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DYNAMICAL COMPENSATOR DESIGN FOR A PWM CONTROLLED FULL-BRIDGE POWER CONVERTER

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Abstract In this article a new approach is proposed for the design of a Pulse Width Modulated (PWM) feedback control scheme regulating a Full Bridge Buck Converter (FBBC) in an output signal stabilization task. The approach is based on the specification of a dynamical feedback control law which accomplishes asymptotic stabilization to a preselected constant operating point, for the average PWM converter model. The approach emphasizes the use of Fliess' Generalized Observability Canonical Form (GOCF) of the average converter model and partial inversion techniques.

A DYNAMICAL PWM FEEDBACK CONTROLLER FOR STABILIZATION IN THE FULL BRIDGE BUCK CONVERTER.

Consider the following model of a FBBC which includes an ideal high frequency isolation transformer (see Figure 1) :

$$\dot{x}_1 = -w_0 x_2 + ub \qquad \dot{x}_2 = w_0 x_1 - w_1 x_2 \tag{1}$$

where x_1 is the normalized input inductor current, defined as $x_1 = I_L\sqrt{L}$, x_2 is the normalized output capacitor voltage given by $x_2 = V_0 N\sqrt{C}$, with N being the transformer's winding turn ratio and V_0 is the primary winding voltage drop. The constant w_0 represents the natural oscillating frequency of the LC input circuit, $w_0 = 1/(N\sqrt{LC})$, with the capacitance being referred to the primary of the transformer, and w_1 is the inverse time constant associated to the RC output circuit, i.e., $w_1 = 1/RC$. The constant b is the normalized input source voltage, $b = V_s/\sqrt{L}$, assumed to be constant. The variable u is the control input taking values in the discrete set {-1, 0, 1} in accordance with the positions of the switches. The available control inputs are realized as in the following table (see also [1]):

	S_1	S_2	S_3	S_4
u= 1	ON	OFF	ON	OFF
u= 0	OFF	ON	ON	OFF
u=-1	OFF	ON	OFF	ON

The FBBC may then be naturally controlled by means of an ON-OFF-ON PWM control policy (see the appendix in Sira-Ramirez [2]) of the following type :

$$u = \begin{cases} \text{sign}\, \mu[x(t_k)] & \text{for } t_k \le t < t_k + |\mu[x(t_k)]|T \\ 0 & \text{for } t_k + |\mu[x(t_k)]|T \le t < t_k + T \end{cases} \tag{2}$$

where $\mu : R^n \to R$ is the <u>duty ratio function</u> taking values in the closed interval [-1, 1] of the real line, $x(t_k)$ is the sampled value of the state at time t_k and T represents the constant sampling period. The duty ratio function $\mu(x)$ must be regarded, effectively, as a feedback control input.

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The PWM controller design is carried out on the following, infinite switching frequency, <u>average PWM controlled converter model</u>:

$$\dot{z}_1 = -w_0 z_2 + \mu b \qquad \dot{z}_2 = w_0 z_1 - w_1 z_2 \tag{3}$$

where μ is now regarded as a smooth function of the average state vector z, taking values in the closed interval [-1,1]. For controller design purposes, the above model has been thoroughly justified in several journal publications (see [3]) :

The average equilibrium state of the FBBC, for a constant value U of the duty ratio μ with $U \in [-1,1]$, is simply:

$$z_1 = Z_1(U) = \frac{bw_1}{w_0^2} U \ ; \ z_2 = Z_2(U) = \frac{b}{w_0} U \tag{4}$$

Alternatively, one finds from (4) the relationship

$$Z_1 = \frac{w_1}{w_0} Z_2 \tag{5}$$

Hence, regulation of the average output capacitor voltage z_2 towards constant equilibrium values, specified by (4), can be indirectly accomplished through regulation of the average input inductor current z_1 toward its equilibrium value Z_1. This simple observation also allows for the possibility of specifying a dynamical feedback regulator instead of a static one ([1]).

Stabilization of the FBBC through dynamical PWM control.

Consider the average model (3) of the FBBC with output equation:

$$\dot{z}_1 = -w_0 z_2 + \mu b \qquad \dot{z}_2 = w_0 z_1 - w_1 z_2 \qquad y = z_1 - Z_1 = z_1 - \frac{w_1}{w_0} Z_2 \tag{6}$$

The following invertible duty-ratio (i.e., control) dependent state coordinate transformation :

$$\xi_1 = z_1 - \frac{w_1}{w_0} Z_2 \ ; \ \xi_2 = -w_0 z_2 + \mu b \qquad z_1 = \xi_1 + \frac{w_1}{w_0} Z_2 \ ; z_2 = \frac{-\xi_2 + \mu b}{w_0} \tag{7}$$

takes the average model (6) into Fliess' GOCF [4] :

$$\dot{\xi}_1 = \xi_2 \qquad \dot{\xi}_2 = -w_0^2 \xi_1 - w_0 w_1 Z_2 - w_1 \xi_2 + b w_1 \mu + \dot{\mu} b \qquad y = \xi_1 \tag{8}$$

Stabilization of the transformed coordinates ξ_1 and ξ_2 to zero is equivalent to stabilization of the original average state coordinates z_1 and z_2 to their equilibrium values: Z_1 and Z_2.

The zero dynamics, associated to $\xi_1 = 0$, $\xi_2 = 0$, is obtained by equating the last differential equation in (8) to zero (see Fliess [5]):

$$\dot{\mu} = -w_1(\mu - \frac{w_0}{b} Z_2) = -w_1(\mu - U) \qquad (9)$$

which implies an asymptotically stable solution towards $\mu = U$, i.e., the achieved equilibrium corresponds to a minimum phase behavior.

An unrestricted dynamical feedback controller specifying the required (computed) stabilizing duty ratio function μ is immediately obtained from (8) by simply equating the last differential equation in (8) to a suitable linear combination of the transformed variables. This operation bestows on the closed loop transformed system convenient, stable, pole locations characterized by, say, a damping ratio ζ and a natural undamped frequency w_n:

$$\begin{aligned} \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= -w_0^2\xi_1 - w_0 w_1 Z_1 - w_1\xi_2 + b w_1 \mu + \dot{\mu} b \\ &= -2\zeta w_n \xi_2 - w_n^2 \xi_1 \\ y &= \xi_1 \end{aligned}$$

i.e.:

$$\dot{\mu} = -w_1\mu + \left(\frac{w_1 - 2z w_n}{b}\right)\xi_2 + \left(\frac{w_0^2 - w_n^2}{b}\right)\xi_1 + \frac{w_0 w_1}{b} Z_2 \qquad (10)$$

We denote the solution of (10) as $\hat{\mu}$, and regard it as the computed duty ratio function.

We may interpret the dynamical duty ratio synthesizer (10), as a classical proportional-derivative controller, followed by a low pass filter, with cut off frequency w_1, and a set-point feedforward term.

In original average coordinates one obtains the following dynamical feedback controller:

$$\frac{d}{dt}\hat{\mu} = -2\zeta w_n \hat{\mu} + \left(\frac{w_0^2 - w_n^2}{b}\right) z_1 + \frac{(2\zeta w_n - w_1)}{b} w_0 z_2 + \frac{w_n^2 w_1}{b w_0} Z_2 \qquad (11)$$

The actual duty ratio function μ is simply obtained by bounding the solutions of (11) within the physically meaningful interval [-1,1]. i.e. :

$$\mu = \begin{cases} +1 & \text{if } \hat{\mu} > 1 \\ \hat{\mu} & \text{if } -1 \leq \hat{\mu} \leq +1 \\ -1 & \text{if } \hat{\mu} < -1 \end{cases} \qquad (12)$$

A Simulation Example

Simulations were carried out on a FBBC with parameter values : $R = 1.5\ \Omega$, $C = 2700\ \mu F$, $L = 40\ \mu H$, $V_s = 30$ V and a transformer winding turn ratio N = 10, (i.e., $w_0 = 304.29$, $w_1 = 246.91$, $b = 4.743 \times 10^4$). The chosen damping ratio and natural oscillating frequency of the closed loop system stable complex poles were set, respectively, set at $\zeta = 0.7$ and $w_n = 1000$ rad/s. The required equilibrium value for the normalized output voltage was specified as 7.794, which corresponds to an actual output voltage of 15V. The corresponding equilibrium value for the normalized input current set point is 6.324 (i.e. $I_L = 1$ A). These equilibrium values correspond to a constant steady state value of the duty ratio function: $\mu = U = 0.5$. A sampling frequency of 2KHz was used for the proposed PWM controller. Figures 2a and 2b depict, respectively, the transient response of the average and the discontinuously PWM controlled input current and output voltage of the FBBC. Figure 2c represents the evolution of the dynamically synthesized duty ratio function μ and the corresponding discontinuous PWM control actions u.

CONCLUSIONS

In this article a dynamical PWM control scheme has been presented for the output stabilization in dc-to-dc power converters of the Full Bridge Buck type. The approach was based on synthesizing a dynamical, unrestricted, smooth feedback control law specifying the required duty ratio function for the regulation of the infinite frequency average model of the PWM controlled converter. Such a dynamical scheme corresponds to a pole placement approach which imposes a desirable stable dynamics on Fliess' Generalized Observability Canonical form of the error dynamics associated to the average stabilization problem defined on the bridge converter. Aside from the high response quality of the controlled variables, the approach also allows for an explicit determination of the stabilization limitations exhibited by the treated class of "step down" converters.

REFERENCES

[1] F. Boudjema, M. Boscardin, P. Bidan, J.C. Marpinard, M. Valentin and J.L. Abatut, "VSS Approach to a Full Bridge Buck Converter Used for AC Sine Voltage Generation," *Proc. of the Fiftheenth IEEE Annual Conference of the Industrial Electronics Society* (IECON-89), pp. 82-89. Philadelphia (USA), November 1989.

[2] H. Sira-Ramirez, "Nonlinear Dynamical Discontinuous Feedback Controlled Descent on a Non Atmosphere-Free Planet : A Differential Algebraic Approach," *Control -Theory and Advanced Technology*, Vol. 7, No 2, pp. 301-320, June 1991 .

[3] H. Sira-Ramirez, "A Geometric Approach to Pulse-Width-Modulated Control in Nonlinear Dynamical Systems," *IEEE Transactions on Automatic Control* , Vol. AC-34, No. 2, pp. 184-187. February 1989.

[4] M. Fliess, "Nonlinear Control Theory and Differential Algebra,"in *Modeling and Adaptive Control*, Ch. I. Byrnes and A. Kurzhanski (Eds.). Lecture Notes in Control and Information Sciences, Vol. 105. New York : Springer-Verlag 1990.

[5] M. Fliess, " What the Kalman State Variable Representation is Good For" in Proc. of the 29th IEEE Conference on Decision and Control, Vol.3, pp. 1282-1287, Honolulu, Hawaii, December 1990.

FIGURES

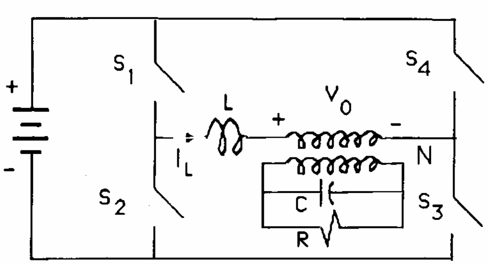

Figure 1. Full Bridge Buck Converter

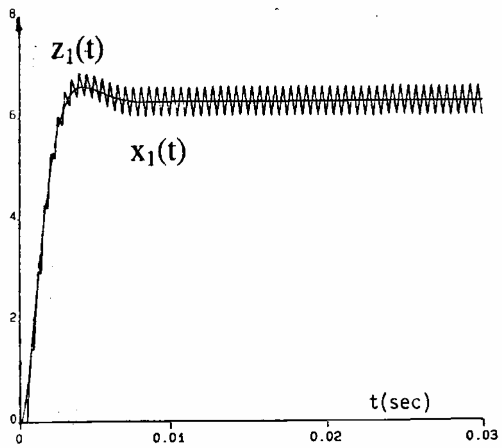

Figure 2(a). Average and Actual Input Inductor Current Response for a Dynamical PWM Stabilization of Full Bridge Buck Converter.

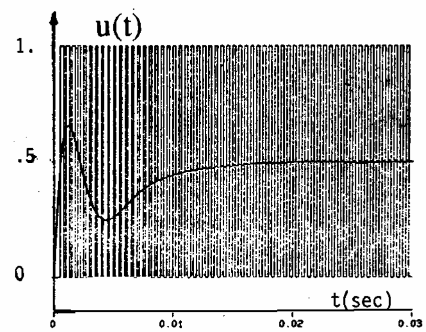

Figure 2(c). Duty Ratio Function and Control Input Responses in Dynamical PWM Stabilization of Full Bridge Buck Converter

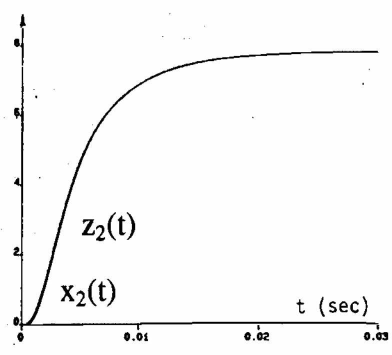

Figure 2(b). Average and Actual Output Capacitor Voltage Response for a Dynamical PWM Stabilization of Full Bridge Buck Converter.