ANALOG SIGNAL ENCODING IN DELTA MODULATION CIRCUITS USING THE THEORY OF VARIABLE STRUCTURE SYSTEMS.

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Abstract

In this article the theory of Variable Structure Systems (VSS), and their associated Sliding Regimes, is proposed as a means of dealing with the problem of analog signal tracking in Delta Modulation Circuits. Classical Delta Modulation schemes (i.e., analog, continuous-time encoding) are treated from this general viewpoint. The results are easiliy extendable to modern analog signal encoding.(i.e, digital, discrete-time)

1. INTRODUCTION

Sliding mode control of systems, on discontinuity surfaces of the state space, has been the subject of sustained research during the last 30 years, mainly by Soviet and East European researchers (Utkin [1],[2] Itkis [3]). Recently, the technique has found renewed interest in the United States in areas such as Robotics (Slotine and Asada [4]), Aerospace problems (Sira-Ramirez and Dwyer [5]) and Power Electronics (Sira-Ramirez [6]).

Sliding mode is the most important controlled behavior of Variable Structure Systems (VSS), taking place about a pre-specified switching manifold. The discontinuous control actions are geared to obtain state trajectories which converge towards the sliding manifold and are forced to adopt such surface as an integral manifold. The essential feature is the robustness of the controlled motions with respect to systems parameters and external perturbations, while special desirable qualitative features (such as stability) can be imposed on the constrained dynamics. The constrained motions are achieved thanks to switchings, among two different feedback laws, triggered by incipient surface overshoot. For a thorough survey about the theoretical and practical achievements of the discipline the reader is referred to a recent survey article by Utkin [7].

Section 2 of this article deals with a reapproachment to classical analog signal encoding in Delta Modulation (DM) circuits (Steele [8]) via Variable Structure Systems theory and their associated sliding modes. On-line analog signal tracking is viewed as a problem of inducing a sliding regime on a time-varying discontinuity surface defined in the extended state space of the circuit. Such surface is directly defined in terms of the codification error signal. The existence conditions for a sliding regime, and the corresponding concept of ideal (average) sliding dynamics, naturally rederive and explain the overload conditions, the overload frequency dependent characteristics, the idling behavior and step responses in a number of practical encoder arrangements including Linear Delta Modulation (LDM), Double Integration Delta Modulation (DIDM), Exponential Delta Modulation (EDM), Delta-Sigma Modulation (DSM), and the Sylabically Companded (i.e., adaptive) Delta Modulation schemes. Section 3 contains the conclusions and suggestions for further research.

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2. SLIDING REGIMES IN DELTA MODULATION CIRCUITS

2.1 Generalities

Linear and adaptive DM schemes have been developed since the end of World War II. A body of general theory was lacking in the existing literature about Delta Modulation. It is our hope to demonstrate that the theory of Variable Structure Systems fills this gap.(See [8],[9]-[12]).

2.2 A Variable Structure Systems Model for Continuous-Time (Classical) Linear Delta Modulation (LDM).

The basic scheme of LDM encoding and decoding circuits is shown in Figure 2.1. In this circuit, the signal to be encoded for transmission, $x(t)$, is continuously compared with the state of the local decoder (integrator) to generate an error signal $e(t)$. The quantizer circuit generates a piecewise constant signal which is sampled during a small sampling interval τ with sampling period T. At the remote decoder, the resulting transmitted signal $L(t)$ is integrated and filtered to recover $x(t)$. The dynamical equations specifying the behavior of the modultation process are :

$$e(t) = x(t) - y(t) = x(t) - \int L(t)dt \qquad (2.1)$$

$$L(t) = \begin{cases} V \text{ sign } e(t) & \text{for } kT \leq t < kT + \tau \\ 0 & \text{for } kT + \tau \leq t < (k+1)T \end{cases} \qquad (2.2)$$

The transmitted signal $L(t)$ is thus a sequence of regularly spaced pulses of duration τ, frequency $f = 1/T$, and amplitude of $\pm V$ according to the sign of the error signal at the moment of sampling. The signal $y(t)$ is thus constituted by the sum of step functions of varying polarity, regularly delayed over the real line, which approximates the encoding signal $x(t)$ within a quantization (granularity) step of value $\pm V\tau =: \pm \gamma$ ($= \Delta x$).

We may recast the tracking problem associated with LDM in terms of a sliding mode creation problem under certain high frequency sampling idealizations. The binary transmission aspects will be hence overlooked and deemed as independent of the tracking aspects. It is assumed that suitable technological means are available to transduce the high frequency outputs of the sliding-mode tracking process into coded pulses suitable for transmission.

Assumptions 2.1: The basic idealizations needed for a Variable Structure Control (VSC) model of the modulation process are summarized below :

1)Pulse duration τ of sampling process is infinitely small.
2)Sampling frequency $f := 1/T$ is infinitely large

3) $\lim_{T\to 0,\ \gamma\to 0} \gamma/T = \lim_{f\to\infty,\ \gamma\to 0} \gamma f = V$

Using these assumptions the basic ideal Variable Structure Control (VSC) model for LDM is given by :

System Dynamics :

$$dy/dt = u \ ; \quad u \in \{-V, V\} \qquad (2.3)$$

Sliding Line :

$$S = \{ (y,t) \in R^2 : \ s(y,t) = e(t) = x(t) - y = 0 \} \qquad (2.4)$$

Existence Conditions:

$$\lim_{s\to +0} ds/dt = dx/dt - u^+ = dx/dt - V < 0 \ ;$$
$$\lim_{s\to -0} ds/dt = dx/dt - u^- = dx/dt + V > 0$$

The Variable Structure control law :

$$u = L(t) = V \text{ sign } s = V \text{ sign } e(t) \qquad (2.5)$$

creates a sliding motion on $s(y,t) = 0$ provided the sliding mode existence conditions, given by (A.4), are satisfied.

Theorem 2.2 Analog signal tracking is possible via a sliding regime creation if and only if for all t :

$$-V < \min_t dx/dt \le dx/dt \le \max_t dx/dt < +V \qquad (2.6)$$

whose violation is the well known **slope overload condition** [3].

Proof : Immediate from above, and the results in the Appendix D

Under ideal sliding conditions, $s = 0$, $ds/dt = 0$, one obtains :

$$x(t) = y(t) \quad ; \quad dx/dt = dy/dt \qquad (2.7)$$

i.e., ideal sliding is equivalent to perfect tracking of the encoding signal.

Summarizing, the basic immediate consequences of a VSC approach to LDM are the following :

1) Signal tracking is possible if and only if the slope overload condition is violated.

2) For x(t) identically zero the existence conditions are trivially satisfied and the tracking of "silence" is always accomplished in a chattering motion known as the **idling response**. If $x(t) = K =$ *constant*, the existence conditions are again trivially satisfied and a sliding motion is reached, from rest conditions, in finite time $t = V/K$. After this, the modulator exhibits again the idling response.

3) A step input violates the existence conditions (slope overload is trivially verified) in the time interval before the idling behavior starts.

4) For sinusoidal inputs $x(t) = E_s \sin 2\pi f_s t$, the existence conditions (2.6) translates into : $2\pi f_s E_s < V \ (= \gamma f)$. This is the well known **overload frequency characteristic** for sinusoidal inputs in LDM ([8] pp. 10).

5) The sliding mode existence conditions demand knowledge of the time derivative of the encoding input signal which can only be known on line. This clearly points the need of an adaptive scheme which guarantees the use of an appropiate on-line time-varying quantization limits, V(t), -V(t), computed on the basis of the derivative of the input signal x(t).

Example 2.3 Figure 2.2 shows a computer simulated LDM response to a sinusoidal input of frequency 2.25 KHz and amplitude $E_s = 0.1$ mV . The chosen quatization limit is V = 2 Volts. The initial state of the local decoder was set to -0.1mV.

2.3 Sliding Mode Aspects of Double Integration Delta Modulation (DIDM)

In order to achieve a faster tracking response of the encoding signal on the part of local feedback decoder a second integrator is added to the basic LDM scheme. The co-dec arrangement is shown in Figure 2.3. The sampling process is still assumed to satisfy the idealizations presented in the previous section.

The dynamical system describing a DIDM scheme is simply :

$$dy_1/dt = y_2 \ ; \ dy_2/dt = u \ ; \ u \in \{-V, V\} \qquad (2.8)$$

In the doble integration delta modulation case it is easy to show that a sliding motion does not exist on a line representing zero tracking error. Indeed, assume the sliding line is given by:

$$S_1 = \{ (y_1,y_2) : \ s_1 = e_1(t) = x(t) - y_1 = 0 \} \qquad (2.9)$$

In this case the system is not relative degree 1 (See appendix) and a sliding regime does not exist. Hence, the DIDM arrangement is definitely not based on ideal sliding-mode tracking of the encoding signal.

Example 2.4 Figures 2.4a-2.4b show computer simulated responses, of a DILDM, to a sinusoidal input of frequency 2.25 KHz, amplitude $E_s = 0.1$ mV and quatization limit V = 2 Volts. A switching line of the form (2.9) was used and the initial state of the local decoder was set respectively to -0.02 mV. and -0.06mV/sec. The figures verify that a tracking sliding regime does not exist in this case but merely a limit-cycle type of oscillatory response of the local decoder output around the encoding signal. The encoding error is smaller as the initial condition for the output integrator of the local decoder is chosen close to the initial value of the encoding signal.

In order to provide a sliding-mode based explanation of DIDM , consider instead a sliding line, defined in R^2, by :

$$S_2 = \{(y_1,y_2) : s_2 = e_2(t) = c[x(t)-y_1] - y_2 = 0, \ c > 0 \} \qquad (2.10)$$

In this case the resulting system with output s_2 is relative degree one. Hence, a sliding mode may exists on S_2 by appropriate switching action. This modification of the DIDM arrangement is widely known as Predictive DIDM. [8]

Existence Conditions:

$$\lim_{s\to +0} ds_2/dt = c\,[dx(t)/dt - y_2] - V < 0 \ ;$$
$$\lim_{s\to -0} ds_2/dt = c\,[dx(t)/dt - y_2] + V > 0$$

i.e., the VSC law :

$$u = V \text{ sign } s_2 = V \text{ sign } e_2(t) \qquad (2.11)$$

creates a sliding motion on $s_2 = 0$ provided the sliding mode existence conditions, given by (A.4), are satisfied.

Theorem 2.5 A sliding regime exists on S_2 if and only if the derivative tracking condition is verified:

$$-V/c < [dx(t)/dt - y_2] < V/c \qquad (2.12)$$

Proof : Immediate from above, and the results in the Appendix □

Under ideal sliding conditions $s_2 = 0$, $ds_2/dt = 0$ one obtains :

$$dy_1/dt = -c[y_1-x(t)] \ , \ dy_2/dt = -c[y_2-dx/dt] \qquad (2.13)$$

i.e., ideally y_1 and y_2 asymptotically track the enconding signal x(t) and its time derivative dx/dt, respectively.

2.4 Sliding Mode Aspects of Exponential Delta Modulation (EDM)

In an EDM arrangement the perfect integrator used in the local decoder of the LDM scheme is replaced by a "leaky" integrator represented by an RC circuit. The encoder output is obtained through a sample and hold circuit usually realized by a D-type Flip-Flop circuit ([8], pp. 57). For this reason, the previous idealizations about the nature of the sampling process is no longer needed. Instead, an infinite frequency clock signal is assumed to drive the Flip-Flop circuit. The basic encoder circuit is shown in Figure 2.5 .

System Dynamics :

$$dy/dt = -(1/RC)\ y + (1/RC)\ u \ ; \quad u \in \{-V,V\} \qquad (2.14)$$

Sliding line :

$$S = \{\ y :\ s = e = x(t) - y = 0\ \} \qquad (2.15)$$

Existence Conditions :

$$\lim_{s \to +0} ds/dt = (\ dx/dt + (1/RC)\ y - (1/RC)\ V\ ) < 0$$

$$\lim_{s \to -0} ds/dt = (\ dx/dt + (1/RC)\ y + (1/RC)\ V\ ) > 0$$

$$(2.16)$$

The variable structure control law $u = V$ sign e creates a sliding regime on S if and only if the following existence condition is satisfied :

$$-V < RC\ dx/dt + x < V. \qquad (2.17)$$

This condition is an immediate consequence of the fact that the ideal sliding conditions are represented by $x(t) = y(t)$ and the equivalent control value is given by $u_{EQ} = RC\ dx/dt + y = RC\ dx/dt + x$.

For sinusoidal inputs $x(t) = E \sin 2\pi ft$ the existence condition (2.17) leads to :

$$-V < \min_t \{\ 2\pi f\ RC\ E \cos (2\pi ft) + E \sin (2\pi ft)\ \} < RC\ dx/dt + x < \max_t \{\ 2\pi f\ RC\ E \cos (2\pi ft) + E \sin (2\pi ft)\ \} < +V$$

In order to find the extreme values of the composite signal, RC dx/dt + x, above, let $m(t) = 2\pi f\ RC\ E \cos (2\pi ft) + E \sin (2\pi ft)$, then taking the time derivative of m(t), equating to zero, solving for the time variable and substituting the result in the expression for m(t), the maximum amplitude of the signal RC dx/dt + x is given by $E \sqrt{1 + (2\pi fRC)^2}$. The overload condition takes the well known form :

$$E\ [\ 1 + (2\pi fRC)^2\ ]^{1/2} < V \qquad (2.18)$$

In terms of the transfer function of the RC circuit, H(s), condition (2.18) is equivalent to : $E < V \mid H(j2\pi f) \mid$ with the complex variable s substituted by $jw = j2\pi f$ ([8], pp. 62).

Example 2.6 Figure 2.6a and 2.6b show, respectively, a computer simulated EDM response to a sinusoidal input of frequency 2.25 KHz and amplitude $E_s = 0.1$ mV, and its corresponding codification error signal. With R = 100 KΩ and C = 10 µF, the quatization limit V may be chosen, according to (2.18), as V = 2 Volts. The initial state of the local decoder was set to -0.2 mV.

2.5 Sliding Mode Aspects of Delta-Sigma Modulation (DSM)

Classical DSM arose naturally from the inconvenience of on-line accurate measurement, or estimation thereof, of the time derivative of the encoding signal (this was possible to accomplish by using Delayed Linear Delta Modulation). DSM proposes to encode the time integral of the input signal i.e., an integrator is thus placed at the input of the LDM encoder (Fig. 2.1). In this manner, the overload conditions (2.6) are now given in terms of the original signal amplitude x(t), which is indeed on-line measurable, rather than its time derivative dx/dt. The input integrator and the local feedback integrator can be replaced by a single integrator preceeding the quantizer unit as shown in Figure 2.7. The remote decoder is thus substantially simplified.

Using the same idealizations developed earlier, the corresponding VSC model to be considered in this case is given by:

System Dynamics :

$$de/dt = x(t) - u\ , \quad u \in \{\ -V,\ V\ \} \qquad (2.19)$$

Sliding surface :

$$S(e,t) = \{\ s(e,t) = e(t) = 0\ \} \qquad (2.20)$$

The VSC law :

$$u = L(t) = V \text{ sign } s(e,t) = V \text{ sign } e(t) \qquad (2.21)$$

creates a sliding motion on $s(e,t) = 0$, provided the sliding mode existence conditions, given by (A.4), are satisfied.

Theorem 2.7 A sliding regime exists on S if and only if the existence conditions:

$$\lim_{s \to +0} ds/dt = \lim_{e \to +0} de/dt = x(t) - V < 0 \ ;$$
$$\lim_{s \to -0} ds/dt = \lim_{e \to -0} de/dt = x(t) + V > 0$$

are satisfied., i.e., for all t :

$$-V < \min_t x(t) < x(t) < \max_t x(t) < V \qquad (2.22)$$

or briefly, for all t , $|x(t)| < V$.

Proof : Immediate from above, and the results in the Appendix □

The slope overload condition (2.6) is thus transformed into an **amplitude overload condition.**

For sinusoidal inputs, $x(t) = E_s \sin 2\pi f_s t$, the overload condition (2.22) is frequency independent and of the form $E_s < V$ ($\neq \gamma f$) (See [8] pp. 18).

The ideal sliding conditions, $s = 0$, $ds/dt = 0$, are represented by :

$$e = 0 \ , \quad u_{EQ}(t) = x(t). \qquad (2.23)$$

Equation (2.23) simply means that the ideally transmitted signal is just the input signal x(t).

The basic consequence of a sliding mode approach to DSM is the rederivation of the folowing known facts [8] :

1) Idling behavior of DSM, corresponding to zero input response, is due to trivial satisfaction of of sliding mode existence conditions (amplitude overload violation). The step response exhibits idle behavior if and only if the step amplitude is within quantizer limits.

2) Overload characteristics are independent of the frequency of the sinusoidal input and hence the DSM coding-decoding process is suitable for a wider range of applications.

Example 2.8 Figure 2.8 shows a computer simulated DSM codification error response to a sinusoidal input of frequency 2.25 KHz, amplitude E_s = 0.1 mV (also shown in the figure). The quatization limit, satisfying 2.25, is V = 2 Volts. The initial state of the local decoder was set to -0.1 mV.

2.6 Sylabically Companded Delta-Sigma Modulation (SCDSM)

SCDSM constitutes an adaptive scheme implemented on a DSM circuit. The local decoder is complemented with a nonlinear circuit which includes a decoder for the transmitted binary signal (just a low-pass-filter), a rectifier and an envelope detector folowed by a circuit that rises the resulting signal to a certain positive integral power n. A constant positive polarization signal, A, is subsequently added and the resulting signal is multiplied by the binary coded error signal. The resulting signal is compared with the analog input signal and the slope error is formed. Figure 2.9 shows the basic SCDSM encoder arrangement. (See [8])

In the DSM encoding the equivalent control, generating perfect tracking of the encoding signal integral, is simply given by :

$$u^{EQ} = x(t) \qquad (2.24)$$

Using the variable structure control law (A.11),

$$u = (\alpha + K\, env^{n} \mid x(t) \mid) \, V \, \mathrm{sign}\, e(t) \qquad (2.25)$$

the conditions for the existence of a sliding mode (A.4) are trivially satisfied and hence tracking of the encoding signal is possible through a sliding regime of the state of the circuit. The SCDSM arrangement is thus a consequence of the realization of the VSC law (2.25).

If instead of a SDM encoder a LDM scheme is used above, x(t) is to be substituted, in (2.24) and (2.25), by the time derivative of x(t). In the dicrete-time case with n = 0, the scheme is of utmost importance and it is intimately related to the celebrated Song Algorithm (See [9] and [8]).

4. CONCLUSIONS

It has been shown that Sliding regimes are relevant in the analysis and conceptual design of analog signal tracking devices such as those performing analog signal encoding for transmission over binary communication channels.

The necessary and sufficient conditions for the existence of a sliding regime determine the capabilities of the dynamic tracking arrangement. In Delta Modulation systems, the slope overload condition is shown to be directly related to well known sliding regime existence conditions. These, in turn, naturally point to the basic need for an adaptive modulation scheme. It was also shown that in Double Integration Delta Modulation the zero error condition, acting as sliding line, does not lead to a sliding regime tracking the input signal. The reason being a global violation of the relative degree condition. In this kind of analog signal modulation, it was shown that a tracking sliding regime indeed existed on the sliding line defined by the predictive method. Known results are also rederived for the Exponential Delta Modulation and Delta-Sigma modulation arrangements. In particular, the overload characteristics are shown to be frequency dependent for the first case and frequency independent for the second case. Finally, the Sylabically Companded schemes are naturally rederived using a general equivalent-control-based variable structure controller.

The discrete-time aspects of classical delta modulation, with finite sampling frequencies, directly lead to a digital realization of the encoding and decoding parts of the modulation process. A complete rederivation of known results and some new ones, obtained as suitable extensions of VSS theory to discrete-time systems, will be presented elsewhere.

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APPENDIX

Generalities about Sliding Regimes

Consider the linear, time invariant dynamical system

$$dy/dt = A\,y + bu \qquad (A.1)$$

with $y \in R^n$. The scalar control function u is assumed to take values on the discrete set $U := \{-V, V\}$. Discontinuous control actions are exercised on the system according to the switching logic :

$$u = V \text{ sign } s(y) \qquad (A.2)$$

where $s(y) = 0$ defines the switching surface:

$$S(y) = \{y \in R^n : s(y,t) = cy = 0\} \qquad (A.3)$$

with c an n-dimensional row vector.

Definition A.1 [1] A local **Sliding Regime** is said to exist on S(y) if and only if, the controlled motion (A.1),(A.2) is such that :

$$\lim_{s \to +0} ds/dt < 0 \quad \text{and} \quad \lim_{s \to -0} ds/dt > 0 \qquad (A.4)$$

The trajectories converge, locally, towards the discontinuity surface S(y) where they locally undergo a constraining chattering motion.

Lemma A.2 [14] If a sliding motion exists on S(y) then the following transversality condition is locally verified:

$$cb \neq 0 \qquad (A.5)$$

i.e., the system is relative degree one with respect to the output s(y).

Ideally, on $s(y) = 0$, the motions of the controlled system can be described as if influenced by a smooth feedback control function known as the equivalent control. Such a function is defined from the following invariance conditions [3]:

$$ds/dt = 0 \quad ; \quad s = 0 \qquad (A.6)$$

From (A.6), the corresponding **equivalent control**, here denoted by $u_{EQ}(x,t)$, is given by :

$$u_{EQ}(y,t) = -(cb)^{-1}[cAy] \qquad (A.7)$$

The ideal sliding dynamics is obtained from use of (A.7) on (A.1), with the formal substitution of u by $u_{EQ}(x,t)$. This is the basis of the **Equivalent Control Mebtod** [1]:

$$dy/dt = [I - bc/cb]Ay \qquad (A.8)$$

Theorem A.3 A sliding motion locally exists on S(y) if and only if locally along $s(y) = 0$, the following condition is satisfied :

$$-V < u_{EQ}(y) < V \qquad (A.9)$$

Proof: (See [13,14]).

It is easy to see from (A.9) and (A.7) that the sliding motion is global whenever c is a left eigenvector of A (i.e., cA is proportional to c). It follows from (A.5) and the PBH test that this holds iff (A,b) is controllable.

Finally, we present prescriptions for a family of variable structure control laws which locally guarantee the existence of a sliding motion on the discontinuity surface S(y).

Proposition A.4 If a local sliding regime exists on S(y) and $u_{EQ}(y)$ is not identically zero on an open set of R^n, then there exists a constant $K > 1$ such that the switching logic $u = -K\,|u_{EQ}(y)|$ sign s(y) creates a local sliding regime on S(y).

Proof: Obvious from the existence condition (A.4). □

Since it is entirely possible that $u_{EQ}(x,t) \equiv 0$, the following prescription, of importance in adaptive delta modulation cases, guard against this possibility. The proof of this proposition is left to the reader.

Let env z(x(t)) denote the function specified by :

$$env\, z(x(t)) = \max\{ \sup_{\tau \leq t} |z(x(\tau))|, |z(x(t))| \} \qquad (A.11)$$

By $env^m z(x(t))$ is denoted the m-th power of the values of env z(x(t))

Proposition A.5 If a local sliding regime exists on S(y,t) then there exists constants $K > 1$, $\alpha > 0$ and an integer $m \geq 1$ such that the switching logic :

$$u = (\alpha + K\; env^m |u_{EQ}(y)|) \text{ sign } s(y) \qquad (A.12)$$

creates a local sliding regime on S(y).

Proof Immediate from the existence condition (A.9).

FIGURES

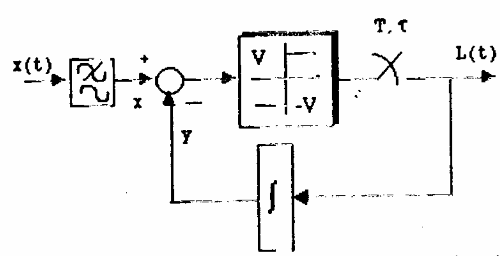

Figure 2.1 Linear Delta Modulation encoder circuit

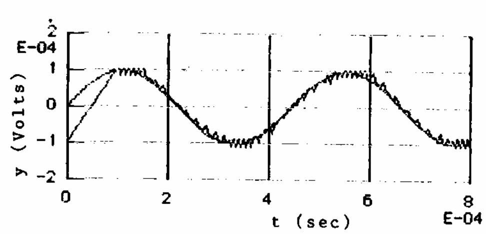

Figure 2.2 Typical sinusoidal output response of a VSC-based LDM circuit.

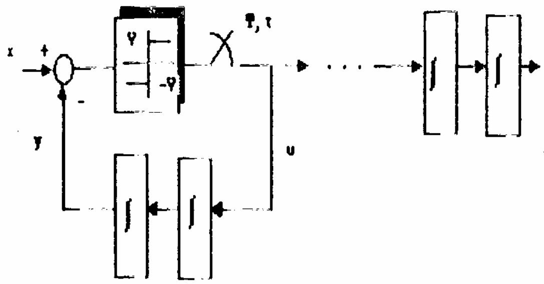

Figure 2.3 Double Integration Delta Modulation encoder and decoder circuits

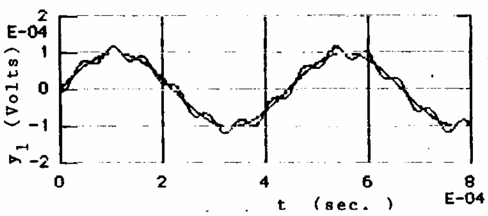

Figure 2.4a Typical output response of DIDM circuit

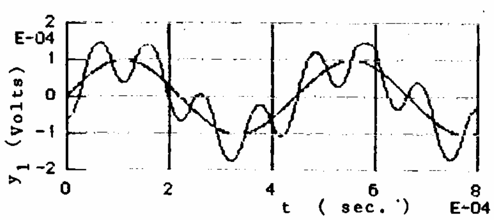

Figure 2.4b Effect of initial conditions on a typical output response of a DIDM circuit

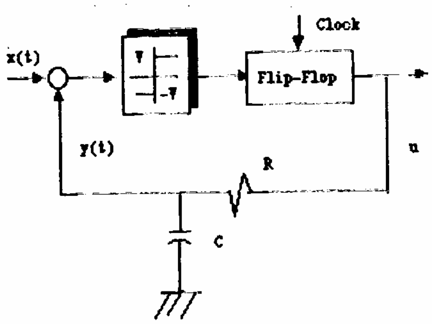

Figure 2.5 Exponential Delta Modulation encoder circuit

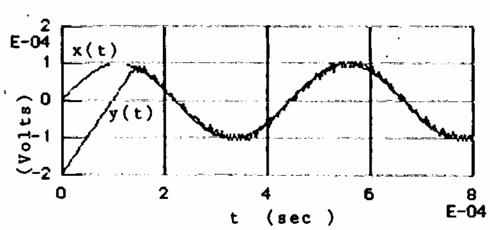

Figure 2.6a Typical VSC-based EDM output response.

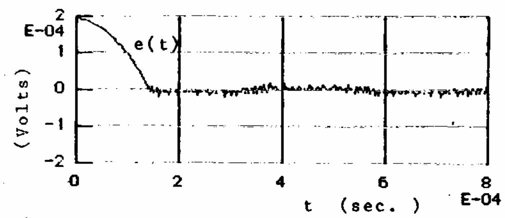

Figure 2.6b Typical VSC-based EDM codification error response.

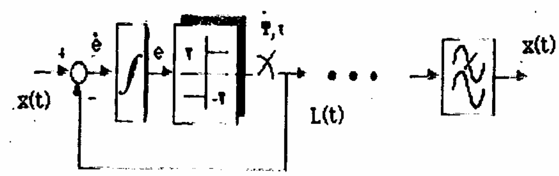

Figure 2.7 Delta-Sigma Modulator and Demodulator Circuits

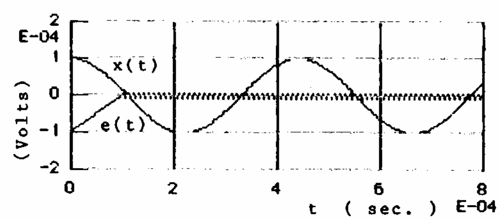

Figure 2.8 Typical VSC-based DSM codification error response

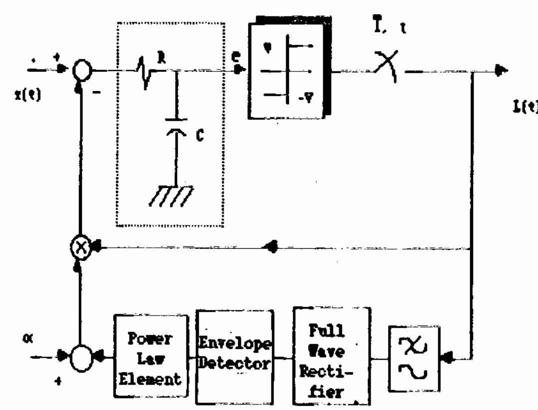

Figure 2.9 Sylabically Companded Delta-Sigma Modulation