SLIDING MODE CONTROL IN AC-TO-AC CONVERTERS[1]

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Abstract

In this article the theory of variable structure systems, and their associated sliding regimes, is proposed as a means of dealing with the design problem of AC-to-AC converters achieving a leading power factor in AC lines. The approach allows for; 1) a direct approach methodology without average approximations based on discrete time considerations, 2) simplicity in hardware implementation over currently employed Pulse-Width-Modulation techniques and, 3) determination of template current tracking capabilities of the line-converter arrangement via sliding mode existence conditions relevant to the design problem.

Keywords : AC Power Converters, Sliding Regimes, PWM Control.

1. INTRODUCTION

Forced-commutated converters are receiving increased attention in the literature thanks to their remarkable capability of controlling the AC power factor and perform harmonic improvements without the need of capacitor or inductor banks. Instead, thiristor converters (Stefanovich, 1979), Csaki *et al*, 1983) and, more recently, pulse-width-modulation-based AC-to-AC converters (Ooi *et al*. 1987, Katauka *et al*, 1979) are used with a substantial savings in both cost and cabinet space. On the other hand, the impressive advances in fast-switching transistors with high-power ratings such as Mosfets and GTO's, have resulted in a gradual replacement of line-commutated thyristor converters by corresponding ones controlled by transistors and GTO's.

In this article, the theory of variable structure systems, and their associated sliding modes (Utkin, 1978), is proposed as a means of achieving a leading power factor in AC-to-AC converters. The approach allows for a rather direct analysis of the switched-controlled converter without local average or discrete time considerations. A variable structure controlled converter challenges, in hardware simplicity, the currently employed controlled-current Pulse-Width-Modulation converters. The sliding mode approach allows for the study of template current waveform tracking capabilities of the converter under the scope of sliding mode existence conditions relevant to the design problem. Finally, added to the intrinsic conceptual and intuitive advantages, all results pertaining to sliding modes can also be translated into corresponding pulse-width-modulation control schemes in a straightforward manner, as it is shown in Sira-Ramirez (1987, 1988a, 1988b).

Section 2 is devoted to formulate the power factor modulation problem of an AC power line as a variable structure control problem defined on a power line provided of an AC-to-AC converter, such as that treated in (Ooi *et al* 1987). The single phase converter model motivates and is the basis of the three-phase converter. Section 3 contains the conclussions and suggestions for further research in the area. Basic results about variable structre systems, and their associated sliding regimes, are presented in Section 4, which qualifies as an Appendix.

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2. A VARIABLE STRUCTURE CONTROLLED AC-TO-AC CONVERTER

2.a Variable Structure Control model of Single-Phase Converter.

The AC-to-AC converter used to improve upon the power factor of the AC line is taken from Ooi *et al* (1987). The operation of any three phase AC-AC converter is better understood through the single-phase converter shown in Figure 1.

The operation of the converter involves the on-off sates of the bipolar transistors T_1 and T_2 . Assuming positive values of the line current, when T_1 is switched on, the current i_1 flows through the transistor T_1 and the line current I rises provided appropriate values of E are used in relation to the voltage amplitude $\sqrt{2}$ V of the line. When T_2 is set on, T_1 is set off and the current now flows through the freewheeling diode D_2 . The line current now decays. The same analysis follows for negative values of the line current but now currents i_1 and i_2 flow through D_1 and T_2 respectively. The branches with the constant voltage sources are known as the DC-links. The sources may be obtained from batteries or from a DC machine-chopper arrangement (See Ooi *et al*, 1987). Usually, the transistors T_1 and T_2 are triggered through a PWM control scheme, reflecting the need of having the line current I track a "template" current waveform i_m obtained from the AC mains. The phase angle of the template is controlled through a three phase phase-shifting transfomer with outputs multiplied by an set point amplitude I_m whose limitations and restrictions will be determined later, i.e., $i_m = \sqrt{2}I_m \mathrm{sen}(vt+\phi)$. If the line current I is made to track i_m, the AC power factor will depend on ϕ and hence, conceivably, one can improve it, or, alternatively, make it adopt a purely capacitive character (static capacitor operation mode).

The mathematical model of the single phase converter is obtained separately for each transistor operating cycle. Thus, when either T_1 or D_1 conducts;

$$L\, di_1/dt = 0.5\, E - v \qquad (2.1)$$

On the other hand, when either T_2 or the diode D_2 conducts;

$$L\, di_2/dt = - 0.5\, E - v \qquad (2.2)$$

At each instant of time, the inductor current I is either i_1 or i_2 , while the other current is zero, i.e., $i_1 = I$ with $i_2 = 0$, and $i_2 = I$ when $i_1 = 0$. The variable structure control model reflecting the time-varying nature of the input signal $v(t) = \sqrt{2}\, V \sin (vt)$ is then given, in extended phase space coordinates (I, θ) (see Appendix) by :

$$L\, dI/dt = u\, 0.5E - \sqrt{2}\, V \sin v\theta \qquad (2.3)$$
$$d\theta/dt = 1$$

with u taking values in the discrete set $U = \{-1,1\}$. The model (2.3) is a suitable variable structure control model, as presented in the Appendix, in which the parameter u is the control function representing an ideal relay (switch) position function. When $u = 1$, T_1, or D_1, is supposed to be on, while D_2, or T_2, is, respectively, off. When $u = -1$, T_1, or D_1, is off and D_2, or T_2, is, respectively, conducting.

The switching logic that guarantees a template current tracking will be determined in the next section from the sliding mode existence conditions.

2.b Sliding Mode Aspects of the Single-Phase Converter.

Let the discontinuity "surface" be represented by the time-varying error signal ($\theta \equiv t$):

$$s(I,\theta) = I - i_m(\theta) = I - \sqrt{2}I_m \sin (v\theta + \phi) \qquad (2.4)$$

where $i_m(\theta) = \sqrt{2}\, I_m \sin (v\theta + \phi)$, is the template current waveform obtained from the AC mains via a phase-shifting transfrormer (See Figure 2).

The conditions for a sliding regime to exist on $s(I,\theta) = 0$ are obtained from the necessary and sufficient conditions (4.4) applied to this particular case. The control u is to be switched in such a form that if we denote by $\underline{u}$ the opposite control position value to that of u :

$$\lim_{s\to+0} d/dt\, s(x,\theta) = \qquad (2.5)$$
$$u\, 0.5(E/L)-[v(\theta)/L]-(di_m/d\theta)d\theta/dt < 0$$

$$\lim_{s\to-0} d/dt\, s(x,\theta) =$$
$$\underline{u}\, 0.5\, (E/L)- [v(\theta)/L]-(di_m/d\theta)d\theta/dt > 0$$

Therefore, if for $s > 0$ we let $u = -1$ then for $s < 0$ $\underline{u} = 1$ and one obtains, from (2.5), that for all t :

$$-(0.5E + v) < L\, d\, i_m(\theta)/d\theta$$
$$(0.5E - v) > L\, d\, i_m(\theta)/d\theta \qquad (2.6)$$

The following **slope overload condition** is obtained as a necesary and sufficient condition for the existence of a sliding motion on $s(x,\theta) = 0$:

$$-\max_\theta (0.5E+v) < \min_\theta L\, d\, i_m/d\theta < L di_m/d\theta < \max_\theta L\, d\, i_m/d\theta < \min_\theta (0.5E-v) \qquad (2.7)$$

From here, using $v = \sqrt{2}\, V \sin(w\theta)$ and $i_m(\theta) = \sqrt{2}\, I_m \sin(w\theta + \phi)$, the following frequency dependent relation is obtained for the existence of a slidng regime:

$$-0.5E - \sqrt{2}\, V < -w L \sqrt{2}\, I_m < w L \sqrt{2}\, I_m \cos(w\theta+\phi) < w L \sqrt{2}\, I_m < 0.5E - \sqrt{2}\, V \qquad (2.8)$$

or, more briefly:

$$|\, w L I_m + V \,| = w L I_m + V < [1/(2\sqrt{2})]E \qquad (2.9)$$

The template current tracking condition (2.9) has been systematically overlooked in the literature.

For fixed V and E, as the angular frequency of the line voltage increases, the converter is only capable of tracking smaller amplitude template waveforms. At the same time, the existence condition (2.9) is independent of the phase shift impossed on the template waveform. Condition (2.9) determines the amplitude and frequency ranges of the allowable harmonic template waveforms.

Figure 2 shows the block diagram representing the synthesis process for the switching logic leading to a sliding motion responsible for the tracking of the template signal. Figure 3 represents a typical phase modulator response.

Under ideal sliding conditions (see Appendix) :

$$s(x,\theta) = 0 \quad \text{and} \quad d/dt\, s(x,\theta) = 0. \qquad (2.10)$$

Hence, from (2.10) we have that, ideally, the line current I is equal to the template current waveform i_m at each instant of time, while the time derivatives of both signals would be, ideally, identical.

$$I(t) = i_m(t) \;\; ; \; dI(t)/dt = di_m(t)/dt \qquad (2.11)$$

Ideal sliding conditions are tantamount to perfect tracking of template waveform by the AC line current.

In terms of Filippov's average dynamics, the second equation of (2.11) is written, according to (2.3) and (4.14), in extended phase space coordinates as :

$$L dI/dt = \mu[-0.5E - v(\theta)] + (1-\mu)[0.5E - v(\theta)] = L d/d\theta\, i_m(\theta)$$
$$d\theta/dt = 1 \qquad (2.12)$$

from where one easily obtains :

$$\mu = [0.5E - v(\theta)]/E - (L/E)\, di_m(\theta)/d\theta = [0.5E - \sqrt{2}V\sin w\theta]/E - (\sqrt{2}I_m wL/E)\cos(w\theta + \phi)$$

while :

$$1-\mu = [0.5E + v(\theta)]/E + (L/E) di_m(\theta)/d\theta = [0.5E + \sqrt{2}V\sin w\theta]/E + (\sqrt{2}I_m wL/E)\cos(w\theta + \phi)$$

From the first of (2.11) it follows trivially that

$$I = \mu i_m + (1-\mu) i_m \qquad (2.13)$$

The function μ is interpreted as the fraction of time on which the controlled direction field of (2.3) is determined by the $u = 1$ switch position. This allows one to redefine, in a more exact fashion than that of Ooi *et al* (1987), the **local average of the DC link currents** as :

$$\mathcal{I}_1 = \mu\, i_m \quad \text{and} \quad \mathcal{I}_2 = (1-\mu)\, i_m \qquad (2.14)$$

i.e :

$$\mathcal{I}_1 = 0.707\, I_m \sin(w\theta + \phi) + (VI_m/E)[\cos\phi - \cos(2w\theta + \phi)] - (I_m^2 wL/E)\sin(2w\theta + 2\phi) \qquad (2.15)$$

$$\mathcal{I}_2 = 0.707\, I_m \sin(w\theta + \phi) - (VI_m/E)[\cos\phi - \cos(2w\theta + \phi)] + (I_m^2 wL/E)\sin(2w\theta + 2\phi) \qquad (2.16)$$

The average delivered DC power, expressed as $P_{DC} = 0.5E(\mathcal{I}_1 - \mathcal{I}_2)$, is given by :

$$P_{DC} = VI_m [\cos\phi - \cos(2w\theta + \phi)] - I_m^2 wL \sin(2w\theta + 2\phi) \quad (2.17)$$

On the other hand, the average power consumed by the AC source (Ooi *et al*, 1987) is :

$$P_{AC} = v(\theta)i(\theta) = 2VI_m \sin w\theta \sin(w\theta+\phi) = VI[\cos\phi - \cos(2w\theta + \phi)] \quad (2.18)$$

which leads to the following average power distribution equation involving the inductive character of the load :

$$P_{AC} = P_{DC} + I_m^2 wL \sin 2(w\theta + \phi) \quad (2.19)$$

2c. Three-phase phase modulator.

The previous results are easily extended to the three-phase phase modulator. For reasons of space such case is not treated here. Only a few remarks will be made.

An interesting property of the three-phase controlled system is that the average DC-link current, computed as the sum of the average currents in the DC-link branches, results in a time-invariant DC-current (Ooi *et al*, 1987). In the Variable Structure approach, this is shown just by addding the three ideal average DC link currents occurring respectively on the upper and lower branches of the arrangement. Also, the instantaneuous average DC power coincides with the instantaneous AC power. It is easy to see that the balanced condition implies a null neutral current. Ideally, then, any center tap connection of the DC sources can be effectively removed. Finally, for $\phi = 270^o$, the phase modulator operates as a static capacitor and the corresponding DC-link currents are ideally zero.

3. CONCLUSIONS

Sliding regimes have been shown to be relevant to the problem of phase current modulation through bipolar transistor-based converters in AC lines with inductive loads.

The necessary and sufficient conditions for the existence of a sliding regime, guaranteeing template phase-shifted current tracking, lead to a new frequency dependent condition involving the template current and the line voltage amplitudes.

Ideal sliding conditions are equivalent to perfect template current tracking on the part of the line current. Using Filippov's average (ideal) sliding dynamics concept, a local average DC-link current definition is proposed as an appropriate and natural time varying fraction of the template current rather than a constant fraction of such quantity. Essential differences arise, in the single phase modulator case, with respect to controlled current PWM based converters such as that treated in Ooi *et al* (1987). These differences are due to inappropriate definitions in Ooi's paper of local average DC-link currents whereby assumptions of constant template current during transistor-diode switchings ignore average load inductance effects and only result in first term approximations to the average currents computed on the basis of ideal infinite frequency switchings. The definition proposed in this article is not only phisically more natural than that of Ooi *et al* (1987), while being mathematically more rigurous in the sense of Filippov (1960), but they also capture the inductive frequency dependent average effects.

The three-phase characteristics of the ideal sliding dynamics associated to the phase modulation process totally rederive known results obtained through PWM-based phase modulators. However, the hardware simplicity associated with a Variable-Structrue-based Base Drive Control is vastly superior than that corresponding to a PWM-based option.

4. APPENDIX

4.1 Generalities about Sliding Regimes on Nonstationary Discontinuity Surfaces

Applications of Sliding Mode control range from Power Electronics, Speech encoding , Robotics, Power Systems Control, Aircraft controller design, and controlled maneuvers in Flexible Agile Spacecrafts (See Utkin 1977, 1983). Here we shall only present some of the relevant aspects of sliding modes of nonlinear dynamical systems in time-varying sliding surfaces. The reader is referred to Sira-Ramirez (1988a, 1988b) for a more thorough exposition for the stationary sliding surface case. The proofs of the theorems are simple extensions of those in Sira-Ramirez (1988b), for this reason they are not presented here.

Consider the nonlinear time-varying dynamical system defined on an open set X x T of R^n x R and expressed in local coordinates of the extended state space (x,θ) as :

$$dx/dt = f(x,\theta) + u\, g(x,\theta)$$
$$d\theta/dt = 1 \quad (4.1)$$

with f and g being locally defined time-varying smooth vector fields. The control function u is assumed to be of discontinuous nature, taking values on the discrete set U = { -1,1 } , according to the switching logic :

$$u = -\text{sign } s(x,\theta) \qquad (4.2)$$

where $s(x,\theta)$ is a smooth function, of time varying nature, with uniformly nonzero gradient on X x T. $s : R^n \times R \to R$, and defines a discontinuity surface, in the extended state space, of the form :

$$S(x,\theta) = \{ (x,\theta) \in R^{n+1} : s(x,\theta) = 0 \} \qquad (4.3)$$

Definition 4.1 (Utkin, 1978)

A local **Sliding Regime** is said to exist on $S(x,\theta)$ if and only if locally on X x T the controlled motion (4.1),(4.2) is such that :

$$\lim_{s \to +0} ds/dt < 0 \quad \text{and}$$
$$\lim_{s \to -0} ds/dt > 0 \qquad (4.4)$$

The trajectories locally, and uniformly in T, converge towards $S(x,\theta)$ where they locally undergo a chattering motion constraining, in the average, the state trajectories to the discontinuity surface. Choosing an appropriate discontinuity surface, the system dynamics is changed at will with charcteristics usually not present in any of the controlled structures represented by (4.1),(4.2) on each side of $S(x,\theta)$.

By a **local property**, it will be henceforth understood that it holds true locally on X and uniformly on the open interval T. It is assumed that X has a nonempty intersection with $S(x,\theta)$ for all θ in T.

Lemma 4.2 (Sira-Ramirez, 1988b)

If a sliding motion exists on $S(x,\theta)$ then the following transversality condition is locally verified on $S(x,\theta)$:

$$[\partial s/\partial x]^T g(x,\theta) > 0 \qquad (4.5)$$

Proof. (See Sira-Ramirez, 1988b)

The solutions of (4.1),(4.2) are locally defined everywhere on X x T, except on the surface $S(x,\theta)$ where the right hand side of (4.1) undergoes discontinuities. Basically, two kinds of solutions have been proposed to describe the resulting motion on $S(x,\theta)$ in an ideal manner which neglects switch inertias and imperfections due to perturbations and modelling errors. The first one is due to Filippov (1964), and the second is due to Utkin (1978). For the linear in the control case, such as (4.1), both approaches are equivalent.

Ideally, on $S(x,\theta)$, the motions of the controlled system can be described as if controlled by a smooth feedback control function known as the equivalent control. Such a function is defined from the following invariance conditions :

$$ds/dt \,|_{s=0} = 0 \qquad (4.6)$$

where the total time derivative of $s(x,\theta)$ is computed along local solutions of (4.1) and satisfied for certain smooth control known as the **equivalent control**, denoted by $u_{EQ}(x,\theta)$. i.e.,

$$[\partial s/\partial x]^T[f(x,\theta)+u_{EQ}(x,\theta)g(x,\theta)]+\partial s/\partial \theta = 0$$

or explicitly :

$$u_{EQ}(x,\theta) = -([\partial s/\partial x]^T g(x,\theta))^{-1}[\partial s/\partial x]^T f(x,\theta)$$
$$-([\partial s/\partial x]^T g(x,\theta))^{-1}[\partial s/\partial \theta] \qquad (4.7)$$

From (4.7) and Lemma 4.2 it easily follows that a necessary condition for the existence of a local sliding motion on $S(x,\theta)$ is that $u_{EQ}(x,\theta)$ locally exists. It is also straightforward to show that, ruling out the trivial case $[\partial s/\partial x]^T f(x,\theta) = 0$ and $[\partial s/\partial x]^T g(x,\theta) = 0$, the equivalent control, if it exists, is also unique (Sira-Ramirez, 1988b).

The ideal sliding dynamics is obtained from using (4.7) on (4.1) with the formal substitution of u by $u_{EQ}(x,\theta)$. This is the basis of the **Equivalent Control Mehtod** (Utkin, 1978).

$$\dot{x} = \{I-g(x,\theta)[(\partial s/\partial x)^T g(x,\theta)]^{-1}(\partial s/\partial x)^T\}$$
$$f(x,\theta)-g(x,\theta)[(\partial s/\partial x)^T g(x,\theta)]^{-1}(\partial s/\partial \theta) \qquad (4.8)$$

Theorem 4.3

A sliding motion locally exists on S if and only if on S(t) :

$$-1 < u_{EQ}(x,\theta) < 1 \qquad (4.10)$$

Proof: See (Sira-Ramirez, 1988b)

Notice furthermore, that it follows from (4.4) and (4.6) that on $S(x,\theta)$ one has :

$$ds/dt|_{s=0,\,u=+1} < ds/dt|_{s=0,\,u=u_{EQ}}$$

$$< ds/dt|_{s=0,\,u=-1} \qquad (4.11)$$

Because of its similarity with the tracking condition in **Delta Modulation Systems** (Steele, 1975), the violation of this inequality is known as the slope overload condition.

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6. FIGURES

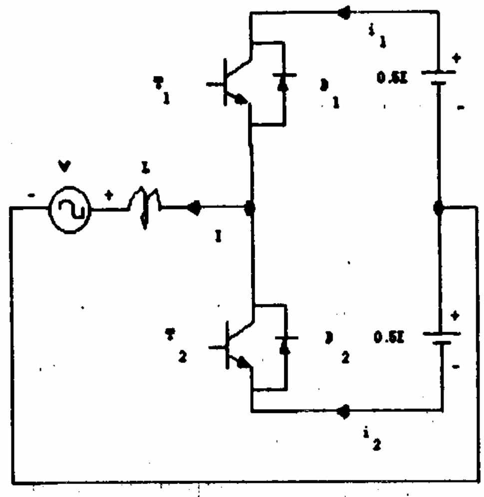

Figure 1: Single-Phase Converter

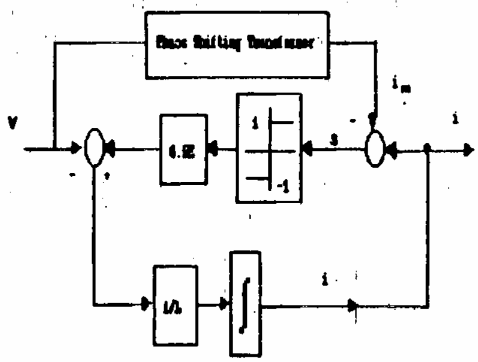

Figure 2 : Variable Structure Model of Phase Modulator

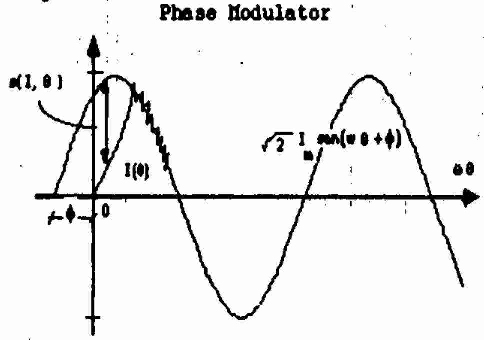

Figure 3: Typical Phase Modulator Sliding Response