An Extended Linearization Approach to Sliding Mode Control of Nonlinear Systems

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Abstract

A new approach, based on the method of Extended Linearization is proposed for the synthesis of Variable Structure feedback control strategies leading to stable linearizing Sliding Regimes for nonlinear dynamical controlled systems. The class of systems, to which this technique is applicable, is assumed to posses a continuum of constant equilibrium points in the input-output space. The approach is illustrated and evaluated by means of a design example.

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1. Introduction

In this article we illustrate by means of a physically meaningful example the use of the *Extended Linearization* technique, developed by Rugh and his coworkers [1]-[2], in the specification of nonlinear *sliding manifolds* for nonlinear variable structure systems. The obtained sliding manifold is such that the *ideal sliding dynamics* (see Utkin [3]) results in a linear asymptotically stable motion toward preselected constant equilibrium points. The method also exhibits certain degree of *self scheduling* properties inherited from well known merits of parametrized controllers based on the extended linearization approach. In [4], the same approach was used for designing nonlinear sliding manifolds for nonlinear systems in which no restrictions in the availability of feedback control laws was assumed. In this article we extend, only by means of an example, the results in [4] to the case of *proper* variable structure systems regulated by switched control actions between two available, fixed, feedback strategies, or "structures". To this class of systems belong : switchmode dc-to-dc power supplies, on-off jet-controlled satellites and, in some instances, certain valve controlled fluidic systems and robotic manipulators.

The proposed approach constitutes an attempt to have a systematic means of sliding mode control design for nonlinear systems and is based on designing a family of parametrized linear sliding surfaces for the linearization, around a generic constant equilibrium point, of the ideal sliding system dynamics. On the proposed linear switching surfaces, the tangent system - appropriately placed in controller canonical form - is forced to adopt an asymptotically stable behavior characterized by imposed eigenvalues which are independent of the operating point. The obtained family of switching manifolds may be regarded as an integrable distribution which is tangent to a certain nonlinear sliding manifold. The standard extended linearization process indicates that one should find a solution manifold by means of direct integration. Assessment of the existence of a sliding regime by means of switching among the given constraining feedback structures is easily accomplished by evaluation of the equivalent control and its well known intermediacy condition with respect to the given feedback laws (which can always be reduced to take numerical values in the discrete set $\{0,1\}$).

In section 2 we present a simple but illustrative sliding mode controller design example, of the on-off type, for a nonlinear system. We evaluate, by means of computer simulations, the performance of the proposed discontinuous controller. The parametrized sliding mode controller is shown to be particularly well suited to handle abrupt changes in the desired equilibrium point without "rescheduling" of the controller. Section 3 contains the conclusions of the article.

2. A Design Example

Consider a system consisting of two identical tanks containing a fluid which escapes from the first tank into the second through an orifice. The second tank also empties to the environment through an identical orifice. The first tank receives fluid controlled by an on-off valve. A nonlinear dynamical model for this system is represented by (see Rugh [2]):

$$\dot{x}_1 = -\frac{c}{A}\sqrt{x_1} + \frac{1}{A}u \quad ; \quad \dot{x}_2 = -\frac{c}{A}\sqrt{x_2} + \frac{c}{A}\sqrt{x_1} \qquad (2.1)$$

where x_1 and x_2 represent the height of the liquid on each tank and u is the feed rate, taking values in the set $\{0,U_{max}\}$. The constants c and A are assumed to be known. If a sliding regime exists for system (2.1) on certain smooth sliding manifold $s(x) = 0$, thanks to the use of the available control set $\{0,U_{max}\}$, the *ideal sliding dynamics* is simply obtained by formally replacing in (2.1) the *equivalent control*, denoted by $\mu(x)$ on (2.1). The equivalent control is obtained from the invariance condition $ds(x)/dt = 0$. The obtained autonomous dynamics represents a description of the *average behavior* taking place on a generic representative of the family of switching surfaces $s(x) =$ *constant* and. If we denote by z_1 and z_2 the states corresponding to such an average behavior, the model (2.1) can be formally replaced by the piecewise continuous model ($0 < \mu < U_{max}$):

$$\dot{z}_1 = -\frac{c}{A}\sqrt{z_1} + \frac{1}{A}\mu \quad ; \quad \dot{z}_2 = -\frac{c}{A}\sqrt{z_2} + \frac{c}{A}\sqrt{z_1} \qquad (2.2)$$

On such an average system a linearization procedure around a given average equilibrium point indeed makes sense. This is in clear contradistinction to the case of system (2.1), which is discontinuous and, hence, non-linearizable! The average equilibrium state of (2.2), associated to a constant average input $\mu = U$, is represented by: $z_1(U) = z_2(U) = U^2/c^2$. Linearization about this generic equilibrium point yields:

$$\dot{z}_{1\delta} = -\frac{c^2}{2AU}z_{1\delta} + \frac{1}{A}\mu_\delta \quad ; \quad \dot{z}_{2\delta} = \frac{c^2}{2AU}z_{1\delta} - \frac{c^2}{2AU}z_{2\delta} \quad (2.3)$$

where $z_{i\delta} = z_i - Z_i(U)$; $i = 1, 2$, $\mu_\delta = \mu - U$. The sliding surface design for the tangent model (2.3) is greatly facilitated if the linearized system (2.3) is transformed to controller canonical form by means of the following linear incremental state coordinate transformation:

$$\xi_{1\delta} = \frac{2A^2U}{c^2}z_{2\delta} \quad ; \quad \xi_{2\delta} = A\,z_{1\delta} - A\,z_{2\delta} \qquad (2.4)$$

One then obtains:

$$\dot{\xi}_{1\delta} = \xi_{2\delta} \quad ; \dot{\xi}_{2\delta} = -\frac{c^4}{4A^2U^2}\xi_{1\delta} - \frac{c^2}{AU}\xi_{2\delta} + \mu_\delta \qquad (2.5)$$

In the new coordinates, the proposed stabilizing sliding surface for the tangent model is simply taken as:

$$\sigma_\delta(\xi_\delta) = \xi_{2\delta} + c_1\xi_{1\delta} \quad ; \quad c_1 > 0 \qquad (2.6)$$

Notice that a sliding motion on such a sliding surface yields an incremental state trajectory which asymptotically converges to the origin of coordinates (i.e., to the operating point) in a manner which is only dependent upon the design constant c_1. In original average coordinates the sliding surface (2.6) results in a parametrized switching manifold of the form:

$$s_\delta(z_\delta,U) = \frac{c^2}{2AU}z_{1\delta} + \left(c_1 - \frac{c^2}{2AU}\right)z_{2\delta} \qquad (2.7)$$

The extended linearization approach suggests now to find a nonlinear smooth switching manifold, $s(z,U) = 0$, parametrized by

U, such that: 1) the linearization of the surface around the equilibrium point $z_1(U)$, $z_2(U)$ yields back (2.7) and 2) the obtained sliding surface contains $z_1(U)$, $z_2(U)$. In other words the following relations must hold true:

$$\left.\frac{\partial s(z,U)}{\partial z_1}\right|_{z_1(U)\,=\,z_2(U)\,=\,\frac{U^2}{c^2}} = \frac{c^2}{2AU}\ ;\qquad \left.\frac{\partial s(z,U)}{\partial z_2}\right|_{z_1(U)\,=\,z_2(U)\,=\,\frac{U^2}{c^2}} = \left(c_1 - \frac{c^2}{2AU}\right) \tag{2.8}$$

with the "boundary" condition : $s(z_1(U),z_2(U),U) = 0$. Integration of (2.8) is straightforward upon replacing U by $c\sqrt{z_1}$ in the first relation and by $c\sqrt{z_2}$ in the second relation in (2.8). The following nonlinear sliding manifold yields the solution to the (distribution) integration problem:

$$s(z,U) = \frac{c}{A}\sqrt{z_1} - \frac{c}{A}\sqrt{z_2} + c_1\left[z_2 - \frac{U^2}{c^2}\right] = 0 \tag{2.9}$$

It should be remarked that the solution to the integration problem represented by (2.8) is by no means unique. The proposed solution (2.9), however, has the enormous advantage of yielding a linear ideal sliding dynamics. Indeed, it is easy to see that if a sliding regime exists on (2.9), the algebraic relation satisfied by the state variables implies a constrained motion which satisfies the following reduced order asymptotically stable linear dynamics :

$$\dot z_2 = -c_1\left[z_2 - \frac{U^2}{c^2}\right] = 0 \tag{2.10}$$

As it can be easily checked, after some tedious but straightforward algebraic manipulations, a different solution for the required sliding manifold is represented by :

$$s(z,U) = A\left(z_1 - z_2\right) + \frac{4A^2c_1}{3c}\left[\sqrt{z_2^3} - \frac{U^3}{c^3}\right] = 0 \tag{2.11}$$

This sliding surface results in a nonlinear ideal sliding regime which could be linearized by a nontrivial diffeomorphic state coordinate transformation.

The equivalent control associated to (2.9) is given by:

$$\mu = \frac{cz_1}{\sqrt{z_2}} + 2Ac_1\left(\sqrt{z_1z_2} - z_1\right) \tag{2.12}$$

A sliding regime exists on the region of the sliding surface limited where the following two relations are valid :

$$\frac{cz_1}{\sqrt{z_2}} + 2Ac_1\left(\sqrt{z_1z_2} - z_1\right) < 1 \ \text{ and } \ \frac{cz_1}{\sqrt{z_2}} + 2Ac_1\left(\sqrt{z_1z_2} - z_1\right) > 0 \tag{2.13}$$

It is easy to see that the switching strategy which accomplishes such a local sliding regime on the original system is given by $u = U_{max}$ for $s < 0$ and $u = 0$ for $s > 0$.

Simulations Simulations were performed using the nonlinear facilities of the MATLAB package for the two tank system controlled by the above switching strategy and the sliding surface (2.9). The chosen parameters were $c = 1$, $A = 0.5$, $U_{max} = 7$. A family of sliding mode controlled state trajectories, stabilized to the equilibrium point $z_1(3) = z_2(3) = 9$, are shown in figure 1. A sudden change from the equilibrium point $z_1(3) = z_2(3) = 9$ to the equilibrium pint $z_2(4) = z_2(4) = 16$ is also shown in figure 1, with the corresponding sliding surfaces S_1 and S_2 . The time history of a typical trajectory undergoing such an abrupt change in the operating conditions is shown in figure 2.

3 Conclusions

In this article we have shown, through a simple example, that, for the class of systems possessing constant equilibrium points, the extended linearization method constitutes a most convenient, and rather general, design technique for specifying a parametrized family of nonlinear sliding manifolds with linearizing properties. Although the treated example corresponds to one easily transformed to regular canonical form (see Utkin and Luk'yanov [5] the approach is applicable to systems with no particular control input structure. The multivariable version of the approach seems an interesting topic to be pursued in the future.

4. References

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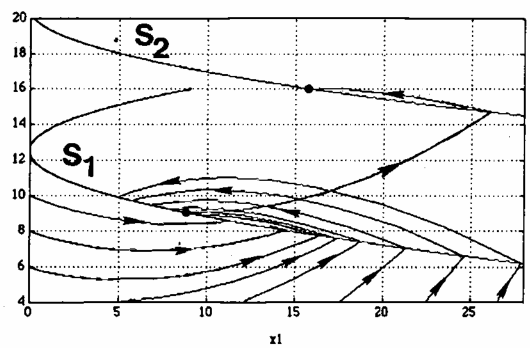

Figure 1. Family of sliding mode controlled state trajectories undergoing a sudden change in the operating conditions.

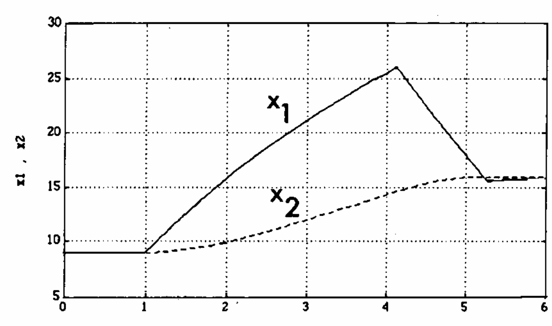

Figure 2. Time response of typical sliding mode controlled trajectory undergoing a sudden change in the operating condition.