DYNAMICAL DISCONTINUOUS FEEDBACK CONTROL OF CHEMICAL PROCESSES

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Abstract In this article, the use of dynamical discontinuous feedback control strategies, such as sliding modes, pulse width modulation, and pulse frequency modulation, are proposed for the asymptotic stabilization of nonlinear dynamical systems describing chemical processes. Illustrative Continuously Stirred Tank Reactor controller design examples, using the various proposed schemes, are provided including computer simulations.

1. INTRODUCTION

Recently, results from the *differential algebraic* approach to control theory, pioneered by Prof. Michel Fliess [1]-[2], have greatly improved the applicability of discontinuous feedback strategies, specially those of the *sliding mode* (SM) type, leading to asymptotic stabilization, and tracking, in nonlinear systems (see Sira-Ramirez [3]-[4] for applications to mechanical and electro-mechanical systems). Some of the traditional disadvantages of sliding mode control policies are fundamentally related to the chattering of input and state variables response signals (See Utkin [5]). These difficulties are easily circumvented via *dynamical* sliding mode controllers while retaining the outstanding robustness, and simplicity, of this class of feedback control schemes.

In this article, Fliess's Generalized Observability Canonical Form (GOCF) is shown to naturally allow for dynamical feedback controller design based on pulse-width-modulation (PWM) strategies and pulse-frequency-modulation (PFM) policies. As in the SM control case, the obtained control input signals are substantially smoothed and, hence, the possibility clearly exists for chattering-free controlled responses. The obtained PWM and PFM controller designs do not resort to traditional approximation schemes, based on (infinte frequency) *average* models, of the discontinuously controlled systems (see, Sira-Ramirez [6]). These features are particularly important in chemical process control problems, in which large and fast input vibrations, or jump discontinuities, cannot be simply allowed on the actuators, while a need still exists for certain degree of robustness and precision of the proposed control scheme. This is particularly the case of pneumatic and mechanically driven valves and dispensers, extensively used by many chemical industries today.

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The synthesis of the several dynamical discontinuous regulators, here proposed, is entirely based on Fliess' *Generalised Observability Canonical Form* (GOCF) for nonlinear systems (See [2]). In Section 2 of this article, we briefly review the dynamical SM control solution to the output stabilization problem and present the PWM and PFM controller design schemes. In section 3, we present an application example on which we test each one of the proposed discontinuous control techniques mentioned above. The application example, taken from Kravaris and Palanki [7], is concerned with the total concentration regulation in an isothermal Continuously Stirred Tank Reactor (CSTR). The presented design examples include computer simulations. Concluding remarks, and proposals for further work, are collected at the end of the article, in Section 4.

2. DYNAMICAL DISCONTINOUS FEEDBACK CONTROL OF NONLINEAR SYSTEMS

The results of this section are easily extended to tracking problems (see [3],[4]) and to multivariable cases.

2.1 Fliess's Generalised Observability Canonical Form.

It has been shown in [2] (see also Conte *et al* [8]) that a nonlinear, single-input single-output n-dimensional analytic system of the form:

$$\dot{x} = f(x,u) \qquad y = h(x) \tag{2.1}$$

can be locally transformed, via an input dependent state coordinate transformation of the form :

$$z = \Phi(x,u,\dot{u},\ldots,u^{(\alpha-1)}) \tag{2.2}$$

into a system of the form:

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ &\cdots \\ \dot{z}_n &= c(z,u,\dot{u},\ldots,u^{(\alpha)}) \\ y &= z_1 \end{aligned} \tag{2.3}$$

provided the following "observability" matrix of the system (2.1) is full rank:

$$\begin{bmatrix} \frac{\partial h(x)}{\partial x} \\ \frac{\partial h^{(1)}(x)}{\partial x} \\ \cdots \\ \frac{\partial h^{(n-1)}(x,u,\dot{u},\ldots,u^{(\alpha-1)})}{\partial x} \end{bmatrix} \tag{2.4}$$

In (2.3), the integer α is assumed to be a strictly positive integer (for extension of the results to *exactly linearizable systems by static state feedback*, i.e., for those systems in which $\alpha = 0$, the reader is refered to Sira-Ramirez [9]). It should be remarked, however, that, in general, (2.3) may not be, necessarily, n-dimensional. Our assumption, thus corresponds to one of a *minimal realization* on (2.1).

The state coordinate transformation (2.2) is evidently given by the local diffeomorphism:

$$z = \Phi(x,u,\dot{u},\ldots,u^{(\alpha-1)}) = \begin{bmatrix} h(x) \\ h^{(1)}(x) \\ \cdots \\ h^{(n-1)}(x,u,\dot{u},\ldots,u^{(\alpha-1)}) \end{bmatrix} \tag{2.5}$$

Suppose $u = U$, $x = X(U)$ describes a constant equilibrium point for the original system (2.1), such that $h(X(U))$ is zero, then $z = 0$ is an equilibrium point of (2.3). The autonomous dynamics described by:

$$c(0,u,\dot{u},\ldots,u^{(\alpha)}) = 0 \tag{2.6}$$

is the *zero dynamics* (see Fliess [10]). The stability nature of an equilibium point $u = U$ of (2.6) determines the *minimum* or *non-minimum phase* character of the system at the corresponding equilibrium point. We denote the above constant equilibrium point for system (2.1) as $(X(U),U,0)$.

2.2 A GOCF Approach to Dynamical Discontinuous Feedback Controller Design for Nonlinear Systems.

Consider the following auxiliary output function $\sigma : R^n \to R$, defined in terms of the transformed variables z,

$$\sigma(z) = \left(\sum_{i=1}^{n-1} \gamma_i z_i\right) + z_n \tag{2.7}$$

such that the following corresponding polynomial in the complex variable s is Hurwitz:

$$\sum_{i=1}^{n-1} \gamma_i s^{i-1} + s^{n-1} \tag{2.8}$$

It is easy to see that, provided the system is locally minimum phase, and if (2.7) is forcefully constrained to zero (whether in finite time, or in an asymptotically stable fashion) by means of appropriate control actions (possibly of discontinuous nature), the resulting controlled dynamics locally evolves in accordance with:

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ &\cdots \\ \dot{z}_{n-1} &= -\sum_{i=1}^{n-1} \gamma_i z_i \\ y &= z_1 \end{aligned} \tag{2.9}$$

which is asymptotically stable to zero.

The various dynamical discontinuous feedback controller design schemes here proposed rely on inducing an asymptotically stable linear time invariant controlled dynamics such as (2.9), with eigenvalues placeable at will, by driving the proposed auxiliary output function $\sigma(z)$ to zero. SM controllers can always accomplish such a task in finite time, PWM and PFM controllers, on the other hand, can only accomplish this task in an asymptotially stable fashion.

Dynamical Sliding Mode Control of Nonlinear Systems

Proposition 2.1 Let W be a strictly positive quantity and let "sgn" stand for the *signum* function. The one dimensional discontinuous system :

$$\dot{\sigma} = -W \operatorname{sgn} \sigma \tag{2.10}$$

globally exhibits a sliding regime on $\sigma = 0$. Furthermore, any trajectory starting on the value $\sigma = \sigma(0)$, at time 0, reaches the condition $\sigma = 0$ in finite time T, given by : $T = W^{-1} | \sigma(0) |$.

Proof Immediate upon checking that globally: $\sigma\, d\sigma/dt < 0$ for $\sigma \neq 0$, which is a well known condition for sliding mode existence [5]. The second part follows trivially from the fact that $| \sigma(t) | = -Wt + | \sigma(0) |$ ■

Proposition 2.2 A minimum phase nonlinear system of the form (2.1) is locally asymptotically stabilizable to the equilibrium point $(U,X(U),0)$ if the control action u is specified as a dynamical SM control policy given by the solution of the following implicit, time-varying, nonlinear discontinuous differential equation :

$$c(z,u,\dot{u},\cdots,u^{(\alpha)}) = -\sum_{i=1}^{n} \gamma_{i-1} z_i - W \operatorname{sgn}\left[\sum_{i=1}^{n-1} \gamma_i z_i + z_n\right] \tag{2.11}$$

where $\gamma_0 = 0$.

Proof Immediate upon imposing on the auxiliary output function $\sigma(z)$ in (2.7) the discontinuous dynamics defined by (2.10). ■

We assumed that in (2.11) the quantity $\partial c/\partial u^{(\alpha)}$ is locally nonzero and, hence, no singularities need to be locally considered.

Controller (2.11) is easily represented in terms of the original state space coordinates x by using the input dependent coordinate transformation (2.5).

Dynamical PFM Control of Nonlinear Systems

Consider the scalar PFM controlled dynamical system, in which the constants r_1, r_2, r_3 and W, are all strictly positive quantities.

$$\dot{\sigma} = -W v$$

$$v = PFM_{T,\tau}(\sigma) = \begin{cases} \operatorname{sgn} \sigma(t_k) & \text{for } t_k \le t < t_k + \tau[\sigma(t_k)]T[\sigma(t_k)] \\ 0 & \text{for } t_k + \tau[\sigma(t_k)]T[\sigma(t_k)] \le t < t_k + T[\sigma(t_k)] \end{cases}$$

$$\tau[\sigma(t)] = \begin{cases} 1 & \text{for } |\sigma(t)| > \frac{1}{r_1} \\ r_1 |\sigma(t)| & \text{for } |\sigma(t)| \le \frac{1}{r_1} \end{cases}$$

$$T[\sigma(t)] = \begin{cases} T_{max} & \text{for } |\sigma(t)| \geq \frac{1}{r_2} \\ T_{min} + \frac{r_2 r_3}{r_3 - r_2}[T_{max} - T_{min}](\sigma(t) - \frac{1}{r_3}) & \text{for } \frac{1}{r_3} < |\sigma(t)| < \frac{1}{r_2} \\ T_{min} & \text{for } |\sigma(t)| \leq \frac{1}{r_3} \end{cases}$$
$$k = 0,1,2,\dots \quad ; \quad t_{k+1} = t_k + T[\sigma(t_k)]. \tag{2.12}$$

where it is assumed that $r_2 < r_1 < r_3$. The t_k's represent irregularly spaced sampling instants, determined by the sampled values of the *duty cycle function*, denoted here by $T[\sigma(t_k)]$. The duty cycle function, $T[\sigma(t)]$, takes values on the closed interval $[T_{min}, T_{max}]$ and it varies linearly with respect to $\sigma(t)$ in the region $|\sigma| < 1/r_2$. The duty cycle, or sampling period, saturates to T_{max} for large values of σ, and remains fixed at the constant lower bound T_{min} for small values of σ. At each sampling instant, t_k, the value of the width of the sign-modulated, unit amplitude, control pulse is determined by the sampled value of the *duty ratio function*, represented by $\tau[\sigma(t_k)]$. In general, the duty cycle and the duty ratio functions may be quite independent of each other. The function "sgn" stands for the *signum* function.

The following proposition establishes a sufficient condition for the asymptotic stability to zero of the PFM controlled system (2.12).

Proposition 2.3 The PFM controlled system (2.12) is asymptotically stable to $\sigma = 0$, if

$$0 < r_3 W T_{max} < 2 \tag{2.13}$$

Proof Due to the piecewise constant nature of the control inputs and the linearity of the continuous system, it suffices to study the stability of the discretized version of (2.12) at the sampling instants. An exact discretization of the PFM controlled system (2.12) yields:

$$\sigma(t_k+T) = \sigma(t_k) - W\,\text{sgn}[\sigma(t_k)]\,\tau[\sigma(t_k)]\,T[\sigma(t_k)] \tag{2.14}$$

Suppose the initial condition $\sigma(0)$ is chosen deep into the region $|\sigma| > 1/r_2$. The evolution of the sampled values of $\sigma(t)$ obey, according to (2.14):

$$\begin{aligned} \sigma(t_k+T) &= \sigma(t_k) - W\,T_{max} \quad \text{for } \sigma(t_k) > 0 \\ \sigma(t_k+T) &= \sigma(t_k) + W\,T_{max} \quad \text{for } \sigma(t_k) < 0 \end{aligned} \tag{2.15}$$

Hence, given an arbitrary initial condition $\sigma(0)$ for σ, it is obvious from (2.15) that the condition : $0 < r_3 W T_{max} < 2$ is sufficient to ensure that the value of $\sigma(t_k)$ will be eventually found within the bounded region $|\sigma| < 1/r_2$. This is due to the fact that the controlled increments taken by $\sigma(t_k)$, in the considered region $|\sigma| > 1/r_2$, are of width WT_{max} and, therefore, the condition: $WT_{max} < 2/r_3$ also guarantees that $WT_{max} < 2/r_2$. It follows that $\sigma(t_k)$ can not "jump" over the band $|\sigma| < 1/r_2$ and, hence, $\sigma(t_k)$ will land on this region for sufficiently large k. Two possibilities arise then : either $\sigma(t_k)$ is found in the "band" $1/r_3 < |\sigma(t_k)| < 1/r_2$, or $\sigma(t_k)$ satisfies $|\sigma(t_k)| < 1/r_3$. Suppose first that: $1/r_3 < |\sigma(t_k)| < 1/r_2$, for some k. In this region, the value of $|\sigma(t_k)|$ can only further decrease, as it is easily seen from (2.13). Indeed, the increments: $\Delta\sigma(t_k) = \sigma(t_{k+1}) - \sigma(t_k)$, taken by σ in the region $1/r_3 < |\sigma| < 1/r_2$, satisfy: $WT_{min} < |\Delta\sigma(t_k)| < WT_{max}$ Since, by assumption, $WT_{max} < 2/r_3$, then one has : $WT_{min} < |\Delta\sigma(t_k)| < WT_{max} < 2/r_3 < 2|\sigma(t_k)|$. It follows that $|\sigma(t_k)|$ further decreases and that the controlled evolution of $\sigma(t_k)$ will eventually reach the region: $|\sigma(t_k)| < 1/r_3$. In this last region the sampled values of σ evolve satisfying :

$$\begin{aligned} \sigma(t_k+T) &= \sigma(t_k) - r_1 W\,T_{min}\,\text{sgn}[\sigma(t_k)]|\sigma(t_k)| \\ &= (1 - r_1 W\,T_{min})\,\sigma(t_k) \end{aligned}$$

which is asymptotically stable to zero, if and only if : $0 < r_1 W T_{min} < 2$. This last condition is evidently equivalent to $WT_{min} < 2/r_1$. Notice, however, that from the assumptions about the parameters in (2.12) : $WT_{min} < WT_{max} < 2/r_3 < 2/r_1$. i.e., the condition (2.13) implies the asymptotical stability requirement for (2.12). The result follows. ■

Proposition 2.4 A minimum phase nonlinear system of the form (2.1) is locally asymptotically stabilizable to the equilibrium point $(U, X(U), 0)$ if the control action u is specified as a dynamical PFM control policy given by the solution of the following implicit, time-varying, nonlinear discontinuous differential equation :

$$c(z, u, \dot{u}, \cdots, u^{(\alpha)}) = -\sum_{i=1}^{n} \gamma_{i-1} z_i - W\,\text{PFM}_{T,\tau}[\sum_{i=1}^{n-1} \gamma_i z_i + z_n] \tag{2.16}$$

where $\gamma_0 = 0$.

Dynamical PWM Control of Nonlinear Systems

Consider the scalar PWM controlled system, in which $r > 0$ and $W > 0$:

$$\dot{\sigma} = -W v$$
$$v = \text{PWM}_T(\sigma) = \begin{cases} \text{sgn}\,\sigma(t_k) & \text{for } t_k \leq t < t_k + \tau[\sigma(t_k)]T \\ 0 & \text{for } t_k + \tau[\sigma(t_k)]T \leq t < t_k + T \end{cases} \tag{2.17}$$
$$\tau[\sigma(t)] = \begin{cases} 1 & \text{for } |\sigma(t)| > \frac{1}{r} \\ r|\sigma(t)| & \text{for } |\sigma(t)| \leq \frac{1}{r} \end{cases}$$
$$k = 0,1,2,\dots \quad ; \quad t_{k+1} = t_k + T.$$

where the t_k's represent regularly spaced sampling instants and "sgn" stands for the *signum* function.

It is easy to see that (2.17) is just a particular case of the PFM controlled system (2.12) in which the duty cycle function $T(\sigma(t_k))$ is now taken as a constant of value T. The following results follow immediately from this fact.

Proposition 2.5 The PWM controlled system (2.17) is asymptotically stable to $\sigma = 0$ if and only if:

$$0 < rWT < 2 \tag{2.18}$$

Proof Sufficiency is clear from the preceeding proposition. Necesity follows from the fact that (2.18) is necessary to have $\sigma(t_k)$ lie in the region $|\sigma(t_k)| \leq 1/r$, for some k, independently of the initial condition. In this region, the PWM controlled dynamics adopts the form $\sigma(t_{k+1}) = (1 - rWT)\,\sigma(t_k)$. The result follows. ■

Proposition 2.6 A minimum phase nonlinear system of the form (2.1) is locally asymptotically stabilizable to the equilibrium point $(U, X(U), 0)$ if the control action u is specified as a dynamical PWM control policy given by the solution of the following implicit, time-varying, nonlinear discontinuous differential equation :

$$c(z,u,\dot{u},\cdots,u^{(\alpha)}) = -\sum_{i=1}^{n} \gamma_{i-1} z_i - W\, PWM_\tau\Big[\sum_{i=1}^{n-1} \gamma_i z_i + z_n\Big] \tag{2.19}$$

where $\gamma_0 = 0$.

Proof. Immediate upon imposing on the auxiliary output function $\sigma(z)$ in (2.7) the asymptotically stable discontinuous dynamics defined by (2.17). ■

3. AN APPLICATION EXAMPLE

3.1 Dynamical Discontinuous Controller Design for Regulation of Total Concentration in a Continuously Stirred Tank Reactor.

Consider the following simple nonlinear dynamical model of a controlled CSTR in which an isothermal, liquid-phase, multicomponent chemical reaction takes place (see [7]):

$$\begin{aligned} \dot{x}_1 &= -(1+D_{a1})\,x_1 + u \\ \dot{x}_2 &= D_{a1}\,x_1 - x_2 - D_{a2}x_2^2 \\ y &= x_1 + x_2 - Y \end{aligned} \tag{3.1}$$

Where x_1 represents the normalized (dimensionless) concentration C_P/C_{P0} of a certain species P in the reactor, with $Y = C_{P0}$ being the desired concentration of the species P and Q measured in mol.m^{-3}. The state variable x_2 represents the normalized concentration C_Q/C_{P0} of the species Q. The control variable u is defined as the ratio of the per-unit volumetric molar feed rate of species P, denoted by N_{PF}, and the desired concentration C_{P0} . i.e., $u = N_{PF}/(FC_{P0})$ where F is the volumetric feed rate in m^3 s^{-1}. The constants D_{a1} and D_{a2} are respectively defined as k_1V/F and k_2VC_{P0}/F with V being the volume of the reactor, in m^{-3}, and k_1 and k_2 are the first order rate constants. in s^{-1}.

It is assumed that the species Q is highly acidic while the reactant species R is neutral. In order to avoid corrosion problems in the downstream equipment, it is desired to regulate the total concentration y to a prescribed set-point value specified by the constant Y. It is assumed that the control variable u is naturally bounded in the closed interval $[0, U_{max}]$ reflecting the bounded (physical) limits of molar feed rate of the species P.

It is easy to verify that for the given system (3.1), the rank of the following 2 by 2 matrix:

$$S = \begin{bmatrix} \dfrac{\partial y}{\partial x} \\ \dfrac{\partial \dot{y}}{\partial x} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & -(1+2D_{a2}x_2) \end{bmatrix} \tag{3.2}$$

is everywhere equal to 2, except on the line $x_2 = 0$ which is devoid of practical significance.

A stable constant equilibrium point for this system is given by :

$$\begin{aligned} u &= U\ ;\ x_1(U) = \frac{U}{(1+D_{a1})}\ ; \\ x_2(U) &= \frac{1}{2D_{a2}}\left[-1+\sqrt{1+\frac{4D_{a1}D_{a2}U}{(1+D_{a1})}}\right] \end{aligned} \tag{3.3}$$

It is easy to verify, by computing the linearized transfer function on the given equilibrium point, that the above system is indeed minimum phase. The following input-dependent state coordinate transformation :

$$\begin{aligned} z_1 &= x_1 + x_2 - Y \\ z_2 &= -x_1 - x_2 - D_{a2}x_2^2 + u \end{aligned} \tag{3.4}$$

allows one to obtain a GOCF for the system in the form given by (2.3). The inverse of this transformation is simply written as :

$$\begin{aligned} x_1 &= z_1 + Y - \sqrt{\frac{u-(z_1+z_2+Y)}{D_{a2}}} \\ x_2 &= \sqrt{\frac{u-(z_1+z_2+Y)}{D_{a2}}} \end{aligned} \tag{3.5}$$

Notice that the quantity inside the square root in (3.5) is never smaller than zero.

In transformed coordinates, the system is given by:

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= -2(1+D_{a1})(z_1+Y)-(3+2D_{a1})z_2 - 2D_{a1}D_{a2}(z_1+Y)\sqrt{\frac{u-(z_1+z_2+Y)}{D_{a2}}} \\ &\quad + 2D_{a2}^2\sqrt{\left(\frac{u-(z_1+z_2+Y)}{D_{a2}}\right)^3} + 2(1+D_{a1})u + \dot{u} \\ y &= z_1 \end{aligned} \tag{3.6}$$

which is in GOCF.

The hidden, or *zero dynamics* associated to the output nulling in (3.6) is given, according to (2.5), by:

$$\dot{u} + 2(1+D_{a1})(u-Y) - 2D_{a1}D_{a2}Y\sqrt{\frac{u-Y}{D_{a2}}} + 2D_{a2}^2\sqrt{\left(\frac{u-Y}{D_{a2}}\right)^3} = 0 \tag{3.7}$$

It is easy to show that the constant equilibrium point, $u=U$, corresponding to $Y = X_1(U)+X_2(U)$, as computed from (3.3), is an asymptotically stable one. The system is, hence, minimum phase around this point. On the other hand $u = Y$ is an unstable equilibrium point for (3.7) which corresponds with a non-minimum phase point.

Consider the following auxiliary output function, with $\gamma_1 > 0$:

$$\sigma = z_2 + \gamma_1 z_1 \tag{3.8}$$

Notice that if σ is zeroed by means of a discontinuous control strategy, then, the response of the output function $y = \xi_1$ is ideally governed by the asymptotically stable linear autonomous dynamics :

$$\dot{z}_1 = -\gamma_1 z_1 \tag{3.9}$$

Dynamical Sliding Mode Controller Design

Imposing on σ the asymptotically stable discontinuous dynamics (2.10) one readily obtains, in general, the following stabilizing dynamical sliding mode controller is obtained:

$$\begin{aligned} \dot{u} = &-2(1+D_{a1})u + 2(1+D_{a1})\,z_1 + (3+2D_{a1}-\gamma_1)z_2 \\ &+ 2D_{a1}D_{a2}(z_1+Y)\sqrt{\frac{u-(z_1+z_2+Y)}{D_{a2}}} \\ &- 2D_{a2}^2\sqrt{\left(\frac{u-(z_1+z_2+Y)}{D_{a2}}\right)^3} - W\,\mathrm{sign}(z_2+\gamma_1 z_1) \end{aligned} \tag{3.10}$$

In original coordinates, the sliding surface $\sigma = 0$ is, evidently, a control input dependent manifold given by:

$$\sigma(z) = \sigma[\Phi\ (x,u)] = \left[-(x_1+x_2) - D_{a2}x_2^2 + u + \gamma_1(x_1+x_2-Y)\right] \tag{3.11}$$

The proposed dynamical sliding mode controller (3.10) adopts, in original coordinates, the following expression:

$$\begin{aligned}\dot{u} = & -(1-\gamma_1)u - (1-\gamma_1)(x_1+x_2) + 2D_{a1}D_{a2}\,x_1x_2 - (3-\gamma_1)D_{a2}\,x_2^2 \\ & - 2D_{a2}^2 x_2^3 - W\,\mathrm{sign}\left[-(x_1+x_2) - D_{a2}x_2^2 + u + \gamma_1(x_1+x_2-Y)\right]\end{aligned} \tag{3.12}$$

Dynamical PFM Controller Design

Imposing on σ the asymptotically stable discontinuous dynamics (2.12) one obtains, in original coordinates :

$$\begin{aligned}\dot{u} = & -(1-\gamma_1)u - (1-\gamma_1)(x_1+x_2) + 2D_{a1}D_{a2}\,x_1x_2 - (3-\gamma_1)D_{a2}\,x_2^2 \\ & - 2D_{a2}^2 x_2^3 - W\,PFM_{\tau,T}\left[-(x_1+x_2) - D_{a2}x_2^2 + u + \gamma_1(x_1+x_2-Y)\right]\end{aligned} \tag{3.13}$$

Dynamical PWM Controller Design

The dynamical PWM controller has precisely the same form as (3.13) except for the fact that a PWM control function, as defined in (2.17), is used.

$$\begin{aligned}\dot{u} = & -(1-\gamma_1)u - (1-\gamma_1)(x_1+x_2) + 2D_{a1}D_{a2}\,x_1x_2 - (3-\gamma_1)D_{a2}\,x_2^2 \\ & - 2D_{a2}^2 x_2^3 - W\,PWM_{\tau}\left[-(x_1+x_2) - D_{a2}x_2^2 + u + \gamma_1(x_1+x_2-Y)\right]\end{aligned} \tag{3.14}$$

3.2 Simulation Results

Simulations were performed for a reactor characterized by the following parameters:

$$D_{a1} = 1.0 \quad ; D_{a2} = 1.0$$

The simulated control task considered an output stabilization problem for the total normalized concentration y in system (3.1) to a constant reference value, Y = 3. This was accomplished by means of the three dynamical discontinuous feedback controllers proposed in this article.

Figure 1a. portrays the time responses of the dynamical SM controlled output y, the chattering-free input signal u and the corresponding controlled state trajectories x_1 and x_2 . These variables are seen to converge to their equilibrium values: $y = 0$, $u = U = 4$, $x_1 = X_1(4) = 2$, $x_2 = X_2(4) = 1$. Figure 1b. shows the evolution of the sliding surface coordinates function σ. The parameter γ_1 in the sliding surface (3.8) was set to be $\gamma_1 = 1.0$. The dynamical variable structure controller parameter was set, in accordance to (2.10), to W = 1.

Figure 2a. portrays the time responses of the dynamical PFM controlled output, the smoothed input signal and the corresponding controlled state trajectories. As before, these variables are seen to converge to their equilibrium values. Figure 2b. shows the evolution of the auxiliary output function σ, with defining parameter $\gamma_1 = 1.0$, and the time responses of the duty cylce function, $T(\sigma(t))$, and the duty ratio function $\tau(\sigma(t))$. The dynamical PFM controller parameters were set, in accordance to (2.12), to $W = 1$, $r_1 = 1$, $r_2 = 0.5$, $r_3 = 1.5$, $T_{max} = 0.6$, $T_{min} = 0.2$.

Figure 3a. depicts the time responses of the output, states and (smoothed) input variables for the dynamical PWM controlled system. Figure 3b. shows the evolution of the auxiliary output function σ, with $\gamma_1 = 1.0$, and the time responses of the duty ratio function $\tau(\sigma(t))$. The dynamical PWM controller parameters were set, in accordance to (2.17), to W = 1, r= 1, and sampling period : T= 0.5.

4. CONCLUSIONS

The feasibility of chattering-free discontinuous feedback controllers has been demonstrated via dynamical feedback strategies based on stabilization of suitably specified auxiliary output functions defined on the basis of Fliess GOCF. Stabilizing SM, PWM and PFM controller design procedures for nonlinear plants are unified via this technique which is derived from basic facts of the differential algebraic approach to system dynamics. The results are easily extendable to multivariable plants.

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FIGURES

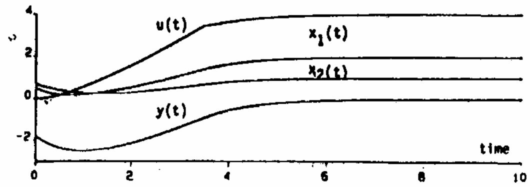


Figure 1a. Output, states and input variables trajectories of dynamical Sliding Mode controlled CSTR.

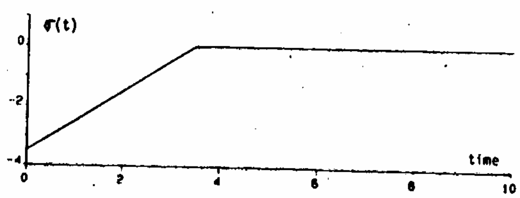


Figure 1b. Sliding Surface Coordinate evolution for dynamical Sliding Mode controlled CSTR.

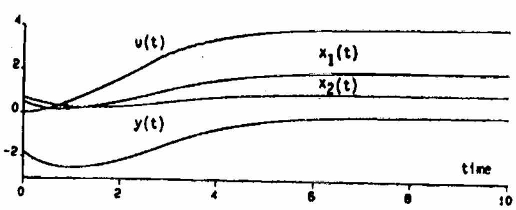
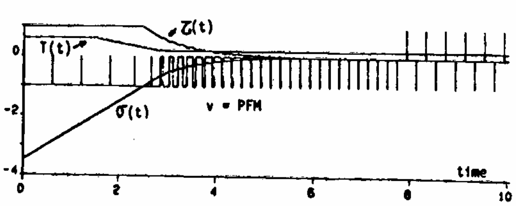


Figure 2a. Output, states and input variables trajectories of dynamical PFM controlled CSTR.



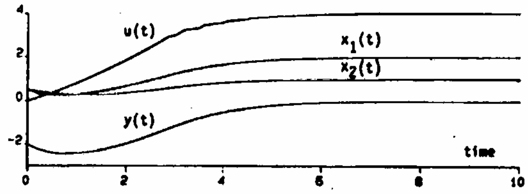


Figure 3a. Output, states and input variables trajectories of dynamical PWM controlled CSTR.

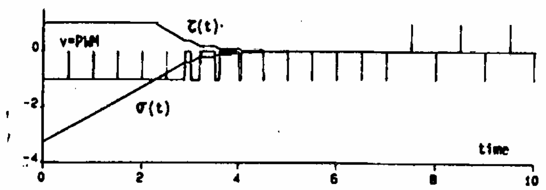


Figure 3b. Evolution of Auxliary Output Function, and Duty Ratio Function for dynamical PWM controlled CSTR.