Proceedings of the 29th Conference
on Decision and Control
Honolulu, Hawaii • December 1990

PULSE WIDTH MODULATED CONTROL OF ROBOTIC MANIPULATORS

Hebertt Sira-Ramírez
Control Systems Department
Universidad de Los Andes
Mérida-VENEZUELA

Mohamed Zribi and Shaheen Ahmad
School of Electrical Engineering
Purdue University
West-Lafayette, Indiana
U.S.A.

Abstract

In this article a robust discontinuous feedback control scheme of the Pulse Width Modulation (PWM) type is proposed for the regulation of angular positions of links in robotic manipulators. The discontinuous PWM controller design is carried out on the basis of the extended linearization approach for a suitably defined average PWM controlled model of continuous nature.

1. INTRODUCTION

A Pulse-Width-Modulated (PWM) feedback control scheme is proposed for the regulation of link positions in robotic manipulators. The discontinuous PWM stabilizing controller design is carried out on the basis of a continuous feedback stabilizing controller design, performed for the nonlinear average model of the PWM controlled system, using extended linearization (Bauman [1]). The average model is obtained by formally imposing an infinite sampling frequency assumption on the PWM actuator. The average (piece-wise smooth) controlled system model has been shown to capture all the essential qualitative features (i.e., stability) of the actual (discontinuous) PWM controlled system, provided a sufficiently high sampling frequency is used (See Sira-Ramirez [2]-[3] and Sira-Ramirez *et al* [4]). The actual PWM controller design is easily obtained from the average PWM closed loop stabilizing design.

2. GENERALITIES ABOUT NONLINEAR MULTIVARIABLE PWM CONTROLLED SYSTEMS

Consider the n-dimensional nonlinear PWM controlled system with multiple inputs:

$$\frac{dx(t)}{dt} = f(x(t)) + G(x(t))u(t) \qquad (2.1)$$

regulated by the following multivariable feedback PWM control switching strategy defined for each input component as:

$$u_i = \begin{cases} u_i^+(x) & \text{for } t_k \le t < t_k + \mu_i[x(t_k)]T \\ u_i^-(x) & \text{for } t_k + \mu_i[x(t_k)]T \le t < t_k + T \end{cases} \qquad (2.2)$$

where μ_i (i=1,2,...m) is the i-th piecewise smooth feedback synthesized *duty ratio function* determining the pulse width on which the smooth feedback control laws $u_i^+(x)$ and $u_i^-(x)$ are alternatively enabled within fixed inter-sampling periods of value T (also known as the *duty cycle*). The sampling instants are assumed to be regularly spaced accroding to: $t_{k+1} = t_k + T$. Also, $0 < \mu(t) < 1$ for all t.

This work was supported by the Consejo de Desarrollo Científico, Humanístico y Tecnológico (CDCHT) of the Universidad de Los Andes, under Research Grant I-325-90, by the Consejo Nacional de Investigaciones Científicas y Tecnológicas (CONICIT) of VENEZUELA and by the School of Electrical Engineering of Purdue University.

The *average PWM system* is defined as the smooth dynamical system obtained from (2.1)-(2.2) when the sampling frequency of the PWM actuator, 1/T, grows to infinity. This definition is a mathematical idealization which bears the same connotation and consequences as the *Ideal Sliding Dynamics* in Variable Structure Sytems undergoing *Sliding Regimes* (See Utkin [5], and [2]).

The *average PWM system* is represented by ([4]) :

$$\frac{dz(t)}{dt} = f(z(t)) + \sum_{i=1}^{m} \left[\mu_i u_i^+ + (1-\mu_i)u_i^-\right] g_i(z(t)) \qquad (2.3)$$

where z(t) denotes the *averaged state vector* x(t), and the g_i 's are the smooth column vectors comprising the nxm matrix $G(\cdot)$.

It may be shown, in full generality (See [4] for details), that the *integral equation* associated to the average PWM model (2.3) constitutes a *regular second order perturbation* of the corresponding integral equation representing the discontinuous controlled system (2.1),(2.2). Hence, for identical initial conditions, $x(t_0) = z(t_0)$, as the sampling frequency 1/T tends to infinity in (2.1),(2.2), the actual PWM controlled solutions continuously converge towards that of the average system (2.3). The stability characteristics impossed by a feedback design on the average model (2.3) may, therefore, be completely inherited by (2.1),(2.2), provided a sufficiently high sampling frequency 1/T is enforced on the PWM actuator regulating the actual plant behavior.

A design procedure for nonlinear multivariable PWM controlled plants which synthesizes a stabilizing feedback duty ratio function $\mu(x(t))$ may, thus, be proposed for the actual discontinuously controlled system (2.1),(2.2), on the basis of a stabilizing design prescribed for the average PWM controlled model (2.3) :

1) Obtain the average model (2.3) for the nonlinear multivariable PWM controlled system (2.1)-(2.2).

2) Design a nonlinear stabilizing multivariable feedback control scheme, $\mu[z(t)]$, for the average PWM controlled system (2.3). A constraint must be impossed bounding, between 0 and 1, the values of each duty ratio function component μ_i.

3) Implement, from the derived stable average PWM closed loop system, the actual (discontinuous) PWM stabilizing feedback control law through sampling of the synthesized multivariable duty ratio signal. Furnish a sufficiently high sampling frequency for the PWM actuator, until a desirable agreement is obtained among the actual and the average PWM controlled responses.

3. APPLICATIONS TO THE PWM CONTROL OF ROBOTIC MANIPULATORS

In this section the PWM regulation of a two link robotic manipulator is presented. As a feedback design scheme, synthesizing the appropriate multivariable duty ratio function for the average PWM controlled model, we use an already existing smooth stabilizing feedback control scheme presented by Baumann in [1].

Consider the following state space model of a two link robotic manipulator [1] :

$$\frac{dx_1}{dt} = x_2$$

$$\frac{dx_2}{dt} = \frac{1}{1+\sin^2(x_3)}\left[x_4^2\sin(x_3)+2x_2x_4\sin(x_3)-g\cos(x_1+x_3)-2g\cos(x_1)\right] +$$

$$\frac{1+\cos(x_3)}{1+\sin^2(x_3)}\left[x_2^2\sin(x_3)+g\cos(x_1+x_3)\right]+\frac{u_1}{1+\sin^2(x_3)}-\frac{[1+\cos(x_3)]\,u_2}{1+\sin^2(x_3)}$$

$$\frac{dx_3}{dt} = x_4$$

$$\frac{dx_4}{dt} = -\frac{1+\cos(x_3)}{1+\sin^2(x_3)}\left[x_4^2\sin(x_3)+2x_2x_4\sin(x_3)-g\cos(x_1+x_3)-2g\cos(x_1)\right] +$$

$$\frac{3+2\cos(x_3)}{1+\sin^2(x_3)}\left[-x_2^2\sin(x_3)-g\cos(x_1+x_3)\right]-\frac{[1+\cos(x_3)]\,u_1}{1+\sin^2(x_3)}+\frac{[3+2\cos(x_3)]u_2}{1+\sin^2(x_3)} \qquad (3.1)$$

where g is the acceleration due to gravity, x_1 and x_3 are the angular positions of the links, while x_2 and x_4 are the corresponding angular velocities. The u's represent the input torques. All masses and lengths are taken as unity.

A state-scheduled continuous multivariable stabilizing feedback controller design, based on Extended Linearization, is given by (See [1]):

$$u_1 = [2\cos(x_1)+\cos(x_1+x_3)]\,g - 32[1+\cos(x_3)]\,x_2 - 8[1+\cos(x_3)]\,x_4 +$$

$$16[1+\cos(x_3)](x_{1d}-x_1) + [21+22\cos(x_3)](x_{2d}-x_3)$$

$$u_2 = g\cos(x_1+x_3) - 32x_2 - 8x_4 + 16(x_{1d}-x_1)+[23-\cos(x_3)](x_{2d}-x_3) \qquad (3.2)$$

with x_{1d} and x_{2d} representing , respectively, desired angular positions of links 1 and 2, and acting as reference inputs.

Let us assume one is to constrain the torque control actions u_1 and u_2 to piecewise constant torque specifications of magnitude $\pm u_{1max}$ and $\pm u_{2max}$ for each joint, in a PWM fashion. The given continuous controller design (3.2) may still be utilized on an on-off PWM switching strategy. For this, one simply replaces the control inputs u_1, u_2 in the system model (2.1) by :

$$u_1 = u_{1max}(2w_1-1)\,;\quad u_2 = u_{2max}(2w_2-1) \qquad (3.3)$$

and considers the signals w_1 and w_2 as switch position functions constrained to the discrete set {0,1}. The average model of such a resulting PWM controlled system would be simply obtained by directly replacing the discontinuous signals w_1 and w_2 by the duty ration functions μ_1 and μ_2 , respectively. The duty ratio functions can now be obtained from the continuous controller (3.2) as :

$$\mu_i = \inf\{\,1,\ \sup\{0,\ \bar{\mu}_i(t)\}\,\}\ ;\ i=1,2 \qquad (3.4)$$

with:

$$\bar{\mu}_i = 0.5\left[1+\frac{u_i(x)}{u_{i\,max}}\right]\ ;\ i=1,2. \qquad (3.5)$$

The phase plane behavior, for both links, of the average and the actual PWM controlled manipulator are portrayed in figure 1. A sampling frequency of 10 samples per second was used with u_{1max} = 40.0 Nm and u_{2max} = 20.0 Nm.

4. CONCLUSIONS

A systematic method has been proposed for the design of multivariable nonlinear feedback regulators of discontinuous nature such as those required in systems including PWM actuators. The method is based on the feedback stabilization of a multi-input continuous average model easily derived from the original discontinuous system model. The approximation characteristics of the responses of the actual PWM discontinuous design improve, with respect to those of the average design, as the sampling frequency of the PWM actuator is increased. A PWM feedback stabilization of a two link robotic manipulator was presented using a known state-scheduled feedback controller design based on Extended Linearization. Simulations were performed for both the average and the actual PWM controlled robotic manipulator models .

REFERENCES

[1] W.T.Baumann, "Feedback Control of Multiinput Nonlinear Systems by Extended Linearization," IEEE Transactions on Automatic Control, Vol. AC-33, No. 2, February 1988.

[2] H. Sira-Ramirez, "A Geometric Approach to Pulse-Width-Modulated Control in Nonlinear Dynamical Systems," IEEE Transactions on Automatic Control, Vol. AC-34, No. 2, pp. 184-187. February 1989.

[3] H. Sira-Ramirez, "Design of Nonlinear PWM Controllers : Aerospace Applications," in Variable Structure Control for Robotics and Aerospace Applications K.K. David Young (Ed.) Elsevier Science Publishers. Amsterdam, The Netherlands (to appear).

[4] H. Sira-Ramirez, M. Zribi and S. Ahmad, "Pulse Width Modulated Control of Robotic Manipulators," Technical Report TR-EE-90-16, School of Electrical Engineering, Purdue University. West Lafayette, IN 47907. February 1990.

[5] V. Utkin, Sliding Regimes and Their Applications in Variable Structure Systems, MIR, Moscow, 1978.

Acknowledgement The first author is grateful to Prof. Shaheen Ahmad for his generosity in making possible a short visit to Purdue University, in the winter of 1989, where this research was carried out.

FIGURES

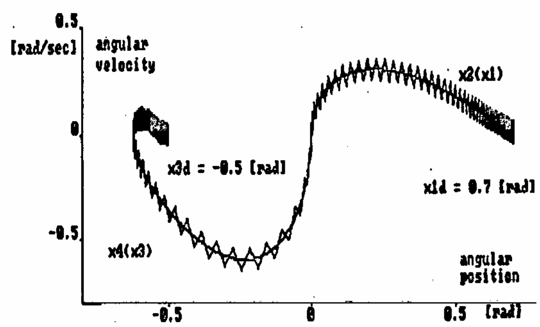

Figure1. Average and Acutal Phase Plane Trajectories of PWM Controlled Manipulator for both links.