ON THE DESIGN OF NONLINEAR PULSE-WIDTH-MODULATION CONTROLLERS[1]

WA-4-2

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Abstract In this article a design method is proposed for the specification of a variety of Pulse-Width-Modulation (PWM) feedback controllers acting on plants described by Nonlinear Dynamical Systems. It is shown that an average model of the PWM controlled system, derived by an infinte sampling frequency assumption, is simply obtained by replacing the discontinuous PWM controller by a memoryless nonleariry completely specified by the feedback duty ratio function. The average model captures the essential qualitative stability properties of the controlled system and, thus, can be used to considerably simplify the PWM controller design task. The case of PWM regulators for linear plants can be systematically treated by traditional input-output design methods. Some examples are provided.

1. INTRODUCTION

PWM controllers have been extensively used and studied in the past. Fundamental work in the area was carried out by many authors (See references in Sira-Ramirez[1]). For background purposes we shall only mention the work of Skoog and Blankenship[2], a recent article by La Cava *et al.*[3] and p. 591 of Czaki's book (ref. 4). The underlying feature of all the works in the area is the exploitation of the discrete-time aspects of PWM schemes. Such an approach fits the problem quite naturally, due to the inherent sampling process associated to every PWM control scheme. The method, however, also leds to tortuous calculations where the essential simplicity of the PWM scheme is hoplessly lost in analytical considerations pursuing an approximation scheme that obscures the essential aspects of the average control action.

In recent articles, hoewever, Sira-Ramirez[5-7] has explored a different design approach by using the geometric properties of average PWM controlled responses (obtained by an infinite frequency sampling assumption). The advantage of the method is that the design can be carried on the basis of exact considerations about the average plant while the actual responses generally exhibit sliding motions arbitrarily closely approximating the average designed behavior. Applications of these results to the control of DC to DC power converters are found in Sira-Ramirez[8], Sira-Ramirez and Ilic-Spong[9].

An interesting class of switch controlled systems is represented by those in which the control variable takes values in the discrete set {-1,0,+1} (ref. 2). Typically, torque actuators, used for control of joint poisitions in robotic manipulators are of this type. Gas reaction jets controlling reorientation, or detumbling, maneuvers in artificial satellites are also expressible as systems of this class. The controlled switch, in this case, is addressed as an ON-OFF-ON switch. In this article we first extend the results of Sira-Ramirez[1,7] to general nonlinear PWM controlled systems of the ON-OFF-ON type, and then proceed to particularize the results to several PWM schemes. As a general design method, we propose to carry out the PWM controller specification (duty ratio function) on the basis of the corresponding features of the static nonlinearity producing a desirable performance of the average PWM controlled system. The required PWM sampling frequency may be obtained in accordance with the degree of similarilty one would like to impose on the actual PWM closed loop performance with respect to that of the average design responses.

The average PWM model is obtained by replacing the PWM regulator by means of a nonlinear memoryless controller totally specified by the particular form of the feedback duty ratio function. The average model is then used for design purposes and the required features of the nonlinearity are determined. This design scheme has been shown to be valid even in the case of dealy-differential plants controlled by a PWM loop (See Sira-Ramirez[10]). For the case of linear controlled plants, a vast amount of input-output design methods are readily available for PWM controller specification: Small Gain Theorem, Circle Criterion, describing functions, Nyquist stability criterion. Popov criterion, etc. The technical problems associated with the unbounded character of the PWM operator and the need for introducing low pass multipliers cascading the PWM controller (ref. 2) are entirely circumvented with our proposed analysis and design technique.

In Section 2 general considerations are presented about PWM controlled systems. A general average model is derived which substitutes the PWM controller block under infinte frequency samnpling of the error signal. A particular family of PWM controller schemes are introduced in that section. The general design procedure is also outlined in section 2. Design examples are furnished in Section 3. Section 4 contains the conclusions and suggestions for futher work in the area.

II DEFINITIONS AND BASIC RESULTS

2.1 Definition of Average PWM model.

Consider a nonlinear feedback PWM controlled system described in R^n, by :

$$dx/dt = f(x) + g(x)u$$
$$y = h(x)$$
$$e = y_d(t) - y$$
$$u = M\ PWM_\tau [e(t_k)] \qquad (2.1)$$

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with f and g smooth vector fields and h a smooth scalar output function. The control input u is a discontinuous scalar control function obtained as the output of a Pulse-Width-Modulator excited by the error signal e. The sampling process is assumed to take place at regularly spaced time instants, t_k, with a constant frequency of value 1/T, i.e., $t_{k+1} = t_k + T$. M is a positive constant gain.

The PWM control operator, $PWM_\tau[e]$, is usually characterized by (See ref. 2):

$$PWM_\tau[e(t_k)] = \begin{cases} \mathrm{sign}[e(t_k)] & \text{for } t_k \le t < t_k + \tau[e(t_k)]T \\ 0 & \text{elsewhere.} \end{cases} \qquad (2.2)$$

where $\tau[e(t_k)]$ is known as the duty ratio function, assumed to be a picewise smooth function of even nature (i.e., $\tau[e(t_k)] = \tau[-e(t_k)]$). We denote the picewise smooth function $\tau[e(t_k)]\mathrm{sign}[e(t_k)]$ by $N(e(t_k))$. N(e) is evidently odd ($N(e) = -N(-e)$).

Proposition 2.1 As the sampling frequency $F := 1/T$ tends to infinity, the description of the nonlinear controlled system (2.1)-(2.2) coincides with :

$$\begin{aligned} dx/dt &= f(x) + g(x)v \\ y &= h(x) \\ e &= y_d(t) - y \\ v &= M\, N(e(t)) \end{aligned} \qquad (2.3)$$

System (2.3) will be henceforth addressed as the average PWM controlled system while (2.1)-(2.2) as the actual PWM controlled system.

Proof. Let $I_\phi(t)$ dentote the vector-valued function $\int_0^t \phi(x(\sigma))d\sigma$, with $\phi(x)$ being a given smooth vector field on R^n defining the flow of $dx/dt = \phi(x)$. Notice that the limit: $\lim_{T\to 0} [I_\phi(t+\alpha T)-I_\phi(t)]/T = \alpha\, dI_\phi(t)/dt = \alpha\,\phi(x(t))$. The sampled state-response of the nonlinear feedback system (2.1) can be written, in terms of an equivalent integral equation, as :

$$x(t_k+T) = x(t_k) + I_f(t_k+T) - I_f(t_k) + M\,[I_g(t_k+\tau[e(t_k)]T) - I_g(t_k)]\mathrm{sign}\,[e(t_k)]$$

Therefore, according to the above expression, the limit :

$$\lim_{T\to 0,\, t_k\to t} T^{-1}[x(t_k+T) - x(t_k)] =: dx(t)/dt$$

$$= \lim_{T\to 0,\, t_k\to t} T^{-1}\,[I_f(t_k+T)-I_f(t_k)] + \lim_{T\to 0,\, t_k\to t} \{ T^{-1}M\,[I_g(t_k+\tau[e(t_k)]T)-I_g(t_k)]\}\,\mathrm{sign}[e(t_k)]$$

$$= f(x(t)) + g(x(t))\, M\, \tau[e(t)]\,\mathrm{sign}\,[e(t)]$$

$$= f(x(t)) + g(x(t))\, M\, N[e(t)] = f(x) + g(x)v \qquad \square$$

Remark The behavior of the infinite frequency sampled system is described by the original nonlinear plant fedback via a nonlinear piece-wise smooth block N(e), excited by the error signal e. The scalar input u is generated as the output of the memoryless nonlinear function N(e). In other words, to evaluate the average behavior of the PWM controlled system, the PWM controller block is simply substituted by a nonlinear memoryless controller totally specified by the duty ratio function (See Figures 1 and 2). As the sampling frequency grows to infinity, the average and the actual PWM responses coincide. Thus a large sampling frequency can always be specified which keeps the actual PWM closed loop response of (2.1)-(2.2) within a prespecified bound defined about the corresponding average response of (2.3).

The qualitative stability characteristics of the PWM controlled system (2.1)-(2.2) have been shown to be captured by its infinite-frequency sampling average model (2.3) (See ref. 1). In particular, a necessary condition for asymptotic stability towards the e = 0 manifold of the actual PWM system is constituted by the asymptotic stability of the average model towards such a manifold. It was also established in reference 1, that if the response of the average model (2.3) is asymptotically stable towards the e = 0 manifold, then the actual PWM controlled response can be made to follow, arbitrarily close, the response of the average model by suitably increasing of the sampling frequency of the PWM controller. Thus, modulo sufficiently large -but finite- sampling frequency, the following theorem holds true :

Theorem 2.2 The closed loop PWM controlled system (2.1)-(2.2) is asymptotically stable towards the e =0 manifold if and only if the average PWM system (2.3) is asymptotically stable towards such a manifold.

Proof (See ref. 1).

2.2 A Family of PWM Controllers

The PWM controller is exclusively determined by the nature of the duty ratio function $\tau(e)$. Unless the duty ratio is a constant, as it is, quite commonly, the case in switch-regulated power electronics devices, it actually acts as a feedback function. The nature of such feedback is one based in administering the length of intersampling time on which one of two available control actions are enabled to affect the system, while the rest of the sampling period the control action is switched off (to zero input). The available control actions are generally of fixed nature and oposite signs but they could also be amplitude modulated by the error or the state signals.

Here we propose several PWM control configurations which lead to well known nonlinearities for the average PWM block N(e). Some of them, particularly the one leading to a saturation nonlinearity has been extensively studied in refs. 1,7,10 the rest of the schemes presented here are believed to be new.

Saturation PWM controller

This controller is defined by the following even duty ratio function.

$$\tau[e(t_k)] = \begin{cases} b|e(t_k)| & \text{for } |e(t_k)| \le 1/b \\ 1 & \text{for } |e(t_k)| > 1/b \end{cases} \qquad (2.4)$$

Evidently, the average block subsituting a PWM controller characterized by (2.2) and (2.4) is given by $N(e) = \tau[e(t)]\mathrm{sign}[e(t)] = \mathrm{sat}[e(t)]$, i.e., it is given by the saturation function of the error signal $e(t)$.

The gain of the saturation function is easily shown to coincide with the parameter b of the PWM controller.

The nature of the actual PWM controlled response with respect to the average response in a feedback scheme using a saturation PWM controller is such that -under the same initial conditions- the state responses coincide in the saturation region $|e(t)| > 1/b$. In the boundary layer region, $|e(t_k)| \le 1/b$, the actualstate responses slide about integral manifolds of the average response (ref. 7).

Relay PWM Controller

A duty ratio that remains constant regardless of the value of the error signal, evidently excercises a periodic bang-off-bang action on the controlled plant. Evidently the constant value characterizing the duty ratio function must be smaller than one.

$$\tau[e(t_k)] = k \qquad (0 < k < 1) \qquad (2.5)$$

The average block subsituting a PWM controller characterized by (2.2) and (2.5) is given by $N(e) = \tau[e(t)]\mathrm{sign}[e(t)] = k\ \mathrm{sign}[e(t)]$, i.e., it is given by the ideal relay function of the error signal $e(t)$.

The gain of the relay function has been shown to be infinte.

The actual PWM controlled response exhibits, for sufficiently large sampling frequency, a sliding regime about the integral manifolds of the average system on each side of the $e = 0$ manifold. i.e., on $e < 0$ the actual state response slides about integral manifolds of $dx/dt = f(x) - kg(x)$ and for $e > 0$ about integral manifolds of $dx/dt = f(x) + kg(x)$. The actual response slides about $e = 0$ if such a manifold is attractive.

Dead-band PWM Controller

This controller is defined by the following even duty ratio function.

$$\tau[e(t_k)] = \begin{cases} 0 & \text{for } |e(t_k)| \le 1/d \\ 1 & \text{for } |e(t_k)| > 1/d \end{cases} \qquad (2.6)$$

The average block subsituting a PWM controller characterized by (2.2) and (2.6) is given by $N(e) = \tau[e(t)]\mathrm{sign}[e(t)] = \mathrm{dbd}[e(t)]$, i.e., it is given by the dead-band function of the error signal $e(t)$.

The gain of the dead-band function is easily shown to coincide with the parameter d of the PWM controller.

In this case, it is easy to see that the state responses of the actual dead-band PWM controlled totally coincide with the state responses of the average PWM controlled model under the assumption of identical initial conditions.

Saturation with Dead-Band PWM Controller

This controller is defined by the following duty ratio function ($d > b$) .

$$\tau[e(t_k)] = \begin{cases} 0 & \text{for } |e(t_k)| \le 1/d \\ 1 & \text{for } |e(t_k)| > 1/b \\ bd/(d-b)\,[e(t_k) - 1/d] & \text{for } 1/d < |e(t_k)| \le 1/b \end{cases} \qquad (2.7)$$

The average block subsituting a PWM controller characterized by (2.2) and (2.6) is given by $N(e) = \tau[e(t)]\mathrm{sign}[e(t)] = \mathrm{dbd}[e(t)]$, i.e., it is given by the saturation function with dead-band of the error signal $e(t)$.

The gain of the saturation function with dead-band is easily shown to coincide with the parameter b of the PWM controller.

It should be clear that the actual state responses of the PWM controlled sytem coincide with those of the average PWM model under saturation conditions. On the linear portions of the duty ratio characteristic, the actual response slides about integral manifolds of the average PWM model. The responses on the dead band zone of both systems coincide if initial conditions are taken to be the same. Otherwise, the responses locally evolve on the integral manifolds of $dx/dt = f(x)$ containing the particluar initial conditions of each model.

Negative Deficiency Saturation PWM Controller

This controller is defined by the following even duty ratio function.

$$\tau[e(t_k)] = \begin{cases} k + K_N |e(t_k)| & \text{for } |e(t_k)| < 1/b \\ 1 & \text{for } |e(t_k)| \ge 1/b \end{cases} \qquad (2.8)$$

Notice that the parameters defining this duty ratio function are related by $K_N/(1-k) = b$. The average block subsituting a PWM controller characterized by (2.2) and (2.7) is given by $N(e) = \tau[e(t)]\mathrm{sign}[e(t)] = \mathrm{ndef}[e(t)]$, i.e., it is given by the negative deficiency function of the error signal $e(t)$.

As in the case of the ideal realy, the gain of the negative deficiency function is infinte.

As before, under saturation conditions the actual and the average PWM responses totally coincide. In the linear regions the integral manifolds of the average model, on each side of the $e=0$ manifold, qualify as sliding surfaces where the actual PWM controlled state responses evolve.

Granularity PWM Controller

This PWM controller is defined by the following duty ratio function. ($\Delta k = 1/n$)

$$\tau[e(t_k)] = \begin{cases} 0 & \text{for } |e(t_k)| \le \Delta b/2 \\ \Delta k & \text{for } \Delta b/2 < |e(t_k)| \le 3\Delta b/2 \\ 2\Delta k & \text{for } 3\Delta b/2 < |e(t_k)| \le 5\Delta b/2 \\ \cdots & \\ (n-1)\Delta k & \text{for } (2n-3)\Delta b/2 < |e(t_k)| \le (2n-1)\Delta b/2 \\ 1 & \text{for } |e(t_k)| > (2n-1)\Delta b/2 \end{cases} \qquad (2.9)$$

The average block substituting a PWM controller characterized by (2.2) and (2.9) is given by $N(e) = \tau[e(t)]\text{sign}[e(t)] = gr[e(t)]$, i.e., it is given by the granularity function of the error signal $e(t)$.

The gain of the granularity function coincides with the ratio: $2b/\Delta k$ of the PWM controller parameters.

The actual PWM responses exhibit sliding regimes about integral manifolds of the average system $dx\backslash dt = f(x) + i\Delta k\ g(x)$ for each i ($i = 1,2,\ldots n-1$). i.e., sliding mode exist for those regions of the sate space where the duty ratio function is neither saturated nor zero. The actual state trajectories jump from leave to leave of the discrete folliation induced by integral manifolds of the average system system $dx/dt = f(x) + i\Delta kg(x)$ For $i = 0$ or n the average and actual PWM responses coincide under identical initial conditions.

2.3 Linear Controlled Plants

For the case of linear controlled plants, a vast amount of input-output design methods can be directly used for PWM controller design based on the average PWM model. The design problem for a particular member of the family of described PWM controllers is reduced to finding the stabilizing gain corresponding to the nonlinear block in the classical feedback configuration shown in Figure 3. By determining this gain, the defining parameter of the average PWM block is uniquely determined in most cases (granularity and infinite gain cases excluded). These parameters completely define, in turn, the actual stabilizing PWM controller. Among the available methods to solve such a design problem one finds, in general: the Small Gain Theorem, the Circle Criterion, Describing Function methods, Nyquist stability criterion, the Popov criterion (especially for linear plants including delays), etc. All these design techniques, readily found in the literature (See MacFarlane[11]), are based on well-known sufficient conditions for stability and asymptotic stability of linear feedback controlled plants. The sampling frequency required to arbitrarily closely approximate the actual PWM responses to those of the designed average PWM model pretty much depends on the difference one is willing to tollerate among two such responses.

3. SOME APPLICATION EXAMPLES

Example 3.1 Let G(s) be the rational transfer function characterizing a causal linear time-invariant plant. It is easy to see, according to the circle cirterion, that if G(s) is positive real (i.e., G(s) is analytic in $Re(s) > 0$, the Nyquist plot of G(s) lies in the closed right half of the complex plane and those poles of G(s) on the imaginary axis are simple and have a nonnegative residue) or strictly positive real (i.e., $G(s-\varepsilon)$ is positive real for some positive ε), then any PWM controller, of the type described in Section 2, will always stabilize such a plant. This positive real requirement, incidentally, is shared by variable structure systems (in plants controlled by a relay function) where the relay is seen as an infinite-gain limit of an average saturation PWM controlled system.

Example 3.2. Consider the problem of designing a stabilizing PWM controller of the saturation type for a distributed linear plant described by :

$$G(s) = e^{-Ts}(s+p)^{-1} \quad ; \qquad p > 0$$

Let $g_{av}(PWM)$ denote the gain of the average PWM controller. Then, $g_{av}(PWM) = \sup_{e \neq 0} |sat(e)||e|^{-1} = b$. The Small Gain theorem (Zames[12-13]) leads to the following sufficient condition for asymptotic stability :

$$g_{av}(PWM).\sup_w |e^{-Tjw}(p+jw)^{-1}| = bp^{-1} < 1 \Rightarrow b < p$$

One may instead use a particularization of the Circle Criterion (Sandberg[14]) for the design case of the saturation controller. This criterion leads to :

$$\inf_w Re[e^{-jwT}(p+jw)^{-1}] > -b^{-1}$$

If for the above plant, a stabilizing design was required for a PWM controller characterized by a constant duty ratio of value $0 < k < 1$, neither the small gain theorem nor the particularization of Sandberg's criterion are conclusive in asessing the stability features of the closed loop average PWM system. This is due to the infinite gain associated to the relay characteristic, the nonminimum phase character of the delayed plant transfer function and the sufficiency of the referred criteria.

Example 3.3. Consider the kinematic and dynamic model of a single-axis externally controlled spacecraft whose orientation is given in terms of the Cayley-Rodrigues representation of the attitude parameter, denoted by ξ (See Dwyer and Sira-Ramirez[15]) . The angular velocity is represented by ω while I stands for the moment of inertia and τ is the applied external torque, restricted by $\tau \in \{-\tau_{max}, 0, \tau_{max}\}$.

$$d\xi/dt = 0.5(1+\xi^2)\omega \quad ; \quad d\omega/dt = \tau / I \qquad (3.1)$$

Given arbitrary initial conditions, a slewing maneuver is required which brings the attitude parameter to a final desired value ξ_d and the angular velocity to a rest equilibrium. For feedback purposes a nonlinear output of the following form is made available :

$$y = \omega - 2\lambda(\xi - \xi_d)/(1+\xi^2) \quad (3.2)$$

with $\lambda < 0$. Notice that if the output function is ideally driven to zero in finite time and the controlled state trajectory is ideally made to stay in such a manifold $y = 0$, then $\omega = 2\lambda(\xi - \xi_d)/(1+\xi^2)$ and the equation governing the attitude parameter ideally becomes linear and of the form : $d\xi/dt = \lambda(\xi - \xi_d)$. The controlled trajectories thus asymptotically converge to $\xi = \xi_d$ with exponential decay rate set by λ while, simultaneously, ω would tend to zero as desired. Without loss of generality we may assume that $\xi_d = 0$.

A PWM controller, with constant duty ratio, k, leads to an average model represented by a nonlinear controlled plant fedback by an ideal relay and a constant gain block of value $M = k\,\tau_{max}$ preceeding the ideal relay. A sliding regime has been shown to globally exist about the zero error manifold $y = 0$ leading to the prescribed asymptotically stable ideal sliding dynamics (See ref. 15) described above.

A PWM controller of the saturation type can also be proposed to regulate the motions towards the vicinity of the zero error manifold and approximately obtain the desired linear stable dynamics (see ref. 1). The average PWM model for the saturation PWM controller, is governed by a nonlinear saturation type of nonlinearity with saturation values ± 1 and a linear gain of value b. In accordance to model (2.3), in this case, the gain M takes the value $M = b\tau_{max}$. Using a Lyapunov stability analysis, it is relatively straightforward to show that the average model trajectories of the saturation PWM controlled plant may be made to asymptotically evolve towards the stabilizing and linearizing surface $y = 0$.

A dead-band controller is quite common in gas reaction control schemes governing satellite maneuvers. This fact motivates the use of a dead band PWM controller, of the type described in the preceeding section, for the above nonlinear plant.

Simulated state responses of the actual dead-band PWM controlled system (which actually coincides entirely with the average PWM responses) are shown in Figure 4. For this simulation, the following values were used $I = 94$ Kg-m^2, $\lambda = -0.11$ s^{-1}, $\tau_{max} = 1.55$ Kg m^2/s^2, $\xi_d = 0$, $d = 50$.

4. CONCLUSIONS AND SUGGESTIONS FOR RESEARCH

In this article a design procedure has been proposed for the specification of stabilizing feedback PWM controllers. It was shown that, under an infinite sampling frequency assumption, an average model is obtained, for every member of a family of PWM ON-OFF-ON controllers, by simply replacing the discontinuous PWM controller by a memoryless type of compensator model. This average model captures the basic qualitative (i.e. stability) features of the actual PWM controlled system. This allows to treat the PWM design problem in a more exact fashion and, in the case of nonlinear plants, resort to well known Lyapunov design techniques. At the same time, one totally circumvents the technical difficulties associated to the fact that PWM operators are indeed unbounded operators on the Banach space of absolutely integrable functions. For the particular case of linear controlled plants, design techniques based on well-known sufficient conditions for asymptotic satability, are made available for PWM controller design. The results were applied, through simple design examples, to PWM controller specification for linear and nonlinear plants. The results here presented can be easily extended to multivariable PWM controllers.

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FIGURES

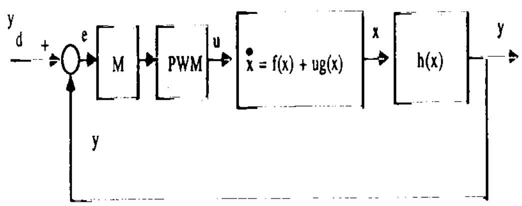

Figure 1. Nonlinear PWM controlled system

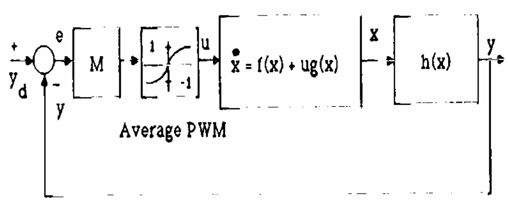

Figure 2. Average Model of PWM Controlled System

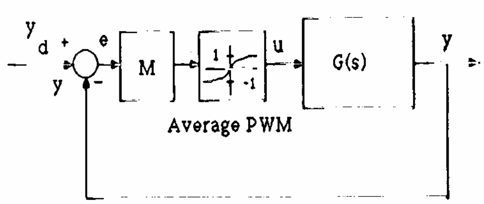

Figure 3. Average Model of Linear Plant Controlled by Pulse-Width-Modulation

Figure 4. State trajectory response of average and actual dead-band PWM Controlled Spacecraft Model