PULSE-WIDTH-MODULATION CONTROLLER DESIGN FOR NONLINEAR DELAY DIFFERENTIAL SYSTEMS

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ABSTRACT

In this article a design method is proposed for the specification of Pulse-Width-Modulation (PWM) feedback controllers of Nonlinear Delay Systems. The design method is based on an average model of the PWM controlled system, derived by an infinte sampling frequency assumption which captures the essential qualitative stability properties of the actual controlled system. Some examples are provided.

1. INTRODUCTION

Early contributions to the study of Pulse-width-modulation (PWM) controllers are those of Tsypkin [1]. Further developments were contributed, later on, by Skoog and Blankenship [2], La Cava *et al.* [3] and many others. In all these works, emphasis was primarily placed on the discrete-time aspects of such controllers

A different design approach [4],[5] has been proposed which uses the geometrical properties of average PWM controlled responses (obtained by an infinite frequency sampling assumption). The results, aside from allowing exact analysis of average responses, found an ideal equivalence with Variable Structure controlled trajectories undergoing sliding motions on the integral manifold of the average PWM system. The class of PWM systems treated in [4]-[5] corresponded to single ON-OFF controlled switch (the control variable takes values in the discrete set {0,1}).

A related class of switch controlled systems is represented by those in which the control variable takes values on the set {-1,0,+1} (See [2]). Typically, gas reaction jets controlling reorientation, or detumbling, maneuvers in artificial satellites are expressible as systems of this class. Some torque actuators, used for control of joint poisitions in robotic manipulators, are also of this type. The controlled switch, in this case, is addressed as an ON-OFF-ON switch. Extending the results of [5] to nonlinear delay differential systems governed by PWM controllers of the ON-OFF-ON type, we propose to carry out the PWM controller specification on the basis of the average model of the PWM controlled system. This model is obtained by replacing the PWM regulator by means of a nonlinear memoryless saturation controller while leaving the system delays unaltered. Generally, the proposed design approach allows to totally circumvent the technical problems associated with the unbounded character of the PWM operator and the need for introducing low pass multipliers cascading the PWM controller ([2]).

In Section 2 the average PWM feedback controlled model is derived for general nonlinear dynamical systems described by delay differential equations. In Section 3 a satellite attitude control problem is presented. Section 4 contains the conclusions and suggestions for futher work.

2. DEFINITIONS AND BASIC RESULTS

2.1 Definition of Average PWM controlled model.

We shall distinguish three kinds of delays affecting the behavior of nonlinear systems. These are: plant delays , sensor delays and actuator delays

Plant delays case

Let τ_p be a strictly positive constant. Consider a nonlinear delay differential system controlled by a PWM scheme based on the error signal. The system is assumed to be globally described in R^n, as :

$$\begin{aligned} dx(t)/dt &= f[x(t)] + \phi[x(t-\tau_p)] + g[x(t)]u \\ y &= h(x) \\ e &= y_d(t) - y \\ u &= M\ PWM_\tau[e(t_k)] \end{aligned} \tag{2.1}$$

with f, g and ϕ globally smooth vector fields. The function h is a smooth scalar output function. The control input u is a discontinuous scalar control function obtained as the output of a Pulse-Width-Modulator excited by the error signal e. The sampling process is assumed to take place at regularly spaced time intervals of fixed duration T, i.e., $t_{k+1} = t_k + T$. M is a positive constant gain representing the maximum allowable input strength.

The PWM control operator, $PWM_\tau[e]$, is characterized by (See [2]):

$$PWM_\tau[e(t_k)] = \begin{cases} \text{sign}[e(t_k)] & \text{for } t_k \le t < t_k + \tau[e(t_k)]T \\ 0 & \text{elsewhere.} \end{cases} \tag{2.2}$$

where $\tau[e(t_k)]$ is known as the duty ratio function defined by :

$$\tau[e(t_k)] = \begin{cases} \beta|e(t_k)| & \text{for } |e(t_k)| \le 1/\beta \\ 1 & \text{for } |e(t_k)| > 1/\beta \end{cases} \tag{2.3}$$

with β being a positive constant. Notice that :

$$\tau[e(t_k)]\text{sign}[e(t_k)] = \text{sat}[e(t_k),\beta] := \begin{cases} \beta e(t_k) & \text{for } |e(t_k)| \le 1/\beta \\ \text{sign}[e(t_k)] & \text{for } |e(t_k)| > 1/\beta \end{cases} \tag{2.4}$$

Proposition 2.1 As the sampling frequency $F := 1/T$ tends to infinity, the description of the nonlinear controlled system (2.1) coincides with :

$$\begin{aligned} dx(t)/dt &= f[x(t)] + \phi[x(t-\tau_p)] + g[x(t)]v \\ y &= h(x) \\ e &= y_d(t) - y \\ v &= M\ \text{sat}(e,\beta) \end{aligned} \tag{2.5}$$

System (2.5) will be henceforth addressed as the average PWM controlled system.

Proof. Let $I_\phi(t)$ dentote the vector-valued function $\int_0^t \phi(x(\sigma))d\sigma$, with $\phi(x)$ being a given smooth vector field on R^n defining the flow of $dx/dt = \phi(x)$. Notice that the limit: $\lim_{T\to 0} [I_\phi(t+\alpha T)-I_\phi(t)]/T = \alpha\, d\, I_\phi(t)/dt = \alpha\, \phi(x(t))$. Similarly, consider $\phi[x(t-\tau)]$ for any fixed positive τ. We let $I_{\phi[x(t-\tau)]}(t) = \int_0^t \phi[x(\sigma-\tau)]d\sigma$. It follows that $\lim_{T\to 0} [I_{\phi[x(t-\tau)]}(t+T)-I_{\phi[x(t-\tau)]}(t)]/T = d\, I_{\phi[x(t-\tau)]}(t)/dt = \phi[x(t-\tau)]$. The sampled state-response of the nonlinear feedback system (2.1) can be written, in terms of an equivalent integral equation, as :

$$x(t_k+T) = x(t_k) + I_f(t_k+T) - I_f(t_k) + [I_{\phi[x(t-\tau_p)]}(t_k+T) - I_{\phi[x(t-\tau_p)]}(t_k)] + M\,[\, I_g(t_k+\tau[e(t_k)]T) - I_g(t_k)]\,\mathrm{sign}\,[e(t_k)]$$

Therefore, according to the above expression, the limit :

$$\lim_{T\to 0,\, t_k\to t} T^{-1}[x(t_k+T) - x(t_k)] =: dx(t)/dt$$
$$= \lim_{T\to 0,\, t_k\to t} T^{-1}\{ [I_f(t_k+T)-I_f(t_k)] + [I_{\phi[x(t-\tau_p)]}(t_k+T)- I_{\phi[x(t-\tau_p)]}(t_k)] + M\,(I_g[t_k+ \tau[e(t_k)]T] - I_g(t_k)]\,\mathrm{sign}\,[e(t_k)]\,)\,\}$$
$$= f[x(t)] + \phi[x(t-\tau_p)] + g[x(t)]\, M\, \tau[e(t)]\,\mathrm{sign}\,[e(t)]$$
$$= f[x(t)] + \phi[x(t-\tau_p)] + g[x(t)]\, M\, \mathrm{sat}[e(t),\beta] = f(x) + \phi[x(t-\tau_p)] + g(x)\, v \qquad \square$$

Remark The behavior of the infinite frequency sampled delay differential system is then described by a nonlinear delay differential system with a continuous piece-wise linear control v, generated as the output of a memoryless nonlinear function of the saturation type. The saturation function, in turn, is excited by the error signal e. In other words, to evaluate the smooth average behavior of the actual PWM controlled system, the PWM controller is simply substituted by a nonlinear memoryless saturating controller independently of the plant delays.

The fundamental qualitative stability characteristics of the actual nonlinear PWM controlled system (2.1) are entirely captured by the average model (2.5). In order to show the validity of this assertion, it will be first analyzed the nature of the discrepancies among the open loop state trajectory responses of both system models, (2.1) and (2.5), when subject to the same arbitrary bounded measurable input signal. Later on, the close loop behavior will also be treated.

Let $e^*(t_k) = x(t_k) - z(t_k)$ be the discrepancy among the state x of the PWM controlled system (2.1) and the state z of the average system (2.5) at the sampling instant t_k. It will be assumed that the vector fields $f(x)$ and ϕ are globally Lipschitz and that the vector field $g(x)$ is globally bounded on R^n. i.e., there exist constants L_1, L_2 and M such that $\|f[x(t)]-f[(z(t)]\| \le L_1 \|x(t)-z(t)\|$, $\|\phi[x(t-\tau_p)]-\phi[z(t-\tau_p)]\| \le L_2\| x(t) - z(t) \|$ and $\|g(x)\| \le G$ for all x and z in R^n.

Theorem 2.2 Under the above assumptions on the vector fields $f(x)$, $\phi(x)$ and $g(x)$, given a small positive constant ε there exists, for any arbitrary finite time interval, $[0,NT]$, a sampling frequency $F_0 = 1/T_0$ such that, if the inital states discrepancy, $e^*(t_0) = x(t_0) - z(t_0)$, of systems (2.1) and (2.5) is norm bounded by δ, then the discrepancy $e^*(t_0+NT)$ of the corresponding state responses, due to the same arbitrary bounded measurable input $r(t)$ with $|r(t)| \le K$ for $t \in [t_0,t_0 +NT]$, is norm bounded by $(1+\varepsilon)\delta$, for any sampling frequency $F > F_0$.

Proof According to (2.1) and (2.5) one has :

$$x(t_k+T) = x(t_k) + \int_{t_k}^{t_k+T} f(x(\sigma))d\sigma + \int_{t_k}^{t_k+T} \phi[x(\sigma-\tau_p)]d\sigma + M \int_{t_k}^{t_k+\tau[r(t_k)]T} g(x(\sigma)) \text{ sign } r(t_k)\, d\sigma$$

$$z(t_k+T) = z(t_k) + \int_{t_k}^{t_k+T} f(z(\sigma))d\sigma + \int_{t_k}^{t_k+T} \phi[z(\sigma-\tau_p)]d\sigma + M \int_{t_k}^{t_k+T} g(z(\sigma))\text{sat}[r(\sigma),\beta]d\sigma$$

subtracting these expressions, one obtains :

$$e^*((t_k+T) = e^*(t_k) + \int_{t_k}^{t_k+T} \{f[x(\sigma)]-f[z(\sigma)]\}d\sigma + \int_{t_k}^{t_k+T}\{ \phi[x(\sigma-\tau_p)]- \phi[z(\sigma-t_p)]\}\, d\sigma + M \{ \int_{t_k}^{t_k+\tau[r(t_k)]T} g(x(\sigma)) \text{ sign } r(t_k)\, d\sigma - \int_{t_k}^{t_k+T} g(z(\sigma))\text{sat}[r(\sigma),\ \beta\ ]d\sigma \}$$

Hence:

$$\| e^*(t_k+T) \| \le \| e^*(t_k)\| + L_1 \int_{t_k}^{t_k+T} \| e^*(\sigma) \|\, d\sigma + L_2 \int_{t_k}^{t_k+T} \| e^*(\sigma) \|\, d\sigma + M \| \int_{t_k}^{t_k+\tau[r(t_k)]T} g(x(\sigma)) \text{ sign } r(t_k)\, d\sigma - \int_{t_k}^{t_k+T} g(z(\sigma))\text{sat}[r(\sigma),\beta]d\sigma \|$$

$$\le \| e^*(t_k)\| + (L_1+L_2) \int_{t_k}^{t_k+T} \| e^*(\sigma) \|\, d\sigma + M \{ \int_{t_k}^{t_k+T} \| g(x(\sigma)) \|d\sigma + \int_{t_k}^{t_k+T} \beta \,\| g(z(\sigma)) \|\, |r(\sigma)|\, d\sigma \}$$

$$\le \| e^*(t_k)\| + (L_1 + L_2) \int_{t_k}^{t_k+T} \| e^*(\sigma) \|\, d\sigma + M\, (1 + \beta\, K\,)\, T\, G$$

By the Gronwall-Bellman lemma one has, after letting $L_3 = L_1 + L_2$, :

$$\|e^*(t_k+T)\| \le [\, \| e^*(t_k) \| + M\, (1+\beta K)T\, G\,]\, \exp\, (\, L_3T\,)$$

Using iteratively the above inequality for $k=0,1,\ldots,N$, the following loose estimate for the norm of the discrepancy $e^*(t_0 + NT)$ is easily obtained,

$$\| e^*(t_0 + NT) \| \le [\, M(1+\beta K)NG\, T + \| e^*(t_0) \|\,]\, \exp\, (\, L_3NT\,)$$

i.e., the discrepancy, at an arbitrary finite time $t_0 + NT$, among the actual open loop PWM state response and that of the average system remains bounded by an amount determined by the initial states discrepancy, the system constants and the sampling interval T. The following trascendental equation, is easily obtained from the previous equation:

$$(1+\varepsilon)\,\delta \exp\, \{\, -L_3N\, T\, \} = [\, M(1+\beta K)NG\, T + \delta]$$

It is easy to see that because the right hand side term of this equation monotonically increases with T from the value δ at $T = 0$, while the left hand side term monotonically decreases with T from the value $(1+\varepsilon)\delta > \delta$ at $T = 0$, it has a unique solution for some $T = T_0 > 0$. Hence, given an initial discrepancy bound, $\| e^*(t_0) \| \le \delta$, and a small positive constant ε, a sampling frequency $F_0 = 1/T_0$ exists for which a preassigned error response bound Δ can be obtained at any finite later time $t_0 + NT$, $\| e^*(t_0+NT) \| \le \Delta$, with Δ being of the form $(1+\varepsilon)\delta$. It follows that for any sampling frequency $F > F_0$ (i.e., $T < T_0$) a states discrepancy strictly bounded by $(1+\epsilon)\delta$ is obtained among the open loop state responses at time $t_0 + NT$. □

The qualitative stability characteristics of the delay differential PWM controlled system (2.1) are captured by its smooth infinite-frequency sampling average model (2.5). In particular, a necessary condition for asymptotic stability of the actual PWM system (2.1) to the zero error manifold, is the asymptotic stability of the average model to such surface. It is also obvious that if the response of the average model (2.5) is asymptotically stable to $e = 0$, the actual PWM controlled response can be made to follow, arbitrarily close, the response of the average model by suitably increasing of the sampling frequency in the PWM controller. Thus, modulo sufficiently large -but finite- sampling frequency, the following theorem holds true.

Theorem 2.3 The closed loop PWM controlled system (2.1) is asymptotically stable to the $e = 0$ manifold if and only if the average PWM system (2.5) is asymptotically stable to such manifold.

Proof The average model of the PWM controller establishes three disctintive regions in the state space of the system. These are : the saturation regions $S_{+1} = \{ x : e = -h(x) > 1/\beta$ and $S_{-1} = \{ x : e = - h(x) < -1/\beta \}$ and the linearity, or boundary layer region $S_\beta = \{ x : -1/\beta \le e = -h(x) \le 1/\beta \}$.

In the saturation regions the actual delay differential PWM controlled system (2.1) totally coincides with the average model (2.5). Hence, under exactly the same functional and static initial conditions defined, respectively in $-\tau_p \le t < 0$ and $t = 0$, the actual PWM system trajectories and those of the average PWM controlled system entirely coincide. By virtue of a straightforward extension of the continuity of solutions in the initial conditions, it follows that for arbitrarily small discrepancies of the initial state functions or initial states, the corresponding state trajectories of the actual and the average PWM systems remain arbitrarily close to each other in such regions.

Within the boundary layer region, the actual PWM system is described by a two position switch (ON-OFF type). Depending on the sign of the error signal the PWM controlled system is described as follows:

For $e > 0$:

$$dx/dt = f(x) + \phi[x(t-\tau_p)] + u\, g(x) ; \qquad u = \begin{cases} +1 & \text{for } t_k < t \le t_k + \tau(e)T \\ 0 & \text{elsewhere} \end{cases}$$

For $e < 0$:

$$dx/dt = f(x) + \phi[x(t-\tau_p)] + u\, g(x) \quad ; \quad u = \begin{cases} -1 & \text{for } t_k < t \le t_k + \tau(e)T \\ 0 & \text{elsewhere} \end{cases}$$

It can be shown that these systems exhibit global sliding motions about integral manifolds of the average PWM controlled system (which in fact corresponds to the ideal sliding dynamics of the equivalent sliding motion). It follows that, within the boundary layer region, the trajectories of the average PWM controlled system are followed, arbitrarily close, by the actual PWM controlled responses in the same manner that a sliding motion follows, on a given surface in the state space, the trajectories of the corresponding ideal sliding dynamics. Increasing of the sampling frequency in the PWM scheme is totally equivalent to discarding imperfections of the equivalent variable structure switch. In the limit, PWM controlled trajectories would coincide with its average description in the same manner that an actual sliding regime coincides with its ideal sliding dynamics. □

Actuator and Sensor Delays

Let τ_a and τ_s be positive constants. We say that (2.1) has actuator delays whenever the control function u is of the form $u(t_k+\tau_a) = M\ \mathrm{PWM}[e(t_k)]$. The controlled system is said to have sensor delays if $e(t) = y_d(t) - y(t-\tau_s)$. It is easy to show that actuator delays and sensor delays, similarly to the case of plant delays, do not affect the form of the average PWM controller. In such cases, the PWM controller is still substituted by the saturation nonlinearity while the blocks representing, or containing, time delays remain unaltered in the average model. This is easily seen to be a valid statement from the following straightforward considerations.

Sensor delays : Let

$$e(t_k) = y_d(t_k) - y(t_k-\tau_s) \quad \text{i.e.,} \quad e(t) = y_d(t) - y(t-\tau_s)$$

then

$$\lim_{T\to 0,t_k \to t} \{ T^{-1} M [I_g(t_k+\tau[e(t_k)]T) - I_g(t_k)] \} \operatorname{sign}[e(t_k)]$$
$$= M\tau[y_d(t) - y(t-\tau_s)] g(x(t)) \operatorname{sign} [y_d(t) - y(t-\tau_s)]$$
$$= M\tau[e(t)] g(x(t)) \operatorname{sign} [e(t)] = M \operatorname{sat}(e(t),\beta) g(x(t)) = g(x(t))v(t)$$

i.e., the form of the average controller is unaffected by the fact that sensor delays are present in the feedback loop.

Actuator delays : In this case, the control function is given by

$$u(t) = \mathrm{PWM}_{\tau}[e(t_k-\tau_a)] = \begin{cases} \operatorname{sign}[e(t_k-\tau_a)] & \text{for} \quad t_k \le t < t_k + \tau[e(t_k-\tau_a)]T \\ 0 & \text{elsewhere.} \end{cases}$$

hence,

$$\lim_{T\to 0,t_k \to t} T^{-1}\int_{t_k}^{t_k+T} M\, g(x(\sigma))u(\sigma)d\sigma$$
$$=\lim_{T\to 0,t_k \to t} T^{-1} M \int_{t_k}^{t_k+\tau[e(t_k-\tau_a)]T} g(x(\sigma))d\sigma \operatorname{sign}[e(t_k-\tau_a)]$$
$$= \lim_{T\to 0,t_k\to t} T^{-1} M \int_{t_k-\tau}^{t_k-\tau_a + \tau[e(t_k-\tau_a)]T} g(x(\sigma+\tau_a))d\sigma \operatorname{sign}[e(t_k-\tau_a)]$$
$$= M\tau[e(t-\tau_a)]g(x(t))\operatorname{sign}[e(t-\tau_a)] = g(x(t)) M \operatorname{sat}[e(t-\tau_a),\beta] = g(x(t))v(t-\tau_a)$$

From the second line to the third line above, a change of the time integration variable σ by $\sigma-\tau_a$ was performed while noting that the duty ratio, $\tau[e(t_k-\tau_a)]$, is a fixed constant during the integration period and precisely determined at time $t_k-\tau_a$. The average model of the PWM controller in a system subject to actuator delays is still a memoryless saturation type of controller excited by delayed errors. It is easy to see that the delay operator and the saturation nonlinearity commute with each other. The result follows. □

2.3 Linear Controlled Plants

For the case of linear controlled plants without delays, a vast amount of input-output design methods can be directly used for PWM controller design based on the average PWM model. The design problem is reduced to finding the stabilizing gain β corresponding to the linear part of the saturation block in the classical feedback configuration shown in Figure 1. Among the available methods to solve such a problem one finds : the Small Gain Theorem, the Circle Criterion, Describing Function methods, Nyquist stability criterion. For linear plants including finite time delays, the Popov criterion is especially suitable for carrying out an input-output stabilizing desing based on the average PWM controlled model. All these classical design techniques, readily found in the literature, are based on well-known sufficient conditions for stability and asymptotic stability of linear feedback controlled plants.

Example. Consider the problem of designing a stabilizing PWM controller for a distributed linear plant described by :

$$\theta(s) = e^{-Ts}(s+p)^{-1} \quad ; \qquad p > 0$$

Let $g_{av}(PWM)$ denote the gain of the average PWM controller. Then, $g_{av}(PWM) = \sup_{e \neq 0} |sat(e,\beta)||e|^{-1} = \beta$. The Small Gain theorem leads to the following sufficient condition for asymptotic stability :

$$g_{av}(PWM).\sup_{w} | e^{-Tjw}(p+jw)^{-1}| = \beta p^{-1} < 1 \Rightarrow \beta < p$$

One may instead use a particularization of the Circle Criterion for the design case of the saturation controller. This criterion leads to :

$$\inf_{w} Re [e^{-jwT}(p+jw)^{-1}] > -\beta^{-1}$$

3.APPLICATIONS TO A SATELLITE ATTITUDE CONTROL PROBLEM

Here we consider a single-axis attitude control problem for a well known linearized benchmark model representing a satellite plant (the model is taken from Howe and Cavanaugh [6]). This model includes fast dynamical rate and position sensors as well as a first order model for the actuator preceeded by a small pure time delay representing the gas jet reaction control system. The pitch angle θ and the pitch rate $d\theta/dt$ are used in a PWM feedback scheme designed to follow a desirable command angle θ_r. The PWM controlled model is shown in a block diagram form in Figure 2. We assume that the reference pitch angle θ_r is 0.09 radians.

The PWM controller design entitles the specification of the constant β and the sampling frequency 1/T. A satisfactory design, based on the average PWM controlled model, necessarily implies the use of high frequency sampling if basic qualitative and quantitative features of the average designed response are to be captured by the actual PWM controlled process, within arbitrarily small discrepancies. The estimation of the necessary sampling frequency can be assessed from the results of theorem 2.2 above. However, this issue, more related to response precision, can be easily handled by extensive simulation and, thus, it will not concern us here. The more relevant task is, in our view, constituted by the choosing of the constant β, due to its direct influence on the stability of the average closed loop system.

For the case at hand, the closed loop system is guaranteed to be asymptotically stable, according to a well known particularization of Sandberg's Circle Criterion, if

$$-1/\beta < \inf_w \mathrm{Re}[\, G(jw)]$$

where G(jw) represents the complex transfer function of the single input-single output open loop system, including the actuator dynamics, actuator delay, and sensor dynamics.

Figure 3. shows a Nyquist diagram of the open loop system for a normalized PWM slope gain of $\beta = 1$. Any positive value of β smaller than, say, 1500 guarantees asymptotic stability. Simulated asymptotically stable plant state average responses are shown in Figure 4 corresponding to $\beta = 50$. The actual PWM responses do not differ significantly from the average responses due to the low pass filter effect of the actuator dynamics.

4. CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

In this article a design procedure is proposed for the specification of stabilizing feedback PWM controllers in linear and nonlinear delay differential systems. It is shown that, under an infinite sampling frequency assumption, an average model for the classical PWM ON-OFF-ON controller is obtained by simply replacing the discontinuous PWM controller by a memoryless saturation type of compensator. This average model was shown to capture the basic qualitative (i.e. stability) features of the actual PWM controlled system. This allows to treat the PWM design problem in a cleaner fashion than using traditional discrete-time approximations. At the same time one totally circumvents the technical difficulties associated to the fact that PWM operators are indeed unbounded operators on the Banach space of absolutely integrable functions. For the particular case of linear controlled plants, the results of this paper make readily suitable for PWM controller design, a vast number of traditional input-output design techniques, based on well-known sufficient conditions for asymptotic satability. The results were applied to PWM controller specification for a seventh order linearized satellite plant, including actuator delays, in a pitch angle attitude control problem of the reorientation type.

The results of the article can be easily extended to multivariable PWM controllers with, or without, delays. Connections arise among high-gain systems and nonlinear average PWM controlled systems when the saturation nonlinearity, replacing the PWM controller, exhibits a large slope. This also explains some equivalences among high gain controlled systems and variable structure systems undergoing sliding motions. Interesting connections may be expected, for the case of linear PWM controlled multivariable plants, with H_∞ control theory.

REFERENCES

[1] Ya.Z. Tsypkin. *Relay Control Systems*, Cambridge University Press, Cambridge, 1984.

[2] R.A. Skoog and G.L. Blankenship, " Generalized Pulse-Modulated Feedback Systems : Norms, Gains, Lipschitz Constants, and Stability," *IEEE Trans. on Auto. Control*, Vol. AC-15, No.3, pp. 300-315, June 1970.

[3] M. La Cava, G. Paletta and C. Picardi, "Stability Analysis of PWM Feedback Control Systems with PID Regulators," *Int. J. of Control*, Vol. 39, No. 5, pp. 987-1005. May 1984.

[4] H. Sira-Ramirez, "Sliding Mode Control on Slow Manifolds of DC-to-DC Power Converters," *Int. Journal of Control*, Vol. 47, No. 5 pp 1323-1340. 1988

[5] H. Sira-Ramirez, " A Geometric Approach to Pulse-Width-Modulated Control in Nonlinear Dynamical Systems," *IEEE Trans. on Auto. Control*. January 1989.

[6] R.M Howe and W.P. Cavanaugh, " AD 100 Simulation of a Satellite Attitude Control System," Application Report. Applied Dynamics International, 3800 Stone School Road. Ann Arbor, MI 48104. 1985.

FIGURES

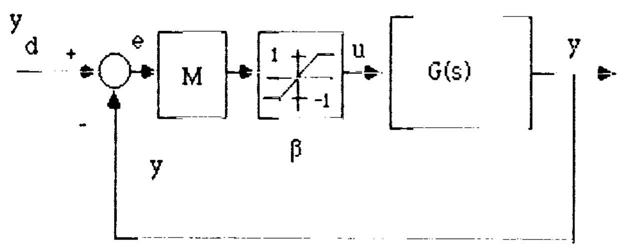

Figure 1. Average Model of Linear Plant Controlled by Pulse-Width-Modulation

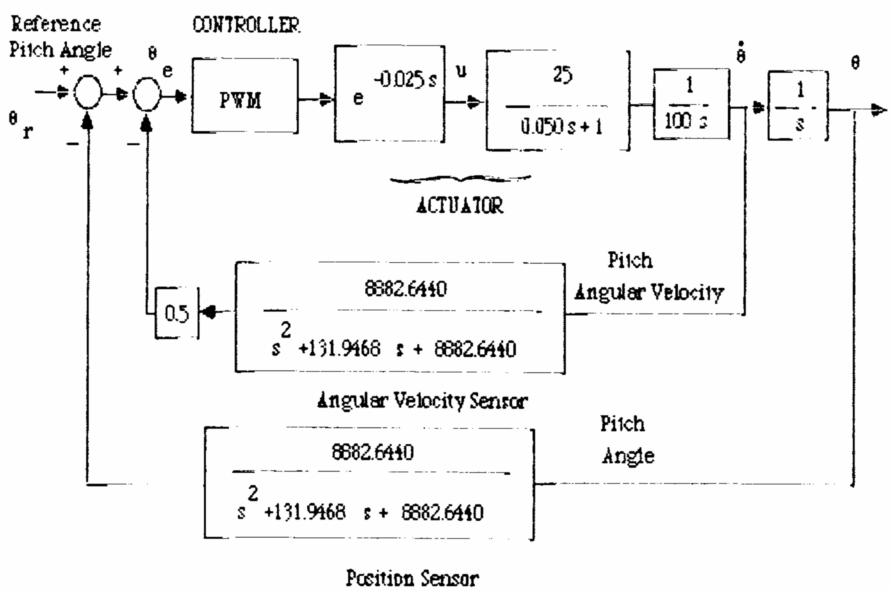

Figure 2. A PWM Satellite Atttitude Feedback Control System

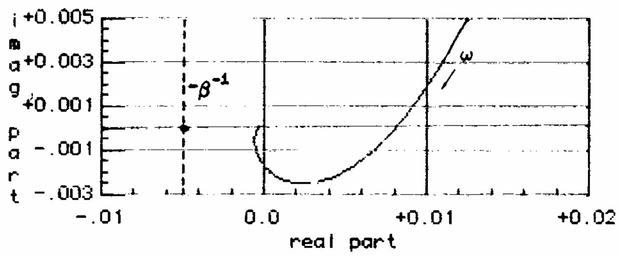

Figure 3. Nyquist plot of the open loop linearized system
(PWM operator gain, β, normalized to the value of 1)

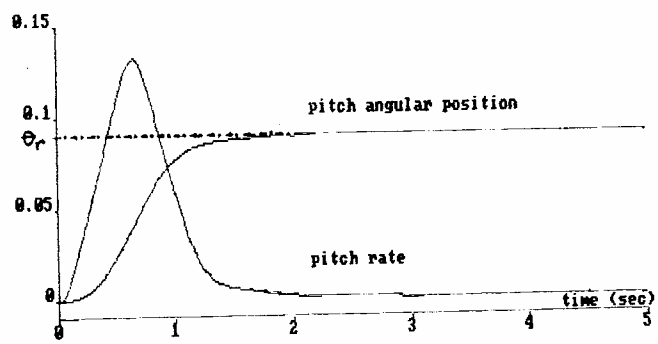

Fig. 4. Average state trajectory responses for β = 50

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