AN EXTENDED LINEARIZATION APPROACH TO SLIDING MODE CONTROL OF DC-TO-DC POWER SUPPLIES

Miguel Rios-Bolivar and Hebertt Sira-Ramirez
Departamento Sistemas de Control
Universidad de Los Andes
Mérida, VENEZUELA

Abstract The method of Extended Linearization is proposed for the systematic solution of sliding mode controller design in DC-to-DC Power Converters of the Boost and the Buck-Boost type. A nonlinear sliding surface with suitable properties is synthesized on the basis of the extension of a linear design carried out on the average incremental model of the converter. The obtained feedback strategies lead to asymptotically stable Sliding Modes with remarkable self-scheduling properties. Simulation examples are presented for illustrative purposes.

1. INTRODUCTION

In this article, a new method is proposed for the synthesis of stabilizing Sliding Modes (Utkin [1]) in bilinear switch-controlled DC-to-DC Power Supplies. The method of *Extended Linearization*, developed by Rugh and his co-workers in [2]-[3] is extensively used for the specification of the nonlinear sliding surface. The design technique entitles resorting primarily to parametrized linearization, about a general constant equilibrium point, of a suitably defined average converter model. Using linear sliding-mode design results (Utkin [1]), a traditional stabilizing sliding hyperplane design is carried out on the basis of the family of parametrized linear systems. A most convenient framework for this purspose consists in placing the average incremental (linearized) model in *controllable canonical form* by means of standard incremental state coordinates transformation. The linear design is led by imposing a set of stable eigenvalues, chosen independently of the constant operating point, on the resulting *ideal sliding dynamics*. The core of the method lies in specifying a suitable *extension* of the sliding hyperplane design and thus obtaining a *nonlinear switching manifold*. The designed surface, which is tangent to the prescribed hyperplane, contains the equilibrium point and it is parametrizable in terms of the nominal operating conditions. A conceptual advantage of this procedure is that the resulting ideal sliding dynamics can always be made locally linear (modulo a suitable local diffeomorphic state coordinate transformation directly derivable from the linearized system model). A direct integration procedure of the synthesized sliding hyperplane results in a sliding surface, which is generally nonuniquely defined. This feature is characteristic in the standard integration schemes used in Extemded Linearization controller design techniques [2]-[3]. The nonlinear sliding mode switching logic is synthesized on the basis of the obtained nonlinear *sliding surface coordinate function*. The region of existence of a stabilizing sliding regime is assessed from knowledge of the parametrized *equivalent control*.

The proposed sliding mode controller exhibits a most important property, aside from those already mentioned, related to adaptability to sudden changes in the nominal operating conditions. Such self-scheduling properties lie, in general, at the heart of the extended linearization method. Thus, if a desirable, or accidental, change of the nominal operating conditions of the converter takes place, the proposed discontinuous control scheme automatically creates a sliding regime which stabilizes the converters trajectories to the new equilibrium point, located on a new corresponding sliding surface. This last property is clearly inherited from well known advantages of the extended linearization technique, and it results in no need for a "scheduling" process of the sliding manifold and of the switching "gains". Sliding regimes, based on Extended Linearization, have been also recently proposed by the authors for a variety of aerospace control problems [4],[5].

[1] This research was supported by the Consejo de Desarrollo Científico, Humanístico y Tecnológico of the Universidad de Los Andes under Research Grant I-325-90.

Section 2 of this article presents a general procedure for synthesizing stable nonlinear sliding manifolds, for a large class of switched controlled systems, via Extended Linearization. Section 3 presents applications of the proposed design procedure to the Boost Converter and the Buck-Boost Converter Models. The results are accompanied by simulations. Section 4 summarizes the conclusions and suggestions for further work.

2. AN EXTENDED LINEARIZATION SYNTHESIS PROCEDURE FOR SLIDING MODE CONTROLLERS IN NONLINEAR SWITCHING SYSTEMS

2.1 Problem Formulation

Consider the n-dimensional *switched controlled dynamical nonlinear system* :

$$\dot{x} = f(x) + u\, g(x) + \eta \qquad (2.1)$$

where $f(\cdot)$ and $g(\cdot)$ are smooth vector fields defined on an open set of R^n. and η is a constant vector. The control input function u takes values on the binary discrete set {0,1}. This general formulation corresponds to the typical situation in bilinear switched controlled circuits as well as in the most common category of switch-mode controlled DC-to-DC power converters (See Sira-Ramirez [6])

Associated to (2.1), and under the assumption of *fast switchings*, we define an *average model* by formally replacing the discontinuous control function u in (2.1) by a continuous piecewise smooth function μ

$$\dot{z} = f(z) + \mu\, g(z) + \eta \qquad (2.2)$$

where the state vector is now denoted by z, just to differentiate it from the actual state x.

One of the main difficulties in using any linearization method on the specification of a controller for switching systems of the form (2.1) lies in the fact that (2.1) cannot be linearized due the discrete nature of u and the high frequency control discontinuities associated with the operation of such class of systems. However, in two important discontinuous feedback control schemes represented by sliding mode controlled systems and PWM control based strategies, an infinite switching frequency average model of (2.1) may be obtained, precisely, in the form of equation (2.2). In both cases the ideally controlled dynamics, or the average model, is obtained by substituting the discontinuous control function u by the piecewise smooth *equivalent control* or by the piecewise smooth *duty ratio function*. In any of the two above cases, the *average control function* μ takes values in the closed interval [0,1]. Notice that a linearization procedure is entirely feasible, possibly in a local fashion, on average models of the form (2.2). This justifies our use of the extended linearization technique in switching systems (see also Sira-Ramirez [7])

The average controlled system (2.2) is assumed to have a continuous family of constant state equilibrium points, Z(U), corresponding to average constant inputs, $\mu = U$, which are neither 0 nor 1, i.e., $0 < U < 1$.The equilibrium points satisfy :

$$f(Z(U)) + Ug(Z(U)) + \eta = 0.$$

The pair of linearized maps, given by :

$$[\, \partial f/\partial x(Z(U)) + U \partial g/\partial x(Z(U)),\ g(Z(U))]$$

is assumed to be *controllable*.

BRAZILIAN POWER ELECTRONICS CONFERENCE, SOBRAEP/UFSC, DEC 1991

It is desired to locally maintain, in a stable fashion, the trajectories of the nonlinear system (2.1) in the vicinity of the constant nominal average equilibrium trajectory, Z(U), by means of a sliding motion suitably induced on a manifold S, which contains such an equilibrium point. In other words, it is required to synthesize 1) a nonlinear sliding surface S, parametrized by the nominal average control input U, of the form:

$$S = \{ x \in R^n,\ s(x,U) = 0 \} \qquad (2.3)$$

such that $s(Z(U),U) = 0$, and 2) an associated variable structure control law :

$$u(x,U) = \begin{cases} 1 & \text{for } s(x,U) > 0 \\ 0 & \text{for } s(x,U) < 0 \end{cases} \qquad (2.4)$$

which automatically forces every small state deviation, from the nominal operating conditions, to zero, via the local creation of a stable sliding regime, taking place on S, and leading the state trajectory to X(U). This stabilization is to be accomplished, of course, modulo small chattering around the prescribed equilibrium point.

In order to specify such a sliding manifold we propose to resort to the method of *Extended Linearization* ([2]-[3]) as indicated in the following paragraphs.

2.2 A Nonlinear Sliding Mode Controller Design based on Extended Linearization .

1) Linearize the average dynamical system (2.2) about each point in the family of average constant operating trajectories, [U,Z(U)], obtaining the following parametrized family of linear systems:

$$\dot{z}_\delta = A(U)z_\delta + b(U)\mu_\delta \qquad (2.5)$$

where, for fixed U, the input and state perturbation variables are defined, respectively, as: $\mu_\delta = \mu(t)-U$, $z_\delta(t) = z(t)-Z(U)$, while the $n \times n$ matrix A(U) and the n-vector b(U) are defined as :

$$A(U) := \frac{\partial f}{\partial x}(Z(U)) + U\frac{\partial g}{\partial x}(Z(U)) \ ; \ b(U) := g(Z(U)) \qquad (2.6)$$

Since the pair [A(U),b(U)] is assumed to be controllable, a similarity transformation exists of the form:

$$\zeta_\delta = P(U)z_\delta =: [p_1(U), p_2(U), \cdots, p_n(U)]z_\delta \qquad (2.7)$$

such that (2.5) may be represented as a *controllable canonical realization.* The nonsingular matrix P(U) is obtained from the well known expression :

$$P^{-1}(U) = \left[b(U),\ A(U)b(U), \cdots, A^{n-1}(U)b(U)\right] M(U) \qquad (2.8)$$

$$M(U) = \begin{bmatrix} \alpha_1(U) & \alpha_2(U) & \cdots & 1 \\ \alpha_2(U) & \alpha_3(U) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{n-1}(U) & 1 & \cdots & \vdots \\ 1 & 0 & \cdots & 0 \end{bmatrix}$$

where:

$$\det[\lambda I - A(U)] = \lambda^n + \alpha_{n-1}(U)\lambda^{n-1} + \alpha_{n-2}(U)\lambda^{n-2} + \ldots + \alpha_0(U).$$

2) Obtain the transformed system in *controllable canonical form* as:

$$\begin{aligned} \dot\zeta_{1\delta} &= \zeta_{2\delta} \\ \dot\zeta_{2\delta} &= \zeta_{3\delta} \\ &\vdots \\ \dot\zeta_{(n-1)\delta} &= \zeta_{n\delta} \\ \dot\zeta_{n\delta} &= -\alpha_{n-1}(U)\zeta_{n\delta} - \alpha_{n-2}(U)\zeta_{(n-1)\delta} - \cdots - \alpha_0(U)\zeta_{1\delta} + u_\delta \end{aligned} \qquad (2.9)$$

3) Use as a *sliding surface* the linear manifold :

$$\Sigma_\delta = \{ \zeta_\delta \in R^n : \sigma_\delta(\zeta_\delta) = \sum_{i=1}^{n} c_i\zeta_{i\delta} = c^T\zeta_\delta = 0 \ ; \ c_n = 1 \} \qquad (2.10)$$

and choose the coefficients c_i, independently of the operating point [Z(U),U], such that the roots of the *characteristic polynomial* :

$$\sum_{i=1}^{n} c_i\lambda^{i-1} = 0 \qquad (2.11)$$

for the (reduced) *linear ideal sliding dynamics* are specified at convenient locations in the open left half of the complex plane. i.e., so that the autonomous ideal sliding mode dynamical system:

$$\begin{aligned} \dot\zeta_{1\delta} &= \zeta_{2\delta} \\ \dot\zeta_{2\delta} &= \zeta_{3\delta} \\ &\vdots \\ \dot\zeta_{(n-1)\delta} &= -c_{n-1}\zeta_{(n-1)\delta} - c_{n-2}\zeta_{(n-2)\delta} - \cdots - c_1\zeta_{1\delta} \end{aligned} \qquad (2.12)$$

is asymptotically stable toward the origin of transformed coordinates.

4) Obtain, on the basis of the previously described design steps, the parametrized sliding hyperplane specification in terms of the average perturbed state coordinates ζ_δ, as follows:

$$S_\delta = \{ z_\delta \in R^n : s_\delta(z_\delta, U) := \sigma_\delta(P(U)z_\delta) = c^TP(U)z_\delta = 0 \} \qquad (2.13)$$

5) Obtain a nonlinear sliding manifold S, characterized by the *parametrized surface coordinate function* $s(z,U) = 0$ such that its corresponding linearization about the operating point [Z(U),U], yields back the sliding hyperplane (2.13). In other words, find a nonlinear switching surface, in average state coordinates z, which is tangent to the sliding hyperplane (2.13) at the equilibrium point. This sliding manifold can be immediately expressed in actual state coordinates x as $s(x,U) = 0$

5a) Sliding Manifold We must, thus, find a nonlinear sliding surface coordinate function s(x,U), parametrized by the constant operating point U, such that the following relations are satisfied:

$$\left.\frac{\partial s(x,U)}{\partial x}\right|_{x=X(U)} = c^TP(U) = \left[c^Tp_1(U), c^Tp_2(U), \cdots, c^Tp_n(U)\right] \qquad (2.14)$$

or, componentwise :

$$\left.\frac{\partial s(x,U)}{\partial x_i}\right|_{x=X(U)} = c^Tp_i(U) \ ; \ i = 1,2,\ldots,n \qquad (2.15)$$

with the additional (boundary) condition : $s(X(U),U) = 0$. Where $X(U) = Z(U)$.

Remark In general, there are many parametrized sliding surface coordinate functions, s(x,U), which satisfy relations (2.14) and the boundary condition. Such a lack of uniqueness of solution may not be totally inconvenient. However, the following *direct integration procedure*, inspired by the results in Rugh [2], allows one to obtain a nonlinear sliding manifold in a systematic manner :

1) Assume, without loss of generality, that the first component $X_1(U)$ of the vector X(U) is invertible, i.e., let there exist a unique solution, $X_1^{-1}(x_1)$, for U in the equation $x_1 = X_1(U)$.

2) It can be verified, after partial differentiation with respect to the components of the vector x and substitution of the equilibrium point, that the following manifold is one possible solution for the required parametrized nonlinear sliding manifold:

$$S = \{\ x \in R^n : s(x,U) = \int_U^{X_1^{-1}(x_1)} c^T P(\upsilon) \frac{dX(\vartheta)}{d\vartheta} d\vartheta$$

$$+ \sum_{j=2}^{n} c^T p_j(X_1^{-1}(x_1)) \left[x_j - X_j(X_1^{-1}(x_1))\right] = 0\ \}$$

(2.16) ∩

5b) Equivalent Control Once the nonlinear sliding surface coordinate function s(x,U) is known, computation of the equivalent control follows by imposing the well known (ideal) invariance conditions, which make of the switching manifold a *local integral manifold* of the smoothly controlled system:

$$s(x,U) = 0 \quad , \quad \frac{d}{dt}\, s(x,U) = 0 \qquad (2.17)$$

5c) Sliding Mode Switching Logic A nonlinear sliding mode switching strategy is usually synthesized such that the sliding mode existence conditions (Utkin [1]) are satisfied, at least, in a local fashion. Such well known conditions are given by:

$$\lim_{s \to 0^+} \frac{ds(x,U)}{dt} < 0 \quad ; \quad \lim_{s \to 0^-} \frac{ds(x,U)}{dt} > 0 \qquad (2.18)$$

It has been shown that, for nonlinear systems which are linear in the scalar control input, a necessary and sufficient conditions for the local existence of a sliding mode is that the equivalent control locally exhibits values which are *intermediate* between the extreme numerical values representing the the switch position values (i.e., $0 < u^{EQ}(x,U) < 1$). The region of existence of such a sliding regime coincides, precisely, with the region where such an *intermediacy* condition is satisfied by the equivalent control. One may, therefore, synthesize the nonlinear sliding mode switching logic from knowledge of the sliding manifold coordinate function, s(x,U), as follows :

$$u(x,U) = \frac{1}{2}[1 + \ \mathrm{sgn}\ s(x,U)] \qquad (2.19)$$

In more general cases, where there is no special input structure to the system, the above switching logic, or any one satisfying the equivalent control intermediacy condition, may still locally create a sliding regime provided the system exhibits a *control foliation property* (See Sira-Ramirez [8]). For the class of application examples we will be presenting in the next section, a switching control law of the form (2.19) suffices.

Notice that due to the discrete nature of the control input set, the equivalent control function is necessarily limited to the closed interval [0,1]. Thus, the region of existence of a sliding mode, on the switching manifold, may not be global in the state space of the system. A necessary and sufficient condition for assessing the region of existence of sliding regimes in the above class of systems was given by Sira-Ramirez in [9]. Such region is simply defined by:

$$0 < u^{EQ}(x,U) < 1 \qquad (2.20)$$

3. APPLICATIONS TO SLIDING MODE CONTROLLER DESIGN FOR BILINEAR SWITCH CONTROLLED CONVERTERS

In this section we present applications of the extended linearization based sliding mode control synthesis procedure, developed in Section 2, for typical bilinear switch-mode DC-to-DC Power converters.

3.1 Boost Converter

Consider the boost converter model shown in figure 1. This converter is described by the following bilinear system of controlled differential equations :

$$\begin{aligned} dx_1/dt &= - w_o x_2 + u\, w_o x_2 + b \\ dx_2/dt &= w_o x_1 - w_1 x_2 - u\, w_o x_1 \end{aligned} \qquad (3.1)$$

where, $x_1 = I\sqrt{L}$, $x_2 = V\sqrt{C}$ represent normalized input current and output voltage variables, respectively. The quantity $b = E/\sqrt{L}$ is the normalized external input voltage and , $w_o = 1/\sqrt{LC}$ and $w_1 = 1/RC$ are, respectively, the LC (input) circuit natural oscillating frequency and the RC (output) circuit time constant. The variable u denotes the switch position function, acting as a control input, and taking values in the discrete set { 0, 1}. System (3.1) is of the same form as (2.1), with $\eta = [\,b \quad 0\,]'$. We now summarize, according to the theory presented in the previous section, the formulae leading to a nonlinear sliding mode controller design for the average model of (3.1) using extended linearization.

Average Boost converter model

$$\begin{aligned} dz_1/dt &= - w_o z_2 + \mu\, w_o z_2 + b \\ dz_2/dt &= w_o z_1 - w_1 z_2 - \mu w_o z_1 \end{aligned} \qquad (3.2)$$

Constant parametrized operating equilibrium points

$$\mu = U\ ; \quad Z_1(U) = b w_1/[w_o^2(1-U)^2]\ ; \qquad Z_2(U) = b/[w_o(1-U)] \qquad (3.3)$$

Parametrized family of linearized systems about the constant operating points

$$\frac{d}{dt}\begin{bmatrix} z_{1\delta}(t) \\ z_{2\delta}(t) \end{bmatrix} = \begin{bmatrix} 0 & -w_0(1-U) \\ w_0(1-U) & -w_1 \end{bmatrix} \begin{bmatrix} z_{1\delta}(t) \\ z_{2\delta}(t) \end{bmatrix} + \begin{bmatrix} b/(1-U) \\ -b w_1/[w_0(1-U)^2] \end{bmatrix} \mu_\delta \qquad (3.4)$$

with :

$$z_{i\delta}(t) = z_i(t) - Z_i(U)\ ; \ i = 1, 2.\ ; \quad \mu_\delta(t) = \mu(t) - U.$$

Transformation of linearized family to controllable canonical form

$$\begin{bmatrix} \zeta_{1\delta} \\ \zeta_{2\delta} \end{bmatrix} = \frac{w_0^2(1-U)^3}{b^2(\,2w_1^2 + w_0^2(1-U)^2)} \begin{bmatrix} \dfrac{b w_1}{w_0^2(1-U)^2} & \dfrac{b}{w_0(1-U)} \\ b & -\dfrac{2bw_1}{w_0(1-U)} \end{bmatrix} \begin{bmatrix} z_{1\delta} \\ z_{2\delta} \end{bmatrix} \qquad (3.5)$$

Prametrized family of linearizations in controllable canonical form

$$\begin{aligned} \dot{\zeta}_{1\delta} &= \zeta_{2\delta} \\ \dot{\zeta}_{2\delta} &= - w_0^2(1-U)^2 \zeta_{1\delta} - w_1 \zeta_{2\delta} + \mu_\delta \end{aligned} \qquad (3.6)$$

Linear sliding surface and ideal sliding dynamics in transformed state coordinates

$$\sigma_\delta(\zeta_\delta) = \zeta_{2\delta} + c_1 \zeta_{1\delta} \qquad (3.7)$$

$$\dot{\zeta}_{1\delta} = - c_1 \zeta_{1\delta}\ ;\ c_1 > 0 \qquad (3.8)$$

Linear sliding surface in original (average) state coordinates

$$s_\delta(z_\delta) = \left[b + \frac{c_1 b w_1}{w_0^2(1-U)^2}\right] z_{1\delta} + \left[\frac{c_1 b}{w_0(1-U)} - \frac{2bw_1}{w_0(1-U)}\right] z_{2\delta} = 0 \qquad (3.9)$$

Nonlinear sliding surface, equivalent control and sliding mode controller

$$s(x,U) = b\,[x_1 - Z_1(U)] + \frac{1}{2} c_1 \left[x_1^2 - Z_1^2(U)\right] + \frac{c_1 - 2w_1}{2}\left[x_2^2 - Z_2^2(U)\right] = 0 \qquad (3.10)$$

$$u^{EQ}(x,U) = 1 - \frac{b(b + c_1 x_1) - w_1(c_1 - 2w_1)x_2^2}{w_0(b + c_1 x_1)x_2 - w_0(c_1 - 2w_1)x_1 x_2} \qquad (3.11)$$

$$u = \frac{1}{2}\,[1 + \mathrm{sgn}\ s(x,U)\,] \qquad (3.12)$$

The region of existence of a sliding mode on the switching manifold, according to (2.20), is given by the zone bounded between the following two curves in the x_1-x_2 coordinate plane (see figure 2).

$$b(b+c_1x_1)-w_1(c_1-2w_1)x_2^2 = 0 \tag{3.13}$$

$$\frac{b(b+c_1x_1)-w_1(c_1-2w_1)x_2^2}{w_0(b+c_1x_1)x_2-w_0(c_1-2w_1)x_1x_2} = 1 \tag{3.14}$$

A local diffeomorphic state coordinate transformation, which can be inferred from the linearized transformation (3.5) takes the average ideal sliding dynamics into an autonomous stable linear system.

$$\xi_1 = \frac{1}{2}(z_1^2 + z_2^2) \quad ; \quad \xi_2 = bz_1 - w_1z_2^2 \tag{3.15}$$

This transformation coincides with the exact linearization transformation found by Sira-Ramirez and Ilic [10] and, not surprisingly, is the same found by pseudolinearization techniques (see Sira-Ramirez [11]). The interpretation of (3.13) in terms of total average energy and average consumed power can be found in [10].

In the new coordinates (3.13), the ideal sliding dynamics is given by:

$$\dot{\xi}_1 = -c_1(\xi_1 - \frac{X_1^2+X_2^2}{2}) \quad ; \quad \dot{\xi}_2 = -c_1(\xi_2 - \frac{bX_1-w_1X_2^2}{2}) = -c_1\xi_2 \tag{3.16}$$

which is evidently linear, as claimed from the outset.

3.2 A simulation example

A boost converter circuit with parameter values : R = 30 Ω. C = 20μF, L = 20mH and E = 15 Volts was considered for sliding mode controller design based on nonlinear switching manifolds computed via extended linearization. The constant operating value of μ was chosen to be U = 0.1619 while the corresponding desirable normalized constant output voltage turned out to be $Z_2(0.1619)$ = 0.08. Figure 2 shows several state trajectories corresponding to different initial conditions set on the ideal boost converter model controlled by a sliding regime (3.10)-(3.12). The average controlled state variables, z_1 and z_2, are shown to converge toward the desirable equilibrium points. Figure 3 shows the effect of a sudden step change in the average equilibrium value of the converter output x_2 from 0.08 to 0.2 (the corresponding change in the operating point of the duty ratio was from 0.1619 to 0.665).

3.3 Buck-Boost Converter

Consider the buck-boost converter model (see figure 4). This device is described by the following constant bilinear state equation model :

$$\begin{aligned} dx_1/dt &= w_0x_2 - u\, w_0x_2 + u\, b \\ dx_2/dt &= -w_0x_1 - w_1x_2 + u\, w_0x_1 \end{aligned} \tag{3.17}$$

where, $x_1 = I\sqrt{L}$, $x_2 = V\sqrt{C}$ represent normalized input current and output voltage variables respectively, $b = E/\sqrt{L}$ is the normalized external input voltage and it is here assumed to be a negative quantity (i.e., reversed polarity) while , $w_0 = 1/\sqrt{LC}$ and $w_1 = 1/RC$ are, respectively, the LC (input) circuit natural oscillating frequency and the RC (output) circuit time constant. The switch position function, acting as a control input, is denoted by u and takes values in the discrete set {0, 1} . System (3.17) is of the same form as (2.1) , with $\eta = 0$ and $g = [\,-w_0x_2+b \quad w_0x_1\,]'$. We now summarize the formulae leading to a nonlinear sliding mode controller design for the buck-boost model (3.17).

Average Buck-Boost converter model

$$\begin{aligned} dz_1/dt &= w_0z_2 - \mu\, w_0z_2 + \mu b \\ dz_2/dt &= -w_0z_1 - w_1z_2 + \mu w_0z_1 \end{aligned} \tag{3.18}$$

Constant equilibrium points

$$\mu = U \; ; \; Z_1(U) = bUw_1/[w_0^2(1-U)^2] \; ; \; Z_2(U) = -bU/[w_0(1-U)] \tag{3.19}$$

Parametrized family of linearized systems about the constant operating points

$$\frac{d}{dt}\begin{bmatrix} z_{1\delta}(t) \\ z_{2\delta}(t) \end{bmatrix} = \begin{bmatrix} 0 & w_0(1-U) \\ -w_0(1-U) & -w_1 \end{bmatrix}\begin{bmatrix} z_{1\delta}(t) \\ z_{2\delta}(t) \end{bmatrix} + \begin{bmatrix} b/(1-U) \\ bw_1U/[w_0(1-U)^2] \end{bmatrix}\mu_\delta \tag{3.20}$$

with :

$$z_{i\delta}(t) = z_i(t) - Z_i(U) \; ; \; i = 1, 2. \; ; \; \mu_\delta(t) = \mu(t) - U.$$

Transformation of linearized family to controllable canonical form

$$\begin{bmatrix} \zeta_{1\delta} \\ \zeta_{2\delta} \end{bmatrix} = \frac{w_0^2(1-U)^3}{b^2\left(w_1^2U(1+U) + w_0^2(1-U)\right)}\begin{bmatrix} \dfrac{bw_1U}{w_0^2(1-U)^2} & -\dfrac{b}{w_0(1-U)} \\ b & \dfrac{bw_1(1+U)}{w_0(1-U)} \end{bmatrix}\begin{bmatrix} z_{1\delta} \\ z_{2\delta} \end{bmatrix} \tag{3.21}$$

Prametrized family of linearizations in controllable canonical form

$$\begin{aligned} \dot{\zeta}_{1\delta} &= \zeta_{2\delta} \\ \dot{\zeta}_{2\delta} &= -w_0^2(1-U)^2\zeta_{1\delta} - w_1\zeta_{2\delta} + \mu_\delta \end{aligned} \tag{3.22}$$

Linear sliding surface and ideal sliding dynamics in transformed state coordinates

$$\sigma_\delta(\zeta_\delta) = \zeta_{2\delta} + c_1\zeta_{1\delta} \tag{3.23}$$

$$\dot{\zeta}_{1\delta} = -c_1\zeta_{1\delta} \; ; \; c_1 > 0 \tag{3.24}$$

Linear sliding surface in original (average) state coordinates

$$s_\delta(z_\delta) = \left[b + \frac{c_1bw_1U}{w_0^2(1-U)^2}\right]z_{1\delta} + \left[\frac{bw_1(1+U)}{w_0(1-U)} - \frac{bc_1}{w_0(1-U)}\right]z_{2\delta} = 0 \tag{3.25}$$

Nonlinear sliding surface, equivalent control and sliding mode controller

$$s(x,U) = b\,[x_1 - Z_1(U)] + \frac{c_1}{2}\left[x_1^2 - Z_1^2(U)\right] + \frac{c_1-2w_1}{2}\left[x_2^2 - Z_2^2(U)\right] - \frac{b}{w_0}(c_1-w_1)[x_2 - Z_2(U)] = 0 \tag{3.26}$$

$$u^{EQ}(x,U) = 1 - \frac{bw_0(b+c_1x_1)-w_1w_0(c_1-2w_1)x_2^2 + bw_1(c_1-w_1)x_2}{w_0b(b+w_1x_1)-w_0^2(b+2w_1x_1)x_2} \tag{3.27}$$

$$u = \frac{1}{2}\,[1 + \operatorname{sgn} s(x,U)\,] \tag{3.28}$$

Bounding curves for the region of existence of a sliding mode

$$bw_0(b+c_1x_1)-w_1w_0(c_1-2w_1)x_2^2 + bw_1(c_1-w_1)x_2 = 0 \tag{3.29}$$

$$\frac{bw_0(b+c_1x_1)-w_1w_0(c_1-2w_1)x_2^2 + bw_1(c_1-w_1)x_2}{w_0b(b+w_1x_1)-w_0^2(b+2w_1x_1)x_2} = 1 \tag{3.30}$$

Linearizing local diffeomorphic state coordinate transformation for average ideal sliding dynamics

$$\xi_1 = \frac{1}{2}\left[z_1^2 + \left(z_2 - \frac{b}{w_0}\right)^2\right] \; ; \; \xi_2 = bz_1 - w_1z_2\left(z_2 - \frac{b}{w_0}\right) \tag{3.31}$$

Ideal sliding dynamics in transformed coordinates

$$\dot{\xi}_1 = -c_1\left[\xi_1 - \frac{X_1^2 + \left(X_2 - \frac{b}{w_0}\right)^2}{2}\right] \; ; \; \dot{\xi}_2 = -c_1\xi_2 \tag{3.32}$$

3.4 A simulation example

A buck- boost converter circuit with the same parameter values as in the previous example, was considered for nonlinear sliding mode controller design. The constant operating value of μ was chosen to be $U = 0.6508$ while the corresponding desirable normalized constant output voltage turned out to be $Z_2(0.6508) = -0.125$. Figure 5 shows several state trajectories corresponding to different initial conditions set on the buck-boost converter model controlled by the sliding mode based regulator of the form (3.26)-(3.28). The average controlled state variables, z_1 and z_2, are shown to converge toward the desirable equilibrium point represented by $Z_1(U) = 0.3773$ and $Z_2(U) = -0.125$ Figure 6 shows the ideal average controlled state variables evolution when subject to a sudden step change in the output desired equilibrum value, from -0.125 to -0.05 (the corresponding change in the operating point of the duty ratio was from 0.6508 to 0.4271).

4. CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

A systematic approach has been proposed for the synthesis of families of nonlinear sliding surfaces, parametrized by constant equilibrium points, defining sliding-mode regulators for DC-toDC power converters. The method entitles the use of the *extended linearization* technique for the specification of the nonlinear switching manifold. On the basis of the proposed parametrized nonlinear manifold, one specifies -in a standard fashion- the associated equivalent control, the required switching strategy and the sliding mode existence region. One of the main advantages of the proposed regulator design scheme resides in the "self-scheduling" properties of the synthesized controller.

The proposed design scheme exhibits the following features: 1) The approach benefits from an extensive list of well known theoretical contributions for design of linear sliding modes, including efficient computer packages already developed for such design tasks. 2) The possibilities of nontrivial applications can be greatly enhanced, and carried out, by means of existing algebraic manipulation systems. 3) The method naturally enjoys rather useful self-scheduling properties when nominal operating conditions are abruptly changed. This is particularly important in the field of control of mechanical manipulators, aerospace systems and other practical nonlinear control application areas. 4) The method developed in this article also constitutes an alternative approach, for approximate linearization of nonlinear systems, to the method developed by Bartolini and Zolezzi in [12].

As a topic for future work, practical implementation of the switching regulators can be attempted on a real converter. Also, automation of the design process via computational algebra packages, such as MACSYMA, REDUCE, or MAPLE, is strongly encouraged.

REFERENCES

[1] V.I. Utkin, Sliding Modes and Their Applications in Variable Structure Systems, Mir Publishers: Moscow 1978.

[2] W.J. Rugh, "The Extended Linearization Approach for Nonlinear Systems Problems," in Algebraic and Geometric Methods in Nonlinear Control Theory, pp. 285-309. M. Fliess and M. Hazewinkel (Eds.) D. Reidel Publishing Company: Dordrecht, 1986.

[3] W.T. Baumann, and W.J. Rugh, "Feedback Control of Nonlinear Systems by Extended Linearization," IEEE Transactions on Automatic Control, Vol. AC-31, No. 1, pp. 40-46, Jan. 1986.

[4] H. Sira-Ramirez and M. Rios-Bolivar, "Synthesis of Sliding-Mode Controllers for Nonlinear Systems via Extended Linearization," Dynamics and Control Vol. 1, No. 3, pp. 321-345, 1991.

[5] H. Sira-Ramirez and M. Rios-Bolivar, "Sliding Mode Controllers for Nonlinear Systems: An Extended Linearization Approach," First IFAC Symposium on Design Methods and Control Systems, Zurich, September 4-6, 1991.

[6] H. Sira-Ramirez, " Sliding Motions in Bilinear Switched Networks, " IEEE Transactions on Circuits and Systems, Vol. CAS-34, No.8 pp. 919-933. August 1987.

[7] H. Sira-Ramirez, "An Extended Linearization Approach to Sliding Mode Control of Nonlinear Systems," 1991 American Control Conference, Boston, June 1991.

[8] H.Sira-Ramirez, "Sliding Regimes in General Nonlinear Systems : A Relative Degree Approach, " International Journal of Control, Vol. 50, No. 4, pp. 1487-1506, October 1989.

[9] H.Sira-Ramirez,"Differential Geometric Methods in Variable Structure Control," International Journal of Control, Vol. 48, No. 4, pp. 1359-1390, Oct. 1988.

[10] H. Sira-Ramirez and M. Ilic, " Exact Linearization in Switch Mode DC-to-DC Power Converters, " International Journal of Control.. Vol. 50, No. 2, pp. 511-524, August 1989.

[11] H. Sira-Ramirez, "Switched Control of Bilinear Converters via Pseudolinearization, " IEEE Transactions on Circuits and Systems. Vol. CAS-36, No. 6. pp. 858-865. June 1989.

[12] G. Bartolini, and T. Zolezzi, "Asymptotic Linearization of Uncertain Systems by Variable Structure Control," Systems and Control Letters, Vol. 10, No. 2, pp. 111-117, Feb. 1988.

FIGURES

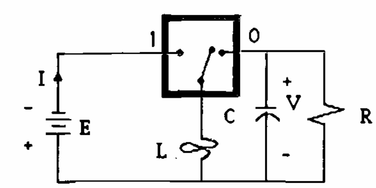

Figure 1. The Boost converter

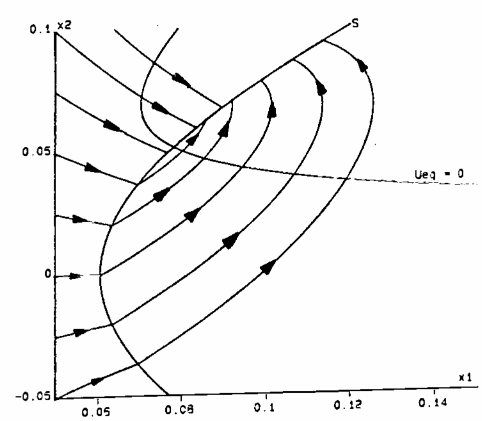

Figure 2. Sliding mode controlled trajectories for the Boost converter with the region of existence of sliding mode.

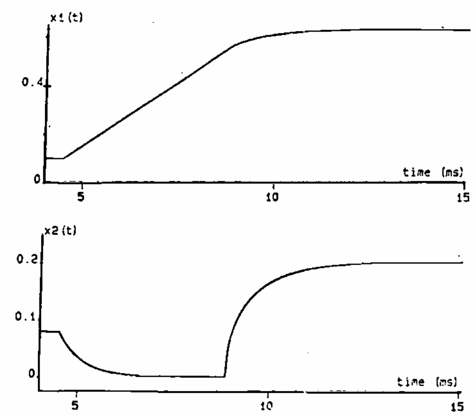

Figure 3. State variables responses to sudden change in the operating point for sliding mode controlled Boost converter.

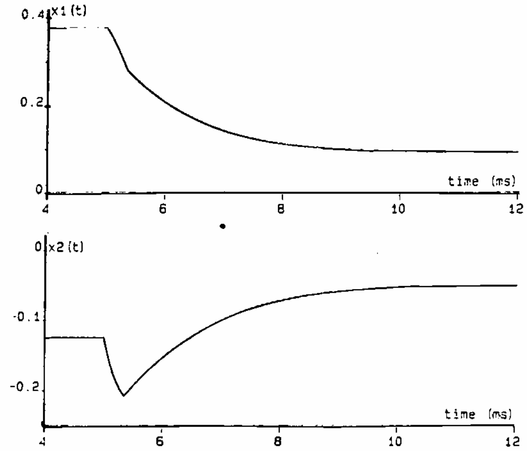

Figure 6. State variables responses to sudden change in the operating point for sliding mode controlled Buck-Boost converter.

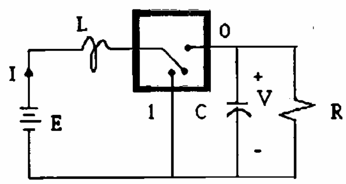

Figure 4. The Buck-Boost converter

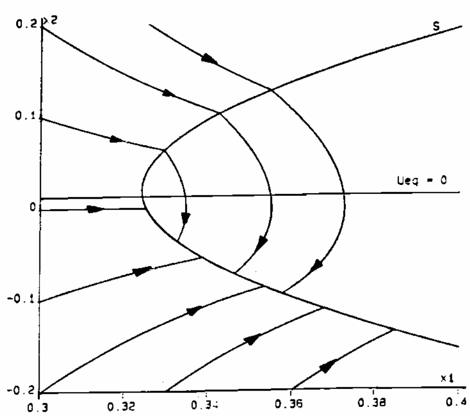

Figure 5. Sliding mode controlled trajectories for the Buck-Boost converter with the region of existence of sliding mode.