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VARIABLE STRUCTURE CONTROL OF GLOBALLY FEEDBACK-DECOUPLED DEFORMABLE VEHICLE MANEUVERS

by

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ABSTRACT

The use of Cayley-Rodrigues attitude parameters as kinematic variables is shown to yield a globally linearized and decoupled model of the equations of motion of a deformable body, wherein the structural deformation state appears only in the coefficients of the inverse transformation. It is then shown how commanded multi-axial attitude maneuvers can be encoded as switching surfaces for a variable structure control implementation of the corresponding computed slew torques, automatically modulated in response only to detected angular rate error signs, for accurate tracking in the presence of separately damped or even uncontrolled (but stable) structural deformations.

INTRODUCTION

The generation of maneuvering commands for nonlinearly coupled dynamical systems by inverting the system dynamics has been gaining increasing attention in recent years, as in [1], and references shown therein.

In particular for rotational dynamics it was shown in [2] that if the vector part of the attitude quaternion used in [3] is divided by its scalar part, the resulting Gibbsian vector of Cayley-Rodrigues parameters can serve as an output vector, which responds in linear and decoupled fashion to an input vector defined as its second time derivative. Moreover, the torque synthesis transformation itself exhibits no singularity whenever the attitude variables themselves are bounded.

One difficulty in implementation is the presence of slew-induced structural deformations that would corrupt the accuracy of any control law designed from the line-of-sight dynamics alone. It has been found in [4], however, that the general methods of global linearization and decoupling yield a partial inversion of the dynamics of a flexible body. These results were adapted from the quaternion formulation in [2] to Rodrigues parameters in [5] and are extended here to include translations.

Another difficulty to be addressed in using such inverse dynamics methods lies in its sensitivity to modeling inaccuracies, or else the deliberate simplification of the inverse transformations, for purposes of real time computation. Nevertheless, it was reported in [4] that simply using the computed torques as reference levels for chattering control yielded good tracking results. That motivated the investigation of the possibility of combining feedback linearization and decoupling by means of Cayley-Rodrigues parameters, with "variable structure" control implementation of the resulting computed torques. In the present paper, that approach is extended from [6] to the case of multiaxial slewing maneuvers for flexible bodies.

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MATHEMATICAL MODEL

In this section the equations of motion of a deformable body will be cast in such a form that the coupling between position, attitude and deformable states is made explicit.

Kinematics

The kinematics is expressed in terms of the location, r, and the velocity, v, of the undeformed center of mass of the vehicle, or another center of the chosen line-of-sight coordinate system when convenient: the angular rate, ω, and an appropriate attitude vector ξ of the commanded line-of-sight; and the vectors of small structural deformations, η, as well as of modal deformation rates, Υ.

The kinematic equations in body coordinates (i.e., undeformed principal axes for r, v, ω, ξ and modal coordinates for η, Υ) are then shown below,

$$\dot{r} = v + \omega \times r, \quad \dot{\xi} = T(\xi)\omega, \quad \dot{\eta} = \Upsilon \qquad (1)$$

where $\omega \times$ denotes the matrix representing the vector product operation expressed in the body coordinates, and where $T(\xi)$ is : a 3 x 3 matrix of trigonometric functions of Euler angles; or a 4 x 3 matrix of linear functions of attitude quaternion parameters; or a 3 x 3 matrix of quadratic functions of Cayley-Rodrigues parameters according to the choice of kinematic variables ξ.

For multiaxial and large angular motion, the parametrization of choice is that given by the Gibbsian vector of Cayley-Rodrigues parameters: that is, the present attitude relative to a reference coordinate frame is described as resulting from a virtual rotation of θ radians about a virtual axis e, with direction cosines e_1, e_2, e_3 having the same values in the reference (e.g. inertial) frame as in the rotated (body) frame (hence the "Gibbsian vector" name). The attitude Gibbs vector is then given as shown:

$$\xi = e \tan\left(\frac{1}{2}\theta\right) \qquad (2)$$

its components are the Cayley-Rodrigues parameters. The time evolution of the resulting attitude vector ξ is governed by Eq. (1) with the matrix $T(\xi)$ as given below:

$$T(\xi) = \frac{1}{2}[I + \xi\xi^T + \xi\times] \qquad (3)$$

In Eq. (3), I denotes the 3 x 3 identity matrix; a superscript "T" denotes transposition so that the second term is the dyadic matrix with (i,j)-th entry given by $\xi_i \xi_j$; while $[\xi x]$ is defined as $[\omega x]$ in Eq. (1). The advantages over quaternion kinematics lie in the invertibility of $T(\xi)$ and unique parametrization by ξ of the change-of-variables matrix $A(\xi)$, from reference frame to rotated frame: one has,

$$T(\xi)^{-1} = 2(1 + \xi^2)^{-1}[I - \xi x] \tag{4}$$

$$A(\xi) = 2(1+\xi^2)^{-1}[I + \xi\xi^T - \xi x] - I \tag{5}$$

where $\xi^2 := \xi^T \xi$. In contrast, if ξ were the full attitude quaternion then $T(\xi)$ would be non-invertible, and ξ as well as its negative $-\xi$ would yield the same rotation matrix A. Moreover, the attitude vector ξ of Eq. (3) is holonomic (i.e., unconstrained), whereas the attitude quaternion is of unit magnitude, making it impossible to be regarded as a linear state variable in the same manner as the translational position vector r. The only caveat is the choice of reference system: a target attitude can be initially regarded as the reference orientation. If the initial orientation were to correspond to a virtual rotation of ±180 degrees about a relative rotation axis e(0) then the initial Gibbs vector would be infinite. In this case the reference orientation for the maneuver can be redefined by a rotation of the inertial frame by ±90 degrees about the initial virtual rotation vector e(0). In the new parametrization the initial attitude will then be modeled by the Gibbs vector $\xi(0) = \pm e(0)$, and the target attitude by the Gibbs vector $-\xi(\text{final}) = \pm e(0)$, as can be easily checked.

Dynamics

Following Wertz [7, Sec. 16.4.2] vehicle motion is commanded by a force f (including Coriolis effects neglected in Wertz, and here) and a torque τ, respectively causing translation and rotation about the undeformed center of mass (or more generally where the body attitude reference frame is fixed. Attitude can be additionally (or alternatively) commanded by changes of angular momentum h produced by reaction wheels or control moment gyros. Direct structural deformation management may also be possible by modal control forces or torques u. Only small deformation modes will be presumed, and squared angular velocities as well as products of translational and rotational velocities (when small compared with acceleration terms) will be neglected: then the dynamic equations in the vehicle-fixed rotating frame take the form given in [7, Sec. 16.4.2] shown below,

$$m\dot{v} + Z\dot{\Upsilon} = f \tag{6}$$

$$J\dot{\omega} + N\dot{\Upsilon} = \tau - \omega \times l - \dot{h} \tag{7}$$

$$Z^T\dot{v} + N^T\dot{\omega} + M\dot{\Upsilon} = u - C\,\Upsilon - K\,\eta \tag{8}$$

where m is the system mass; J its matrix of moments of inertia (resolved along undeformed principal axes); M, C, K are respectively the modal mass, damping and stiffness coefficients of the elastic structure; while Z and N are the coupling matrices between the elastic structure and, respectively, the spacecraft frame's translational and rotational motion. Finally, l stands for the total angular momentum, given below:

$$l = J\,\omega + N\,\Upsilon + h \tag{9}$$

The vector product of l with ω accounts for the rotation of the vehicle frame, and is retained for illustration of dynamic nonlinearity, or for the possibly asymmetric moments of inertia. Overdots denote time derivatives in the body-fixed frame parametrizing the line of sight.

f, τ, h, v, ω are modeled as 3 x 1 matrices in terms of the body frame, while η, Υ are modeled as n x 1 matrices of modal coordinates, in terms of appendage cantilevered mode shapes. If attitude is controlled only by reaction wheels or CMG's it is possible to model the total angular momentum l in terms of the attitude variables ξ, thereby dispensing with the propagation of the wheel dynamics, as follows: the total angular momentum at the time t_o when external torques cease to be applied is given in inertial coordinates by the formula below, where Ω denotes the column matrix of relative angular rates of three orthogonal reaction wheels, and J' denotes the matrix of axial wheel moments of inertia: If $\tau(t) = 0$ for $t \geq t_o$ one will have $l(t, \text{inertial}) = \text{constant}$. It follows that for $t \geq t_o$ one has in body axes:

$$l(t) = l(\xi(t)) := A(\xi(t))A(-\xi(t_o))l(t_o) \tag{10}$$

As shown in [3], Eq. (7) can then be replaced by:

$$(J - J')\dot{\omega} = N\dot{\Upsilon} = -\tau' - \omega \times l(\xi) \tag{11}$$

A more complete model would have all structural parameters coupled with angular velocity or acceleration. Moreover, elastic slew-induced excitation (here given by $N^T\dot{w}$ in Eq. (3)), is in fact a function of angular velocity as well as of acceleration: cf [31], pp. 41-45. It will be seen, however, that such refinements do not affect the applicability thereto of the principle of generating shaped maneuvering commands for line-of-sight model following promoted herein.

DECOUPLED CONTROL

Without either single axis or small angle slews, and without inertial symmetry assumptions, it is still possible to obtain global decoupled linear models of arbitrary vehicle motion, by feedback: this is most simply accomplished by employing the system generalized coordinates given by Eq. (12), but also with redefined system inputs $\tilde{u}$, as shown below,

$$q := (r^T, \xi^T, R^T)^T \tag{12}$$

$$\tilde{u} := (\tilde{u}_1^T, \tilde{u}_2^T, \tilde{u}_3^T)^T \tag{13}$$

with components $\tilde{u}_i$ consisting of linear combinations of up to the first two derivatives of the translation rotation, and deformation variables as shown,

$$\tilde{u}_1 :+ \ddot{r} + A_1\dot{r} + B_1 r \tag{14}$$

$$\tilde{u}_2 := \ddot{\xi} + A_2\dot{\xi} + B_2\xi \qquad (15)$$

$$\tilde{u}_3 := \ddot{\eta} + A_3\dot{\eta} + B_3\eta \qquad (16)$$

with freely chosen coefficients A_i, B_i.

A convenient but inessential choice of coefficients above is one that allows for critically damped step responses to commanded repositioning, reorientation and vibration damping. Presuming a prescribed time constant of λ^{-1} as well as a prenormalized modal mass matrix $M = I_n$ in Eq. (8) (I_k denoting the k x k identity matrix), the equivalent linear system model will have the same form as the single axis or small angles model but with redefined coefficients:

$$\tilde{M}\ddot{q} + \tilde{C}\dot{q} + \tilde{K}q = \tilde{u} \qquad (17)$$

where one has:

$$\tilde{M} = I_{6+n} \qquad (18)$$

$$\tilde{C} = \mathrm{diag}[A_1,\ A_2,\ A_3] = 2\lambda\ \mathrm{diag}\ I_{6+n} \qquad (19)$$

$$\tilde{K} = \mathrm{diag}[B_1,\ B_2,\ B_3] = \lambda^2 \mathrm{diag}\ I_{6+n} \qquad (20)$$

An example of control design using the decoupled model is that of a critically damped, combined vehicle translation to a position r_o, reorientation to an attitude ξ_o, as well as structural relaxation, all with a common time constant of λ^{-1} sec. The required controls are the following step commands,

$$\tilde{u}(t) = \lambda^2(r_o^T,\ \xi_o^T,\ 0^T)^T \qquad (21)$$

with outer loop correction for disturbances by any standard means (e.g. PID, pole placement, LQR).

The required translational forces f, and torques $\tau - \tilde{h}$ by vehicle main drivers as well as structural actuator forces or torques u, are obtained from $\tilde{u}$ by solving Eqs. (14), (15) and (17) for r, ξ, η and insertion thereof in the dynamic equations (6), (7), (8). The resulting computed forces and torques are given below:

$$f = [mI_3;\ 0;\ Z]\,\tilde{u} - [2\lambda mI_3;\ 0;\ 2\lambda\ Z](\dot{r}^T,\ \dot{\xi}^T,\ \dot{\eta}^T)^T + [\lambda^2 I_3;\ 0;\ \lambda^2 Z](\dot{r}^T,\ \dot{\xi}^T\ \dot{\eta}^T)^T \qquad (22)$$

$$\tau = [0;\ J\ T(\xi)^{-1};\ N]\,\tilde{u} - [0;\ 2\lambda JT(\xi)^{-1};\ 2\lambda N](\dot{r}^T,\ \dot{\xi}^T,\ \dot{\eta}^T)^T$$

$$-[0;\ \lambda^2 J\ T(\xi)^{-1};\ \lambda^2\ N](r^T,\ \xi^T,\ \eta^T)^T -$$

$$\{J\ T(\xi)^{-1}\ \dot{\xi}\dot{\xi}^T\ T(\xi)^{-T}\ \xi + (J\ T(\xi)^{-1}\dot{\xi}) \times T(\xi)^{-1}\ \dot{\xi}\} \qquad (23)$$

when only externally applied torques are used for attitude maneuvers; or:

$$-\tau' = [0;\ (J-J')T(\xi)^{-1};\ N]\,\tilde{u}$$

$$-[0;\ 2\lambda(J-J')T(\xi)^{-1};\ 2\lambda\ N](\dot{r}^T,\ \dot{\xi}^T,\ \dot{\eta}^T)^T$$

$$[0;\ \lambda^2(J-J')T(\xi)^{-1};\ \lambda^2 N](r^T,\ \xi^T,\ \eta^T)^T -$$

$$\{(J-J')T(\xi)^{-1}\dot{\xi}\dot{\xi}^T T(\xi)^{-T}\ \xi + l(\xi) \times T(\xi)^{-1}\dot{\xi}\} \qquad (24)$$

when only momentum transfer wheel torques τ' are used for attitude maneuvers; and

$$u = [Z^T;\ N^T\ T(\xi)^{-1};\ I]\tilde{u} -$$

$$[2\lambda Z^T;\ 2\lambda N^T T(\xi)^{-1};\ 2\lambda I - C](\dot{r}^T,\ \dot{\xi}^T,\ \dot{r}^T)^T$$

$$-[\lambda^2 Z^T;\ \lambda^2 N^T\ T(\xi)^{-1};\ \lambda^2 I_n - K](r^T,\ \xi^T,\ \eta^T) -$$

$$N^T T(\xi)^{-1}\ \dot{\xi}\dot{\xi}^T\ T(\xi)^{-T}\ \xi \qquad (25)$$

under the hypothesis of unit modal mass, $M = I_n$. Use was made, in obtaining the formulas above, of the following kinematic identity, derived from Eqs. (2) and (7):

$$\dot{\omega} = T(\xi)^{-1}\ddot{\xi} - \omega\omega^T\xi \qquad (26)$$

Thus the relationship between the generalized system forces u_s given by Eq. (27) below, and the transformed inputs $\tilde{u}$ driving the equivalent linear system given by Eq. (14) is seen to be of the form

$$u_s = (f^T(\tau - \tilde{h})^T,\ u^T)^T = G(q)\tilde{u} - g(q,\ \dot{q}) \qquad (27)$$

with G and g obtained by inspection of Eqs. (23) through (26).

It should be noted that any other linear guidance and control laws besides Eq. (22) can now be directly applied to the nonlinear plant cascaded with the nonlinear equalizer, or linearizing and decoupling transformation, modeled by Eq. (27).

For line-of-sight pointing, the control of structural deformations is required only insofar as instrument line-of-sight is deformed. This can be done even in the absence of structural actuators ($u = 0$), provided the structural deformations η and deformation rates Υ thereby excited can be measured, as done in [4]. It is the complexity of real time computation of controls such as Eqs. (22) through (25), as well as its sensitivity to modeling errors, that motivates the use of sliding modes, as is discussed next.

VARIABLE STRUCTURE CONTROL

A robust and more easily implementable command generation task can be accomplished by implementing only an estimate of the computed torques or forces, but in an average sense by an "overshoot and switch" logic triggered by the detection of deviations from the desired maneuver, called "variable structure control", denoted hereafter by VSC as in [6] and references therein. The equations of motion are best handled if expressed in the form given next.

State Space Model

The VSC method is best described for systems in the "regular form" of Eqs. (29), (30),

$$\dot{x}_1 = F(x_1)x_2 \qquad (28)$$

$$\dot{x}_2 = f(x_1,\ x_2) + G\ \mu \qquad (29)$$

where $x = (x_1^T,\ x_2^T)^T$ is the system state and μ the input control vector. For rotational maneuvers one may use the following state variables in terms of Eqs. (1), (7), (8),

$$x_1 = \begin{bmatrix} \xi \\ \eta \end{bmatrix}, \quad x_2 = \begin{bmatrix} \omega \\ \gamma \end{bmatrix}, \quad \mu = \begin{bmatrix} \tau \\ u \end{bmatrix} \tag{30}$$

obtaining with no reaction wheels, i.e., $h = 0$, the following expressions for the case of external torques and Vernier torques τ,

$$F = \mathrm{diag}\ \{T(x_{11}),\ I_n\} \tag{31}$$

$$f = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}, \quad G = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} \tag{32}$$

where I_n is the identity matrix for n elastic modes, while the entries of f and G are as follows:

$$f_1 = -R\ \{NM^{-1}[K|C]x_2 + x_{21} \times [J|N]x_2\} \tag{33}$$

$$f_2 = -S\ \{[K|C]x_2 + N^T J^{-1}(x_{21} \times [J|N]x_2)\} \tag{34}$$

$$G_1 = R[I_3|-NM^{-1}],\ G_2 = S[-N^T\ J^{-1}|I_n] \tag{35}$$

$$R = [J - NM^{-1}N^T,\ S = [M - N^T J^{-1} N]^{-1} \tag{36}$$

For torques τ' generated by servomotors driving orthogonal reaction wheels, it is enough to replace τ by $-\tau'$ in the definition of the control μ, as well as replacing J by J - J' (J' being given by the wheel inertias), as well as $[J|N]x_2$ by $l(x_{11})$ (the system angular momentum expressed in terms of the attitude variables in accordance with Eq. (21)).

The variable structure control techniques are still applicable when G in Eq. (29) is state-dependent (but invertible as well as F).

Switching Surface

A desired closed loop behavior of the "kinematic" part x_1 of the state of the abstract system given by Eqs. (46), (47) is presumed to be modeled by Eq. (48) below.

$$\dot{x}_1 = f_d(x_1) \tag{37}$$

One example is the result of inserting the step inputs given in Eq. (22) (omitting the first input in the absence of translations) into Eqs. (16) and (17). If their critically damped version is selected, the response of $x_{11} = \xi$ in particular will be exponentially decreasing in magnitude towards a presumed fixed target attitude ξ^*, while $x_{12} = \eta$ will likewise decay exponentially towards the undeformed state ($\eta = 0$). Whichever choice of desired closed loop kinematics, system inputs computed therefrom will not exactly track the motion so commanded, as is discussed below.

It is true that exogenous disturbances can be handled by outer loop filters and regulators. Nevertheless, plant modeling errors still affect the inverse dynamics. The ensuing tracking error is interpreted in VSC as a deviation of the system state $(x_1,\ x_2)$, from the switching surface S defined below:

$$S = \{x|s := x_2 - \omega(x_1) = 0\} \tag{38}$$

$$\omega(x_1) := F(x_1)^{-1} f_d(x_1) \tag{39}$$

In particular, for rigid body motion the model given by Eqs. (28) and (29) is further simplified by setting $x_1 := \xi$, $x_2 := \omega$, $\mu := \tau$ (or $-\tau'$), for which F^{-1} becomes T^{-1} given by Eq. (4), and f becomes $x_2 \times Jx_2$ (or $x_2 \times l(x_1)$). In this case the switching surface variable s in Eq. (38) is the error between the actual and the ideal angular velocity $\omega(x_1)$, for a desired maneuver.

The perturbed system is best studied when expressed in the surface coordinates $(x_1,\ s)$, yielding Eqs. (41) and (42) below, by insertion of the representation for x_2 in terms of x_1 and s given by Eq. (40):

$$x_2 = s + \omega(x_1) = s + F(x_1)^{-1}\ f_d(x_1) \tag{40}$$

The transformed equations of motion then take the form below,

$$\dot{x}_1 = f_d(x_1) + F(x_1)s \tag{41}$$

$$\dot{s} = \tilde{f}(x_1,\ s) + G\ \mu \tag{42}$$

where $\tilde{f}$ is given by Eq. (43):

$$\tilde{f}(x_1,\ s) = f(x_1,\ s + \omega(x_1)) - [\frac{\partial \omega}{\partial x}]F(x_1)(s + \omega(x_1)) \tag{43}$$

If G were state-dependent then $G(x_1,\ x_2)$ would likewise be replaced by $\tilde{G}(x_1,\ s)$ obtained by replacing x_2 by its expression in Eq. (44):

$$\tilde{G}(x_1,\ s) = G(x_1,\ s + \omega(x_1)) \tag{44}$$

Equivalent Control

The system will evolve on ("slide along") the surface S if the ideal sliding mode conditions given by Eqs. (46) are satisfied:

$$s = 0,\ \dot{s} = 0 \tag{45}$$

Intuitively these conditions mean the system state is on the surface S and remains on it.

The reduced order feedback control that would drive the system state along such a sliding mode is called the equivalent control, denoted here by μ^{EQ}. The equivalent control, given below by Eq. (46), is found by application of the sliding mode conditions (45) to Eq. (42),

$$\mu^{EQ} = -\tilde{G}(x_1,\ 0)^{-1}\tilde{f}(x_1,\ 0) \tag{46}$$

where $\tilde{f}$ was defined by Eq. (43) and $\tilde{G} = G$ for present purposes.

It can be easily checked that the equivalent control is exactly the control obtained by feedback decoupling, whenever the sliding surface is defined by the corresponding commanded closed loop trajectories.

Initial conditions might not lie on the ideal surface S. Moreover, if driven by the equivalent control, the system state will generally deviate from the sliding mode regime due to modeling errors in the computation of μ^{EQ}. This motivates the alternative VSC control strategy discussed next.

Additive VSC Correction

VSC is implemented by a choice of control law that counteracts deviations of the surface tracking error s in Eq. (49) from zero. Such a control law can be obtained by a choice of inputs that satisfies the sliding mode existence conditions, which for a single input are given by the intuitively clear relations (47):

$$\lim_{s \to 0^+} \dot{s} < 0, \quad \lim_{s \to 0^-} \dot{s} > 0 \tag{47}$$

For general vector-valued situations the conditions (47) are replaced by the Lyapunov condition below:

$$\frac{d}{dt} ||s||^2 = 2s^T \dot{s} < 0 \tag{48}$$

The condition (48) (hence (49)), is verified by s when a control law of the following form is used in Eqs. (41), (42):

$$\mu = \mu(x_1, s) = -G^{-1}(\tilde{f} + K\text{sign}(s)) \tag{49}$$

Here "sign(s)" is the vector sign function

$$\text{sign}(s) = (\text{sign}(s_1), \text{sign}(s_2), \ldots)^T \tag{50}$$

and K is any positive definite weight matrix; indeed, one then gets:

$$\frac{d}{dt} ||s||^2 = -2s^T K\text{sign}(s) \tag{51}$$

(Again G would be replaced by $\tilde{G}$ from Eq. (44) if G were state-dependent).

The VSC law $\mu(x_1, s)$ is a correction to the equivalent control, to account for errors in its computations: indeed, by setting s approximately zero near the switching surface S, one finds by inspection of Eqs. (46) and (49):

$$\mu(x_1, s) = \mu^{EQ}(x_1) - G^{-1}K\text{sign}(s) \tag{52}$$

The VSC gain K is experimentally set sufficiently high to guarantee the overshooting of the ideal surface S, thereby triggering the switching logic. Only an estimate of the ideal control $\mu^{EQ}(x_1)$ computed from Eqs. (23) or (24) is therefore needed at each instant, although more accurate values of μ^{EQ} require less control.

It should be noted that the commanded state trajectory (i.e., line-of-sight) is continuously tracked, the control switching being precisely the cause of such accurate tracking, inasmuch as control switching is triggered by incipient tracking errors.

Multiplicative VSC Correction

A VSC law can also be constructed as a multiplicative correction, rather than an additive correction, to the equivalent control. To do this it is first to be observed that control laws such as Eq. (52) are of a switching form, generically given by Eq. (53) below:

$$\mu(x_1, s) = \begin{cases} \mu^+(x_1) & \text{if } s > 0 \\ \mu^-(x_1) & \text{if } s < 0 \end{cases} \tag{53}$$

It is easily checked that the ideal sliding mode conditions (46), when applied to the system model (46), (47) excited by control law (54), lead to the following characterization of the off-surface controls μ^+ and μ^-:

$$G\,\mu^+ < G\,\mu^{EQ} < G\,\mu^- \tag{54}$$

A VSC law that can be easily shown to verify the conditions (54) is as follows,

$$\mu(x_1, s) = -k\,|\mu^{EQ}(x_1)|\,\text{sign}\,(s) \tag{55}$$

with k > 1 if G > 0, or with sign reversed if G < 0: indeed it is enough to test the cases $\mu^{EQ} \gtrless 0,\ s \gtrless 0,\ G \gtrless 0.$

Again only an estimate $\hat{\mu}^{EQ}$ of μ^{EQ} is needed in Eq. (55), since errors are detected by the switching logic, and the gain k can be set sufficiently larger than unity to guarantee reachability of the switching surface. Better accuracy in the estimation of μ^{EQ} such as the computed torques has the effect of requiring smaller VSC gain k, hence less control effort required, and in practice less frequent switching.

Decoupled VSC

For vector inputs it is even possible to use an estimate u^{EQ}_i of each scalar component of the equivalent control, and select separate single channel VSC gains k_i for separate reachability of sliding surfaces given by each component ω_i of the ideal ω in Eq. (39),

$$S_i = \{x | s_i := x_{2i} - \omega_i(x_i) = 0\} \tag{56}$$

yielding the following control law,

$$\mu_i(x_1, s) = -k_i\,|\hat{\mu}^{EQ}_i(x_1)|\,\text{sign}(s_i) \tag{57}$$

with the gains k_i "ex post facto" tuned to guarantee surface reachability (not necessarily simultaneously, although one does have S = : S_i)

Interchannel coupling is thereby treated as a disturbance, undifferentiated from plant or parameter modeling errors, inasmuch as tracking accuracy is determined only by the switching logic.

Chattering Suppression

Undesirable chattering, evident in either VSC implementation, can be suppressed at the cost of tracking accuracy. This can be done by replacement of sliding surface reachability by bounder layer reachability: that is, the system state (x_1, x_2) or (x_1, s) is required to be maintained only in a dead zone about the ideal surface, within designer-selected tolerance $\varepsilon_i > 0$ for each coordinated s_i. The required ε-accurate VSC can be obtained by replacement, in Eq. (52) or (57), of the sign(s) function by the ε-saturation function defined below:

$$\text{sat}_\epsilon(s) := \begin{cases} \text{sign}(s) & \text{if } |s| > \epsilon \\ \epsilon^{-1}s & \text{if } |s| \leq \epsilon \end{cases} \qquad (58)$$

The resulting controllers have been successfully employed in precision maneuvering of robotic manipulators under the name of "suction control," in [8] and elsewhere.

APPLICATION TO RETARGETING MANEUVERS

For purposes such as line-of-sight retargeting, a vastly simplified VSC implementation is possible, according to the following procedure.

Retargeting Switching Surfaces

A 3-dimensional switching surface, in the 6-dimensional rotational phase space of (ξ, ω), is selected to correspond to tracking a target attitude ξ_* with exponentially decaying tracking error $\xi - \xi^*$: the surface variables are obtained by insertion of $(\xi - \xi^*)^{\cdot} = -\lambda(\xi - \xi^*)$ into Eq. (1). The retargeting to a selected constant orientation ξ' yields the three sliding surfaces below (where different time constants λ_i per axis are permitted),

$$s_1 = \frac{2}{1+\xi^2}\{\lambda_1(\xi_1 - \xi_1^*) - \lambda_2\xi_3\xi_2^* + \lambda_3\xi_2\xi_3^*\} + \omega_1 \qquad (59)$$

$$s_2 = \frac{2}{1+\xi^2}\{\lambda_1\xi_3\xi_1^* + \lambda_2(\xi_2 - \xi_2^*) - \lambda_3\xi_1\xi_3^*\} + \omega_2 \qquad (60)$$

$$s_3 = \frac{2}{1+\xi^2}\{-\lambda_1\xi_2\xi_1^* + \lambda_2\xi_1\xi_2^* + \lambda_3(\xi_3 - \xi_3^*)\} + \omega_3 \qquad (61)$$

with $\xi^2 := \xi_1^2 + \xi_2^2 + \xi_3^2$.

Retargeting Torque Estimates

The equivalent torques can be chosen as estimates of the corresponding corrected torque profiles.

At the price of possibly unnecessarily high commanded torques, much cruder equivalent torques can be used, by ignoring elastic deformations as well as interchannel coupling, yielding Eq. (62), for variable thruster torque actuators (with $\lambda_1 = \lambda_2 = \lambda_3 = \lambda$ for simplicity),

$$\hat{\tau}_{EQ,\,i}(\xi) = 2\lambda^2 J_i(1 + \xi_i^2)^{-2}(1 - \xi_i^2 + 2\xi_i\xi_i^*)(\xi_i - \xi_i^*) \qquad (62)$$

where $i = 1, 2, 3$. Similar estimates can be derived for reaction wheel actuators. Also different time constants λ_i can be used for each rotation axis in the case of different actuator torque ratings, or highly asymmetric inertias.

Such estimates need not be precise, however, at the cost of the need for experimental tuning of the VSC gains, either $K = \text{diag}|k_i|$ in Eq. (53) (for additive VSC correction) or else in Eq. (56) (for multiplicative VSC correction).

A functional block diagram of such a simplified decoupled, multiplicatively corrected VSC retargeting control system is shown in Figure 1, designed around a model of a deformable vehicle commanded to undergo slewing maneuvers.

Example: VSC Retargeting Maneuver

Decoupled multiplicative VSC was used to execute a reorientation of the vehicle model of [4], [5], repeated below, from the initial attitude $\xi = (1, 0, 0)^T$ in Eq. (2)). The time constant was chosen to be $\lambda^{-1} = 10$ seconds, which should require rigid body torques of the order of 5 Newton meters. The structural parameters were as follows:

$$J = \text{diag}(800, 400, 600)\ \text{Kgm}^2$$

Five structural modes were assumed, with normalized unit mass $M = I_5$ and a common damping ratio of $\zeta = 0.05$, which yields

$$C = \text{diag}\ (0.19, 0.41, 0.58, 0.60, 0.79)$$

and

$$K = \text{diag}(3.61, 16.81, 33.64, 36.00, 62.41)$$

in consistent units, while the coupling matrix from the elastic structural model (Eq. (8) with $v = 0$ for no translation to the line-of-sight torque equation (7) (again with $v = 0$) was:

$$N = \begin{bmatrix} 10.0 & 0.5 & 0.1 & 1.0 & 0.0 \\ 0.5 & 2.0 & 10.9 & 0.5 & 0.0 \\ 0.2 & 0.0 & 0.8 & 0.5 & 0.5 \end{bmatrix}$$

No structural control was presumed, so that slewing maneuver-induced structural deformations such as that shown by Fig. 2 for the first modeled elastic modes occurred. Nevertheless, the commanded attitude variable ξ_1 decayed exponentially to its zero target value, as shown in Figure 3. The corresponding actual (simulated) angular velocity profile is shown in Figure 4, wherein it is seen that the sliding mode was attained in less than 5 seconds. Finally, the corresponding VSC-corrected torque profile is shown in Figure 5, where the switching activity predicted by Eq. (57) is observed. The chosen VSC gain was $k_i = 1.3$.

DISCUSSION

It has been shown how the choice of Cayley-Rodrigues attitude parameters as kinematic variables removes the singularity occurring in the inverse dynamics transformations obtained by feedback linearization with quaternions. It has also been shown that such global linearization and decoupling is still valid for line-of-sight command following in the presence of uncontrolled but damped structural deformations; but require the availability of full state feedback).

Due to the difficulty of precise real time generation of such deformation-corrected computed slew torques, the alternative of variable structure control implementation thereof was imposed: it was shown that estimates of the computed torques can be corrected either additively or multiplicatively by switching the direction of the applied torques in response to changes in the signature pattern of the detected errors between measured and ideal line-of-sight slew rates, for accurate tracking in the presence of torque profile computation errors.

A simulation example was given of variable structure-implemented simplified torque profiles, for accurate commanded exponentially stable attitude maneuvers despite uncontrolled but stable structural deformations.

ACKNOWLEDGMENTS

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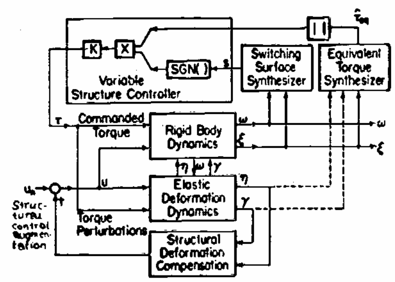

Figure 1

[illegible]

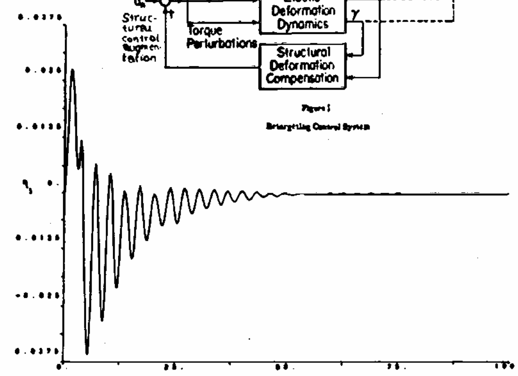

Figure 2

[illegible] Deformation Mode 1 with VSC

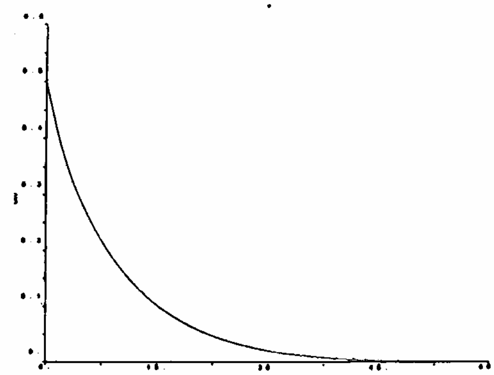

Figure 3

Attitude Parameter ξ_1 with VSC

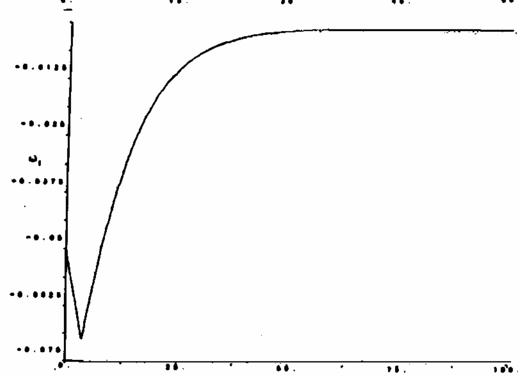

Figure 4

Angular Velocity ω_1 with VSC

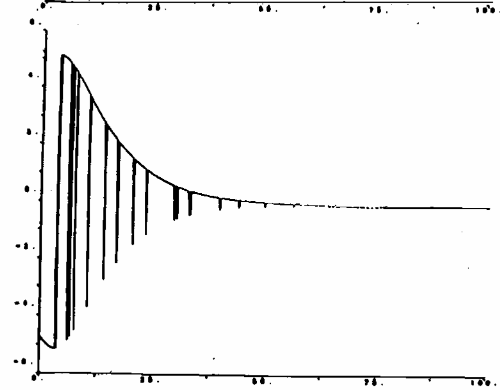

Figure 5

VSC Torque τ_1