DYNAMICAL FEEDBACK REGULATION IN DC-TO-DC POWER CONVERTERS: AN EXTENDED SYSTEM APPROACH

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Abstract

Nonlinear dynamical feedback regulation is proposed for the local asymptotic stabilization of output load voltage in pulse-width-modulation controlled switch-mode DC-to-DC power converters. The controller design is based on linearization of the converters' average extended system model, appropriately placed in Isidori's Normal Canonical Form. The performance of the compensators is evaluated through computer simulations.

1. Introduction

In this article, the notion of the "*extended system*" (Nijmeijer and Van der Schaft [1]) is used for synthesizing nonlinear dynamical feedback regulators which locally asymptotically stabilize, to a preselected desirable constant set point value, the output load voltage of typical configurations of pulse width modulation (PWM) controlled DC-to-DC power converters operating in the *continuous conduction mode*. Two fundamental nonlinear cases are treated: the boost and the buck-boost converter. Due to elementary *zero dynamics* stability considerations the output load voltage must be indirectly controlled, in the above two cases, via an input inductor current control strategy. If direct output load voltage control is attempted, for any of the above mentioned converters, the average dynamic linearizing compensator is unstable around the desired equilibrium point due to the resulting non-minimum phase character of the system.

Feedback control schemes for DC-to-DC power converters have been traditionally restricted to be of a static nature. This is particularly true in the sliding mode control approach (see Venkatarramanan *et al* [2], Sira-Ramirez [3]), and in the PWM control approach associated to: exact static feedback linearization [4], pseudolinearization [5], or some other design schemes within the geometric framework [6],[7]. The dynamical feedback regulation approach here proposed thus constitutes a conceptual departure, from the traditional controller design schemes for DC-to-DC converters, which exploits modern nonlinear controller design techniques. For a closely related approach using *differential algebra*, the reader is referred to Sira-Ramírez and Lischinsky-Arenas [8]. A more detailed exposition of the same topic treated here may be found in [9].

This work was supported by the Consejo de Desarrollo Científico, Humanístico y Tecnológico of the Universidad de Los Andes, under Research Grant I-325-90.

The design procedure used in this article can be summarized as follows : 1) Obtain the average PWM converter model in state space form from the switch controlled model by formally substituting the *switch position function* by the *duty ratio function* (see [7]). 2) Extend the obtained average model by just adding an integrator before the duty ratio input and define an auxiliary input function as the new input to the integrator. The original duty ratio function qualifies now as an added state variable 3) Place the average extended system in *Normal Canonical Form* (see Isidori [10]) and obtain an input-output linearizing -asymptotically stable- static state feedback controller in terms of the defined auxiliary input function, the original duty ratio function, and the original state variables of the converter. 4) Study the stability of the resulting *zero dynamics* and determine the *minimum phase* character of the system. 5) Revert to original state and input coordinates obtaining a dynamical (first order) system for the synthesis of the required (i.e., computed) duty ratio function. 6) Limit the computed duty ratio function to the interval [0,1] to obtain the actual duty ratio function which drives the PWM actuator. 7) Design low pass filters to generate an approximation to the average state variables and close the loop through the dynamical controller (See [9] for further details).

Section 2 presents in detail the derivations leading to the linearizing dynamical compensators for the boost and buck-boost DC-to-DC power converters. The performance of the proposed controllers is evaluated through simulations carried out on a digital computer. Section 3 contains the conclusions. and suggestions for further work in this area.

2. Nonlinear Dynamical Controller Design for DC-to-DC Power Converters.

2.1 Boost Converter

Consider the boost converter model shown in Figure 1. This circuit, also known as the *step up voltage flyback* converter (Williams [11]), is described by the following discontinuously controlled state equation model of the bi-linear type (see Severns and Bloom [12]):

$$\dot{x}_1 = -w_0(1-u)x_2+b$$
$$\dot{x}_2 = w_0(1-u)x_1 - w_1x_2 \qquad (2.1)$$

where, $x_1 = I\sqrt{L}$, $x_2 = V\sqrt{C}$ represent, respectively, *normalized input current* and *normalized output voltage* variables. The quantity $b = E/\sqrt{L}$ is the *normalized external input voltage* and, $w_0 = 1/\sqrt{LC}$ and $w_1 = 1/RC$ are, respectively, the LC (input) circuit natural oscillating frequency and the RC output circuit time constant. The variable u denotes the switch position function, acting as a control input, and taking values in the discrete set {0,1}. We now summarize, according to the design scheme outlined in the previous section, the formulae leading to a nonlinear dynamical stabilizing feedback controller design for the average model (2.1). We let μ denote the *duty ratio function.*

Average PWM boost converter model [7]

$$\dot{z}_1 = -w_0(1-\mu)z_2+b$$
$$\dot{z}_2 = w_0(1-u)z_1 - w_1z_2 \qquad (2.2)$$

Constant average operating equilibrium points

$$\mu = U\, ;\ Z_1(U) = bw_1/[w_0^2(1-U)^2]\, ;$$
$$Z_2(U) = b/[w_0(1-U)] \qquad (2.3)$$

Input inductor current is taken as the output variable, y, to be regulated. One usually pursues constant average normalized input current regulation to indirectly obtain a desirable controlled constant average normalized output voltage at the load. Indeed, given a desirable constant equilibrium value Z_2 for the average normalized output load voltage, one can compute from (2.3), the corresponding set point value for the normalized average input inductor current Z_1. This value is given by:

$$Z_1 = \frac{w_1}{b} Z_2^2 \qquad (2.4)$$

We now propose an *Input Current Control Mode* stabilizing nonlinear dynamical feedback controller scheme.

Average PWM boost converter model for input current control mode

$$\dot{z}_1 = -w_0(1-\mu)z_2+b$$
$$\dot{z}_2 = w_0(1-u)z_1 - w_1z_2 \qquad (2.5)$$
$$y = z_1 - Z_1$$

Average extended boost converter model for input current control mode

$$\dot{z}_1 = -w_0(1-z_3)z_2+b$$
$$\dot{z}_2 = w_0(1-z_3)z_1 - w_1z_2$$
$$\dot{z}_3 = v \qquad (2.6)$$
$$y = z_1 - Z_1$$

Where Z_1 is a desirable constant average normalized input inductor current, possibly computed in terms of the desirable constant average normalized output load voltage Z_2 as in (2.4), or in terms of the constant average operating value U of the duty ratio function μ as in (2.3).

State coordinate transformation for expressing the average extended boost converter model in normal canonical form

The average extended model (2.6) is easily seen to be *relative degree* 2, hence, the *zero dynamics* is one-dimensional [10]. The *Normal Canonical Form* coordinates of the extended average model (2.6) are defined as:

$$\xi_1 = z_1 - Z_1$$
$$\xi_2 = -(1-z_3)w_0z_2 + b \qquad (2.7a)$$
$$\eta_1 = z_3$$

The inverse transformation is simply:

$$z_1 = \xi_1 + Z_1$$
$$z_2 = \frac{b - \xi_2}{(1-\eta_1)w_0} \qquad (2.7b)$$
$$z_3 = \eta_1$$

The average extended boost converter model in normal canonical form

$$\dot{\xi}_1 = \xi_2$$
$$\dot{\xi}_2 = -(1-\eta_1)^2w_0^2\xi_1 - [\frac{v}{(1-\eta_1)} + w_1]\xi_2 +$$
$$b[\frac{v}{(1-\eta_1)} + w_1] - (1-\eta_1)^2w_0^2Z_1 \qquad (2.8)$$
$$\dot{\eta}_1 = v$$
$$y = \xi_1$$

A static linearizing regulator for the extended boost converter model in terms of normal form coordinates

If the second differential equation in (2.8) is equated to a linear combination of the state variables, say : $-a_1\xi_1-a_2\xi_2$, a static controller is obtained which, for suitably chosen constant parameters a_1 and a_2, asymptotically stabilizes the closed loop linear transformed system to the origin of coordinates. i.e., for suitably chosen a_i's, $\lim_{t \to \infty} \xi_i = 0$, (i=1,2). In particular, asymptotic convergence of z_1 to the desirable equilibrium point Z_1 is guaranteed. This linearization procedure on the extended system yields a static controller given by :

$$v = \frac{(1-\eta_1)}{\xi_2-b}\left\{\left[a_1-(1-\eta_1)^2 w_0^2\right]\xi_1 + [a_2-w_1]\xi_2 + bw_1 - (1-\eta_1)^2 w_0^2 Z_1\right\} \qquad (2.9)$$

In original state and average input coordinates z and μ, the above algebraic equation (2.9) for the static controller of the extended system actually represents a time-varying nonlinear differential equation for μ. The solution of this differential equation characterizes, then, a nonlinear dynamical compensator which synthesizes the required stabilizing computed duty ratio μ. Before specifying the dynamical compensator associated to (2.9), we establish the stability properties of the different equilibrium points exhibited by the zero dynamics of the feedback controlled extended average model (2.8),(2.9).

Stability of equilibrium points of the zero dynamics associated to the static linearizing controller for the extended boost model operating in input current mode.

Replacing the expression for the static compensator (2.9) on the extended system equations (2.8), and setting both the output voltage error ξ_1 and its time derivative ξ_2 to zero, one obtains the following autonomous nonlinear differential equation characterizing the zero dynamics :

$$\dot{\eta}_1 = -\frac{w_1}{(1-U)^2}(1-\eta_1)(2-U-\eta_1)(\eta_1-U) \qquad (2.10)$$

with equilibrium points given by : $\eta_1 = 1$, $\eta_1 = U$ and $\eta_1 = 2-U$. Linearization of the autonomous zero dynamics (2.10) around each of the equilibrium points reveals that: $\eta_1 = 1$ is an unstable equilibrium point with eigenvalue located at $w_1 > 0$. The equilibrium values $\eta_1 = U$, and $\eta_1 = 2-U$ are both asymptotically stable equilibrium points for the zero dynamics with eigenvalues located, respectively, at $-2w_1$. However, since $0 < U < 1$, the equilibrium point $\eta_1 = 2-U$ does not correspond to a physically meaningful duty ratio function and, moreover, it corresponds to a saturation condition for the duty ratio limiter.

Nonlinear dynamical feedback compensator in current control mode for the average PWM controlled boost converter

In normal form coordinates:

$$\tilde{\mu} = \frac{(1-\tilde{\mu})}{\xi_2-b}\left\{\left[a_1-(1-\tilde{\mu})^2 w_0^2\right]\xi_1 + [a_2-w_1]\xi_2 + bw_1 - (1-\tilde{\mu})^2 w_0^2 Z_1\right\} \qquad (2.11)$$

$$\mu = \mathrm{sat}_{[0,1]}\,\tilde{\mu}$$

In original coordinates:

$$\dot{\tilde{\mu}} = -\frac{1}{w_0 z_2}\left\{\left[a_1-(1-\tilde{\mu})^2 w_0^2\right](z_1-Z_1(U)) - w_0(a_2-w_1)(1-\tilde{\mu})z_2 + \left[a_2 b-(1-\tilde{\mu})^2 w_0^2 Z_1(U)\right]\right\} \qquad (2.12)$$

$$\mu = \mathrm{sat}_{[0,1]}\,\tilde{\mu}$$

where:

$$\mathrm{sat}_{[0,1]}\,\tilde{\mu} = \begin{cases} 1 & \text{for } \tilde{\mu} \geq 1 \\ \tilde{\mu} & \text{for } 0 < \tilde{\mu} < 1 \\ 0 & \text{for } \tilde{\mu} < 0 \end{cases}$$

The discontinuous PWM control policy for the actual circuit is devised on the *switch position function* u and generated by the PWM actuator according to the value of the *actual duty ratio function* μ, sampled at regularly spaced instants of time t_k, and obeying :

$$u(t) = \begin{cases} 1 & \text{for } t_k \leq t < t_k + \mu(t_k)T \\ 0 & \text{for } t_k + \mu(t_k)T \leq t < t_k + T \end{cases}$$

$$k = 0,1,2,\ldots \quad ; t_{k+1} = t_k + T$$

A simulation example.

A Boost converter circuit with parameter values (taken from Middlebrook [13]) : R = 11.2 Ω, C = 2000μF, L = 195μH and E = 28 Volts, was considered for nonlinear dynamical feedback controller design. The desirable normalized average constant output voltage is $Z_2 = 3.135$, which corresponds to a constant value U = 0.6 for the duty ratio μ. The corresponding set point for the average normalized input inductor current is $Z_1(0.6) = 0.2182$. The poles of the linearized closed loop system were chosen at : -353.55 ±j353.55. This corresponds with a damping coefficient of 0.707 and a natural oscillating frequency of 500 rad/s. Figure 2 shows average normalized state trajectories of the PWM controlled circuit. The average controlled state variables, z_1 and z_2, are shown to converge toward the desirable equilibrium point.

2.2 Buck-Boost Converter

Consider the buck-boost converter model, also known as the *step up/step down voltage flyback converter*, (see [11]) shown in Figure 3. This circuit is described by the time-invariant bilinear state equation model (see [12]) :

$$dx_1/dt = w_0(1-u)x_2 + u\,b$$
$$dx_2/dt = -w_0(1-u)x_1 - w_1 x_2 \qquad (2.13)$$

where, $x_1 = I\sqrt{L}$, $x_2 = V\sqrt{C}$ represent, respectively, normalized input current and output voltage variables. $b = E/\sqrt{L}$ is the normalized external input voltage and it is here assumed to be a negative quantity (for reversed polarity of the output voltage) while, $w_0 = 1/\sqrt{LC}$ and $w_1 = 1/RC$ are, respectively, the LC (input) circuit natural oscillating frequency and the RC output circuit time constant. The switch position function, acting as a control input, is denoted by u and takes values in the discrete set {0,1}. We now summarize the formulae leading to a nonlinear controller design for the average PWM model (2.13).

Average PWM buck-boost converter model in input current control mode. [7]

$$\dot z_1 = w_0(1-\mu)z_2 + \mu b$$
$$\dot z_2 = -w_0(1-\mu)z_1 - w_1 z_2 \qquad (2.14)$$
$$y = z_1 - Z_1$$

Constant equilibrium points

$$\mu = U\,;\quad Z_1(U) = bUw_1/[w_0^2(1-U)^2]\,;$$
$$Z_2(U) = -bU/[w_0(1-U)] \qquad (2.15)$$

Input inductor current error is taken as the output variable to be regulated. Given a desirable constant equilibrium value Z_2 for the average normalized output load voltage, one computes, from (2.15), the corresponding set point value for the normalized average input inductor current Z_1 as:

$$Z_1 = \frac{w_1}{bU} Z_2^2 \qquad (2.16)$$

Average extended buck-boost converter model for input current control mode

$$\dot z_1 = w_0(1-z_3)z_2 + z_3 b$$
$$\dot z_2 = -w_0(1-z_3)z_1 - w_1 z_2$$
$$\dot z_3 = v \qquad (2.17)$$
$$y = z_1 - Z_1$$

State coordinate transformation for expressing the average extended buck-boost Converter model in normal canonical form.

$$\xi_1 = z_1 - Z_1$$
$$\xi_2 = (1-z_3)w_0 z_2 + z_3 b \qquad (2.18)$$
$$\eta_1 = z_3$$

The inverse transformation is:

$$z_1 = \xi_1 + Z_1$$
$$z_2 = \frac{\xi_2 - \eta_1 b}{(1-\eta_1)w_0} \qquad (2.19)$$
$$z_3 = \eta_1$$

The average extended buck-boost converter model in normal canonical form.

$$\dot\xi_1 = \xi_2$$
$$\dot\xi_2 = -(1-\eta_1)^2 w_0^2 \xi_1 - [\frac{v}{(1-\eta_1)} + w_1]\xi_2 + b[\frac{v}{(1-\eta_1)} + \eta_1 w_1] - (1-\eta_1)^2 w_0^2 Z_1 \qquad (2.20)$$
$$\dot\eta_1 = v$$
$$y = \xi_1$$

A static linearizing regulator for the extended buck-boost converter model in terms of normal form coordinates

$$v = \frac{(1-\eta_1)}{\xi_2 - b}\left\{\left[a_1 - (1-\eta_1)^2 w_0^2\right]\xi_1 + \left[a_2 - w_1\right]\xi_2 + b\xi_1 w_1 - (1-\eta_1)^2 w_0^2 Z_1\right\} \qquad (2.21)$$

with the a_i (i=1,2) chosen in such a way that for the linearized system it is satisfied: $\lim_{t\to\infty} \xi_i = 0$, for i=1,2.

Stability of equilibrium points of the zero dynamics associated to the static linearizing controller for the extended buck-boost model operating in input current mode.

The zero dynamics is given by:

$$\dot\eta_1 = -\frac{w_1}{(1-U)^2}(1-\eta_1)\left[\eta_1(1-U)^2 - U(1-\eta_1)^2\right] \qquad (2.22)$$

whose equilibrium points are given by $\eta_1 = 1$, $\eta_1 = U$ and $\eta_1 = 1/U$. Linearization of the zero dynamics around each of the equilibrium points reveals that: $\eta_1 = 1$ is an unstable equilibrium point with eigenvalue located at $w_1 > 0$. The equilibrium values $\eta_1 = U$, and $\eta_1 = 1/U$ are both stable equilibrium points, for the zero dynamics, with eigenvalues located, respectively at $-2w_1$ and $-(1+U^{-1})w_1$. However, since $0 < U < 1$, the equilibrium point $\eta_1 = 1/U$ does not correspond to a physically meaningful duty ratio function and, moreover, it corresponds to a saturation condition for the duty ratio limiter.

Nonlinear dynamical feedback compensator in current control mode for the average PWM controlled boost converter

In normal form coordinates:

$$\dot{\tilde{\mu}} = \frac{(1-\tilde{\mu})}{\xi_2 - b}\left\{ \left[a_1 - (1-\tilde{\mu})^2 w_0^2\right]\xi_1 + \left[a_2 - w_1\right]\xi_2 + b\tilde{\mu} w_1 - (1-\tilde{\mu})^2 w_0^2 Z_1 \right\}$$
$$\mu = \mathrm{sat}_{[0,1]}\tilde{\mu} \qquad (2.23)$$

In original coordinates:

$$\dot{\tilde{\mu}} = \frac{1}{w_0 z_2 - b}\left[a_1 - (1-\tilde{\mu})^2 w_0^2\right]\left[z_1 - Z_1(U)\right] + (a_2 - w_1) w_0 (1-\tilde{\mu}) z_2 + \left[a_2 \tilde{\mu} b - (1-\tilde{\mu})^2 w_0^2 Z_1(U)\right] \qquad (2.24)$$
$$\mu = \mathrm{sat}_{[0,1]}\tilde{\mu}$$

where the saturation function $\mathrm{sat}_{[0,1]}$ is defined as before.

A simulation example

A Buck-Boost converter circuit with the same parameter values as in the previous Boost example, except that, now, E = -28 V, was considered for nonlinear dynamical controller design. The desirable normalized constant output voltage is Z_2 = 1.8783 which corresponds to a constant value U = 0.6 for the duty ratio μ. The corresponding set point for the average normalized input inductor current is $Z_1(0.6)$ = -0.1309. The poles of the linearized closed loop system were chosen at :-353.55 ±j353.55. This corresponds, as before, with a damping coefficient of 0.707 and a natural oscillating frequency of 500 rad/s. Figure 4 shows the average normalized state trajectory step responses of the PWM controlled circuit . The average controlled state variables, z_1 and z_2, are shown to converge toward the desirable equilibrium point.

3. Conclusions and suggestions for further research.

In this article a new design method is proposed for the regulation of output load voltage, in Pulse-Width-Modulated controlled DC-to-DC Power Converters. The stabilizing controller design method is based on nonlinear dynamical feedback linearization of the average PWM extended system model associated to the converter's dynamical equations.

Computing the normal canonical form of the extended system for the different switch-mode controlled converters, such as the boost, and the buck-boost converters, a nonlinear dynamical feedback compensator is immediately suggested which partially linearizes, in a local fashion, the closed loop controlled system with arbitrarily imposed second order stable dynamics. As it is widely known (Isidori [10]), local stability of feedback linearized controller schemes, toward a desirable reference set point, crucially depends upon the minimum-phase character of the zero dynamics associated to the system at the preselected equilibrium point. We explicitly analyze the zero dynamics associated to the proposed linearizing scheme for each case, and determine the stability properties of each design. According to these results, in the two cases studied in this article, the average input inductor current can be effectively regulated by means of the proposed technique thanks to its minimum phase characteristics around the physically meaningful equilibrium points. Thus, average output load voltage regulation is indirectly accomplished via regulation of the average input inductor current.

The dynamical controllers obtained in this article are generally constituted by nonlinear time-varying dynamical compensators of the proportional derivative type. Their structure is invariably associated to a nonlinear set point feedforward term and a nonlinear first order filter.

Practical implementation of the proposed controllers, via nonlinear solid state analog electronics, is a topic that needs to be pursued in the future. The results can also be applied to the Cuk converter case and some of its celebrated variations.

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FIGURES

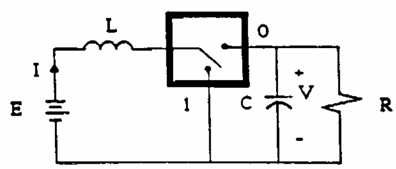

Figure 1. The boost converter.

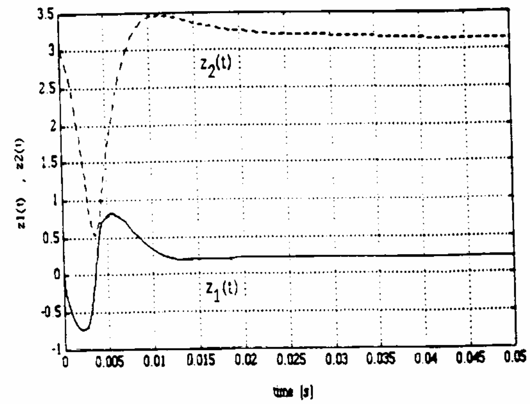

Figure 2 : Average normalized output load voltage and input current step response for a PWM dynamically feedback controlled boost converter.

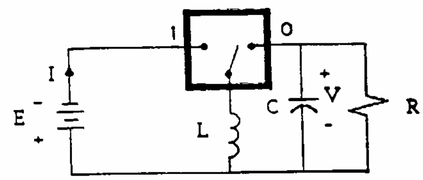

Figure 3. The buck-boost converter.

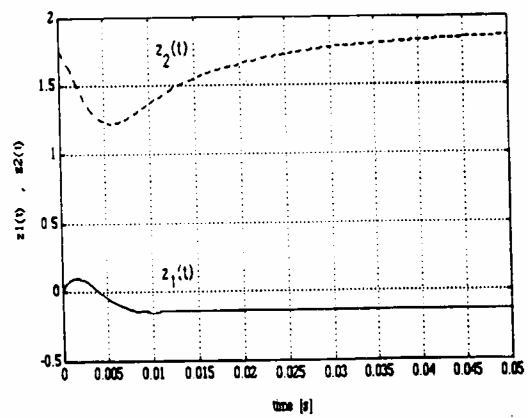

Figure 4 : Average normalized output load voltage and input current step response for a PWM dynamically feedback controlled buck-boost converter.