, inherited from the geometry of the sliding manifold, are obtained in the controlled system for which the sliding manifold is ideally invariant. The control action consists of oportune, drastic, changes among available structures in the feedback loop.

## I INTRODUCTION

The theory of Variable Structure Systems (VSS) and their associated **sliding mode** behavior (Emelyanov [1]) has undergone extensive development in the last quarter of a century. Contributions embrace a wide range of applications that covers from aerospace design problems (Utkin [2], Calise [3], Vadali [4] ) to power systems control (Young and Kwatny [5], Bengiamin and Chan [6], Chan and Hsu [7] ), robot manipulators ( Young [8], Slotine and Sastry [9], Morgan and Ozguner [10] ) and hydropower generation (Ershler *et al* [11] ). Survey articles (Utkin [12], Utkin [13] ), and several books (Itkis [14], Utkin [15], Utkin [16] ), contain lucid expositions on the state of the art and the potentials for the future of this simple and yet robust design methodology.

We shall briefly review a differential geometric approach ( Sira [17] ) to the sliding regime creation problem and present a series of simple examples in which the main features of nonlinear VSS are exposed.

## II PROBLEM FORMULATION AND MAIN RESULTS

Consider the nonlinear dynamic system :

$$\frac{dx}{dt} = f(x) + g(x)u \qquad (2.1)$$

where x is a local coordinate system on a smooth n-dimensional manifold M which we usually take as an open set in $R^n$. The vectors f and g are local representations of smooth vector fields in M. We denote $\Delta_G(x)$ the linear span of $g(x)$ in the tangent space of M ; $T_x M$. Let $x$ be partitioned as $x = col(\underline{x}_1, x_n)$ with $\underline{x}_1 \in R^{n-1}$, $x_n \in R$.

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We are given an n-1 dimensional smooth submanifold of $R^n$ represented by :

$$s = \{x \in R^n : s(x) = x_n - m(\underline{x}_1) = 0\} \qquad (2.2)$$

and refered to as the **sliding manifold S**. $\Delta_S(x)$ will denote the tangent subspace of $S$ in $T_x M$, while $L_h \sigma$ denotes the directional derivative of the scalar function $\sigma$ with repect to the vector field $h$.

It is desired to prescribe a VSC of the form :

$$u(x) = \begin{cases} -u^+(x) & \text{for } s(x) > 0 \\ -u^-(x) & \text{for } s(x) < 0 \end{cases} \qquad (2.3)$$

such that **ideally**, once the state trajectory reaches the surface $S$, the **invariance conditions**:

$$s = 0 \ ; \ \frac{d}{dt}s = L_{f+gu}\, s = 0 \qquad (2.4)$$

are satisfied for an indefinite period of time, thus creating, on the average, an **ideal sliding dynamics** constrained to the surface $S$. Notice that the second invariance condition is equivalent to : $f + gu \in Ker\ ds = \Delta_S$. Notice also that if $g \in \Delta_S$ then, unless $f \in \Delta_S$ the invariance conditions can not be met with finite values of $u$. We assume that the **transversality condition** $g \notin \Delta_S$ is satisfied.

If system (2.1) is in **regular form,** ( Luk'yanov and Utkin [18] ) :

$$\frac{d}{dt}\underline{x}_1 = f_1(\underline{x}_1, x_n)$$

$$\frac{d}{dt}x_n = f_n(\underline{x}_1, x_n) + u\ g_n(\underline{x}_1, x_n) \qquad (2.5)$$

and the surface equation is given by (2.2), the transversality condition takes the form $g_n(\underline{x}_1, x_n) \neq 0$ ,and then, there exists a smooth control function, known as the **equivalent control** and denoted by $u_{EQ}$ for which the invariance conditions are satisfied.

From the definition of $S$ and (2.5) it follows that the equivalent control is :

$$u_{EQ}(\underline{x}_1) = \frac{[-f_n(\underline{x}_1, m(\underline{x}_1)) + \frac{\partial m}{\partial \underline{x}_1} f_1(\underline{x}_1, m(\underline{x}_1))]}{g_n(\underline{x}_1, m(\underline{x}_1))} \qquad (2.6)$$

The reduced order ideal sliding dynamics is governed by :

$$\frac{d}{dt}\underline{x}_1 = f_1(\underline{x}_1, m(\underline{x}_1)) \qquad (2.7)$$

which may be viewed as :

$$\frac{d}{dt}\underline{x}_1 = f_1(\underline{x}_1, v) \quad ; \quad v = m(\underline{x}_1) \qquad (2.8)$$

Since, according to (2.2) the function $m(\underline{x}_1)$ completely defines the sliding manifold S . it is seen that the sliding manifold specification problem, for a required ideal sliding motion, is equivalent to the specification of a desirable feedback controlled behavior on the reduced order equations (2.8). (See Utkin and Young [19] for details concerning sliding surface design methods for linear systems )

Sliding manifold reachability is accomplished, locally, if and only if the resulting velocity vector field of the controlled system is made to point towards the manifold in its immediate vecinity . In terms of the directional derivative of the surface coordinate with respect to the velocity vector field $f + gu$ , these conditions are given by :

$$\lim_{s \to 0^+} L_{f+ug}\, s < 0 \text{ and } \lim_{s \to 0^-} L_{f+ug}\, s > 0 \qquad (2.9)$$

The proof of this fact is obvious from the geometric considerations already furnished above.

Using conditions (2.9) we obtain immediately :

$$u^+(\underline{x}_1) < \frac{\frac{\partial m}{\partial \underline{x}_1} f_1(\underline{x}_1, m(\underline{x}_1)) - f_n(\underline{x}_1, m(\underline{x}_1))}{g_n(\underline{x}_1, m(\underline{x}_1))} = u_{EQ}(\underline{x}_1)$$

$$u^-(\underline{x}_1) > \frac{\frac{\partial m}{\partial \underline{x}_1} f_1(\underline{x}_1, m(\underline{x}_1)) - f_n(\underline{x}_1, m(\underline{x}_1))}{g_n(\underline{x}_1, m(\underline{x}_1))} = u_{EQ}(\underline{x}_1) \qquad (2.10)$$

The stipulation of the variable structure feedback gains is highly dependent upon the equivalent control and takes its value as a reference level in order to produce a convenient " tilting " of the controlled vector field such that surface reachability is guaranteed from any "side " of the sliding manifold.

## III EXAMPLES

**Example 1**

In this example it is shown how to sinthesize a sustained oscillatory motion from two asymptotically stable structures.

Consider the system descirbed by :

$$\frac{d}{dt}x = \frac{1}{2}(1+z)[-y + \alpha x z u]$$

$$\frac{d}{dt}y = \frac{1}{2}(1+z)[x + \alpha y z u]$$

$$\frac{d}{dt}z = \frac{1}{2}(1+z)[x^2 + y^2]u$$

The trajectories of this system evolve on the three dimensional sphere of radius 1 , irrespectively of the value of the control input $u$ . The system has two equilibrium points at $x = y = 0$ and $|z| = 1$. For $u = -1$ the trajectories converge, in an asymptotically stable fashion, towards the "north pole " of the shpere from the unstable " south pole ". For $u = +1$ the trajectories are reversed and the south pole is now a global attractor.( See Fig. 1 )

Consider the possibility of inducing a sliding motion on $z = K = constant$ $-1 < K < 1$ . In order to bring the system to regular form, we proceed in two steps : first we transform the system to stereographic coordinates onto the equatorial plane and then we use a transformation to polar coordinates.

The transformation to equatorial stereographic projection coordinates $\xi$ , $\eta$ is given by : $\xi = \frac{x}{1+z}$ , $\eta = \frac{y}{1+z}$ . It is easy to

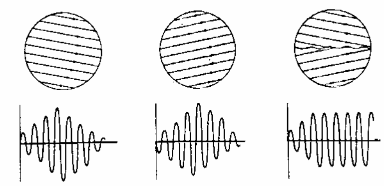

Fig. 1

verify that the inverse transformations are :

$$x = \frac{2\xi}{1+\xi^2+\eta^2} \; ; \; y = \frac{2\eta}{1+\xi^2+\eta^2} \; , \; z = \frac{1-\xi^2-\eta^2}{1+\xi^2+\eta^2}$$

The transformed sytstem is obtained as :

$$\frac{d}{dt}\xi = \frac{-\eta+\alpha\,\xi\,u}{1+\xi^2+\eta^2}, \quad \frac{d}{dt}\eta = \frac{\xi+\alpha\,\eta\,u}{1+\xi^2+\eta^2}$$

A sliding motion on $z = K$ in $R^3$ is equivalent to producing it on a circle of radius $\beta = \frac{1-K}{1+K}$ on the coordinate plane $\xi-\eta$. For equatorial sliding $K = 0$ and $\beta = 1$. However, the system is not yet in regular form. If we further transform the $\xi-\eta$ coordinates to polar coordinates by means of :

$$\rho = \sqrt{\xi^2+\eta^2} \quad , \quad \theta = \tan^{-1}\left(\frac{\eta}{\xi}\right)$$

The system is now represented as :

$$\frac{d}{dt}\theta = 1 \quad ; \quad \frac{d}{dt}\rho = \frac{2\alpha\,\rho}{1+\rho^2}\,u$$

The sliding line equation is now $s = \rho - \beta = 0$. With $\beta < 1$ a sliding line is obtained in the northern hemisphere, contained in a plane parallel to the equator. If $\beta > 1$ then the sliding line is in the southern hemisphere. For equatorial sliding $\beta = 1$. The tangent line to the sliding line is simply : $\Delta_S(\rho,\theta) = span\ \{\frac{\partial}{\partial\theta}\}$. The defining vector field for the system is $f + ug = \frac{\partial}{\partial\theta} + [\frac{2\alpha\rho}{1+\rho^2}u]\frac{\partial}{\partial\rho}$ . If $f + ug \in \Delta_S$ ,ideally, then the equivalent control must anihilate the radial components of the controlled field. We therefore have : $u_{EQ} = 0$ . Reachability of the sliding surface is accomplished by a controller that takes $u = 0$ as a reference level control.

From the fact that the sliding line coordinate evolves according to :

$$\frac{ds}{dt} = \frac{2\alpha(s+\beta)u}{1+(s+\beta)^2}$$

the reachability conditions (2.10) translate into : $u^+ < 0$ *for* $\rho > \beta$ ; $u^- > 0$ *for* $\rho < \beta$ . Translating these conditions in terms of the original cartesian coordinates we obtain the VSC in terms of the z coordinate. $u^+ = +1$ for $z < 0$ and $u^- = -1$ for $z > 0$ . Fig. 1 shows the time responses of the individual structures and those of the controlled system.

**Example 2** (Finite Time Reachability of a Limit Cycle )

This example shows that a limit cycle on the sphere, which strictly speaking is reachable in infinite time, is made reachable in finite time through VSC while ideally preserving the nature of the oscillatory response.

Consider the system described in conventional spherical coordinates $\rho$ ,$\theta$ ,$\psi$ ( radius, longitudinal angle and azimuthal angle ):

$$\frac{d}{dt}\rho = 0 \quad , \quad \frac{d}{dt}\theta = \frac{1}{2}(1+|\cos\psi|) \; :$$

$$\frac{d}{dt}\psi = \frac{1}{2}\sin\psi\,[\,(1-\cos\psi) - u\,(1+\cos\psi)\,]$$

This system has an equatorial limit cycle for $u = 1$ and both poles are unstable equilibrium points for the azimuth coordinate angle $\psi$. The system is already in regular form. For $u = constant < 1$ the limit cycle is shifted to the southern hemisphere, while for $u = constant > 1$ the limit cycle appears in the northern hemisphere. We take as a sliding surface the equatorial line : $s = \psi - \frac{\pi}{2} = 0$ . The tangent subspace to the equatorial line is simply $\Delta_S = span\ \{\frac{\partial}{\partial\theta}\}$ ( Notice $\Delta_s$ is a subspace of the tangent space $T_{(\theta,\psi)}B = span\ \{\frac{\partial}{\partial\theta} + \frac{\partial}{\partial\psi}\ \}$ ). Since on $s = 0$ the defining vector field is given by : $f + ug = \frac{1}{2}(\frac{\partial}{\partial\theta} - \frac{\partial}{\partial\psi} + u\ \frac{\partial}{\partial\psi})$ it follows that the invariance condition (2.4) implies $u_{EQ} = 1$ . From (2.10), it is clear that a sliding motion exists on $s = 0$ for a VSC of the form : $u^+ = \beta > 1$ for $s > 0$ while $u^- = \alpha < 1$ for $s < 0$. With $\alpha, \beta$ constant inputs. Fig. 2. depicts the intervening integral curves for each feedback control structure and the sliding motion.

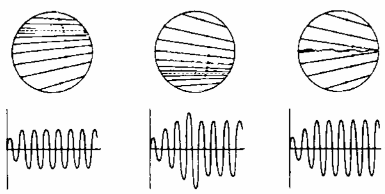

Fig. 2

**Example 3** (Harmonic Van der Pol Oscillator )

This example shows how to create a "perfect" sinusoidal response on a VS controlled Van der Pol Oscillator. i.e., VS Control has the capability of changing the nature of a limit cycle which is periodic but not harmonic.

Consider the structure controlled Van der Pol equation :

$$\dot{x}_1 = x_2 \quad ; \quad \dot{x}_2 = 2\zeta\,\omega_0(1-\mu x_1^2)x_2\,u - \omega_0^2 x_1.$$

When $u = +1$ a stable limit cycle ( unstable origin ) exists surrounding the origin of coordinates. With $u = -1$ an unstable limit cicle ( stable origin ) is obtained . This limit cycle is the mirror image on the $x_2$ axis of the preceeding limit cycle . ( See Fig. 3(a) and 3(b) )

In this case we have :

$$f = x_2\frac{\partial}{\partial x_1} - \omega_0^2 x_1\frac{\partial}{\partial x_2} \ ; \ g = 2\zeta\omega_0(1-\mu x_1^2)x_2\frac{\partial}{\partial x_2}$$

The distribution $\Delta_G(x)$ is given by the $span\ \{(1-\mu x_1^2)x_2\frac{\partial}{\partial x_2}\}$ . We take as the sliding surface a circle of radius $r$ in the normalized coordinates $(x_1, x_2/\omega_0)$ ( an ellipse in the unnormalized coordinates $(x_1, x_2)$ ). The tangent space to the sliding manifold , or sliding distribution,is $:\Delta_S(x) = span\ \{x_2\frac{\partial}{\partial x_1} - \omega_0^2 x_1\frac{\partial}{\partial x_2}\}$ Since in this case $f \in \Delta_S(x)$ , the equivalent control must anihilate the components of the input field along $\Delta_G(x)$. It follows immediately that $u_{EQ} = 0$. The ideal sliding is governed by :

$$\frac{d}{dt}x_1 = x_2 \ ; \ \frac{d}{dt}x_2 = -w_0^2\,x_2$$

whose solution is known to evolve in a circle of radius r in the space of normalized coordinates . Notice that eventhough $g(x) = 0$ on $x_2 = 0$ , the sliding condition is not violated, thanks to the fact that $f(x)$ is tangent to the circle at these points. The variable structure gains which locally guarantee reachability are given by the conditions :

$$\lim_{s\to 0^+} L_{f+gu}\,s = 2\zeta\omega_0(1-\mu x_1^2)x_2\,u^+ < 0$$

$$\lim_{s\to 0^-} L_{f+gu}\,s = 2\zeta\omega_0(1-\mu x_1^2)x_2\,u^- > 0$$

The above conditions imply that the switching logic is simply : $u = -1$ *for* $s > 0$ ; $u = +1$ *for* $s < 0$ . It is also clear, from above, that the sign of the term $(1-\mu x_1^2)$ remains unchanged inside the band $|x_1| < 1$ . This in turn implies that the sliding circle must satisfy the constraint : $r < 1/\sqrt{\mu}$

Fig 3(c) shows that a sinusoidal response is obtained by switching among two Van der Pol systems. The sliding motion is robust inside the region covered by the " reverse time " Van der Pol oscillator limit cycle.

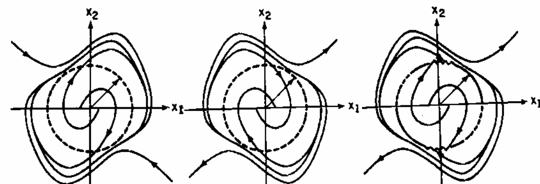


Fig. 3(a) Fig. 3(b) Fig. 3(c)

**Example 4** In this example it is shown how an asymptotically stable system is obtained out of two unstable structures which evolve on the three dimensional sphere.

Consider the system of differential equations :

$$\frac{d}{dt}\xi = \frac{1}{2}\frac{[\xi - u(2\eta - \xi)]}{1+\xi^2+\eta^2} \quad ; \quad \frac{d}{dt}\eta = \frac{1}{2}\frac{[2\xi+\eta+u\,\eta]}{1+\xi^2+\eta^2}$$

representing the equatorial stereographic description of a dynamic system evolving on the sphere. With u = - 1 , the north pole of the sphere is a saddle point for the unstable trajectories that emerge and die at the south pole (See Fig. 4(a)). For u = +1, the south pole of the sphere is a stable equilibrium point of the spiral trajectories arising from the unstable north pole. This is illustrated in Fig. 4(b).

It is easy to show that there exists a meridian line ( obtained as the intersection of the sphere with a plane containing both poles ) on which an asymptotically stable sliding motion can be created. This motion leads all trajectories to the north pole which acts now as a global attractor ( See Fig. 4 (c) ). In stereographic coordinates this meridian line is represented as $S = \{(\xi, \eta) : s = \eta + K\,\xi = 0\ ; 1 < K < \infty\ \}$. In this case we have :

$$\frac{ds}{dt} = \frac{1}{2}\frac{[s + 2\xi + u(s - 2Ks + 2K^2\xi)]}{1+\xi^2+(s - K\,\xi)^2}$$

From above, it follows that the equivalent control is simply :

$$u_{EQ} = -\frac{1}{K^2}$$

and the equivalent dynamics is given by either:

$$\frac{d}{dt}\xi = \frac{1}{2}\frac{[K^2 - 2K - 1]}{1+(1+K^2)\xi^2}\xi \ \ or\ \ \frac{d}{dt}\eta = \frac{1}{2}\frac{[K^2 - 2K - 1]}{K^2+(1+K^2)\eta^2}\eta$$

which represent an asymptotically stable system for any value of $K \in (-0.4142\ , 2.4142)$, i.e., our design may use any value of $K \in (1\ , 2.4142) = (-0.4142\ , 2.4142) \bigcap (1, \infty)$ . The switching logic is sinthesized as :

$$u = \begin{cases} +1 & for\ \ s\,\eta > 0 \\ -1 & for\ \ s\,\eta < 0 \end{cases}$$

If we propose a state coordinate transformation of the form : $z_1 = T_1(\xi,\eta)$ ; $z_2 = T_2(\xi,\eta)$. The system will be in regular form whenever :

$$\frac{\partial T_1}{\partial \xi}(\xi - 2\eta) + \frac{\partial T_1}{\partial \eta}\eta = 0$$

The solution to this partial differential equation is given by $z_1 = \eta^2 e^{\frac{\xi}{\eta}}$ . If we further let : $z_2 = \xi^2 e^{\frac{\xi}{\eta}}$ The inverse transformation is easily seen to be :

$$\xi = \sqrt{z_2}\; e^{-\frac{1}{2}\sqrt{\frac{z_2}{z_1}}} \quad ; \quad \eta = \sqrt{z_1}\; e^{-\frac{1}{2}\sqrt{\frac{z_2}{z_1}}}$$

Using these transformations, the differential equation describing the system is put in regular form. One may proceed as in previous examples.

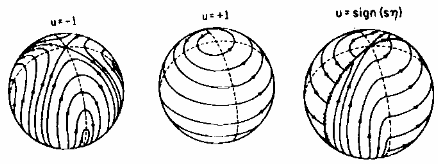


Fig. 4(a) Fig. 4(b) Fig. 4(c)

**Example 5** ( Reduced Order Feedback Linearization )

Using a nonlinear sliding surface, and VS Feedback control, this example shows how to obtain a reduced order linear model for an externally controlled spacecraft undergoing a single axes reorientation maneuver.

Consider the kinematic and dynamic model of a single-axis externally controlled spacecraft whose orientation is given in terms of the Cayley-Rodrigues representation of the attitude parameter, denoted by $\xi$. The angular velocity is represented by $w$ while I stands for the moment of inertia :

$$\frac{d}{dt}\xi = \frac{1}{2}(1+\xi^2)\, w \quad ; \quad \frac{d}{dt}w = \frac{1}{I}\tau$$

Given arbitrary initial conditions a slewing maneuver is required which brings the attitude parameter to a final desired value $\xi_d$ and the angular velocity to a rest equilibrium. For this, a nonlinear sliding surface of the form $s = w - 2\lambda \frac{(\xi - \xi_d)}{1+\xi^2} = 0$ is proposed with $\lambda < 0$. The reduced ideal sliding dynamics is given by $\dot{\xi} = \lambda(\xi - \xi_d)$ i.e., $\xi_d$ is an asymptotically stable equilibrium point for the attitude parameter differential equation. Moreover, the proposed surface yields $w = o$ when the attitude parameter converges to the desired value. The tangent space to the sliding surface $\Delta_S$ is given in this case by:

$$\Delta_S(\xi, w) = span\left\{\frac{\partial}{\partial \xi} + 2\lambda \frac{(1-\xi^2+2\xi\,\xi_d)}{(1+\xi^2)^2}\frac{\partial}{\partial w}\right\}$$

and the vector field defining the controlled motions of the system are given by $f + g\tau = \frac{1}{2}(1+\xi^2)w\frac{\partial}{\partial \xi} + \frac{\tau}{I}\frac{\partial}{\partial w}$ It then follows that the ideal sliding condition $f + \tau g \in \Delta_S$ is satisfied with the equivalent control given by :

$$\tau_{EQ} = \frac{2\lambda^2 I\,(\xi - \xi_d)(1 - \xi^2 + 2\xi\,\xi_d)}{(1+\xi^2)^2}$$

The reachability conditions (2.10) are satisfied whenever a controller of the form :

$$\tau = -k\,|\tau_{EQ}|\; sign(s) \; ; \; k > 1$$

is used. In practise a saturated controller is used which avoids jet thruster sign torque switchings and the associated chattering of the controlled trajectory. Figs. 5(a),5(b) depict the time responses of the attitude parameters and the angular velocity, clearly showing the linear nature of the sliding motion.

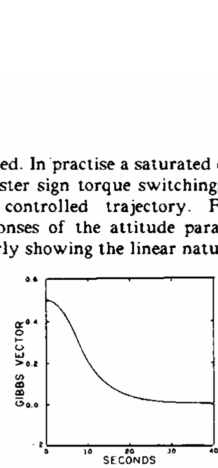


Fig. 5(a)

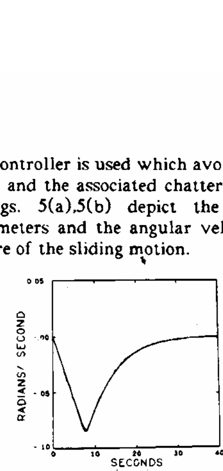


Fig. 5(b)

## IV CONCLUSIONS AND SUGGESTIONS FOR RESEARCH

In this article we have explored the possibilities of addressing the problem of inducing sliding regimes in nonlinear systems governed by variable structure feedback controllers, using notions from differential geometry.

A number of interesting applications may come out as the result of using VSS theory on nonlinear dynamic systems. Aerospace applications of control theory usually deals with systems naturally described on differentiable manifolds ( tumbling satellites, rest to rest maneuvers etc. ). The Van der Pol oscillator limit cycle has been used in the induction of periodic coordinated motions in joints space for the biped locomotion problem. The simple example addressed in this paper shows that some robustness may be gained for that particular application if a VS Controlled Van der Pol Oscillator is used.

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